On the Structure and Scale of Cosmic Ray Modified Shocks

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Abstract. Strong astrophysical shocks, diffusively accelerating cosmic rays (CR) ought to develop CR precursors. The length of such precursor L_p is believed to be set by the ratio of the CR mean free path λ to the shock speed, i.e., $L_p \sim c\lambda/V_{sh}$, which is formally independent of the CR pressure P_c. However, the X-ray observations of supernova remnant shocks suggest that the precursor scale may be significantly shorter than L_p even if one inserts $\lambda = \lambda_{min} = r_g$, the gyroradius (i.e. the Bohm limit). We argue that while the CR pressure builds up ahead of the shock, the acceleration enters into a strongly nonlinear phase in which an acoustic instability, driven by the CR pressure gradient, dominates other instabilities (at least in the case of low β plasma). In this regime the precursor steepens into a strongly nonlinear front whose size scales with *the CR pressure* as $L_f \sim L_p \cdot (L_S/L_p)^2 \, (P_c/P_g)^2$, where L_s is the scale of the developed acoustic turbulence, and P_c/P_g is the ratio of CR to gas pressure. Since $L_S \ll L_p$, the precursor scale reduction may be strong in the case of even a moderate gas heating by the CRs through the acoustic and (possibly also) the other instabilities driven by the CRs.

Keywords: acceleration of particles – cosmic rays – shock waves — supernova remnants — turbulence – nonlinear phenomena

1. Introduction

Significant progress in observations of galactic supernova remnant shocks (SNR) over the passed decade have furthered our understanding of the particle acceleration mechanism which is deemed responsible for both the individual SNR emission and for the galactic cosmic ray (CR) production as a whole. The mechanism is known as the first order Fermi or diffusive shock acceleration (DSA) [11]. The shocks are typically observed as surprisingly thin filaments, particularly well resolved in the X-ray band. The working hypothesis is that this emission is due to the super-TeV shock accelerated, synchrotron radiating electrons, in some cases also seen in gamma-rays (through the inverse Compton (IC) up-scattering of the background photons). The gamma emission may in some cases be contaminated or even dominated by the accelerated protons via π^0 decay [1, 2]. A convincing demonstration of namely the latter scenario would, of course, be a *prima facie* evidence for the acceleration of also the main, i.e. hadronic component of galactic CRs in SNRs.

The key element of the DSA is multiple crossing of the shock front with a $\sim V_{sh}/c$ energy gain after each crossing. In doing so particles diffusively escape from the shock, on average to a distance $L_p \sim \kappa(p)/V_{sh}$. Here κ is the momentum dependent diffusion coefficient and V_{sh} is the shock velocity. An obvious morphological consequence of this process should be and extended $\sim L_p$ shock precursor filled with synchrotron radiating electrons. A disturbing fact of the high-resolution X-ray observations is that there is no supporting evidence for such extended precursors [23, 7, 6, 5]. Note that L_p grows with particle momentum as almost certainly does $\kappa(p)$, so particles of higher energy should make thicker emission filaments than do the lower energy particles. This trend does not seem to be supported by the observations either, see [6, 13].

According to another widely accepted view, the particle diffusion coefficient κ should be close to the Bohm value, $\kappa \sim cr_g(p)/3$, which requires strong magnetic fluctuations $\delta B_k \sim B_0$ at the resonant scale $k \sim 1/r_g(p)$. The high level of fluctuations is achieved through one of the instabilities driven by accelerated particles. The following three CR driven instabilities have been suggested to generate magnetic field fluctuations. The first one is the ion cyclotron resonant instability of a slightly anisotropic (in pitch angle) CR distribution ahead of the shock [8]. The free energy source of this instability has the potential to generate very strong field fluctuations [24]

$$(\delta B/B_0)^2 \sim M_A P_c/\rho V_{sh}^2. \tag{1}$$

where $M_A \gg 1$ is the Alfvenic Mach number, P_c is the CR pressure, ρ is the gas density and u_s is the shock velocity. However, the actual turbulence level was thought to remain moderate, $\delta B \sim B_0$(e.g., [24, 4]).

The second instability, is a nonresonant instability driven by the CR current. The advantage of this instability seems to be twofold. First, it cannot be stabilized by the quasilinear deformation of the CR distribution function since in the upstream plasma frame the driving CR current persists once the CR cloud is at rest in the shock frame. Second, it generates a broad spectrum of waves, and the longest ones were claimed to be stabilized only at the level $\delta B \gg B_0$, due to the lack of an efficient stabilization mechanism at such scales. The stabilization mechanism was assumed to be the magnetic tension [9]. The potential of this instability to generate strong magnetic fields was first emphasized in refs. [10, 9]. In particular Bell pointed out that in the most interesting regime, the instability is driven by a fixed CR return current through the Ampere force $\mathbf{J}_c \times \mathbf{B}$. It should be noted, however, that the dissipation of the return current due to the anomalous resistivity still needs to be addressed.

The third instability is an acoustic instability (also known as Drury's) driven by the pressure gradient of accelerated CRs upstream [19]. This instability was also studied numerically by Dorfi [17]. The pressure gradient is clearly a viable source of free energy for the instability. So, among quantities varying across a shock, the pressure jump is the most pronounced one in that sense that it does not saturate with the Mach number, unlike the density or velocity jumps.

Curiously enough, the acoustic instability has received much less attention than the first two. Moreover, in many numerical studies of the CR shock acceleration, special care is taken to suppress it. The suppression is achieved by using the fact that a change of stability occurs at that point in the flow where $\partial \ln \kappa / \partial \ln \rho \simeq -1$ (for both stable and unstable wave propagation directions). Here κ is the CR diffusion coefficient, and ρ is the gas density. Namely, one requires this condition to hold identically all across the shock precursor, i.e., where the CR pressure gradient $\nabla P_c \neq 0$. Not only is this requirement difficult to justify physically, but, more importantly, an *artificial* suppression of the instability eliminates its *genuine* macroscopic and microscopic consequences, as briefly discussed below.

Among the macroscopic consequences an important one is the vorticity generation through the baroclinic effect (missalignment of the density and pressure gradients $\nabla \rho \times \nabla P \neq 0$). Here ∇P may be associated with a quasi-constant macroscopic CR-gas pressure gradient ∇P_c, generally directed along the shock normal. Variations of $\nabla \rho$ are locally decoupled from P_c, unlike in the situation in a gas with a conventional equation of state where $P = P(\rho)$ and where the baroclinic term vanishes. The vorticity generation obviously results (just through the frozen in condition) in magnetic field generation, so that the field can be amplified by the CR pressure gradient. More importantly, this process amplifies the *large scale field*, required for acceleration of *high energy particles*. Furthermore, the amplification takes place well ahead of the gaseous subshock. The both requirements are crucial for improving high energy particle confinement and making the shock precursor shorter in agreement with the observations. Large scales should be present in the ambient plasma as a seed for their amplification by the acoustic instability and could be driven/seeded by wave packet modulations. Apart from that, they result from the coalescence of shocks formed in the instability, and from the scattering of Alfven waves in k-space by these shocks to larger scales [16]. Note that the Bell instability is essentially a short scale instability (the maximum growth rate is at scales much smaller than the gyro-radii of accelerated particles). At larger scales the growth rate decreases and the instability transitions into the conventional cyclotron instability. It should be noted that, as well known [22, 20], vorticity (and thus magnetic field) can be efficiently generated also at the subshock. This would, of course, be too late for improving particle confinement and reducing the scale of the shock precursor.

Now the question is which instability dominates the CR dynamics? Given the finite precursor crossing time, it is reasonable to choose the fastest growing mode and consider the development of a slower one on the background created by the fast mode after its saturation. The resonant cyclotron instability is likely to dominate at the outskirt of the shock precursor where both the CR current and pressure gradient (driving the other two instabilities) are weak, whereas the pitch angle anisotropy is strong enough to drive the resonant instability. Recall that the anisotropy is typically inversely proportional to the local turbulence level which must decrease with the distance from the shock. In this paper, however, we focus on the main part of the shock precursor where both the CR-pressure gradient and CR current are strong, and consider the Bell and Drury instabilities as the strongest candidates to govern the shock structure. In fact, these instabilities are coupled, not only by the common energy source but also dynamically. The coupling of the magnetic and the density perturbations, akin to the modulational instability will be the subject of a separate publication. This paper will be limited to a simpler situation in which one of the instabilities dominates. In particular we will identify conditions under which the acoustic instability grows more rapidly. Then, we determine the shock structure resulting from the nonlinear development of the acoustic instability.

2. Comparison of the growth rates

The derivations of the linear dispersion relations of the Bell and Drury modes can be found in refs.[19, 9]. For the magnetic field and density perturbations of the form, $\tilde{B} \propto \exp(-ikx \pm i\omega t)$ and $\tilde{\rho} \propto \exp(ikx \pm i\omega t)$, in an approximate symmetric representation, in which the diffusive CR damping of acoustic mode and the resonant CR contribution to the nonresonant magnetic instability are neglected, these relations can be written as follows

$$\omega^2 = k^2 C_A^2 - 2\gamma_B k C_A \tag{2}$$

$$\omega^2 = k^2 C_s^2 - 2i\gamma_D k C_S \tag{3}$$

Here C_A and C_S are the Alfven and the sound speeds, respectively while

$$\gamma_B = \sqrt{\frac{\pi}{\rho_0}} J_c/c \text{ and } \gamma_D = -\frac{1}{2\rho_0 C_S} \frac{\partial \overline{P}_c}{\partial x} \tag{4}$$

The last expressions approximately represent the maximum growth rates of the nonresonant B-field (Bell) perturbations and acoustic density (Drury) perturbations, driven by the CR return current and by the CR pressure gradient, respectively. Note that the instabilities are different in that the Bell mode is unstable in the limited k-band where $0 < k < 2\gamma_B/C_A$, and γ_B is only the maximum growth rate that is obviously achieved at $k = \gamma_B/C_A$, whereas the acoustic mode saturates with k at $k \gtrsim \gamma_D/C_S$ at the level γ_D.

Next, we compare the growth rates of the two instabilities. The comparison should be done for the same equilibrium distribution of CRs upstream of the subshock, i.e., in the CR precursor. Apparently, there is a difficulty here. The acoustic instability analysis presumes a certain level of magnetic fluctuations (e.g. $\delta B \sim B$) to pondermotively [3] couple CRs to the gas flow, and thus ensure the equilibrium. In the Bell's stability analysis, it is assumed that the undisturbed B-field is along the shock normal and $\mathbf{J_{CR}}$points into the same direction. Then, the CR pressure gradient cannot be statically compensated since $\mathbf{J_{CR}} \times \mathbf{B} = 0$. Implicitly though, one may assume that the resonant instability provides Alfven waves which balance the CR pressure gradient along the main field through the wave pondermotive pressure at larger scales. Then, the Bell instability develops at the scales much shorter than the gyro-radii of the current-carrying particles. Note that such treatment cannot be extended to the larger scales without further considerations [16, 12]. In a strongly modified shock, the equilibrium distribution upstream in a steady state is given by [25]:

$$f = f_0(p) \exp\left[\frac{q(p)}{3\kappa(p)} \phi(x)\right], \quad x \geq 0 \tag{5}$$

where ϕ is the flow potential, $u = \partial\phi/\partial x$, $f_0(p)$ is the CR distribution and $q(p) = -\partial \ln f_0 / \partial \ln p$ the spectral index at the subshock. On the other hand, if the shock modification is negligible, the equilibrium is simply

$$f = f_0(p) \exp\left[\frac{1}{\kappa(p)} \phi(x)\right], \quad x \geq 0 \tag{6}$$

One sees that the only difference between the last two representations of the particle distribution upstream is the $q/3$ factor in the exponent of eq.(5). This quantity is well constrained in the nonlinear solution given by eq.(5): $3.5 < q(p) < q_{sub}$, where $q_{sub} = 3r_{sub}/(r_{sub}-1)$ with r_{sub} being the subshock compression ratio. For the ratio of acoustic to magnetic growth rates, eq.(4), we obtain

$$\frac{\gamma_D}{\gamma_B} = \frac{C_A}{C_S}\frac{c^2}{3\omega_{ci}}\left\langle \frac{p^2}{\sqrt{1+p^2}}\frac{q}{3\kappa}\right\rangle \tag{7}$$

where p is in units of mc. We have introduced the spectrum averaged quantity above as

$$\langle\cdot\rangle \equiv \frac{\int(\cdot)\, f_0(p)\, p^2 \exp(q\phi/3\kappa)\, dp}{\int f_0(p)\, p^2 \exp(q\phi/3\kappa)\, dp} \tag{8}$$

and where for the test particle solution, given by eq.(6) one should replace $q/3 \to 1$. For the Bohm diffusion coefficient $\kappa = r_g(p)\, c/3$, we obtain

$$\frac{\gamma_D}{\gamma_B} = \left\langle \frac{q}{3}\right\rangle \frac{C_A}{C_S} \tag{9}$$

From the last formula we may conclude that the acoustic instability dominates the magnetic one in the case of low $\beta \equiv C_S^2/C_A^2 \ll 1$ upstream, regardless of the degree of nonlinearity of acceleration. The reason for such a counter-intuitive relation between the growth rates of these two instabilities is that a stronger magnetic field (low-beta plasma) supports a stronger CR pressure gradient which drives the acoustic instability. Note, that plasma heating upstream would increase the role of magnetic instability unless the large scale magnetic field is also strengthened, e.g. through an inverse cascade of the turbulent magnetic energy [16]. On the other hand, the development of the acoustic instability makes (Sec.4) the precursor shorter. This boosts the gradient driven acoustic instability and leaves less room for the current driven magnetic instability. Next, we consider the CR transport in a developed acoustic turbulence with an admixture of Alfven waves, presumably generated by cyclotron instability at a distant part of the shock precursor and convected into its core.

3. CR transport in Shock Precursor

We assume that magnetic perturbations in the precursor are of the following two types. First, there are conventional shear Alfven waves, stemming from the CR cyclotron instability. Second, there are compressible magnetosonic perturbations generated by the Drury instability. The pitch angle scattering of CRs in the both wave fields has been calculated in many publications. We can use the expressions from [14], eqs.(6,7) here. After summing the Bessel function series and retaining only the magnetic parts of the scattering wave fields, we obtain

$$\begin{aligned} D_\mu^A = & -\left(1-\mu^2\right)\sum_{\mathbf{k}}\frac{1}{\xi^2}\int_0^\infty I^A\left(k_\parallel, k_\perp, \tau\right) \\ & \times\ \cos\left(k_\parallel c\mu\tau\right) d\tau \frac{\partial^2}{\partial\tau^2} J_0\left(2\xi\sin\frac{\Omega\tau}{2}\right) \end{aligned} \tag{10}$$

$$D_\mu^S = \frac{1}{3}\left(1-\mu^2\right)\sum_{\mathbf{k}}\frac{1}{\xi^2}\int_0^\infty I^S\left(k_\parallel,k_\perp,\tau\right)\frac{k_\parallel^2}{k_\perp^2}d\tau$$
$$\times\left[\Omega^2\cos\left(k_\parallel c\mu\tau\right)-\xi^{-2}\frac{\partial^2}{\partial\tau^2}\right]J_0\left(2\xi\sin\frac{\Omega\tau}{2}\right). \tag{11}$$

Here I^A and I^S are the spectral densities of magnetic fluctuations of Alfven and slow magnetosonic wave components of the upstream turbulence, μ is the cosine of particle pitch angle, $\xi = k_\perp c\sqrt{1-\mu^2}/\Omega$, J_0 is the Bessel function, and Ω is the relativistic gyrofrequency. The spatial diffusion coefficient can be evaluated as follows

$$\kappa = \frac{c^2}{8}\left\langle\frac{1-\mu^2}{\mathscr{D}_\mu}\right\rangle \tag{12}$$

where $\mathscr{D}_\mu = \left(D_\mu^A + D_\mu^S\right)/\left(1-\mu^2\right)$ and $\langle\cdot\rangle$ denotes here the pitch angle averaging. It is reasonable to assume that the scattering frequency $\mathscr{D}_\mu$ peaks at $|\mu| = 0, 1$. Indeed, the scattering is known to be strongly suppressed for $\xi \gg 1$ (high frequency perturbation of particle orbits), so that particles with $|\mu| \approx 1$ are subjected to a more coherent wave field. Furthermore, particles with $\mu \approx 0$ are effectively mirrored by the compressible component of magnetic turbulence. Therefore, $\mathscr{D}_\mu$ has minima at $|\mu| = \mu_0$. Upon writing

$$\left\langle\frac{1-\mu^2}{\mathscr{D}_\mu}\right\rangle = \int_0^1\left(1-\mu^2\right)d\mu\int_0^\infty\exp\left(-\mathscr{D}_\mu t\right)dt$$

and evaluating the integral in μ by using the steepest descent method, we obtain

$$\kappa = \frac{2\pi c^2\left(1-\mu_0^2\right)}{\sqrt{\mathscr{D}_\mu\left(\mu_0\right)\mathscr{D}_\mu''\left(\mu_0\right)}}$$

where the double prime denotes the second derivative. Assuming that the compressible part of turbulence, which originates from the acoustic instability, dominates ($D^A \ll D^S$), we can represent the momentum averaged $\bar\kappa$ as

$$\bar\kappa = \frac{\int\kappa\left(p\right)f\left(p\right)p^2dp}{\int f\left(p\right)p^2dp} = \frac{\bar\kappa_B}{F_S+\alpha} \tag{13}$$

Here $\bar\kappa_B$ is the momentum averaged Bohm diffusion coefficient $\kappa_B = cr_g\left(p\right)/3$, α is the normalized level of Alfvenic turbulence $\alpha \sim \left(\delta B_A\right)^2/B_0^2$, (originating from D^A), and F_S is the level of compressible turbulence (originating from D^S). The latter can be calculated in the simplest 1D model assuming steepening of acoustic waves unstably growing at a rate γ_D into an ensemble of shocks (shocktrain) [26, 28]

$$F_S = \sigma\left(\frac{\partial P_c}{\partial x}\right)^2 \quad \text{with} \quad \sigma = \left(\frac{L_S}{\rho C_S^2}\right)^2\mathscr{F}\left(\vartheta_{nB}\right) \tag{14}$$

where L_S is the average distance between the shocks in the ensemble, and $\mathscr{F} \sim 1$ depends predominantly on the shock obliquity and on the specific model for the shocktrain formation [29, 30].

4. Turbulent CR Front Solution

In this section we derive the equations for the shock transition from the kinetic convection-diffusion equation. Using the expressions (13) and (14) for the momentum averaged diffusion coefficient, we can write the convection-diffusion equation in a steady state as follows

$$\frac{\partial}{\partial x}\left(ug+\frac{\bar{\kappa}_B}{\sigma\left(\partial P_c/\partial x\right)^2+\alpha}\frac{\partial g}{\partial x}\right)=\frac{1}{3}\frac{\partial u}{\partial x}p\frac{\partial g}{\partial p} \tag{15}$$

where $g = fp^3$ and u is the (positive) magnitude of the flow speed. We assume that the CR pressure integral is dominated by particles with $p \gg mc$, but there is a spectral break at some $p = p_*$ which makes the integral finite without introducing a cut-off momentum [26]. Integrating the appropriately weighted convection-diffusion equation for in momentum [18], supplementing the result with the conservation of the mass and momentum fluxes across the shock transition, we can write the closed system of equations that governs the shock transition as follows

$$\begin{aligned}
&\frac{\partial}{\partial x}\left(uP_c+\frac{\bar{\kappa}_B}{\sigma\left(\partial P_c/\partial x\right)^2+\alpha}\frac{\partial P_c}{\partial x}\right)=-\frac{1}{3}\frac{\partial u}{\partial x}P_c\\
&\rho u=\rho_1 u_1=const\\
&\rho u^2+P_c=\rho_1 u_1^2=const
\end{aligned} \tag{16}$$

From these equation we obtain the following equation for the shock front structure

$$\left(\frac{\partial P_c}{\partial x}\right)^2+\frac{L_B}{\sigma P_c\left(1-P_c/P_{c2}\right)}\frac{\partial P_c}{\partial x}+\frac{\alpha}{\sigma}=0$$

where $P_{c2} = (6/7)\rho_1 u_1^2$, $L_B = \bar{\kappa}_B/u_1$ and we assumed that thermal gas is not heated appreciably, so it can be neglected in eq.(16) [25]. The shock solution can be obtained from the last equation in a closed form as $x = x(P_c)$. Introducing a dimensionless coordinate $z = xL_B/\sigma P_{c2}^2$ and $\Phi = 2P_c/P_{c2} - 1$ we can rewrite the last equation describing the front transition as

$$\left(\frac{\partial \Phi}{\partial z}\right)^2+\frac{8}{1-\Phi^2}\frac{\partial \Phi}{\partial z}+4a=0$$

where the transition is governed by a single parameter $a = \alpha\sigma\left(P_{c2}/L_B\right)^2$. Obviously, a smooth transition exists only for $0 < a < 4$. If $a \ll 1$ $\Phi = -\tanh(az/2)$, so that the scale of the front is the familiar $L_B = \bar{\kappa}_B/u_1$. If, however, $a \sim 1$, the scale of the shock (front) transition reduces to

$$L_f=\frac{\sigma}{4L_B}P_{c2}^2\simeq\frac{1}{4}\frac{L_S^2}{L_B}\left(\frac{P_{c2}}{\rho C_s^2}\right)^2.$$

5. Conclusions

We have obtained a one parameter family of smooth, strongly nonlinear shock front transitions which are generally significantly shorter than the conventional κ/V_{sh} scaling shock precursors. The phenomenon in general is somewhat similar to the transport bifurcation phenomenon, which is a subject of active research in magnetic fusion (L-H bifurcations, [21, 15, 27]).

The critical parameter allowing for such transitions (parameter a, Sec.4) depends (through the growth rate of the front supporting acoustic turbulence, γ_D) on the thermal gas pressure inside the front, de facto on the turbulent heating efficiency. The smooth transition exists for $a < 4$, so that in the case $a > 4$ a gaseous subshock must form. This however, should not result in a longer precursor. The determination of the heating efficiency, and thus a more complete study of the shock fronts obtained in this paper will be the subject of a separate publication.

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