Mean shear flows, zonal flows, and generalized Kelvin–Helmholtz modes in drift wave turbulence: A minimal model for L→H transition[a]

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The dynamics of and an interplay among structures (mean shear flows, zonal flows, and generalized Kelvin–Helmholtz modes) are studied in drift wave turbulence. Mean shear flows are found to inhibit the nonlinear generation of zonal flows by weakening the coherent modulation response of the drift wave spectrum. Based on this result, a minimal model for the L→H (low- to high-confinement) transition is proposed, which involves the amplitude of drift waves, zonal flows, and the density gradient. A transition to quiescent H-mode sets in as the profile becomes sufficiently steep to completely damp out drift waves, following an oscillatory transition phase where zonal flows regulate drift wave turbulence. The different roles of mean flows and zonal flows are elucidated. Finally, the effect of poloidally nonaxisymmetric structures (generalized Kelvin–Helmholtz mode) on anomalous transport is investigated, especially in reference to damping of collisionless zonal flows. Results indicate that nonlinear excitation of this structure can be potentially important in enhancing anomalous transport as well as in damping zonal flows. [DOI: 10.1063/1.1559006]

I. INTRODUCTION

Regulation of anomalous transport by sheared flows is conceived as a most promising mechanism for accessing a high confinement regime (the so-called H-mode) in fusion plasmas. This regulation is brought about by the decorrelation of drift wave turbulence by shearing, which reduces the spatial scales of turbulent eddies, eventually tearing them apart. The importance of this effect was first recognized for a (time-averaged) mean $\mathbf{E}\times\mathbf{B}$ shear flow (see Ref. 1, and references therein), which is mainly driven by the density and/or temperature gradient. The shearing by a mean flow is coherent in time and thus can be very efficient in reducing transport. Furthermore, the damping of drift waves by a mean shear flow steepens the profile as a result of the reduced turbulent transport, and thus can even further boost the drift wave damping. This process appears to be a crucial ingredient in the transition to H-mode, although the causal relation between the profile steepening and turbulence damping has not yet been established experimentally.

In addition to mean flows, the suppression of turbulence can occur by zonal flows which are self-generated from turbulence via Reynolds stress.[2] Unlike a mean flow, these are random $\mathbf{E}\times\mathbf{B}$ poloidal flows, with finite, but low frequency. The reduction in anomalous transport by zonal flows has been clearly demonstrated in computer experiments; it is also shown in laboratory experiments.[3] Zonal flows may be potentially as important as mean flows in regulating turbulence because of their radial localization on small scales (which enhances the shearing rate), even if their finite correlation time (or frequency) may reduce the shearing effect.[4] In particular, the presence of zonal flows around the L→H transition was indicated by a strong (three wave) coupling between zonal flows and drift waves through a bicoherence analysis.[5] It is interesting that in contrast to the case of a mean shear flow, drift wave turbulence and zonal flows are self-regulating.[6] That is, the damping of drift wave turbulence by zonal flows does not enhance the growth of zonal flows, but weakens it by depleting the very source of zonal flow generation. As a consequence of this difference, mean shear and zonal flows are likely to play a somewhat different role in L→H transition at different times.

The forgoing observation then suggests that an understanding of the dynamics of a combined system of a mean shear flow, zonal flows, and drift wave turbulence and the interplay among them is necessary for the prediction of anomalous transport, especially for a comprehensive picture of the L→H transition. The purpose of this paper is to study some of these issues in detail. Specific problems that we address are:

(i) What is the effect of a mean shear flow on zonal flows?
(ii) What roles do a mean shear and zonal flows play in the L→H transition?
(iii) What are the other structures contributing to anomalous transport? In particular, what is the impact of a poloidally nonaxisymmetric mode on collisionless zonal flow damping?

Let us elaborate on these three issues in the following. First, one of the interesting problems regarding the interplay among a mean shear flow, zonal flows, and drift waves, which has not been studied so far, is the effect of a mean shear flow on the generation of zonal flows. Zonal flows are excited by a long wave length modulation of drift wave turbulence. During this modulation, a mean flow shears under-

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lying drift wave turbulence, which can effectively decorrelate the modulation, thereby inhibiting the growth of zonal flows. Our result indeed indicates that this is the case. Thus, the effect of a mean shear flow may offer an alternative damping mechanism of collisionless zonal flows, in addition to nonlinear spectral feedback and linear Kelvin–Helmholtz instability.

Second, a minimal model for L→H transition should incorporate the dynamics of a mean shear flow, zonal flows, and drift wave turbulence, since as noted previously, both mean and zonal flows are likely to participate during transition. Thus, based on the result for the effect of mean shear on zonal flow, we propose a simple model for L→H transition to facilitate the study of the importance of mean shear and zonal flows during this transition. We find that zonal flows regulate drift wave turbulence before the transition to H mode and damp away together with the drift wave turbulence as the transition to H mode is completed by pressure profile steepening. Moreover, bursty temporal behavior due to self-regulating nature of zonal flows and drift waves is recovered in our simple model.[6]

Third, in addition to mean shear flows and zonal flows, there may be more structures affecting anomalous transport. One obvious example is a streamer, which is radially extended and poloidally localized and is thought to enhance the radial transport.[7] Another example, which has received less attention, is the *generalized Kelvin–Helmholtz* mode (referred to as GKH hereafter).[8] This is a poloidally nonaxisymmetric mode $(m\neq 0)$, in contrast to the case of the zonal flow. This structure may arise as a result of linear Kelvin–Helmholtz instability of zonal flows, which breaks up the zonal flows and thus serves as a damping mechanism for them in the collisionless limit. However, as is well known, magnetic shear tends to make the linear Kelvin–Helmholtz mode rather feeble.[9] An alternative route to generating this structure is via modulational instability of drift wave turbulence,[8] in a similar way to the generation of zonal flows. This nonlinear mechanism is particularly important as a linear picture of the Kelvin–Helmholtz instability is valid only in an exceptional case, where the amplitude of drift wave is negligible (such as in a Dimits shift regime[10]) compared to zonal flows and also when there is a clear time scale separation between zonal flows and the Kelvin–Helmholtz mode. In the last part of the paper, we thus explore the nonlinear generation of GKH mode. The specific goals here are to compare nonlinear with linear generation and to find out when the linear picture of the GKH mode is valid. We propose that nonlinearly generated GKH mode can enhance the anomalous transport.

This remainder of the paper is organized as follows. In Sec. II, the effect of a mean flow on zonal flows are studied in the context of modulational instability. Based on the result obtained here, a minimal model for L→H transition is provided in Sec. III. The nonlinear generation of GKH mode is studied in Sec. IV. Our conclusion and discussions are provided in Sec. V.

II. THE EFFECT OF A MEAN SHEAR FLOW ON ZONAL FLOWS

To study the effect of a mean shear flow on modulational instability of zonal flows, we assume that the equilibrium background consists of a mean shear flow $\langle V_E\rangle$ and mean drift wave quanta density $\langle N_k\rangle$. Here, N_k is the drift wave quanta density. To leading order, the mean drift wave quanta satisfies $\gamma\langle N_k\rangle=\Delta\omega\langle N_k\rangle^2$, where γ and $\Delta\omega$ are the linear growth and nonlinear damping rate of drift waves. The modulation of drift wave quanta $\tilde{N}$ by zonal flows $\tilde{V}_E$ (in the form of $\exp\{ipx\}$) is then governed by the following linearized wave kinetic equation

$$\frac{\partial}{\partial t}\tilde{N}_k+ipv_{gx}\tilde{N}_k-k_\theta\langle V_E'\rangle\frac{\partial}{\partial k_r}\tilde{N}_k+\gamma\tilde{N}_k =ipk_\theta\tilde{V}_E\frac{\partial}{\partial k_r}\langle N_k\rangle, \tag{1}$$

where γ is the linear growth rates and v_g is the group velocity (measured in the moving frame with $\mathbf{E}\times\mathbf{B}$ velocity) of drift waves, respectively. Note that the use of (kinematic) linear growth rate in Eq. (1) is just an approximation. By using a quasilinear closure, we can express the evolution of a mean drift wave quanta density $\langle N_k\rangle$ as

$$\frac{\partial}{\partial t}\langle N_k\rangle-k_\theta\langle V_E'\rangle\langle N_k\rangle+\frac{\partial}{\partial k_r}\Gamma_k=\gamma\langle N_k\rangle-\Delta\omega\langle N_k\rangle^2. \tag{2}$$

Here, $\Gamma_k=-\langle(\partial_x(k_\theta\tilde{V}_E)\tilde{N}\rangle$ is the quasilinear flux, which represents diffusion of $\langle N_k\rangle$ in k_r space, ultimately leading to the damping of drift wave turbulence in the presence of dissipation. Note that this is a self-regulating term. The effect of a mean shear flow $\langle V_E\rangle$ on $\tilde{N}_k$ is explicitly shown in the third term on the left-hand side of Eq. (1). In order to incorporate this effect, we solve Eq. (1) along a nonperturbed orbit by introducing a total time derivative D_t as

$$D_t=\frac{\partial}{\partial t}-k_\theta\langle V_E'\rangle\frac{\partial}{\partial k_r}. \tag{3}$$

In this coordinate, the shearing effect by $\langle V_E'\rangle$ is explicitly reflected in the linear increase of k_r in time as

$$D_t k_r=-k_\theta\langle V_E'\rangle. \tag{4}$$

Equation (1) can be integrated along this nonperturbed orbit from an initial time t_0 to final time t as

$$\tilde{N}_k(p,t)=\int_{t_0}^{t}dt'\exp\left\{-\gamma(t-t')-ip\int_{t'}^{t}dt''v_{gx}(t'')\right\}\times ipk_\theta\tilde{V}_E(p,t')\frac{\partial\langle N_k(t')\rangle}{\partial k_r(t')}, \tag{5}$$

where a term depending on the initial condition is dropped by assuming $(t-t_0)\gamma\gg 1$. The shearing effect by a mean flow is embedded in the time dependent group velocity v_{gx} and equilibrium wave quanta density spectrum $\partial\langle N_k(k_r(t'))\rangle/\partial k_r(t')$. For instance, the result of a usual modulational instability in the absence of a mean shear can be recovered from Eq. (5), by taking $k_r(t')$ and $v_g(t'')$ to be constant. For simple drift wave turbulence with $\omega=\omega_*/(1+k^2)$, $v_{gx}=-2k_r v_*/(1+k^2)^2$. Here, $\omega_*=v_* k_\theta$, $v_*=c_s/L_n$ is the electron diamagnetic velocity, $c_s=\sqrt{T_e/m_i}$, and $L_n=-(\partial_x \ln n_{i0})^{-1}$ is the scale length of the background density; the length is measured in unit of ρ_s. Thus, the dependence of v_{gx} on $\langle V_E'\rangle$ through k_r represents the slowdown of the propagation of drift waves due to enhanced inertia (i.e., large k_r) via mean shearing. On the other hand, the second term $\partial\langle N_k(k_r(t'))\rangle/\partial k_r(t')$ $(=-D_{t'}\langle N_k(k_r(t'))\rangle/k_\theta\langle V_E'\rangle)$ captures the shearing effect of a mean flow on $\langle N_k\rangle$.

For a modulational analysis to be valid, the equilibrium $\langle N_k\rangle$ should vary on a time scale much longer than the characteristic time scale of a drift wave. Therefore, we shall assume that the time variation in v_{gx} is much faster than $\partial\langle N_k(k_r(t'))\rangle/\partial k_r(t')$, and also that $\tau_\gamma=1/\gamma$ is much smaller than the characteristic time scale of zonal flows $\tau_c=1/\Omega$ $(\tau_\gamma<\tau_c)$. Furthermore, we limit our analysis to the weak shearing case where τ_γ is smaller than the shearing time scale of the mean flow $\tau_s=1/\langle V_E'\rangle$ $(\tau_\gamma<\tau_s)$. This is because in the opposite limit of strong shear, the mean wave quanta density $\langle N_k\rangle$ is likely to vary too rapidly in time to justify our quasilinear analysis. We thus approximate $\partial\langle N_k(k_r(t'))\rangle/\partial k_r(t')\sim\partial\langle N_k(k_r(t))\rangle/\partial k_r(t)$ and substitute the time dependence of $\exp\{-i\Omega t\}$ for $\tilde{N}_k$ and $\tilde{V}_E$ in Eq. (5). By noting that the fastest time scale in Eq. (5) resides in the argument of an exponential function, we evaluate the time integral by integration by parts up to $O(\langle V_E'\rangle^2)$. The result of this straightforward algebra is

$$\tilde{N}_k(p,\Omega)\sim-p^2 k_\theta \phi_{\rm ZF} R\frac{\partial\langle N_k\rangle}{\partial k_r}, \tag{6}$$

where $\tilde{V}_E=ip\phi_{\rm ZF}$ and the real part of R is given by

$$\begin{aligned}{\rm Re}(R)\sim&\frac{\gamma}{\gamma^2+(\Omega-pv_{gx})^2}-\frac{12k_\theta^2\langle V_E'\rangle^2\omega_*^2p^2}{(1+k_\perp^2)^4}\\&\times\left[\frac{\gamma^5(1+k_\theta^2-3k_r^2)^2}{(\gamma^2+(\Omega-pv_{gx})^2)^5}-\frac{16\gamma^3k_r^2(1+k_\theta^2-k_r^2)^2}{(\gamma^2+(\Omega-pv_{gx})^2)^4}\right].\end{aligned} \tag{7}$$

Note that this result is valid in the weak shear limit. For $k_\perp<1$ and $\gamma>pv_{gx}>\Omega$, Eq. (7) is simplified to

$${\rm Re}(R)\sim\frac{1}{\gamma}\left[1-\frac{12p^2\langle V_E'\rangle^2\omega_*^2k_\theta^2}{\gamma^4}\right]. \tag{8}$$

The sign of ${\rm Re}(R)$ is always positive since R was obtained by treating the effect of $\langle V_E'\rangle^2$ as a small perturbation. Equation (8), derived in a weak shearing limit, is not valid around L→H where the mean shearing rate becomes comparable to the linear growth rate. Thus, Eq. (8) shall be extrapolated to a strong shear case for a simple model for L→H transition in Sec. III.

On the other hand, the evolution of zonal flows $\phi_{\rm ZF}$, driven by Reynolds stress, can be written in terms of the modulation of wave quanta density $N_k=(1+k^2)^2|\phi_k'|^2$ as

$$-i\Omega\phi_{\rm ZF}(p,\Omega)=\int d^2k\frac{k_\theta k_r}{(1+k^2)^2}\tilde{N}, \tag{9}$$

where ϕ' is the electric potential of drift waves. Thus, the frequency (Ω) of zonal flows follows from Eqs. (6) and (9) as

$$\Omega\sim ip^2\int d^2k\frac{k_\theta^2k_r}{(1+k^2)^2}R\left(-\frac{\partial\langle N_k\rangle}{\partial k_r}\right). \tag{10}$$

The substitution of Eq. (8) in Eq. (10) clearly shows that a mean shear flow *suppresses* the growth rate of zonal flows! This reduction arises due to the time variation of v_{gx}, which can be understood as related to the decorrelation of drift wave propagation by a shear flow, which in turn weakens the (coherent) modulation response of the drift wave spectrum. Note that zonal flow growth rate may also be reduced by ion Compton scattering,[11,12] which has not been taken into account in our analysis.

As a mean shear flow reduces the growth of zonal flows, it should also decrease the self-regulating flux in k_r space, which we denoted as Γ_k. This can be easily checked by computing Γ_k as follows. We substitute Eq. (6) in Γ_k and then evaluate the correlation (or, average) by assuming that the statistics of zonal flows is homogeneous in x, i.e.,

$$\langle\tilde{V}_E(p_1,\Omega)\tilde{V}_E(p_2,\Omega)\rangle=\delta(p_1+p_2)|\tilde{V}_E(p_1,\Omega)|^2. \tag{11}$$

The result is

$$\Gamma_k=-k_\theta^2\int dp\,p^2R(-p)|\tilde{V}_E(p,\Omega)|^2\frac{\partial\langle N_k\rangle}{\partial k_r}. \tag{12}$$

For $k_\perp<1$ and $\gamma>pv_{gx}>\Omega$, the use of Eq. (8) in Eq. (12) confirms the reduction in the real part of Γ_k due to a mean shear flow, as expected.

III. A MINIMAL MODEL FOR L→H TRANSITION

In Sec. II, a mean shear flow is found to inhibit zonal flow generation. This effect is likely to be important during the transition to a high confinement regime where the mean **E**×**B** shear flow becomes strong as the profile steepens. Note here that while it is possible that a mean shear flow can be driven by Reynolds stress, its effect is negligible compared to that of the pressure profile, when the latter has a steep gradient. Furthermore, Reynolds stress drive for a mean flow is likely to be weaker than that for zonal flows as the former has larger (radial) scale than the latter. Thus, by ignoring Reynolds stress drive for a mean flow, we consider a minimal model for the L→H transition which consists of the amplitude of drift wave turbulence $\epsilon\propto\langle N\rangle$, zonal flow shear $V_{\rm ZF}\propto\tilde{V}_E'$, and a (ion) pressure p_i, together with the momentum balance equation which relates a mean shear flow $\langle V_E\rangle$ to a profile. For simplicity, we take the following momentum balance relation

$$\langle V_E\rangle=-\frac{1}{eB_z}\left(\frac{1}{n}\frac{dp_i}{dr}\right). \tag{13}$$

Note that a toroidal flow, which can be potentially important in L→H transition, has been neglected in Eq. (13). Note also that the pressure profile should evolve on time scale much larger than than zonal flows to justify the neglect of $V_{\rm ZF}$ in Eq. (13). By assuming a constant ion temperature profile, Eq.

(13) can be written in terms of mean flow shear $V=\langle V_E'\rangle$ and the gradient of density profile $\mathcal{N}=-(L_n/n)\partial_r n$ as

$$V=d\mathcal{N}^2, \tag{14}$$

with a constant d.[13] We now propose a simple 0D model (depending only on time) for the evolution of ϵ, $V_{\rm ZF}$, and $\mathcal{N}$, as follows:

$$\partial_t\epsilon=\epsilon\mathcal{N}-a_1\epsilon^2-a_2V^2\epsilon-a_3V_{\rm ZF}^2\epsilon, \tag{15}$$

$$\partial_t V_{\rm ZF}=b_1\frac{\epsilon V_{\rm ZF}}{1+b_2V^2}-b_3V_{\rm ZF}, \tag{16}$$

$$\partial_t\mathcal{N}=-c_1\epsilon\mathcal{N}-c_2\mathcal{N}+Q. \tag{17}$$

Here, a_i, b_i, and c_i are constant, which depend on a specific model. The meaning of various terms are as follows. From the left, the terms on the right-hand side of Eq. (15) represent the generation of drift wave from density gradient (i.e., linear drift wave instability), nonlinear saturation of drift waves, and suppression of drift waves by V and $V_{\rm ZF}$, respectively. Note that $a_2\simeq a_3$ in the limit where the frequency of zonal flows is much smaller than the decorrelation time of turbulence.[4] The first and second terms on the right-hand side of Eq. (16) capture the generation of $V_{\rm ZF}$ by Reynolds stress and zonal flow damping, respectively. The growth inhibition by a mean shear, which is valid even for a strong shear, is modeled by a term $1/(1+b_2V^2)$. The three terms on the right-hand side of Eq. (17) represent, from the left, the turbulent diffusion of the profile by drift wave turbulence, neoclassical transport, and input power. Note that the major difference between this model and those that were previously studied[13] lies in the self-consistent treatment of the dynamics of zonal flows.

For fixed values of parameters a_i, b_i, and c_i, the evolution of this system [Eqs. (15)–(17)] is determined by the input power Q, which serves as a control parameter for this system. For sufficiently large Q, the profile becomes steep to completely damp drift waves, and the system evolves to a quiescent H mode, where

$$\epsilon=V_{\rm ZF}=0, \quad \mathcal{N}=\frac{Q}{c_2}. \tag{18}$$

In this regime, the slope of the profile is determined by neoclassical transport. Note that we have ignored a magnetohydrodynamic (MHD) instability which can be caused by further steepening of pressure profile in this H-mode state. To study how this system evolves to H mode, we make Q as a linear function of time ($Q=0.01t$) and solve Eqs. (15)–(17), by taking the values of parameters a_i, b_i, and c_i to be order of unity. The evolution of ϵ (solid line), $V_{\rm ZF}$ (dotted line), and $\mathcal{N}/5$ (dashed line) are shown in Fig. 1. As can be seen clearly, there are three distinct stages. The early stage is characterized by growing drift waves by linear instability (from $\mathcal{N}$), followed by rapidly growing self-generated zonal flows. As the shearing by zonal flows becomes efficient to damp drift waves, a system evolves into a transition regime where zonal flows and drift waves are self-regulating and exhibit oscillatory behavior: ϵ and $V_{\rm ZF}$ grow as they draw energy from $\mathcal{N}$ and ϵ, respectively while ϵ and $\mathcal{N}$ damp by growing $V_{\rm ZF}$ and ϵ, respectively. Note that the oscillatory behavior is a generic feature of a self-regulating system of drift waves and zonal flows.[6,4,15] Note also that this oscillatory transition phase may explain dithering observed in ASDEX Upgrade[16] although there could be other causes leading to oscillatory behavior.[17] In addition to oscillation, there is a gradual increase in ϵ. This is caused by the reduction in the zonal flow growth by the mean shear flow, which in turn promotes the growth of drift waves. The behavior of this envelope is given by a stationary solution $\epsilon=b_1(1+b_2V^2)/b_3$ [see Eq. (16)], which increases as the profile steepens ($V=d\mathcal{N}^2$). Finally, for sufficiently large Q, drift waves are completely damped by strong mean flow shearing, entering deep into H mode (a quiescent H mode). As drift waves damp, zonal flows die out, and the profile steepens linearly with Q, consistent with Eq. (18).

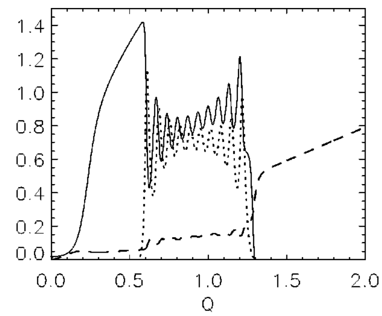

FIG. 1. Evolution of ϵ (solid line), $V_{\rm ZF}$ (dotted line), and $\mathcal{N}/5$ (dashed line) as a function of input power $Q=0.01t$. Parameter values are $a_1=0.2$, $a_2=a_3=0.7$, $b_1=1.5$, $b_2=b_3=1$, $c_1=1$, $c_2=0.5$, and $d=1$.

The inhibition of zonal flow growth by a mean shear flow [term with b_2V^2 in Eq. (16)] prolongs the oscillatory transition phase as it effectively reduces drift wave damping (by zonal flows). This can be clearly seen by comparing Fig. 1 with Fig. 2, which was obtained by using the same parameter values as Fig. 1, but with $b_2=0$; the transition to a quiescent H mode occurs at $Q\sim1.3$ in Fig. 1 while it happens at smaller $Q\sim1.2$ in Fig. 2. In this case, the oscillation of ϵ is about a roughly constant value, in contrast to Fig. 1. This constant value is again given by a stationary solution $\epsilon=b_3/b_1=2/3$ with $b_2=0$ [see Eq. (16)]. A slight decrease in the amplitude of oscillation is due to nonlinear damping of drift waves ($a_2\epsilon^2$). Therefore, the effect of mean shearing on

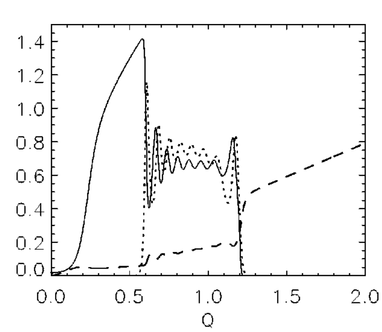

FIG. 2. The same as Fig. 1 besides $b_2=0$.

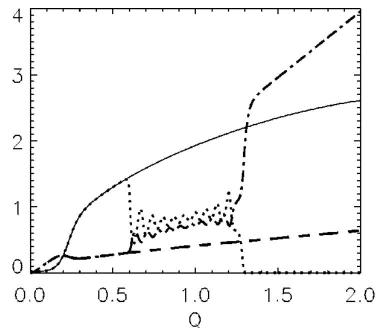

FIG. 3. Evolution of ϵ (solid line) and $\mathcal{N}$ (dashed line) as a function of input power $Q=0.01t$ with $V_{\rm ZF}=0$. Parameter values are the same as Fig. 1 ($a_1=0.2$, $a_2=a_3=0.7$, $c_1=1$, $c_2=0.5$, and $d=1$). For comparison, ϵ (dotted line) and $\mathcal{N}$ (dotted–dashed line) in Fig. 1 are superimposed.

zonal flow generation is manifested in the longer duration of oscillatory transition phase and a slow rise in the envelope of the oscillation.

These results indicate that zonal flows help (or "trigger") the transition to a high confinement regime by regulating drift wave turbulence before the transition to a quiescent H mode. That is, zonal flows lower the threshold of Q for the transition. To demonstrate this effect, we depict the evolution of ϵ (solid) and $\mathcal{N}$ (dashed line) in Fig. 3, by solving Eqs. (15) and (17) with $V_{\rm ZF}=0$ for the same parameters values as in Fig. 1. It clearly illustrates that due to the absence of zonal flows, the amplitude of drift waves is too large to reach a quiescent H mode for the values Q up to 2; the transition will occur at higher value, i.e., $Q>2$. For comparison, the Q dependence of ϵ and $\mathcal{N}$ in Fig. 1 are superimposed by dotted and dotted–dashed lines, respectively, in Fig. 3.

Our results imply that the duration of the transition regime, where drift waves and zonal flows are self-regulating and exhibit oscillatory (bursty temporal) behavior, sensitively depends on how rapidly the input power Q is raised. For instance, too rapid an increase in Q would cause the disappearance of this regime, i.e., its duration would become arbitrarily short, and certainly less than the experimental resolution time. Therefore, to experimentally test the role of zonal flows in L$\rightarrow$H transition, input power should be ramped up slowly. One of the most recent experiments, which was successful in accessing this transition regime, was reported in Ref. 14, where periodic bursts were observed before the transition to quiescent H mode. There a slow transition was achieved by slowly increasing the input power. Our results obtained from a minimal model Eqs. (15)–(17) not only offer a simple theoretical explanation for this bursty temporal behavior during the transition regime (which is referred to as IM mode in Ref. 14) but also provides a concrete route leading to a quiescent H mode by a pressure profile steepening. Furthermore, it can also predict the correlation between pressure profile and zonal flows. Note that the theoretical explanation for bursts provided in Ref. 14 invoked a self-regulation between drift waves and a poloidal flow which is assumed to be driven by Reynolds stress alone, ignoring the dynamics of zonal flows and those of the pressure profile.

IV. GENERALIZED KELVIN–HELMHOLTZ MODE

We have so far assumed that large-scale structures, influencing anomalous transport in plasmas, are radially localized mean shear and zonal flows. This is, of course, an oversimplified picture, and there are other structures, that can be potentially important in transport. As noted in the Introduction, one such structure is a streamer, which is poloidally localized but radially extended. Other example, that we are going to focus on in this paper, is a poloidally nonaxisymmetric mode with a finite $m\neq 0$ (GKH). In toroidal geometry, this can be viewed as a mode with a small but finite m. GKH modes arise naturally when zonal flows (or mean shear flow) become linearly unstable to classical shear (Kelvin–Helmholtz) instability due to inflection point. It is well known that in a torus with a magnetic shear, this linear instability becomes energetically unfavorable, as the exchange of two vortices around an inflection point requires the energy to realign them with magnetic fields.[9] In ion temperature gradient (ITG) turbulence, the linear instability may be excited with a small growth rate ($<|\partial_{xx}\phi_{\rm ZF}|$) in a radially localized regime, provided that the zonal flow $\phi_{\rm ZF}$ has an opposite phase to zonal temperature $T_{\rm ZF}$ (i.e., $\phi_{\rm ZF}T_{\rm ZF}<0$).[18] By assuming $\phi_{\rm ZF}T_{\rm ZF}<0$, Ref. 18 invoked the linear Kelvin–Helmholtz instability as a mechanism which terminates the Dimits up shift regime.

In this section, we explore an alternative mechanism for generating GKH mode, which also provides indirect zonal flow damping. In the case when GKH mode evolves on time scales much larger than the characteristic time scale of drift wave turbulence (i.e., ω, γ), drift waves are adiabatically modulated by the former. This modulation can lead to the growth of GKH mode (i.e., modulational instability). Note that this is the very mechanism for the generation of zonal flows. That is, GKH modes can be excited nonlinearly by modulational instability (or, Reynolds stress). As the energy source of these modes is drift wave turbulence, their excitation effectively weakens the growth of zonal flows.

The nonlinear generation of GKH modes are particularly interesting for the following reasons. First, the Dimits up shift regime[10] (where drift wave turbulence is very weak) is an exceptional case with no collisional damping of zonal flows and with a sufficiently weak turbulence drive (e.g., very close to marginality, small q, etc.). Note that the Dimits up shift regime is based on local picture in which a nonlocal effect induced by the turbulence spreading[19] is ignored. Second, the components of zonal temperature $T_{\rm ZF}$ and zonal flow $\phi_{\rm ZF}$ with negative phase shift (i.e., $T_{\rm ZF}\phi_{\rm ZF}<0$) (which is required for unstable linear Kelvin–Helmholtz instability), may not be robust. In fact, by a self-consistent modulational instability calculation for a curvature driven ITG model, this phase was shown to be likely to be "positive," rather than negative.[8] Third, a linear picture of Kelvin–Helmholtz instability of zonal flows is invalid when zonal flows themselves evolve on time scale shorter than that of the instability. Finally, GKH modes (with a finite radial flow) contribute to radial transport, and their excitation may provide an effective damping of zonal flows (see below).

In the following, we estimate the growth rate of GKH modes by modulational instability and then discuss its implication, including the validity of the linear picture of GKH. To this end, we use a simple model with cold ions in two dimensional slab geometry and consider a flute-like perturbation ($k_\parallel \simeq 0$) for GKH modes. The electric potential for GKH mode is denoted by ϕ, and that for drift waves by ϕ'. Nonlinear generation of ϕ by Reynolds stress is then given by

$$\frac{\partial}{\partial t}\nabla_\perp^2 \phi = (\partial_{xx} - \partial_{yy})\langle v_x' v_y' \rangle + \frac{1}{2}\partial_{xy}\langle v_y'^2 - v_x'^2 \rangle, \quad (19)$$

where $\mathbf{v}' = -\nabla \times \phi' \hat{z}$. We assume a homogeneous and isotropic drift wave turbulence background, which satisfies $\gamma\langle N_k\rangle = \Delta\omega \langle N_k \rangle^2$, and consider harmonic modulation of N_k and ϕ around this background as $\tilde{N}_k \sim \phi \sim \exp\{-i(\Omega t - px - qy)\}$. The linearized wave kinetic equation then gives

$$\frac{\delta \tilde{N}_k}{\delta \phi} = -i\frac{(pk_y - qk_x)(p\partial_{k_x} + q\partial_{k_y})\langle N_k\rangle}{\Omega - (pv_{gx} + qv_{gy}) + i\gamma}. \quad (20)$$

The nonlinear growth rate [i.e., $\gamma_{\mathrm{NL}} = \mathrm{Im}(\Omega)$] is obtained from Eqs. (19) and (20), with the help of $N_k = (1 + k^2)^2 |\phi_k'|^2$, as

$$\Omega = \frac{p^2 - q^2}{p^2 + q^2}\int d^2k \frac{k_x k_y}{(1+k_\perp^2)^2}\frac{(pk_y - qk_x)(p\partial_{k_x} + q\partial_{k_y})\langle N_k\rangle}{[\Omega - (pv_{gx} + qv_{gy}) + i\gamma]}. \quad (21)$$

Note that in the limit of $q \to 0$, Eq. (21) simply leads to the growth rate of zonal flows as

$$\gamma_{\mathrm{ZF}} \sim \int d^2k \frac{k_x^2 k_y^2}{2(1+k^2)^2}\frac{p^2\gamma}{(pv_{gx})^2 + \gamma^2}\left(-\frac{\partial\langle N_k\rangle}{\partial k_x^2}\right), \quad (22)$$

which is positive for a normal drift wave spectrum $\partial_{k_x}\langle N\rangle < 0$. To estimate the growth rate for GKH mode, we assume $v_{gy} > v_{gx}$, $\gamma/|v_{gy}q| \sim \delta k_y/q < 1$, $\gamma/|v_{gx}p| \sim \delta k_x/p(\rho_s k)^2 \gtrsim 1$, $p \sim q$ (so $(p-q)^2 \sim (p+q)^2$), and $\rho_s k \lesssim 1$, which reduces the imaginary part of Eq. (21) to

$$\gamma_{\mathrm{NL}} \sim \int d^2k \frac{k_x^2 k_y^2}{2(1+k^2)^2}\frac{\gamma}{v_{gy}^2}\left(-\frac{\partial\langle N_k\rangle}{\partial k_x^2}\right). \quad (23)$$

The ratio among γ_{ZF}, γ_L, and γ_{NL} can be obtained by using the growth rate of linear Kelvin–Helmholtz instability $\gamma_L \sim pv_{\mathrm{ZF}}$,[8] $\gamma = \delta\omega$, and the mixing-length estimate $\langle N_k\rangle \sim 1/(k_0 L_n)^2$ (k_0 is the characteristic wave number of drift waves) as

$$\frac{\gamma_{\mathrm{ZF}}}{\gamma_{\mathrm{NL}}} \sim \left(\frac{v_{gy}q}{\gamma}\right)^2 \sim \left(\frac{p}{k_0\delta}\right)^2, \quad (24)$$

$$\frac{\gamma_{\mathrm{NL}}}{\gamma_L} \sim \frac{k_0}{q}\frac{c_s}{v_{\mathrm{ZF}}}\frac{\rho_s}{L_n}\delta, \quad (25)$$

$$\frac{\gamma_{\mathrm{ZF}}}{\gamma_L} \sim \frac{p}{\delta k_0}\frac{c_s}{v_{\mathrm{ZF}}}\frac{\rho_s}{L_n}. \quad (26)$$

Finally, the substitution of estimates $v_{\mathrm{ZF}} \lesssim 10^{-2}c_s$ (e.g., see Ref. 20) and $\rho_s/L_n \sim 0.01$ in Eqs. (24)–(26) leads us to the following conclusion: (1) the nonlinear generation of zonal flow can be more effective than both nonlinear and linear generation of GKH (i.e., $\gamma_{\mathrm{ZF}} \gtrsim \gamma_{\mathrm{NL}}, \gamma_L$) near marginality, with $\delta < q/k_0$, and (2) the nonlinear generation rate of GKH can be comparable to its linear generation rate (i.e., $\gamma_{\mathrm{NL}} \sim \gamma_L$) away from marginality $\delta > p/k_0$. Therefore, near marginal stability, the linear analysis of GKH, which treats a zonal flow as an equilibrium background for the evolution of GKH, is invalid, with no clear distinction between secondary (zonal flow) and tertiary mode (GKH) possible (cf. Ref. 18). Furthermore, far away from marginality, the nonlinear excitation of GKH modes, with the amplitude comparable to zonal flow, provides an effective damping of zonal flows since the former draws the energy from drift waves. This then effectively enhances the radial transport (e.g., χ_i) by reducing shearing. In addition, GKH modes, with finite radial flows, may directly contribute to χ_i. Therefore, the nonlinear excitation of GKH modes offers an alternative route to damping zonal flows and increasing transport.

V. CONCLUSIONS

One of the most interesting questions in the prediction of anomalous transport is the interplay among turbulence and the various structures it generates. While small scale turbulence, such as drift waves, causes anomalous transport, large-scale structure can either enhance or reduce transport, depending on its spatial structure. Two of the important structures, which regulate turbulent transport, are mean shear flows and zonal flows. Despite this common effect on turbulent transport, they differ in the mechanism for their generation as well as their spatial and temporal behavior. Another structure that is potentially important to transport is the GKH mode, which is poloidally nonaxisymmetric with small m. This can be excited not only linearly by shear (Kelvin–Helmholtz) instability of zonal flows (or mean shear flows), but also nonlinearly, through modulational instability. The purpose of this paper was to study the dynamics of, and interplay among, these large-scale structures and drift waves.

We first investigate the effect of mean shear flows on zonal flows, by incorporating the mean shearing effect in the modulational instability of zonal flows. By focusing on the weak shear case, we found that mean flows reduce the growth rate of zonal flows by decorrelating the propagation of drift waves during modulation.

Based on this result, we proposed a minimal model for L→H transition which consists of the evolution of drift wave amplitude, zonal flows, and density profile, together with a momentum balance relation. This model differs from those previously studied[13] in that it includes the self-consistent evolution of the zonal flows. Our results revealed the different role of mean shear flows and zonal flows: zonal flows "trigger" or "help" the transition by damping drift waves until the shearing by mean shear flows is sufficiently strong so as to damp both drift waves and zonal flows. The initial transition stage is marked by oscillatory behavior (because of self-regulation of drift waves and zonal flows) until the transition to a steep profile (quiescent H mode) sets in. This result may provide a simple theoretical explanation for the periodic bursts which were observed before the transition to quiet H mode in a recent tokamak experiment.[14] Our result also implies that the critical input power for the transition is likely to be lowered by zonal flows. The inhibition of zonal flow growth is found to prolong the initial oscillatory transition phase as it effectively reduces the drift wave damping.

The last part of the paper was devoted to the study of the nonlinear excitation of GKH mode (yet another structure) by modulational instability, in view of its ''weak'' linear instability. We found that nonlinear excitation can be as important as linear excitation away from marginality. The nonlinearly excited GKH mode enhances the radial transport directly by its finite radial flow and indirectly by damping (collisionless) zonal flows. Thus, the former may serve as another structure that plays an important role in transport. In experiments, the distinction between the linear and nonlinear origin of GKH mode may be determined by bicoherence analysis.[5]

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