A simple model of interactions between electron temperature gradient and drift-wave turbulence

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A *self-consistent* theory for the interaction between electron temperature gradient (ETG) and drift-ion temperature gradient (DITG) turbulence is presented. Random shear suppression of ETG turbulence by DITG modes is studied, as well as the back-reaction of the ETG modes on the DITG turbulence via stresses. It is found that ETG dynamics can be sensitive to shearing by short-wavelength DITG modes. DITG modulations of the electron temperature gradient are also shown to be quite significant. Conversely, the back-reaction of the ETG on the DITG turbulence is found to be weak. The importance of different interactions is quantified via scalings which sensitively depend upon the electron–ion mass ratio. The findings are used to motivate a discussion of the development of a "super-grid" model for the effects of DITG turbulence on the ETG turbulence. [DOI: 10.1063/1.1646675]

I. INTRODUCTION

One of the key challenges in magnetic confinement research is to understand the underlying causes of anomalous particle and heat transport. In most theories and models of turbulent transport, there is assumed to be one dominant kind of instability [such as ion temperature gradient-driven drift-waves[1,2] (DITG) or resistive interchange modes[3]] which is taken as the sole driver of all of the transport channels. In reality, multiple instabilities on different scales may coexist [i.e., DITG on $\rho_s = c_s/\Omega_{ci}$ scales, and electron temperature gradient[4–7] (ETG) driven modes on $\rho_e = v_{Te}/\Omega_{ce}$ scales]. Previously, it has been argued (or more often, tacitly assumed) that the separation in temporal and spatial scales meant that interactions between instabilities on different scales were generally negligible relative to the nonlinear "self" interactions of a particular instability. For instance, the effects of ETG turbulence on DITG turbulence and *vice versa* (such as the shearing of ETG eddies by the DITG flow field, or the Reynolds stress of the ETG turbulence on the DITG turbulence) were ignored. However, with the rise in interest in the ETG mode as a source of electron heat transport,[8–12] and given the possibility of the simultaneous presence of DITG turbulence, it is important to consider their interactions in a more quantitative fashion. For instance, it is important to determine whether the shearing action of the DITG turbulence on the ETG turbulence can strongly suppress the ETG turbulence, thereby suppressing the associated turbulent electron heat transport. In particular, it has been suggested that ETG turbulence can drive experimentally relevant levels of transport through the formation of "streamers,"[8,13–16] which are radially extended convective cells, or through electromagnetic inverse cascade processes which lead to the accumulation of energy on collisionless skin depth scales.[9,17,18] In either of these cases, the scales relevant for transport are much greater than ρ_e, thus reducing the effective separation between the ETG-driven transport and DITG scales, and thereby increasing the likelihood of significant shearing interactions between streamers or skin-depth scale ETG fluctuations and the DITG turbulence. The question of whether the ETG might have a significant back-reaction on the DITG turbulence is also important. For instance, Li and Kishimoto have recently argued that the presence of ETG driven zonal flows may impact the dynamics of DITG turbulence.[19] However, in their study, the ETG-driven zonal flow had a fixed amplitude and the effects of DITG shearing on the zonal flow were not included (essentially, the equilibrium is modified to include a small-scale zonal flow); in this paper, we study the couplings between fields in a more self-consistent fashion. More generally, the problem of how different different scales of turbulence interact is also of intrinsic interest as a novel problem in nonlinear dynamics. The generic structure of this problem has been considered by Itoh and Itoh, and collaborators.[20,21]

In this paper, we investigate the interactions between DITG and ETG turbulence using simple models for the individual instability dynamics. The paper is organized as follows: in Sec. II, we consider how random shearing by DITG turbulence might affect the ETG turbulence, via adiabatic theory. A generalized k-space diffusion tensor is derived to represent the random shearing action of the DITG turbulence. It is then shown that the shearing due to short-wavelength DITG modes can significantly impact large-scale ETG structures such as streamers. In Sec. III, the more subtle effect of DITG-induced fluctuations of the electron temperature gradient on ETG turbulence is investigated. It is found that this effect will be significant when the amplitude of the gradient modulation is comparable to the equilibrium deviation from marginality; a simple mean-field estimate is used to show that fluctuations of this magnitude are quite likely. It is important to note that these ∇T_e modulations depend upon

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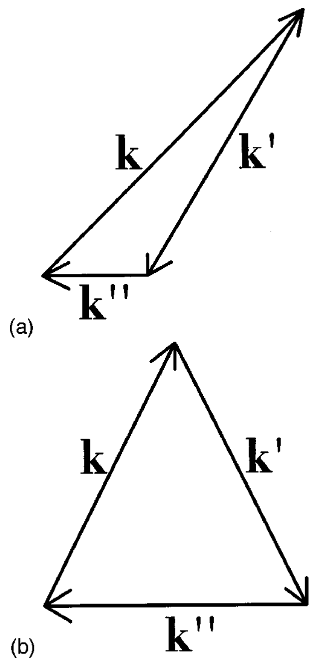

FIG. 1. Triads for (a) disparate-scale interactions (ETG–DITG) and (b) like-scale interactions (ETG–ETG or DITG–DITG).

the poloidal and toroidal angles θ and ϕ, and thus differ from the quasi-stationary flux-surface averaged ∇T_e profile modulations observed in some simulations.[22] A renormalization-based approach to DITG–ETG interactions is presented in Sec. IV, where the back-reaction of the ETG on the DITG is found to be weak (in contrast with those of Ref. 19). The results of these investigations are combined into a zero-dimensional extended predator–prey model for the joint interactions of ETG and DITG turbulence along with DITG driven zonal flows which is given in Sec. V. In addition, the findings are used to motivate the idea of the *supergrid-scale* model for the effects of the DITG on the ETG turbulence.

II. RANDOM SHEARING OF ETG TURBULENCE BY DRIFT-ITG TURBULENCE

We first consider the question of how random shearing by DITG modes might affect ETG turbulence. Throughout this paper, the drift-ion temperature gradient (DITG) label applies to both long-wavelength curvature-driven ion temperature gradient instabilities (which have characteristic radial scales of approximately several ρ_s), as well as instabilities with slightly shorter characteristic scales, such as the "universal instability"[23] or collisionless trapped electron mode.[24–27] We exploit the separation of space and time scales between the DITG and ETG modes [as illustrated in Fig. 1(a)] to describe the evolution of the ETG turbulence in the presence of the DITG modes by the wave-kinetic equation (WKE),[28]

$$\frac{\partial N}{\partial t}+\frac{\partial}{\partial \vec{k}}(\omega_k+\vec{k}\cdot\vec{V}_{\rm DITG})\cdot\frac{\partial N}{\partial \vec{x}}-\frac{\partial}{\partial \vec{x}}(\vec{k}\cdot\vec{V}_{\rm DITG})\cdot\frac{\partial N}{\partial \vec{k}}$$
$$=2\gamma_k N-\Delta\omega\langle N\rangle^2. \tag{1}$$

Here, $N=(\tau+k_\perp^2\rho_e^2)^2|\phi_k^{\rm ETG}|^2+|T_k|^2$ is the potential enstrophy ($\tau=T_e/T_i$ and $T=\tilde{T}_e/T_{e0}$), which is the adiabatic invariant[29] associated with the ETG turbulence, while ω_k and γ_k are the linear frequency and growth rate of the ETG modes, respectively, and the $\Delta\omega N^2$ term represents an simplified model for nonlinear self-damping of the ETG turbulence (i.e., turbulent mixing or decorrelation). $\vec{V}_{\rm DITG}=v_{Te}\hat{z}\times\rho_e\vec{\nabla}\phi^{\rm DITG}$ is the flow field of the DITG turbulence (that is, on ρ_s scales and thus large compared to the ρ_e scale ETG turbulence); $\hat{z}=\vec{B}/|B|$ is the unit vector in the direction of the local magnetic field. For both the ETG and DITG modes, $\phi=|e|\tilde{\phi}/T_e$. It is important to note that here $\vec{V}_{\rm DITG}$ represents the flow field of the *entire* DITG turbulence spectrum, not just the flow field associated with the zonal flows driven by that turbulence (as in previous studies which have used the adiabatic theory approach). In particular, we include the fluctuations associated with trapped electron modes and other effects which may be on slightly smaller scales than the fluctuations associated with traditional curvature-driven ITG turbulence (i.e., turbulence with length scales $l\simeq\rho_s$ as well as on scales $l\geqslant\rho_s$).

The most direct way to estimate the effects of the DITG turbulence is to use a quasi-linear closure of the WKE to derive the k-space diffusion coefficient for ETG modes due to a spectrum of DITG modes $|\phi_q|^2$; this calculation is analogous to that of Diamond *et al.*[30] in determining the effects of zonal flow shearing on the turbulence which generates the flow. We also note that if there are spatial gradients of the ETG turbulence intensity, the DITG turbulence will induce spatial diffusion. Using this approach, we find

$$\frac{\partial\langle N\rangle}{\partial t}=\frac{\partial}{\partial x_\alpha}D^X_{\alpha\beta}\frac{\partial\langle N\rangle}{\partial x_\beta}+\frac{\partial}{\partial k_\alpha}D^k_{\alpha\beta}\frac{\partial\langle N\rangle}{\partial k_\beta}+2\gamma_k\langle N\rangle$$
$$-\Delta\omega\langle N\rangle^2, \tag{2}$$

$$D^X_{\alpha\beta}=\rho_e^2 v_{Te}^2\sum_q (\vec{q}\times\hat{z})_\alpha(\vec{q}\times\hat{z})_\beta R(\Omega_q)|\phi_q|^2, \tag{3}$$

$$D^K_{\alpha\beta}=\rho_e^2 v_{Te}^2\sum_q q_\alpha q_\beta|(\vec{k}\times\vec{q})\cdot\hat{z}|^2 R(\Omega_q)|\phi_q|^2, \tag{4}$$

$$R(\Omega_q)=\frac{1}{2\gamma_k-i(\Omega_q-\vec{q}\cdot\vec{v}_g)}\simeq\frac{1}{2\gamma_k}. \tag{5}$$

Here $D^X_{\alpha\beta}$ is a tensor which describes the *spatial* diffusion of turbulent intensity on large ($>\rho_e$) scales, and $D^K_{\alpha\beta}$ is a tensor wavenumber–space diffusivity coefficient which represents the generalized random shearing action of the DITG spectrum on the small-scale ETG turbulence spectrum. These diffusion tensors represent a direct generalization of previous works (such as Refs. 14, 30) which have used a similar approach to quantify the effects of zonal flow shearing on the underlying turbulence. The tensor structure follows from the fact that the DITG turbulence is a function of both the radius r and polodial angle θ, whereas in the previous approaches the diffusion arose only from the zonal flow spectrum, which is independent of θ. The $R(\Omega_q)$ term [Eq. (5)] represents the response function of the ETG turbulence to mode q of the DITG turbulence; $\Omega_q \simeq q_\theta \rho_s c_s / L_n$ is the real frequency of the DITG mode, while $\vec{v}_g \simeq -(\rho_e v_{Te}/L_n)\hat{\theta} + O((k\rho_e)^2)$ is the group velocity of the ETG turbulence (the minus sign arises from the fact that ETG modes propagate in a direction *opposite* to DITG modes; $\hat{\theta}$ is the unit vector in the polodial direction). Using the equivalence of $\rho_s c_s = \rho_e v_{Te}$ $(= cT_e/|e|B)$, one has $\Omega_q - \vec{q}\cdot\vec{v}_g \simeq 2 q_\theta \rho_e v_{te}/L_n$. However, this term is small relative to the real term $2\gamma_k \simeq 2k_\theta \rho_e v_{Te}/L_n$ because of the scale separation $(k_\theta \gg q_\theta)$, and so one can simply estimate $R(\Omega_q) \simeq 1/(2\gamma_k)$. One can also note that $R(\Omega_q)$ will be strongly influenced by the triad interaction coherence time, which is in turn set by the shortest correlation time, namely that of the ETG turbulence.

To estimate the importance of the k-space DITG shearing, one can identify three time scales, $\gamma_{\mathrm{lin}} = 2\gamma_k$, $\gamma_D \sim D^K_{\alpha\beta}/k_\alpha k_\beta$, and the self-damping/nonlinear decorrelation rate $\gamma_{\mathrm{self}} = \Delta\omega\langle N\rangle$. One can then rewrite the evolution equation for the $\langle N\rangle$ as

$$\frac{\partial\langle N\rangle}{\partial t} = \gamma_{\mathrm{lin}}\langle N\rangle - \gamma_D\langle N\rangle - \gamma_{\mathrm{self}}\langle N\rangle. \tag{6}$$

When $\gamma_{\mathrm{self}} \gg \gamma_D$ the DITG shearing is unimportant, and the turbulence saturates at the level given by $\gamma_{\mathrm{lin}} = \gamma_{\mathrm{self}} \rightarrow \langle N\rangle = \gamma_{\mathrm{lin}}/\Delta\omega$. However, if $\gamma_{\mathrm{self}} \ll \gamma_D$ then the shearing due to the DITG turbulence overwhelms the self-damping, and the ETG turbulence saturates at a reduced level given by the balance of γ_{lin} and γ_D. The relative importance of DITG shearing is then determined by the ratio of $\gamma_{\mathrm{self}}/\gamma_D$. We can estimate γ_{self} by noting that in the absence of DITG shearing, one must have $\gamma_{\mathrm{self}} = \gamma_{\mathrm{lin}}$, and since γ_{self} is set purely by ETG self-interactions, it should not be strongly affected by the presence of the DITG turbulence. Therefore, one can still estimate $\gamma_{\mathrm{self}} \simeq \gamma_{\mathrm{lin}}$ in the presence of DITG turbulence, which makes the relevant ratio to calculate $\gamma_{\mathrm{lin}}/\gamma_D$. Alternatively, one could characterize the strength of the DITG turbulence shearing by arguing that if $\gamma_{\mathrm{lin}} \ll \gamma_D$ then the k-space diffusion rapidly carries energy to high k where it damps, which effectively means the ETG is strongly suppressed. More colorfully, the DITG shearing field will "rip" the ETG turbulence apart before it can grow to a significant intensity level. In the other limit, the random shearing cannot overcome the linear drive of the ETG, which must then saturate by self-damping. If $\gamma_D \sim \gamma_{\mathrm{lin}}$, then one would have a situation in which the DITG shearing was strong enough to significantly lower the saturation level of the ETG turbulence, but would not necessarily completely "quench" it. This regime is particularly relevant for streamers, as in such a case the shearing could reduce the radial correlation length (which serves as a sort of radial step size for the turbulent thermal diffusivity) of the streamers enough to prevent them from driving significant levels of transport, even if they were not entirely suppressed (i.e., a "weak" streamer case).

Having identified the relevant time scales, one can make a more quantitative estimate for the importance of the DITG shearing. However, such a calculation requires a specific model for the DITG spectrum. One way of estimating this is to note that as the DITG turbulence is driven by the temperature gradient, an upper bound for the DITG saturation level is roughly at a mixing length level given by

$$T_q = \frac{\tilde{T}_i}{T_{i0}} = \frac{1}{qL_{Ti}}, \tag{7}$$

where $L_{Ti} = -d\ln T_{i0}/dx$ (L_{Ti} rather than L_n is used because the mode is driven unstable by the ion temperature gradient). Another approach for estimating the importance of DITG shearing on ETG turbulence is through a generalized predator–prey model; such a model is presented in Sec. V. Continuing with the mixing length estimate, one can then use quasi-linear theory to relate T_q to ϕ_q as $T_q = \Sigma_{Ti}\phi_q$, one can then estimate the DITG spectrum as

$$|\phi_q|^2 = \frac{1}{|\Sigma_{Ti}|^2}\frac{1}{q^2 L_{Ti}^2}. \tag{8}$$

It is also easy to show via linear theory that

$$|\Sigma_{Ti}|^2 \simeq \frac{(k_\theta V^*_{Ti})^2}{\omega_q^2 + \gamma_q^2} \simeq \frac{\eta_i}{\epsilon}, \tag{9}$$

where $\eta_i = L_n/L_{Ti}$ and $\epsilon = L_n/L_B$. With this model of the DITG spectrum, one can at last estimate the ratio of $\gamma_{\mathrm{lin}}/\gamma_D$ as

$$\gamma_D \sim \frac{\rho_e^2 v_{Te}^2}{\gamma_{\mathrm{lin}}}\sum_q q^4|\phi_q|^2 \sim \frac{\rho_e^2 v_{Te}^2}{\gamma_{\mathrm{lin}}}\frac{1}{|\Sigma_{Ti}|^2}\frac{\bar{q}^2}{L_{Ti}^2}$$
$$\rightarrow \frac{\gamma_{\mathrm{lin}}}{\gamma_D} = \left(\frac{L_n\gamma_{\mathrm{lin}}}{v_{Te}}\right)^2 \frac{|\Sigma_{Ti}|^2}{\eta_i^2(\bar{q}\rho_e)^2} \simeq \frac{M}{m}\left(\frac{k_\theta\rho_e}{\bar{q}\rho_s}\right)^2 \frac{\eta_e - \eta_e^c}{\tau\eta_i}. \tag{10}$$

In the above estimates, the characteristic wavenumber of the DITG turbulence is given by $\bar{q}$ and the fact that for curvature-driven ETG modes, the linear growth rate can be written as $\gamma_k \simeq (v_{Te}/L_n)k_\theta\rho_e\sqrt{\epsilon(\eta_e - \eta_e^c)/\tau}$ has been used. Here $\tau = T_{e0}/T_{i0}$. In a similar vein, one can define a time scale for DITG induced spatial diffusion $\gamma_X \sim D^X_{\alpha\beta}/L^2_{\mathrm{ETG}}$, where $L_{\mathrm{ETG}} = d\ln\langle N\rangle/dx$ is a characteristic spatial length scale of the ETG population density $\langle N\rangle$. It can then be shown via an analogous estimate that

$$\gamma_X \sim \frac{\rho_e^2 v_{Te}^2}{\gamma_{\mathrm{lin}}}\frac{1}{L^2_{\mathrm{ETG}}}\sum_q q^2|\phi_q|^2$$
$$\sim \frac{\rho_e^2 v_{Te}^2}{\gamma_{\mathrm{lin}}}\frac{1}{L^2_{\mathrm{ETG}}}\frac{1}{|\Sigma_{Ti}|^2}\frac{1}{L^2_{Ti}}$$
$$\rightarrow \frac{\gamma_X}{\gamma_{\mathrm{lin}}} = \left(\frac{v_{Te}}{L_n\gamma_{\mathrm{lin}}}\right)^2 \frac{\rho_e^2}{L^2_{\mathrm{ETG}}}\frac{\eta_i^2}{|\Sigma_{Ti}|^2} \simeq \frac{\tau\eta_i}{\eta_e - \eta_e^c}\left(\frac{1}{k_\theta L_{\mathrm{ETG}}}\right)^2 \tag{11}$$

which indicates that the spatial diffusion will be weak when $k_\theta L_{\mathrm{ETG}} > 1$, a limit which is always satisfied.

As described in the Introduction, there are two particularly significant cases of structure formation in ETG turbulence, as they are believed to be the most likely sources of experimentally relevant levels of electron thermal transport. These are as follows:

(1) large-scale streamers, which are observed in simulations to have $k_\theta \rho_e \simeq 1/10$;
(2) electromagnetic effects, which may drive an inverse cascade of energy, causing energy to accumulate at collisionless electron skin depth $\delta_e = c/\omega_{pe}$ scales, such that $k_\theta \rho_e \simeq \rho_e/\delta_e = \sqrt{\beta_e}$.

In either case, the shearing from DITG modes with $\bar{q}\rho_s < 1$ will still be weak; for instance, it is generally found that in simulations of curvature-driven ITG turbulence that the spectrum peaks near $\bar{q}\rho_s \simeq 0.1$. However, consideration of shorter wavelength modes (such as CTEM modes), which can produce fluctuations with $\bar{q}\rho_s \simeq 1$ would then suggest a shearing ratio for streamers,

$$\frac{\gamma_D}{\gamma_{\mathrm{lin}}} \simeq \frac{1}{k_\theta^2 \rho_e^2} \frac{m}{M} \frac{\tau \eta_i}{\eta_e - \eta_e^c} \simeq 100 \frac{m}{M} \frac{\tau \eta_i}{\eta_e - \eta_e^c}, \tag{12}$$

or (assuming $\beta_e \sim 10^{-2}$)

$$\frac{\gamma_D}{\gamma_{\mathrm{lin}}} \simeq \frac{1}{\beta_e} \frac{m}{M} \frac{\tau \eta_i}{\eta_e - \eta_e^c} \simeq 100 \frac{m}{M} \frac{\tau \eta_i}{\eta_e - \eta_e^c}, \tag{13}$$

for δ_e-scale ETG fluctuations. In either case, it is clear that the shearing ratio could approach unity for some parameters (such as a weak deviation from marginality for the ETG modes).

Thus, it seems that in general while the shearing of ρ_e scale ETG by DITG is fairly weak (i.e., $\gamma_D \ll \gamma_{\mathrm{lin}}$), the shearing due to short-wavelength DITG modes could have a significant impact on larger ETG structures, such as streamers on scales greater than ρ_e. This result confirms the basic intuition that for suppression by a shear flow to be effective, the scale of the turbulence or structure must be close to the scale of the shear flow.[31] It should be noted that while these DITG fluctuations are not generally considered, the primary sources of turbulent transport (and thus often neglected), they constitute the relevant shearing field for the ETG turbulence and structures. This result is particularly important for ETG streamers as it could significantly impact their saturation level and spatial structure, and thus the overall relevance of ETG turbulence as a source of experimentally relevant electron thermal transport. Thus, ETG modes should be studied in the presence of a CTEM (or another short-wavelength component of DITG turbulence) background. In this regard, it is important to note that the shearing effect depends explicitly on mass ratio, therefore any simulations which use artificially high values of m/M to study these interactions must take extra care in quantifying the observed scalings with the mass ratio range explored. In addition, this effect introduces a new way for geometry to affect electron transport, as the shearing can arise from physics such as trapped particles, the population of which has a strong radial dependence. It is also clear that there is the potential for some very interesting nonlinear dynamics for sufficiently strong DITG flows, the most obvious of which would be the localized trapping of ETG turbulence within a particularly strong DITG eddy. The key criterion for such an effect is that the magnitude of the flow field due to the DITG turbulence must exceed or be comparable to the group velocity of the ETG turbulence, as can be seen from the ray-tracing equations for the ETG turbulence,

$$\frac{d\vec{x}}{dt} = \vec{v}_g + \vec{V}^{\mathrm{DITG}}, \tag{14}$$

$$\frac{d\vec{k}}{dt} = -\frac{\partial(\omega_k + \vec{k}\cdot\vec{V}^{\mathrm{DITG}})}{\partial \vec{x}}. \tag{15}$$

Although a simple mean field estimate suggests $|\vec{V}^{\mathrm{DITG}}| \leq \vec{v}_g^{\mathrm{ETG}}$, it is possible that a sufficiently intense DITG eddy could trap the ETG turbulence. This idea has been considered previously for the case of a DITG wavepacket trapped in an DITG-driven zonal flow.[32–34] Such work could be generalized in a manner analogous to that used above for estimating the DITG shearing rate to investigate trapping and straining of ETG wavepackets in DITG eddies.

III. EFFECTS OF MODULATIONS IN η_e

In the previous section, direct interactions between the velocity fields associated with coexisting ETG and DITG turbulence were considered. However, there is at least one more interaction of interest: the convection of electron temperature fluctuations by the DITG turbulence. The impact of this interaction will have a somewhat different character than those of the previous discussions, as the ρ_s scale convection of T_e by DITG modulations will appear as modulations of L_{Te}, or equivalently, $\eta_e = L_n/L_{Te}$, to ETG modes. Such modulations of η_e represent an effective modulation of the ETG growth rate, which scales as $\gamma^{\mathrm{ETG}} \propto \sqrt{\eta_e - \eta_e^c}$, where η_e^c represents a critical value of η_e needed for instability. The effective modulation of equilibrium parameters for small-scale fluctuations due to convection by a larger-scale turbulent spectrum has been previously investigated by Itoh and Itoh and co-workers using a general model of renormalized multi-scale turbulence;[20,21] here, we focus specifically on the effects of DITG-induced ∇T_e modulations on ETG turbulence via a different approach than was used in Refs. 20 and 21. Specifically, we again exploit the fact that the DITG time scale is much slower than the ETG time scale, and treat the problem in the context of a wave-kinetic description of the ETG turbulence. One can write $\eta_e = \eta_e^0 + \delta\eta$, where $\delta\eta$ is the effective modulation due to the DITG turbulence, and we assume $\delta\eta/\eta_e^0 \ll 1$. Linearization of the wave-kinetic equation [Eq. (1)] provides

$$R^{-1}(\Omega_q)\,\delta N_q = 2 \left.\frac{\partial \gamma_k}{\partial \eta_e}\right|_{\eta_e = \eta_e^0} \delta\eta_q \langle N \rangle, \tag{16}$$

where $R(\Omega_q)$ is defined in Eq. (5), and we have expressed $\delta\eta$ as $\delta\eta = \Sigma_q \delta\eta_q \exp(i(\vec{q}\cdot\vec{x} - \Omega_q t))$.

We can then use quasi-linear theory to write the evolution equation for $\langle N \rangle$ as

$$\frac{\partial \langle N \rangle}{\partial t} = 2(\gamma_k + \gamma_{\mathrm{NL}})\langle N \rangle + O(\langle N \rangle^2), \tag{17}$$

$$\gamma_{\mathrm{NL}} = 2\left(\frac{\partial \gamma_k}{\partial \eta_e}\bigg|_{\eta_e=\eta_e^0} \right)^2 \sum_q R(\Omega_q)|\delta\eta_q|^2. \tag{18}$$

Since $\gamma_k \sim (\eta_e - \eta_e^c)^{1/2}$, and one can estimate $R(\Omega_q) \sim 1/2\gamma_k$, the ratio of γ_{NL} to γ_k can be estimated as

$$\frac{\partial \gamma_k}{\partial \eta_e}\bigg|_{\eta_e=\eta_e^0} = \frac{\gamma_k}{2(\eta_e^0 - \eta_e^c)} \tag{19}$$

$$\rightarrow \gamma_{\mathrm{NL}} \simeq \frac{\gamma_k}{4(\eta_e^0 - \eta_e^c)^2} \sum_q |\delta\eta_q|^2 \tag{20}$$

$$\Rightarrow \frac{\gamma_{\mathrm{NL}}}{\gamma_k} = \frac{\Sigma_q |\delta\eta_q|^2}{4(\eta_e^0 - \eta_e^c)^2} = \frac{1}{4}\left(\frac{\delta\eta}{\eta_e^0 - \eta_e^c} \right)^2. \tag{21}$$

Thus, when the magnitude of the DITG modulations of η_e is comparable to the deviation of η_e^0 from the critical value η_e^c (i.e., the deviation from marginality), the gradient modulation effect will be important. One can estimate the magnitude of the fluctuations through a mean-field theory of DITG turbulence; it should also be straightforward to calculate the modulation amplitude using existing numerical simulations. A particularly simple way to estimate the root mean square (rms) amplitude of the modulation is to again use a mixing length estimate for the gradient perturbation driven by DITG turbulence, which gives

$$\begin{aligned} \overline{\delta\eta_e} &= \langle |\delta\eta|^2 \rangle^{1/2} = L_n \langle |\nabla_r \tilde{T}_e|^2 \rangle^{1/2} \\ &= L_n \left(\sum_q \left| q_r \Sigma_{Te} \phi_q^{\mathrm{ITG}} \right|^2 \right)^{1/2} \\ &= L_n \left(\sum_q \left| \frac{\Sigma_{Te}}{\Sigma_{Ti}} \frac{q_r}{q L_{Ti}} \right|^2 \right)^{1/2} \end{aligned} \tag{22}$$

$$\rightarrow \overline{\delta\eta_e} = \eta_e \left(\sum_q \frac{q_r^2}{q^2} \right)^{1/2} = \frac{\eta_e}{\sqrt{2}}, \tag{23}$$

where we have used the fact that $|\Sigma_{Te}/\Sigma_{Ti}| = k_\theta V_{Te}^* / k_\theta V_{Ti}^* = \eta_e/\eta_i$. Equation (23) shows that the relative rms deviation can then be quite large:

$$\frac{\overline{\delta\eta_e}}{\eta_e^0 - \eta_e^c} \simeq \frac{1}{\sqrt{2}} \frac{\eta_e^0}{\eta_e^0 - \eta_e^c}. \tag{24}$$

It is important to note that the preceding analysis implicitly assumes that the net deviation from marginality $\eta_e^0 + \delta\eta_e - \eta_e^c$ is always greater than zero; that is, that the modulations of η_e are never strong enough to stabilize the ETG modes (which occurs when the net deviation is negative). However, common sense and the estimate shown in Eq. (24) suggests that such a situation is entirely possible. One can also turn this caveat around, and note that there could just as easily be a situation in which the equilibrium profile indicated ETG stability, but fluctuations of η_e could *nonlinearly* excite the ETG turbulence; in this case, one would find *sub-marginal* ETG turbulence. Such a situation would most likely induce highly intermittent or ''bursty'' behavior in the electron thermal flux, as η_e rose above or fell below the critical level for instability. While both are interesting questions, treatment of these issues would require a more sophisticated analysis which is beyond the scope of this work. Another important issue to address is that only the effect of temperature gradient modulations have been considered here, while a more complete analysis would also include the effects of DITG-induced modulations of the density scale length L_n on η_e. In particular, whether the modulations of L_n enhance or reduce the estimate for $\overline{\delta\eta_e}$ given in Eq. (24) should be investigated. We reiterate that the fluctuations in η_e considered here are dependent on poloidal and toroidal angle, and are not the quasi-stationary modifications of the flux-surface averaged electron temperature profile observed in some simulations (generally associated with rational surfaces).[22] We also note that the scale separation between the DITG and ETG turbulence suggests that the modulations of η_e could induce significant nonlocal behavior in the ETG dynamics, as the DITG modes would allow coupling of the ETG dynamics across many ρ_e.

The simple analysis presented here has shown that the convection of electron temperature fluctuations by DITG turbulence will manifest itself as a significant nonlinear amplification of the ETG growth rate, which will in turn cause a variation of the thermal transport due to the ETG turbulence. However, this transport will still be occurring on the characteristic scale of the ETG turbulence, which is much smaller than that of the DITG turbulence. Such an effect naturally suggests that it could be included in simulations of DITG turbulence via a subgrid model. One could envision representing this effect as an effective nonlinear hyperdiffusivity in T_e, the strength of which depended upon the local (ρ_s scale) gradients in η_e and the deviation of the electron temperature profile, as a whole, from criticality. Including such a term into simulations of DITG and TEM turbulence could reveal interesting dynamics in the electron heat flux induced by those modes.

IV. RENORMALIZATION APPROACH TO ETG STRESSES ON DITG TURBULENCE

Having considered the effects of DITG flow shear on ETG turbulence via an adiabatic theory analysis, we now undertake the complementary calculation, and study the effects of ETG modes on the DITG turbulence, by a renormalization analysis of cross-scale interactions.[20,35] We aim here to determine the ETG stresses on the DITG modes. Here, the essence of the analysis is to decompose the various effects in to coherent terms (''turbulent viscosities'') and incoherent terms (''noise'' terms), and investigate the relative magnitudes of various interactions. We model the system as two interacting fields with different characteristic scales, each essentially described by a Hasegawa–Mima type equation,

$$\frac{\partial \phi_k^<}{\partial t}+(i\omega_k^< - \gamma_k^<)\phi_k^< = \sum_{k'} \Lambda_{k,k'}^<(\phi_{k'}^<\phi_{k-k'}^< + \phi_{k'}^>\phi_{k-k'}^>), \tag{25}$$

$$\frac{\partial \phi_k^>}{\partial t}+(i\omega_k^> - \gamma_k^>)\phi_k^> = \sum_{k'} \Lambda_{k,k'}^>(\phi_{k'}^<\phi_{k-k'}^> + \phi_{k'}^>\phi_{k-k'}^>). \tag{26}$$

Here, the $<$ superscript refers to the DITG mode (or more generally, the large/ρ_s scale fluctuations), and the $>$ superscript to the ETG (small/ρ_e scale) turbulence. The first term on the rhs of Eq. (25) represents the self-interaction of the DITG mode, and the second term represents the Reynolds stress on the DITG mode due to the ETG turbulence. In Eq. 26, the first term on the rhs represents the shearing of the ETG mode by the DITG turbulence, and the second represents the ETG self-interaction. The coupling coefficients are defined as

$$\Lambda_{k,k'}^< = \rho_s^3 c_s \frac{\hat{z}\cdot(\vec{k}\times\vec{k}')}{1+k^2\rho_s^2}(|\vec{k}'|^2 - |\vec{k}-\vec{k}'|^2), \tag{27}$$

$$\Lambda_{k,k'}^> = \rho_e^3 v_{Te} \frac{\hat{z}\cdot(\vec{k}\times\vec{k}')}{1+k^2\rho_e^2}(|\vec{k}'|^2 - |\vec{k}-\vec{k}'|^2). \tag{28}$$

A more accurate model would be to include separate equations for the ion and electron temperature fluctuations, which were coupled to the flow fields. However, since we are primarily interested in quantifying the strength of direct flow–flow interactions, we use a quasi-linear approximation for the pressure fluctuations, and neglect the cross-field couplings between flows and pressure fluctuations (e.g., the ρ_s scale fluctuations in electron temperature induced by the DITG flow field, or ρ_e scale fluctuations in ion temperature due to the ETG flow), relative to like-scale couplings which are represented by the growth rates terms γ_k. The effects of the DITG modulations of the electron temperature were considered in Sec. III; also see Diamond *et al.*[14] for an alternate approach to this issue (in the context of a nonlinear buoyancy drive for streamers). In addition, the effects of DITG-generated zonal flow shearing on the DITG turbulence have been neglected. Such effects are easy to include, but do not affect the discussion here and so are omitted for the sake of clarity.

Following the usual procedures, one can recast the various terms as a combination of self- and cross-viscosities and noises, to construct an evolution equation for $I_k^< = \langle|\phi_k^<|^2\rangle$,

$$\frac{\partial I_k^<}{\partial t} = 2\gamma_k^< I_k^< + 2\,\mathrm{Re}\left(\sum_{k'} \Lambda_{k,k'}^<\langle\phi_{-k}^<\phi_{k'}^<\phi_{k-k'}^<\rangle + \sum_{k'} \Lambda_{k,k'}^<\langle\phi_{-k}^<\phi_{k'}^>\phi_{k-k'}^>\rangle\right). \tag{29}$$

The first term in the parentheses, $\langle\phi_{-k}^<\phi_{k'}^<\phi_{k-k'}^<\rangle$, is related to the DITG self-interaction, and can be rewritten as a self-viscosity $\nu_k^<$ and a self-noise $S_k^<$. The second term, $\langle\phi_{-k}^<\phi_{k'}^>\phi_{k-k'}^>\rangle$, describes the coupling of the ETG turbulence to the DITG flows, can also be decomposed in a coherent term $\nu_k^> I_k^<$ and incoherent term $S_k^>$.

The renormalized evolution equation for $I_k^<$ is then given by

$$\frac{\partial I_k^<}{\partial t} = 2\gamma_k^< I_k^< - \nu_k^< I_k^< + S_k^< + \nu_k^> I_k^< + S_k^>. \tag{30}$$

Note that the coherent term for the ETG–DITG interactions, $\nu_k^> I_k^<$, is similar to a negative viscosity, in that it transfers energy *into* the DITG turbulence; this directionality follows from the inverse cascade properties of the vorticity advection nonlinearity. This interpretation differs from that used in Refs. 20 and 21, which described the coherent effect of small-scales on large in terms of an "eddy viscosity," or *sink* for the larger-scale turbulence, rather than as a *source*. The case of a negative viscosity then demands resolution of the nagging question of large-scale damping in order to reach a stationary state; the inclusion of DITG-driven zonal flows (which are linearly damped by collisional friction between trapped and circulating ions[36]) could resolve this issue.

More specifically, one can rewrite the terms due to the ETG turbulence as

$$\nu_k^> = \sum_{k'} \Lambda_{k,k'}^< \Lambda_{k-k',k'}^> \Theta_{k,k'}^> I_{k'}^>, \tag{31}$$

$$S_k^> = \sum_{k'} (\Lambda_{k,k'}^<)^2 \Theta_{k,k'}^> I_{k'}^> I_{k-k'}^>. \tag{32}$$

The $\nu_k^<$ and $S_k^<$ terms will have similar forms, except for a different triad interaction time $\Theta_{k,k'}^<$ (along with the replacement of ETG coupling coefficients and spectral intensities with their DITG counterparts). One can write $\Theta_{k,k'}^>$ as (with ν_k being the nonlinear decorrelation rate)

$$\Theta_{k,k'}^> = \mathrm{Re}\frac{1}{-i(\omega_k^< - \omega_{k'}^> - \omega_{k-k'}^>) + (\nu_k^< + \nu_{k'}^> + \nu_{k-k'}^>)} \tag{33}$$

$$\simeq \frac{\nu_k^< + \nu_{k'}^> + \nu_{k-k'}^>}{(\omega_k^< - \vec{k}\cdot\vec{v}_g^>|_{k'})^2 + (\nu_k^< + \nu_{k'}^> + \nu_{k-k'}^>)^2}$$

$$\simeq \frac{1}{\nu_k^< + \nu_{k'}^> + \nu_{k-k'}^>}. \tag{34}$$

One might similarly estimate the decorrelation rate which appears in the DITG self interactions as

$$\Theta_{k,k'}^< \simeq \frac{1}{\nu_k^< + \nu_{k'}^< + \nu_{k-k'}^<}. \tag{35}$$

The key question is to determine the relative strength of the terms due to the ETG turbulence as compared to the terms representing the DITG self-interaction. One can make such an estimate as follows. First, from the underlying time scales, one can estimate that $\nu_k^< \propto c_s/L_n$, while $\nu_k^> \propto v_{Te}/L_n$, and so, $\Theta_{k,k'}^> \propto L_n/v_{Te}$, while $\Theta_{k,k'}^< \propto L_n/c_s$. Second, the structure of the coupling coefficients shows that $\Lambda_{k,k'} \propto k^2 k'^2$ (assuming $k \leqslant k'$). Finally, one must have an estimate for the spectral intensity I_k; here the mixing length estimate $I_k \simeq (|\Sigma_T| k L_T)^{-2}$ is again used for both the ETG and DITG turbulence. Then, the ratio of coherent terms, $\nu_k^> / \nu_k^<$, can be estimated as

$$\nu_k^> \sim \rho_s^3 c_s \rho_e^3 v_{Te} \sum_{k'} k^4 k'^4 \frac{L_n}{v_{Te}} \frac{1}{(|\Sigma_{Te}| k' L_{Te})^2}$$

$$= k^4 \rho_s^3 c_s L_n \frac{\rho_e}{|\Sigma_{Te}|^2 L_{Te}^2} \sum_{k'=k_{\min}^>}^{k_{\max}^>} (k' \rho_e)^2, \tag{36}$$

$$\nu_k^< \sim \rho_s^6 c_s^2 \sum_{k'} k^4 k'^4 \frac{L_n}{c_s} \frac{1}{(|\Sigma_{Ti}| k' L_{Ti})^2}$$

$$= k^4 \rho_s^3 c_s L_n \frac{\rho_s}{|\Sigma_{Ti}|^2 L_{Ti}^2} \sum_{k'=k_{\min}^>}^{k_{\max}^>} (k' \rho_s)^2, \tag{37}$$

$$\rightarrow \frac{\nu_k^>}{\nu_k^<} \sim \frac{\rho_e}{\rho_s} \left(\frac{|\Sigma_{Ti}|}{|\Sigma_{Te}|} \frac{L_{Ti}}{L_{Te}} \right)^2 = \left(\frac{m}{M} \right)^{1/2} \left(\frac{\eta_e}{\tau \eta_i} \right), \tag{38}$$

where $k_{\max}^> \rho_e$ and $k_{\max}^< \rho_s$ are taken to be of $O(1)$, and the relationships $|\Sigma_{Ti}|^2 \simeq \eta_i / \epsilon$ and $|\Sigma_{Te}|^2 \simeq \tau \eta_e / \epsilon$ have been used. The ratio of incoherent terms $S_k^> / S_k^<$ is estimated as (using $I_{k-k'} \simeq I_{k'}$)

$$S_k^> \sim \rho_s^6 c_s^2 \sum_{k'} k^4 k'^4 \frac{L_n}{v_{Te}} \frac{1}{(|\Sigma_{Te}| k' L_{Te})^4} = \frac{k^4 \rho_s^6 c_s^2 L_n}{v_{Te} |\Sigma_{Te}|^4 L_{Te}^4}, \tag{39}$$

$$S_k^< \sim \rho_s^6 c_s^2 \sum_{k'} k^4 k'^4 \frac{L_n}{c_s} \frac{1}{(|\Sigma_{Ti}| k' L_{Ti})^4} = \frac{k^4 \rho_s^6 c_s^2 L_n}{c_s |\Sigma_{Ti}|^4 L_{Ti}^4}, \tag{40}$$

$$\rightarrow \frac{S_k^>}{S_k^<} \sim \frac{c_s}{v_{Te}} \left(\frac{|\Sigma_{Ti}|}{|\Sigma_{Te}|} \frac{L_{Ti}}{L_{Te}} \right)^4 = \left(\frac{m}{M} \right)^{1/2} \left(\frac{\eta_e}{\tau \eta_i} \right)^2. \tag{41}$$

The result of the analysis is that the main effect of direct cross-flow interactions is to effectively add small ($\propto \sqrt{m/M}$) additional damping and noise terms to the DITG evolution equation. However, since these additional terms are smaller than the "self"-damping and noise terms by the square root of the mass ratio, it is not clear that they will play a significant role. This distinction is particularly important in light of the fact that the effects of zonal flow shearing on the DITG turbulence (which would be expected to be stronger than the DITG self-interaction terms) have been omitted. On the other hand, a number of assumptions were needed to reach this conclusion, so it should be regarded as an initial estimate, rather than a definitive result. It should also be reemphasized that these results, like those of the previous section, demonstrate that the effectiveness of cross-field effects will be determined by the effective separation of the respective spatial and temporal scales, which will manifest themselves as mass ratio scalings in the ratios of various terms. Therefore, any computational studies of cross-field effects must use care in choosing artificially large values of m/M. In this context, it should be noted that our results indicate that different effects may well scale differently with the mass ratio. Namely, the importance of shearing of the ETG turbulence by DITG modes, given by Eq. (10), scales linearly with the mass ratio, while the importance of the back reaction of the ETG on the DITG turbulence [Eqs. (38) and (41)] scales as the square root. Thus, simulations with unphysical values of m/M can artificially distort the relative importance of different cross-field interactions processes!

V. INTEGRATED MODELS OF JOINT ETG–DITG TURBULENCE

Having investigated a number of different interactions between ETG and DITG turbulence in the previous sections, we now integrate the results into a unified, self-consistent model. We achieve this unification by generalizing the zero-dimensional predator–prey models previously used[30] to describe zonal flow–drift-wave interactions to three fields (e.g., adding in an ETG field). Let $\mathcal{E}^>$ represent ETG intensity, $\mathcal{E}^<$ DITG intensity, and Ω the DITG-driven zonal flow intensity. A set of coupled equations can then be written as

$$\frac{\partial \mathcal{E}^>}{\partial t} = \gamma^> \mathcal{E}^> - \beta^> (\mathcal{E}^>)^2 - \rho \mathcal{E}^< \mathcal{E}^> + \gamma^{\mathrm{NL}} \mathcal{E}^< \mathcal{E}^>, \tag{42}$$

$$\frac{\partial \mathcal{E}^<}{\partial t} = \gamma^< \mathcal{E}^< - \beta^< (\mathcal{E}^<)^2 + \rho \mathcal{E}^< \mathcal{E}^> - \alpha \Omega \mathcal{E}^<, \tag{43}$$

$$\frac{\partial \Omega}{\partial t} = \alpha \mathcal{E}^< \Omega - \nu \Omega. \tag{44}$$

Here, the γ terms represent the linear growth rates, the β terms represent nonlinear self-damping, $\rho \mathcal{E}^< \mathcal{E}^>$ represents coherent flow interactions between the ETG and DITG turbulence (shear suppression of ETG by DITG/negative viscosity for the DITG due to ETG), $\gamma^{\mathrm{NL}} \mathcal{E}^< \mathcal{E}^>$ represents the nonlinear ETG growth rate due to DITG modulations of η_e, $\alpha \mathcal{E}^< \Omega$ represents shear suppression of DITG by zonal flows/zonal flow generation, and $\nu \Omega$ describes the linear damping of zonal flows. For simplicity, the various noise/incoherent terms have been neglected. These equations have the nontrivial solution for $\mathcal{E}^<$ and $\mathcal{E}^>$,

$$\mathcal{E}^< = \frac{\nu}{\alpha}, \tag{45}$$

$$\mathcal{E}^> = \frac{\gamma^>}{\beta^>} + \frac{(\gamma^{\mathrm{NL}} - \rho)}{\beta^>} \frac{\nu}{\alpha}. \tag{46}$$

The key point is that while the intensity level of the ETG turbulence is affected by the presence of the DITG turbulence and zonal flows, the DITG intensity is independent of the ETG intensity level. In particular, the DITG intensity is the same as found in Ref. 30 (i.e., reflecting the balance of zonal flow generation and damping). In the previous sections, it was found that the nonlinear growth rate term was strong [$\gamma_{\mathrm{NL}} \sim \gamma_k$, Eq. (21)], while the shearing of ETG by DITG was weak, except for certain limited cases [$\gamma_D \leqslant 2\gamma_k$, Eq. (10)]. Therefore, in terms of the model presented here, these findings could be expressed as $\gamma^{\mathrm{NL}} \geqslant \rho$ (where ρ represents the strength of coherent ETG–DITG flow couplings). The dominance of "growth rate enhancement" over shear suppression for DITG–ETG interactions can be contrasted to the findings of Ref. 21, where consideration of interactions between DITG and current-diffusive interchange mode[37] (CDIM) turbulence (which has a characteristic scale of the collisionless electron skin depth) suggests that shear suppression of the CDIM turbulence is the dominant effect. Also note that the equations contain the possibility for submarginal ETG turbulence (as discussed at the end of Sec. III), and the nonlinear excitation of DITG turbulence via the "negative viscosity" due to the ETG turbulence.

One conclusion that the previous sections seem to point towards is that while the effects of DITG turbulence on ETG turbulence may be significant, the back-reaction is generally weak for the DITG dynamics. More plainly, while the presence of DITG modes will affect the ETG dynamics, the ETG modes do not appreciably affect the DITG dynamics. Such a situation motivates one to consider the idea of a "super-grid" model for the numerical simulation of ETG turbulence, in which a specific realization of the large-scale shear flow and temperature fluctuation spectra (taken from a simulation of DITG turbulence) is applied to the ETG turbulence. One way of implementing this model would undertake a simulation of DITG turbulence, and record "snapshots" of the spectra of potential and temperature fluctuations from the fully nonlinear state separated by one or more turbulent correlation times, perhaps windowed to include only smaller scales (such $k\rho_i \geqslant 1$). These snapshots would represent independent realizations of the DITG-induced shearing and temperature modulation profiles for the ETG turbulence. Then, for each realization of the DITG turbulence, a simulation of ETG turbulence in the presence of the DITG fields would be carried out; the spectra resulting from these ETG simulations, averaged over the DITG realizations, could then be used to statistically quantify the effects of DITG flow shearing and temperature modulation on ETG turbulence. Note that in each ETG simulation, the separation of DITG and ETG time scales would allow the input DITG field to be held fixed. Thus, each DITG realization essentially represents a modification to the ETG equilibrium parameters. Thus, the approach proposed here amounts to a model of the "supergrid" scales for the ETG turbulence, in contrast to the more familiar "sub-grid" models used to treat the effects of small scales on large. Alternatively, one might couple a numerical evolution of the ETG wave-kinetic equation to the DITG simulation (either directly or via the "snapshot" approach), rather than the basic ETG equations. Although either method would require a significant amount of computational resources, they may still be preferable to "joint" simulations which include both ETG and DITG dynamics at the expense of reduced mass ratios, particularly as these joint simulations must encompass many DITG correlation times while including the very short ETG space and time scales to determine the statistical steady state of the system. However, the results presented in this paper strongly suggest that understanding the effects of DITG shearing and profile modification on ETG turbulence is critical to determining the true importance of ETG-driven transport in magnetic confinement devices. Note that either scheme suggests that developing some quantitative understanding of the statistics of strong shearing and ∇T_e (or more generally, η_e) profile perturbation events in DITG turbulence would be important in developing a high fidelity supergrid-scale model. This understanding is needed because it is precisely the "peaks" and "high ridges" in the DITG shear and ∇T_e-perturbation landscape which are the regions that will most dramatically impact the ETG turbulence. In particular, it is the population density of large events (e.g., the tail of the perturbation probability distribution function), and not the rms or average values of the DITG fields, which is of interest here. Therefore, understanding the interactions between DITG and ETG turbulence provides yet another motivation for moving from mean-field type studies of turbulence and transport to probabilistic models of turbulent transport (see Refs. 14, 38–40 for initial investigations in this direction). Finally, we note that understanding the interactions between ETG and DITG turbulence is particularly important for understanding transport barrier physics, as the small-scale DITG modes which drive less transport than the large-scale DITG modes, but provide the relevant shearing field for the ETG turbulence, are less likely to be suppressed by the equilibrium $\vec{E}\times\vec{B}$ shear flow, and may therefore still significantly impact the ETG dynamics inside the transport barrier.

VI. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper, interactions between DITG and ETG turbulence have been studied via simple models. It was found that while the random shearing of "generic" ρ_e scale ETG turbulence by DITG modes was weak, shearing of large-scale streamers and collisionless skin-depth fluctuations by short-wavelength ($\bar{q}\rho_s \simeq 1$) DITG modes could be significant. This result should also apply to other large-scale structures such as ETG-driven zonal flows (calling into question the results of Li and Kishimoto[19]) or zonal magnetic fields.[41,42] We emphasize the importance of the DITG shearing of streamers because streamers represent a prominent potential mechanism for allowing ETG to drive experimentally relevant levels of transport. Therefore, their suppression directly impacts the status of ETG as a relevant source of significant transport. It is also important to note that it is the short-wavelength portion of the DITG spectrum which provides the relevant shearing field. This fact may have important ramifications for understanding transport physics in the presence of transport barriers, as these short-wavelength modes will be less affected by the presence of the equilibrium shear flow than the larger scale DITG modes. In addition, a primary source of such short-wavelength DITG modes (such as the CTEM) will be trapped electrons, which suggests that the importance of DITG shearing will vary with minor radius in the confinement device (e.g., as the fraction of trapped electrons increases with normalized radius, DITG shearing effects should become stronger). In Sec. III, a novel mechanism for cross-field coupling is detailed, in which the DITG induced fluctuations of electron temperature gradient are manifested as a nonlinear modulation of the ETG growth rate. This effect was found to scale as $(\delta\eta_e/(\eta_e^0-\eta_e^c))^2$, which can be quite significant. What is particularly intriguing about this effect is that it can work to *enhance* the ETG intensity level, and *oppose* the effect of random shearing by the DITG turbulence. Understanding the competition between these effects would be particularly interesting for streamers and other large-scale ETG structures. In contrast to the effects of DITG on ETG turbulence, the ''back-reaction'' of the ETG turbulence on the DITG modes was found to be weak, as shown by both the renormalization analysis of Sec. IV and the extended predator–prey model of Sec. V. In both investigations of direct flow–flow interactions (either DITG shearing of ETG, or the ETG back-reaction on DITG), explicit mass-ratio scalings for the importance of the various effects were derived. Therefore, particular care must be taken in any direct simulations of ETG–DITG turbulence which use artifical mass ratios, to ensure that the effects of the interactions are not over-estimated, and that they are given proper relative weightings (as different effects scale differently with mass ratio).

These results suggest a number of interesting directions for future study. The most important of these would be to gain a better understanding of how the competing effects of DITG shearing and growth rate enhancement will affect streamer dynamics, as mentioned above. A particularly interesting question would be to determine the relative effectiveness of DITG shearing versus Kelvin–Helmholtz breakup as streamer saturation mechanisms. This study also naturally leads to the idea of examining an ETG wavepacket trapped in a DITG eddy as a kind of ''coherent structure.'' DITG-induced spatial diffusion of ETG turbulence, as well nonlocal effects which could be induced via the DITG modulations of η_e also suggest interesting mechanisms for nonlinear spreading ETG turbulence.[43,44] Sections III–V also indicate that the possibility of nonlinear excitement of DITG and ETG turbulence should be investigated further. A related investigation of this issue has been undertaken by Itoh and Itoh.[20] Especially interesting would be whether ETG turbulence could excite DITG turbulence in the presence of an ion transport barrier. Finally, the development of a supergrid model for ETG turbulence in the presence of a DITG spectrum would not only present insight into ETG dynamics, but would represent a fundamentally new and interesting development in the study of nonlinear dynamics and turbulence. As the first step in developing such a model, more detailed investigations on the *statistics* of DITG shearing and modulations of η_e (due to both L_n *and* L_{Te} modulations) should be undertaken.

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