Ion-temperature gradient modes affected by helical magnetic field of magnetic islands

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Ion temperature gradient mode (ITG) affected by static magnetic field of magnetic islands is investigated numerically by means of Landau fluid model. The ITG is localized around O-points of magnetic islands, and the localization in poloidal direction is similar to the poloidal localization of toroidal ITG. This is because the helical magnetic field of magnetic islands causes geometrical coupling, and thus Fourier modes that have the same helicity as the islands are coupled together. The strength of coupling is characterized by the square of island width, and it corresponds to the fact that the strength of mode coupling of toroidal ITG is characterized by the inverse aspect ratio of torus in reduced fluid models. © *2010 American Institute of Physics*. [doi:10.1063/1.3460346]

Ion temperature gradient modes (ITGs) drive turbulence and cause anomalous heat transport in magnetically confined plasmas,[1,2] and suppression of the instability by zonal flow is a hot topic.[3] The analysis of ITG was carried out for equilibrium that has nested magnetic surfaces. In reality, some magnetic surfaces can be broken by tearing modes[4] or by externally applied magnetic field perturbation,[5] and then magnetic islands can appear. Interactions between drift-wave turbulence and magnetic islands are studied analytically and numerically.[6–14] Magnetic islands can influence ITG through helical magnetic field, flattening of temperature and density profiles, and the flow caused by the rotation of islands. In this brief communication, we focus only on the effect of magnetic field of the islands, which is one of the main effects of islands. We examine effects of static helical magnetic field of magnetic islands on linear growth of ITG in cylindrical plasma by means of Landau fluid model. The effects of magnetic field of the islands are elucidated by adopting cylindrical geometry which excludes torus effects causing toroidal mode coupling and by neglecting profile flattening around the islands. It is found that the ITG is localized in the poloidal direction and appears around O-points of the islands. The effect is similar to toroidal curvature effects on toroidal ITG, which appears in the bad curvature region of torus plasma. The localization in poloidal direction is caused by mode coupling through helical magnetic field of islands and its strength is characterized by square of the island width. It corresponds to the inverse aspect ratio which characterizes the strength of coupling of toroidal ITGs or ballooning modes in reduced fluid models.[15]

Our analysis is based on the Landau fluid model in cylindrical geometry.[16] We consider the effect of helical magnetic field of islands and neglect the effects of islands on temperature and density profiles, and thus only the parallel gradient in the model is modified by the presence of the islands. The presence of helical magnetic field of magnetic islands is described by using helical magnetic flux and a coordinate reflecting helical symmetry ζ. Since we consider large aspect ratio torus plasma with strong and uniform toroidal field in the z-direction $B_z=B_0$, the helical magnetic flux function is written as

$$\psi_h(r,\zeta)=\epsilon\left(h\frac{r^2}{2}-\int\frac{r}{q(r)}dr\right)+\psi_M(r,\zeta)$$

(see Sec. 5.2 in Ref. 17), where $\psi_M=\Psi_M(r)\cos(M\zeta)$ is the magnetic flux representing the $(m,n)=(M,N)$ magnetic islands and $\zeta=\theta-hz$, where $h=N/M$, m and n are helicity, poloidal, and toroidal mode numbers, respectively. The linearized equations for the perturbations are

$$\begin{aligned}\frac{\partial}{\partial t}(\tilde{n}-n_{\mathrm{eq}}\nabla_\perp^2\tilde{\Phi}) &= -n_{\mathrm{eq}}\nabla_\parallel\tilde{v}_\parallel+\frac{1}{\rho_*}\frac{dn_{\mathrm{eq}}}{dr}\nabla_\zeta\tilde{\Phi}+\frac{T_{\mathrm{eq}}}{\rho_*}\frac{dn_{\mathrm{eq}}}{dr}\\ &\quad\times(1+\eta_i)\nabla_\zeta\nabla_\perp^2\tilde{\Phi}+\nu\nabla_\perp^4\tilde{\Phi},\end{aligned} \tag{1}$$

$$n_{\mathrm{eq}}\frac{\partial\tilde{v}_\parallel}{\partial t}=-n_{\mathrm{eq}}\nabla_\parallel\tilde{T}_i-(1+\tau)T_{\mathrm{eq}}\nabla_\parallel\tilde{n}+\nu\nabla_\perp^2\tilde{v}_\parallel, \tag{2}$$

$$\frac{\partial\tilde{T}_i}{\partial t}=-(\Gamma-1)(T_{\mathrm{eq}}\nabla_\parallel\tilde{v}_\parallel+\kappa_L\tilde{T}_i)+\frac{1}{\rho_*}\frac{dT_{\mathrm{eq}}}{dr}\nabla_\zeta\tilde{\Phi}+\nu\nabla_\perp^2\tilde{T}_i, \tag{3}$$

where

$$\tilde{n}=\frac{n_{\mathrm{eq}}}{\tau T_{\mathrm{eq}}}\tilde{\Phi},$$

$\nabla_\parallel f=-[\psi_h,f]$, $\Phi=\rho_*\tilde{\Phi}$, $n=n_{\mathrm{eq}}+\rho_*\tilde{n}$, $T_i=T_{\mathrm{eq}}+\rho_*\tilde{T}_i$, $T_e=\tau T_{\mathrm{eq}}$, $\rho_*=\rho_i/a$, and

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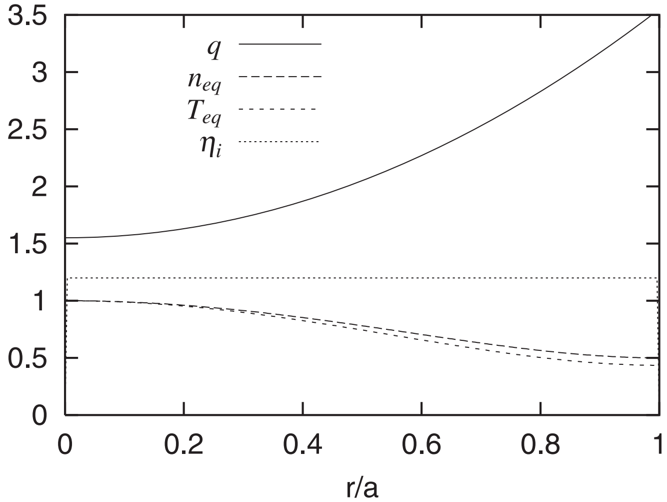

FIG. 1. Equilibrium profiles in the absence of magnetic islands.

$$\kappa_L = \sqrt{\frac{8T_{\rm eq}}{\pi}}|\nabla_\parallel|.$$

In these equations n, $v_\parallel$, Φ, T_i, $\Gamma=5/3$, ρ_i, and a are the electron density, the parallel ion velocity, the electrostatic potential, the ion temperature, the ratio of specific heats, Larmor radius, and the minor radius, respectively. The normalizations are $(tv_{ti}/a, x/\rho_i, \rho_i\nabla_\perp, a\nabla_\parallel, e\Phi/T_0, \times\psi_h/(B_0\rho_i), n/n_0, T/T_0, v_\parallel/v_{ti}) \to (t, x, \nabla_\perp, \nabla_\parallel, \Phi, \psi_h, n, T, v_\parallel)$.

Linear growth of ITG in the presence of static magnetic field of islands is investigated for following profiles. Profiles of safety factor, density, and temperature are $q=1.55+2(r/a)^2$, $n_{\rm eq}=0.5+0.5(1-(r/a)^2)^2$, and $T_{\rm eq}=n_{\rm eq}^{1.2}$, respectively. Notice that the instability parameter $\eta_i\equiv L_n/L_T=1.2$ is uniform, where $L_n=-d\ln n/dr$ and $L_T=-d\ln T/dr$ are density and temperature gradient lengths, respectively. Figure 1 shows temperature, density, and q profiles. Parameters are set to be $\rho_*=1/80$, $\nu=m^4 10^{-7}$, and $\tau=1$. In our analysis $(M,N)=(2,1)$ magnetic islands are adopted. The magnetic flux ψ_M is the eigenfunction of a tearing mode calculated by numerically solving a reduced set of two-fluid equations.[9] Equicontours of the helical magnetic flux ψ_h are shown in Fig. 2.

Linear behavior of ITG that has the helicity $h=n/m=1/2$ is examined for the static magnetic field including the $(M,N)=(2,1)$ magnetic islands. This helical-ITG with zero helical mode number has the same helicity as the islands, and thus the perturbation can be written as $\tilde{f}(r,\theta,z,t)=\Sigma_m\hat{f}_m(r,t)\exp(im\zeta)$. Figure 3 shows the color map of electrostatic potential of ITG (a) in the presence of magnetic field of islands and (b) in the absence of magnetic field of islands. In the presence of islands, the perturbation of ITG appears around O-points of the islands in Fig. 2, while there is no poloidal localization of perturbation in the absence of islands. This poloidal localization is similar to the poloidal localization of toroidal ITG that appears at the outer side of torus plasma. This can be understood by examining the equations including magnetic islands, Eqs. (1)–(3). The difference of the equations from the equations without islands is

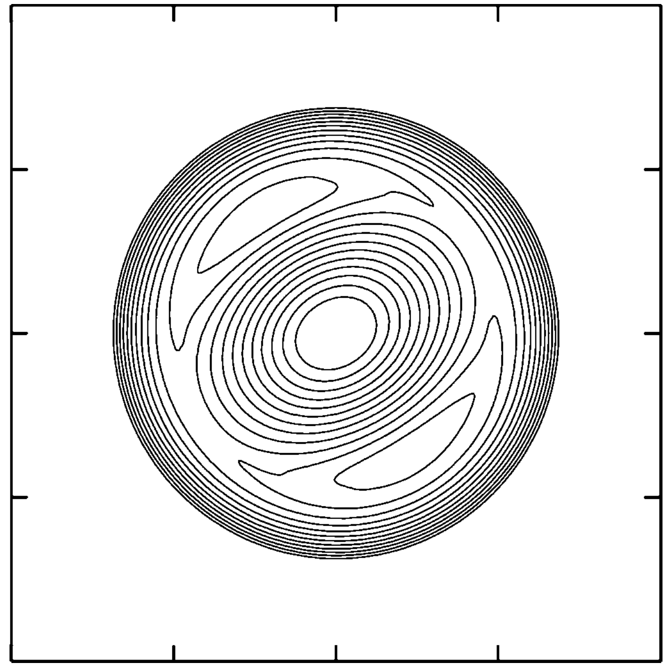

FIG. 2. Equicontours of magnetic flux representing $(m,n)=(2,1)$ magnetic islands.

$-[\Psi_M(r)\cos(M\zeta), f]$ in $\nabla_\parallel f$. This is similar to toroidal curvature term $\epsilon[r\cos\theta, f]$ in reduced fluid models,[15] and $-\cos(M\zeta)$ in the former corresponds to $\cos\theta$ in the latter, where $\epsilon=a/R$ is the inverse of aspect ratio and R is the major radius. Hence, O-points, $M\zeta=\pi$, of magnetic islands in the former correspond to the bad curvature region $\theta=0$ in the latter. The difference in $\nabla_\parallel$ also implies the strength of the coupling. The strength of coupling between modes that have the same helicity as the island is $\Psi_M(r)$ in the presence of islands, while the strength of coupling between modes that have the same toroidal mode is ϵr in torus plasma. Thus, the square of magnetic island width $W^2/2$ corresponds to the inverse aspect ratio ϵr_s of toroidal ITG or ballooning modes because of the relation between the width and magnetic flux $W^2/2=\Psi_M(r_s)$, where r_s is the radius of the rational surface, which satisfies $q(r_s)=M/N=1/h$.

Figure 4 shows the growth rate of the helical-ITG mode with zero helical mode number (i.e., having the same helicity as the island). Due to the coupling caused by the helical

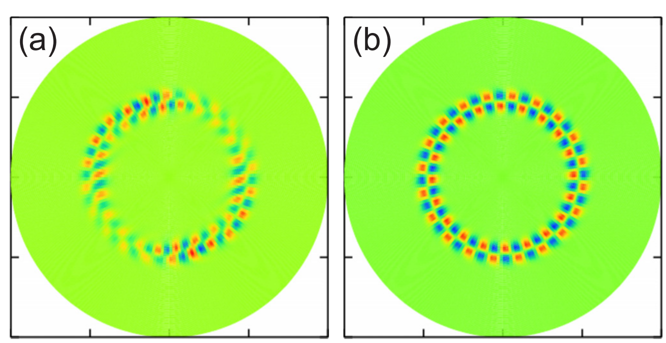

FIG. 3. (Color online) Electrostatic potential profile on a poloidal section, (a) ITG in the presence of magnetic field of islands and (b) ITG in the absence of magnetic field of islands.

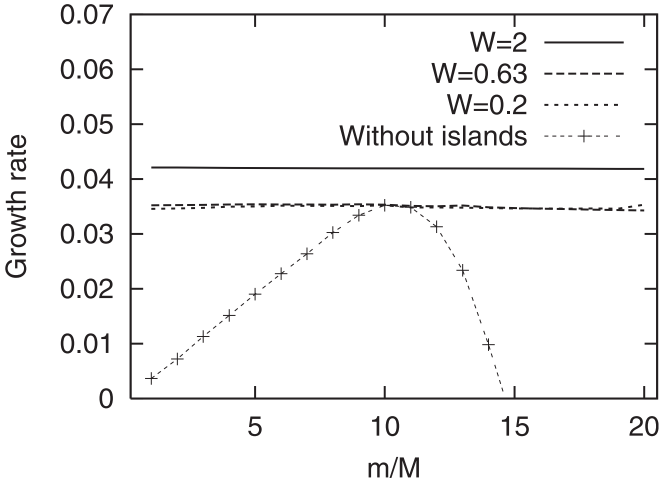

FIG. 4. Growth rate of the helical-ITG mode with zero helical mode number for each island width, where m is the poloidal mode number and $M=2$ is the poloidal mode number of magnetic islands.

perturbation, this mode contains several coupled poloidal mode numbers. In the figure, the island width is normalized by the radial width of ITG without islands. In the absence of islands the growth rates for poloidal modes are independent of each other, and $m=20$ mode is the most unstable mode. On the other hand, the growth rates of all modes are almost the same in the presence of islands because of coupling between modes that has the same helicity $m/n=2/1$ through the $(m,n)=(2,1)$ magnetic field. When the island width is smaller than 1, the growth rate in the presence of island is the same as the most unstable mode of ITG without islands. The growth rate is larger than that for without the islands when the island width is larger than the radial width of ITG without islands.

In summary, we have found that helical magnetic field of magnetic islands causes mode coupling between Fourier modes of ITG which have the same helicity as the islands. The mode coupling is similar to the toroidal mode coupling of ITG or ballooning modes. The coupling causes the localization of mode in the poloidal direction, and ITG appears around O-points of magnetic islands. This localization is similar to the localization around bad curvature region of toroidal ITG in torus plasma. The strength of coupling is characterized by the square of the width $W^2/2$. This corresponds to the fact that the strength of toroidal mode coupling of ITG in torus is characterized by the inverse aspect ratio of torus plasma ϵr_s in reduced fluid models. When the island width is larger than the radial width of ITG without islands, the growth rate of ITG in the presence of islands is larger than that of ITG without islands. On the other hand, when the island width is smaller than the radial width of ITG without islands, the growth rate is the same as the most unstable mode of ITG without islands. Our analysis suggests that a similar analysis of the ballooning mode is useful to analyze ITG in the presence of magnetic islands. Finally, we remark other effects of magnetic islands on the growth of ITG. One is that magnetic islands affect temperature and density profiles around islands. The profiles can be flattened inside the separatrix of magnetic islands, and the flattening can affect the growth of ITG. The other effect on ITG is that of shear flow caused by island rotation, which is related to the polarization current around the separatrix. These effects will be studied in future work.

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