# Kinetic theory of the turbulent energy pinch in tokamak plasmas

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(Dated: August 21, 2012)

**Abstract**

The turbulent energy fluxes, including up-gradient 'energy pinch' effects, are derived by using the nonlinear bounce-kinetic equation for trapped electrons and the nonlinear gyrokinetic equation for ions in toroidal geometry. The quasi-universal type of inward turbulent equipartition (TEP) energy pinch is recovered for both ions and trapped electrons, with different field dependency coefficient due to toroidal effects. A contribution from the density gradient to an outward convective energy flux is also obtained. The direction of the total energy convection is primarily determined by the competition between the TEP energy pinch and the outward density gradient driven energy convection. The magnetic shear dependence of the electron energy pinch is discussed. The energy pinches can provide possible explanations for some puzzling experimental observations.

## I. INTRODUCTION

Tokamak plasmas exhibit a remarkable tendency to self-organize, so as to maintain certain classes of profile structure. Temperature profiles react weakly to changes of auxiliary heating power deposition profile in experiments.[1,2] This property is known as "profile consistency",[3] "profile resilience", or "profile stiffness". Attempts to explain profile stiffness have appealed to a tendency to maintain marginal stability,[4] to critical gradient paradigms,[5] and to a possible energy pinch.[6–8] However, we note that in contrast to the now familiar density and momentum pinches in which (for many, but not all, turbulence drive mechanisms) an outward heat flux drives inward particle and/or momentum transport, the energy pinch is fundamentally a more subtle entity as it can produce a (partially) up-gradient heat flux in a heat flux driven system. This seemingly puzzling aspect of the heat pinch will be discussed at length in the conclusion. For this reason, the microscopic theory of the energy pinch is not well developed.

Experimentally, inward electron heat convection which is proportional to the temperature rather than to its gradient has been observed in some tokamaks such as DIII-D,[1,2] RTP,[7,9] and ASDEX-Upgrade[10] by using localized off-axis electron cyclotron heating (ECH) experiments. In the steady off-axis ECH plasmas, a net inward energy flow for electrons obtained by power balance analysis was used to identify the electron energy pinch,[1,2] and thereafter, modulated ECH provided a useful tool to identify electron heat pinch more directly.[7,10] An inward ion energy flow might exist in the discharges on DIII-D with off-axis neutral beam injection (NBI), although the evidence for the ion energy pinch is not as strong as that of the electron energy pinch.[11,12]

Theoretically, both particle and momentum pinches which consist of turbulent equipartition (TEP) and thermoelectric (temperature gradient driven) pieces have been studied.[13–16] Based on a set of fluid equations for density and temperature, expressions for the turbulent heat pinch have also been presented.[8,17–19] The turbulent energy convection includes a TEP piece and a density gradient driven piece, instead of the temperature gradient driven piece in the particle or momentum pinches. The TEP part results from the inhomogeneous magnetic field and was introduced[15] based on the existence of Lagrangian invariants.[13,14] The TEP convection fluxes are always directed inward. The up-gradient fluxes do not contradict the second law of thermodynamics because of the positive definite entropy production rate.[6,8] In particular, more entropy is produced by the diffusive flux of energy down $\nabla T_s$ ($T_s$ is temperature for species $s$) than is destroyed by the up-gradient pinch flux. This is clear, since instability requires $L_{Ts}/L_B \ll 1$ ($L_{Ts}$ is the temperature scale length, and $L_B$ is the nonuniformity length scale for background magnetic field). The density gradient driven heat pinch obtained by Weiland's model[17] is also inward for a peaked density profile.[19] Thus, it can not explain the hollow temperature profile observed in RTP,[9] which indicated the presence of an outward convective heat flux for a case of density profile peaked on axis.

In this work, we derive quasilinear expressions for energy fluxes by using the nonlinear bounce-kinetic equation for trapped electrons and the nonlinear gyrokinetic equation for ions in toroidal geometry. We only consider electrostatic fluctuations. In the energy fluxes, in addition to diffusive terms, we also find TEP pinch terms and density gradient driven convection terms for both electron and ion species. The field dependency coefficient of the TEP pinch is different from that obtained by using a two-dimensional plasma model in Ref. 15, due to the toroial effect considered here. The trapped electron TEP energy pinch is different from the ion TEP energy pinch because the trapped electron distribution function is bounce averaged. The most important difference between our results and previous findings[19] is that we predict that the density gradient driven energy convection is outward for both species when the density profile is peaked on axis. Therefore, whether the total energy convection in our theory is inward or outward is determined by the competition between the TEP piece and the density gradient driven piece, and so is very sensitive to density profile structure. Note that, in this work, energy pinch means inward convective energy flux but not net inward energy flux.

The rest of this paper is organized as follows. Sec. II presents the derivation of ion and trapped electron energy fluxes. The details of the calculation can be found in Appendixes A and B. The physical mechanisms for convective energy fluxes and some comparison with experimental observations are discussed in Sec. III. Finally, we summarize our work and discuss its possible implications for experiments in Sec. IV.

## II. DERIVATION OF THE ION AND TRAPPED ELECTRON ENERGY FLUXES

In this section, we present the derivation of quasilinear expressions for the ion and trapped electron energy fluxes, respectively. They are applied to either ion temperature gradient (ITG) or trapped electron mode (TEM) turbulence. Thus, the turbulence frequency in this work is of the order of ion or electron diamagnetic drift frequency. In a low-mode (L-mode) plasma without internal transport barrier (ITB), the effects from parallel shear flow, mean $\mathbf{E} \times \mathbf{B}$ flow and the turbulence driven zonal flow are expected to be weak. We don't consider those effects in this work.

### A. Energy flux for ions

We start from the perturbed version of nonlinear electrostatic gyrokinetic equation in toroidal geometry[20] for ions

$$\frac{\partial \delta f_i}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \nabla \delta f_i + \frac{dv_\parallel}{dt} \frac{\partial \delta f_i}{\partial v_\parallel} = -\frac{d\mathbf{R}^{(1)}}{dt} \cdot \nabla F_{i0} - \frac{dv_\parallel^{(1)}}{dt} \frac{\partial F_{i0}}{\partial v_\parallel}, \tag{1}$$

with the gyrocenter equations of motion

$$\frac{d\mathbf{R}}{dt} = v_\parallel \frac{\mathbf{B}^*}{B^*} + \frac{c\mathbf{b}}{eB^*} \times \left(\mu \nabla B + e\nabla \langle\langle \delta\phi \rangle\rangle\right), \tag{2a}$$

$$\frac{dv_\parallel}{dt} = -\frac{\mathbf{B}^*}{m_i B^*} \cdot \left(\mu \nabla B + e\nabla \langle\langle \delta\phi \rangle\rangle\right), \tag{2b}$$

and the perturbed part of gyrocenter equations of motion

$$\frac{d\mathbf{R}^{(1)}}{dt} = \frac{c\mathbf{b}}{B^*} \times \nabla \langle\langle \delta\phi \rangle\rangle, \tag{3a}$$

$$\frac{dv_\parallel^{(1)}}{dt} = -\frac{e\mathbf{B}^*}{m_i B^*} \cdot \nabla \langle\langle \delta\phi \rangle\rangle, \tag{3b}$$

where $\mathbf{R}$ is the gyrocenter position, $\mu$ is the gyrocenter magnetic moment, $v_\parallel$ is the gyrocenter parallel velocity, $\mathbf{b} = \mathbf{B}/B$ is the unit vector along the background magnetic field, $\mathbf{B}^* = \mathbf{B} + (m_i c/e) v_\parallel \nabla \times \mathbf{b}$, $B^* = \mathbf{b} \cdot \mathbf{B}^*$, $\delta\phi$ is the fluctuating electrostatic potential, the double bracket $\langle\langle \cdots \rangle\rangle$ denotes a gyrophase average, and $F_{i0}$ is assumed to be a Maxwellian equilibrium distribution function

$$F_{i0} = n_0 \left(\frac{m_i}{2\pi T_{i0}}\right)^{3/2} \exp\left(-\frac{m_i v_\parallel^2}{2T_{i0}} - \frac{\mu B}{T_{i0}}\right). \tag{4}$$

By linearizing Eq. (1), we can obtain the perturbed distribution function for ions in Fourier space,

$$\delta f_{ik} = i \frac{\frac{T_{i0}}{m_i} \frac{\mathbf{B}^*}{B^*} \cdot \mathbf{k} \frac{\partial \ln F_{i0}}{\partial v_\parallel} + \frac{cT_{i0}}{eB^*} \mathbf{b} \times \nabla \ln F_{i0} \cdot \mathbf{k}}{-i\left(\omega_k - v_\parallel \frac{\mathbf{B}^*}{B^*} \cdot \mathbf{k} - \frac{c\mu}{eB^*} \mathbf{b} \times \nabla B \cdot \mathbf{k} + i|\triangle\omega_k|\right)} \tau \tilde{\Phi}_k F_{i0}, \tag{5}$$

where $\tilde{\Phi}_k = e\delta\phi_k/T_{e0}$, $\tau = T_{e0}/T_{i0}$, $\mathbf{B}^*/B^* \simeq \mathbf{b} + m_i c/(eB^*) v_\parallel \mathbf{b} \times (\mathbf{b} \cdot \nabla \mathbf{b})$, and the finite Larmor radius effects are neglected. Here, we consider stationary turbulence, i.e., the linear growth rate $\gamma_k = 0$. Thus, $\triangle\omega_k$ in the denominator is the $\mathbf{E} \times \mathbf{B}$ nonlinearity-induced self-decorrelation rate, and the absolute value is required by causality. $\triangle\omega_k$ plays an important role in irreversibility. From Eq. (4), we have

$$\nabla \ln F_{i0} \;=\; \nabla \ln n_0 \left[1 + \frac{\nabla \ln T_{i0}}{\nabla \ln n_0}\left(\frac{m_i v_\parallel^2}{2T_{i0}} + \frac{\mu B}{T_{i0}} - \frac{3}{2}\right)\right] - \frac{\mu B}{T_{i0}}\nabla \ln B, \tag{6}$$

and

$$\frac{\partial \ln F_{i0}}{\partial v_\parallel} = -\frac{m_i v_\parallel}{T_{i0}}. \tag{7}$$

Then, Eq. (5) can be written as

$$\delta f_{ik} = -i\frac{k_\parallel v_\parallel + \omega_{di}(2\hat{v}_\parallel{}^2 + \hat{v}_\perp^2) - \omega_{*i}[1 + \eta_i(\hat{v}_\parallel^2 + \hat{v}_\perp^2 - 3/2)]}{-i[\omega_k - k_\parallel v_\parallel - \omega_{di}(2\hat{v}_\parallel^2 + \hat{v}_\perp^2) + i|\triangle\omega_k|]}\tau\tilde{\Phi}_k F_{i0}, \tag{8}$$

where $\hat{v}_\parallel = v_\parallel/\sqrt{2T_{i0}/m_i}$, $\hat{v}_\perp = \sqrt{\mu B/T_{i0}}$, $\eta_i = \nabla \ln T_{i0}/\nabla \ln n_0$, $L_n = -(\nabla \ln n_0)^{-1}$ is the density gradient length, $\omega_{*i} = cT_{i0}/(eB)\mathbf{b} \times \nabla \ln n_0 \cdot \mathbf{k}$ is the ion diamagnetic drift frequency, and $\omega_{di} = cT_{i0}/(eB)\mathbf{b} \times \nabla \ln B \cdot \mathbf{k}$ is the ion magnetic drift frequency. Here, $\mathbf{b} \times (\mathbf{b} \cdot \nabla \mathbf{b}) \simeq \mathbf{b} \times \nabla \ln B$ is used, and is justified for a low $\beta$ plasma. The denominator of Eq. (8) is the ion propagator. For simplicity, we neglect $k_\parallel v_\parallel$ which is related to the ion acoustic dynamics. This approximation requires $\omega_{di} \gg k_\parallel v_\parallel$. We list the conditions for its validity in various cases in Table I. Dropping $k_\parallel v_\parallel$ means that the drift resonance but not the transit resonance is considered in this work. Then, the real part of the inverse ion propagator can be written as

$$\Re\{-i[\omega_k - \omega_{di}(2\hat{v}_\parallel^2 + \hat{v}_\perp^2) + i|\triangle\omega_k|]\}^{-1} \simeq \pi\delta[\omega_k - \omega_{di}(2\hat{v}_\parallel^2 + \hat{v}_\perp^2)] + \frac{|\triangle\omega_k|}{\omega_k^2}. \tag{9}$$

Here, the approximation of $|\triangle\omega_k| \ll |\omega_k|, |\omega_{di}|$ is used which justifies the quasilinear theory. By a rough estimation, one can take $|\triangle\omega_k| \sim |\gamma_{lin,k}|$. For finite amplitude turbulence, the self-decorrelation time $|\triangle\omega_k|^{-1}$ is estimated to be of the order of the autocorrelation time $|\triangle[\omega_k - \omega_{di}(2\hat{v}_\parallel^2 + \hat{v}_\perp^2)]|_{res}|^{-1}$ (for weak turbulence) or the eddy turn-over time (for strong trubulence), which can be calculated by using renormalized theory.[21] Such a calculation is beyond the scope of this work. Here, the $\delta$ function denotes the contribution from ion magnetic drift wave-particle resonance, and $|\triangle\omega_k|/\omega_k^2$ comes from the non-resonant particles. It follows that $\delta f_{ik} = \delta f_{ik}^{Res} + \delta f_{ik}^{NR}$. Then the ion energy flux can be written as

$$Q_i = \langle \delta v_r^* \delta P_i \rangle = \left\langle \delta v_r^* \int d^3 v E \left(\delta f_i^{Res} + \delta f_i^{NR}\right) \right\rangle. \tag{10}$$

There is no contribution from resonant ions for TEM turbulence. However, the contribution from non-resonant ions is always present for both TEM and ITG turbulence. The details of the calculation are presented in Appendix A.

The total ion energy flux including the contributions from both resonant and non-resonant ions is

$$Q_i = -\chi_i n_0 \nabla T_{i0} + V_i n_0 T_{i0}, \tag{11}$$

where

$$\begin{aligned} \chi_i \; = \; & \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \right\rangle \\ & +2 \left(\frac{3}{4}\right)^{7/2} \sqrt{\pi}\tau R_0 \left(\frac{R_0}{L_n}\right)^{3/2} \left(\frac{R_0}{L_n} - 2\right) \exp\left(-\frac{3}{4}\frac{R_0}{L_n}\right) \sum_k |k_\theta| c_s \rho_s \left\langle |\tilde{\Phi}_k|^2 \right\rangle, \end{aligned} \tag{12}$$

and

$$V_i = V_i^{\nabla B} + V_i^{\nabla n}, \tag{13}$$

with

$$V_i^{\nabla B} = -\frac{10}{3}\frac{1}{R_0} \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \right\rangle, \tag{14a}$$

$$V_i^{\nabla n} = \frac{1}{L_n} \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \right\rangle. \tag{14b}$$

Here, $\chi_i$ is the ion thermal diffusivity, $V_i^{\nabla B}$ and $V_i^{\nabla n}$ are ion energy convective velocity driven by the inhomogeneity of the magnetic field and the density gradient, respectively. Note that the second term in $\chi_i$ is from resonant ions' contribution (see Appendix A), which is absent for TEM turbulence, as mentioned above. However, the first term in $\chi_i$ and both the convective components coming from non-resonant ions' contribution are always present for both ITG or TEM turbulence. The physical mechanisms for convective energy flux will be discussed in the next section.

### B. Energy flux for trapped electrons

We start from the nonlinear bounce-kinetic equation for trapped electrons,[22,23]

$$\left(\frac{\partial}{\partial t} + i\omega_{de}\right) h_e + \left\{\frac{\partial}{\partial t} + i\omega_{*e}\left[1 + \eta_e\left(\frac{E}{T_{e0}} - \frac{3}{2}\right)\right]\right\} \frac{e\left\langle\delta\phi\right\rangle_b}{T_{e0}} F_{e0} = \frac{e}{T_{e0}} \nabla \left\langle\delta\phi\right\rangle_b \times \mathbf{b} \cdot \nabla h_e, \tag{15}$$

where $h$ is the nonadiabatic part of the perturbed distribution function for trapped electrons, i.e., $\delta f_e = h_e + F_{e0} e\delta\phi/T_{e0}$, $\eta_e = \nabla \ln T_{e0}/\nabla \ln n_0$, $\omega_{*e} = k_\theta c_s \rho_s/L_n$ is the electron diamagnetic drift frequency, $\langle \cdots \rangle_b$ means bounce average, $\omega_{de} = \omega_{*e} G L_n E/(R_0 T_{e0})$ is the orbit averaged trapped electron precession drift frequency, with $G$ is a function of magnetic shear $\hat{s}$ and azimuthal angle $\theta_0$ of the turning point of a trapped electron, and $F_{e0} = n_0(m_e/2\pi T_{e0})^{3/2} \exp(-E/T_{e0})$. Linearization of Eq. (15) yields the nonadiabatic part of the perturbed distribution function for trapped electrons in Fourier space,

$$h_{ek} \; = \; i\frac{\omega_k - \omega_{*e}\left[1 + \eta_e\left(\hat{\hat{E}} - \frac{3}{2}\right)\right]}{-i\left[\omega_k - \omega_{*e}\frac{L_n G}{R_0}\hat{\hat{E}} + i|\triangle\omega_k|\right]}\tilde{\Phi}_k F_{e0}, \tag{16}$$

where $\hat{\hat{E}} = E/T_{e0}$. Here, the finite orbit width effects of trapped electrons are neglected, $\langle \Phi_k \rangle_b \cong \Phi_k$. As in the ion case, the denominator in the preceding equation is the trapped electron propagator. The real part of the inverse propagator can be written as

$$\Re\left[-i\left(\omega_k - \omega_{*e}\frac{L_n G}{R_0}\hat{\hat{E}} + i|\triangle\omega_k|\right)\right]^{-1} \simeq \pi\delta\left(\omega_k - \omega_{*e}\frac{L_n G}{R_0}\hat{\hat{E}}\right) + \frac{|\triangle\omega_k|}{\omega_k^2}\left(1 + 2\frac{\omega_{*e}}{\omega_k}\frac{L_n G}{R_0}\hat{\hat{E}}\right). \tag{17}$$

Here, the $\delta$ function denotes the trapped electron precession drift resonance. We retain the next order term in the expansion for the non-resonant part of the inverse of the electron propagator. This term contributes to the trapped electron TEP energy pinch which will be shown in Appendix B. Similar to ion species, the energy flux for trapped electron can be written as

$$Q_e = \langle \delta v_r^* \delta P_{e,tr} \rangle = \left\langle \delta v_r^* \int^{tr} d^3 v E\left(h_e^{Res} + h_e^{NR}\right)\right\rangle. \tag{18}$$

The resonant trapped electrons do not contribute to the trapped electron energy flux for ITG turbulence. However, non-resonant trapped electrons can contribute to trapped electron energy flux for both TEM and ITG turbulence. The details of the calculation are presented in Appendix B.

The total energy flux for trapped electrons including the contributions from both resonant and non-resonant trapped electrons is

$$Q_e = -\chi_e n_0 \nabla T_{e0} + V_e n_0 T_{e0}, \tag{19}$$

where

$$\chi_e = \frac{3}{2} f_t \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \right\rangle$$
$$+2 f_t \sqrt{\pi} \frac{R_0}{G} \left(\frac{R_0}{L_n G}\right)^{3/2} \left(\frac{R_0}{L_n G} - \frac{3}{2}\right) \exp\left(-\frac{R_0}{L_n G}\right) \sum_k |k_\theta| c_s \rho_s \left\langle |\tilde{\Phi}_k|^2 \right\rangle, \quad (20)$$

and

$$V_e = V_e^{\nabla B} + V_e^{\nabla n}, \quad (21)$$

with

$$V_e^{\nabla B} = -\frac{15}{2} \frac{G}{R_0} f_t \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \right\rangle, \quad (22a)$$

$$V_e^{\nabla n} = \frac{3}{2} \frac{1}{L_n} f_t \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \left(1 - \frac{\omega_k}{\omega_{*e}}\right) \right\rangle. \quad (22b)$$

Here, $\chi_e$ is the electron thermal diffusivity, $V_e^{\nabla B}$ and $V_e^{\nabla n}$ are trapped electron energy convective velocity driven by the inhomogeneity of the magnetic field and the density gradient, respectively. Similar to ion species again, the second term in $\chi_e$, which comes from the resonant trapped electrons' contribution (see Appendix B), vanishes for ITG turbulence. However, the first term in $\chi_e$ and both the convection components, which come from the non-resonant trapped electrons' contribution, are present in both ITG and TEM turbulence.

At last, all the pieces of ion and trapped electron energy fluxes are listed in Tabble. II.

## III. PHYSICS OF THE CONVECTIVE ENERGY FLUXES

In this section, we discuss the physics of the energy convection and compare with some experimental observations.

The first column in Table II lists the TEP ion and trapped electron energy pinches, both driven by nonthermodynamic forces. They are proportional to $\nabla \ln B \sim -1/R_0$ the inverse scale length for the toroidal magnetic field. The physical origin of the $\nabla B$ driven piece of energy pinches is similar to that of momentum pinch, which has been illustrated by showing that the magnetically weighted parallel ion momentum density is a locally advected scalar.[16] The TEP energy pinch is always inward for ions. An explicit expression for $G = 0.64\hat{s} + 0.57$, obtained by averaging over the azimuthal angle of the trapped electron turning point, can be used for simplicity.[24,25] It is order of one for normal magnetic shear. Thus, the TEP energy pinch is also inward for electrons if the magnetic shear is positive or weakly reversed (i.e. $\hat{s} \gtrsim -0.89$). This quasi-universal TEP energy pinches is determined by magnetic geometry rather than by thermodynamic forces. The up-gradient energy flux has been investigated in Ref. 15 in the context of TEP theory,[13,14] for slab geometry. In our work, the magnetic curvature driven energy pinch which is absent in the slab geometry, is the same as the magnetic gradient driven energy pinch. Therefore, the coefficient of the energy pinch dependency on $\nabla B$ is twice that of Ref. 15. The TEP piece of the trapped electron energy pinches is different from that of ion energy pinches, because the trapped electron distribution function used is obtained by bounce averaging. The magnetic shear dependence of the trapped electron TEP energy pinch comes from the orbit averaged trapped electron precession drift. Although the inward trapped electron TEP energy pinch explicitly decreases with decreasing the magnetic shear, it does not contradict electron thermal transport barrier formation for reverse magnetic shear. This is because TEM turbulence usually saturates at a relatively low level for reversed magnetic shear. Thus, the resulting thermal transport level is small as well, i.e., the total TEM driven heat flux decreases quickly for $\hat{s} < 0$.

The second column of Table II lists the density gradient driven pieces of the ion and the trapped electron convective energy fluxes, respectively. Obviously, the ion convective energy flux driven by the density gradient is directed outward. The direction of the density gradient driven trapped electron energy pinch is determined by the sign of $\omega_{*e} - \omega_k$. For ITG turbulence, the sign of the frequency is the same as that of the ion diamagnetic frequency, which is negative. The frequency of the TEM turbulence is smaller than the electron diamagnetic frequency due to the downward frequency shift coming from the ion polarization density. Therefore, the trapped electron convective energy flux driven by density gradient is also outward. This prediction is different from previous results.[19]

From the above analysis of the TEP and density gradient driven pieces, we can see that the direction of the total energy convection is mainly determined by the competition between those two pieces of the convective flux. The total ion energy convection velocity is

$$V_i = \frac{1}{R_0}\left(\frac{R_0}{L_n} - \frac{10}{3}\right)\sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2}\right\rangle . \tag{23}$$

If the density profile is steep enough, i.e., $R_0/L_n > 10/3$, one can get an outward ion convective energy flux. On the contrary, if $R_0/L_n < 10/3$, one obtains an inward ion energy pinch. The density profile for the discharges on DIII-D with off-axis NBI heating is rather flat[12] so an inward ion energy pinch is predicted by our theory for such flat-n discharges. This provides a possible explanation for the stiffness of the ion temperature profile observed in experiments in this regime.[12] Although a net negative ion energy flux was not obtained in experiment, the inward ion energy pinch (i.e., an inward convective component of the flux) cannot be ruled out. This is because the positive ion energy diffusive flux can exceed the absolute value of the inward ion energy pinch term.

The total trapped electron energy convection velocity is

$$V_e = \frac{3}{2}\frac{1}{R_0} f_t \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2}\left(1-\frac{\omega_k}{\omega_{*e}}\right)\left(\frac{R_0}{L_n} - \frac{5G}{1-\omega_k/\omega_{*e}}\right)\right\rangle . \qquad (24)$$

Here, $1-\omega_k/\omega_{*e}$ is positive, as discussed above. As for the ion energy convection velocity, if $R_0/L_n > (<) 5G/(1-\omega_k/\omega_{*e})$, we predict an outward convective (inward) electron energy flux (pinch). Note that, to explain the observation of hollow electron temperature profile on RTP, an outward convective electron energy flux is required.[9] One of the theoretical models adopted in Ref. 9 can explain this phenomena, but the other one can not. Interestingly, the independent calculation in this work reaches the contrasting conclusion that an outward trapped electron convective energy flux is at least possible for the case of a steep density profile. Note that the density profile in that experiment is indeed quite steep.[9] In addition, the more peaked electron temperature profile with off-axis NBI heating (than with on-axis NBI heating) observed on DIII-D[12] may be related to an inward trapped electron energy pinch, due to the relatively flat density profile in those DIII-D H-mode discharges.

## IV. CONCLUSIONS

In the present work, we have derived the ion and trapped electron energy fluxes by quasilinear theory based on the nonlinear bounce-kinetic equation for trapped electrons and the nonlinear gyrokinetic equation for ions in toroidal geometry. The convective energy fluxes each contain an inward TEP piece driven by $\nabla B$ and an outward density gradient driven piece. The TEP pinch in our work is twice as large as that obtained in slab geometry. The difference results from the fact that the comparable contribution from the magnetic curvature drift as that from magnetic gradient drift is considered for toroidal geometry in our work. The TEP energy pinch is always inward for ions, and is inward for electrons if the magnetic shear is positive or weakly reversed. The density gradient driven convective energy fluxes are outward for both species. We present a detailed analysis of the competition between those two pieces for both species, which determines the direction of the total energy convection. We find some qualitative agreements with certain puzzling experimental observations.

So far, there are very few experimental studies of the ion heat pinch. This may be due to the unavailability of a corresponding tool to ECH, which can heat ions locally in space and time. However, one can not rule out the ion heat pinch just according to the positive ion heat flux. From our theory, an inward ion energy pinch is always predicted for a flat density profile. Therefore, we encourage experimentalists to explore evidence for an ion energy pinch. In addition, our results predict a strong density profile dependence of both the energy pinches. This may be checked by a density profile scan in experiments or by density perturbation experiments (i.e. pellet injection).

Since the energy pinch has, for same time, been a subject of some confusion, it is worthwhile to discuss the clear distinction and contrast between:

1. the role of the electron energy pinch in $\nabla T_e$-driven turbulence, such as collisionless TEM (CTEM) turbulence

2. the role of the energy heat pinch in $\nabla T_i$-driven (i.e. $Q_i$ driven) turbulence, such as ITG turbulence

3. the nature of the energy pinch as opposed to that of anomalous electron-ion energy transfer

First, in $\nabla T_e$-driven turbulence (i.e. CTEM), net entropy production requires that $Q_e$, the turbulent energy flux, be outward (i.e. $Q_e > 0$ and we assume $T_e(r)$ peaked on axis). For $Q_e = -\chi_e n_e \nabla T_e + V_e n_e T_e$, it follows that $-\chi_e \nabla T_e > |V_e T_e|$ is required, so that relaxation of $\nabla T_e$ can outweigh the inward pinch. Note that for this instance, the main effect of the pinch is to regulate $T_e(r)$ structure and to partially set the shape of $Q_e$ vs $1/L_{Te}$ curve, particularly for low but finite values of $Q_e$. The effect of the pinch would, in principle, be detectable in transient transport experiments.

For $\nabla T_i$ and $Q_i$ driven turbulence, such as ITG, net total entropy production requires that the entropy produced by ion relaxation outweigh any entropy destruction by the electron transport. Thus, in this case, it is at least in principle possible to drive a net $Q_e < 0$(!)-i.e. to drive a net inward electron energy flux-using (outward) ion energy flux driven ITG turbulence. Of course, the state of $Q_e < 0$ would persist only until $\nabla T_e$ built up to the point where $-\chi_e n_e \nabla T_e + V_e n_e T_e \approx 0$, i.e. till the gradient in temperature steepened sufficiently to render $Q_e \cong 0$. We remark that this scenario somewhat resembles recent simulation results from studies of intrinsic rotation. For that closely related problem, namely the formation of a non-trivial rotation profile by non-diffusive transport processes, flux driven ITG turbulence (obviously with $Q_i > 0$) has been observed to drive net inward co-momentum transport (by residual stress) in simulations.[26–28] This inward transport persists until $\nabla V_\phi$ becomes sufficient for $-\chi_\phi \nabla V_\phi + \Pi_{r,\phi}^{resid} \approx 0$, which then defines a stationary state of intrinsic co-rotation. The correspondence to $Q_i$-driven build-up of $T_{e0}$ profiles is reasonable and, in our view, instinctive, but should not be taken too far. We note that this scenario for energy should be regarded as something of a gedanken experiment. In practice, very weak outer-species energy transfer is required to actually realize such an idea. Indeed, it may be easier to first test this suggestion in a digital tokamak (i.e. gyrokinetic simulation) than in an analogue experiment.

Finally, we remark that the energy pinch and the anomalous electron-ion coupling are two different, distinct and independent effects. Indeed, the electron heat equation takes the form:

$$\frac{3}{2}\partial_t \left(n_e T_e\right) = -\nabla \cdot Q_e + P_{ei} + S_e, \tag{25}$$

with

$$Q_e = -\chi_e n_e \nabla T_e + V_e n_e T_e.$$

Thus, we see that the pinch is a component of the electron energy flux, while the coupling $P_{ei}$ is a local source or sink. In principle, both are present and both may influence temperature profiles. Collisionless electron-ion coupling via $P_{ei}$ may be especially important in very hot, collisionless plasmas, such as ITER.[29]

Base on the microscopic foundations of the energy pinch, our ongoing work focuses on the synergy and compatibility of heat, particle and momentum pinches, and understanding non-local phenomena, i.e., the fast core heating response to edge temperature perturbations.

We also plan to investigate the relationship between the energy pinch and the mechanism for triggering ITB formation.

## Acknowledgments

We thank T. C. Luce and X. Garbet for stimulating discussions. This work was supported by Ministry of Education, Science and Technology of Korea via the WCI project 2009-001, the U.S. Department of Energy Grant No. DE-FG02-04ER54738, Grant No. DE-FC02-08ER54983 and Grant No. DE-FC02-08ER54959.

## APPENDIX A: ENERGY FLUX FOR IONS

From Eqs. (8) and (9), the resonant and non-resonant perturbed ion distribution functions are

$$\begin{aligned}\delta f_{ik}^{Res} &= -i\left\{\omega_{di}(2\hat{v}_{\|}{}^2+\hat{v}_{\perp}^2)-\omega_{*i}[1+\eta_i(\hat{v}_{\|}^2+\hat{v}_{\perp}^2-3/2)]\right\} \\ &\quad \times\pi\delta[\omega_k-\omega_{di}(2\hat{v}_{\|}^2+\hat{v}_{\perp}^2)]\tau\tilde{\Phi}_k F_{i0}, \qquad \text{(A1)}\\ \delta f_{ik}^{NR} &= -i\left\{\omega_{di}(2\hat{v}_{\|}{}^2+\hat{v}_{\perp}^2)-\omega_{*i}[1+\eta_i(\hat{v}_{\|}^2+\hat{v}_{\perp}^2-3/2)]\right\}\frac{|\triangle\omega_k|}{\omega_k^2}\tau\tilde{\Phi}_k F_{i0}. \qquad \text{(A2)}\end{aligned}$$

Note that the sign of ion magnetic drift frequency is the same as that of ITG turbulence, but opposite to that of TEM turbulence. Therefore, there is no contribution from resonant ions to the ion energy flux for TEM turbulence. However, both resonant and non-resonant ions make contributions to the ion energy flux for ITG turbulence.

The energy flux coming from resonant ions can be written as

$$Q_i^{Res}=\left\langle \delta v_r^* \int d^3v E\delta f_i^{Res}\right\rangle, \qquad \text{(A3)}$$

where, $\delta v_r=-ik_\theta c_s\rho_s\tilde{\Phi}_k$ is the fluctuating $\mathbf{E}\times\mathbf{B}$ drift along the radial direction which is assumed to be responsible for the energy transport, and the bracket $\langle\cdots\rangle$ means flux-surface average. Following the constant energy resonance approximation,[30] we have $2\hat{v}_{\|}^2+\hat{v}_{\perp}^2\simeq$

$(4/3)\hat{E}$ with $\hat{E} = E/T_{i0}$. Then, Eq. (A3) can be written as

$$\begin{aligned}
Q_i^{Res} &= \left\langle \sum_k k_\theta c_s \rho_s |\tilde{\Phi}_k|^2 \tau \frac{3}{2}\sqrt{\pi} n_0 T_{i0} \frac{1}{|\omega_{di}|} \int_0^\infty d\hat{E}\hat{E}^{3/2} \exp(-\hat{E}) \delta\left(\hat{E} - \frac{3\omega_k}{4\omega_{di}}\right) \right. \\
&\quad \left. \times \left\{ \frac{4}{3}\omega_{di}\hat{E} - \omega_{*i}\left[1 + \eta_i\left(\hat{E} - \frac{3}{2}\right)\right]\right\} \right\rangle \\
&= \left\langle \sum_k k_\theta c_s \rho_s |\tilde{\Phi}_k|^2 \tau \frac{3}{2}\sqrt{\pi} n_0 T_{i0} \frac{1}{|\omega_{di}|} \left(\frac{3\omega_k}{4\omega_{di}}\right)^{3/2} \exp\left[-\left(\frac{3\omega_k}{4\omega_{di}}\right)\right] \right. \\
&\quad \left. \left\{ \omega_k - \omega_{*i}\left[1 + \eta_i\left(\frac{3\omega_k}{4\omega_{di}} - \frac{3}{2}\right)\right]\right\} \right\rangle . \qquad \text{(A4)}
\end{aligned}$$

For resonant ions, the sign of turbulence frequency must be the same as that of ion diamagnetic drift frequency. So we assume $\omega_k \approx \omega_{*i}$, i.e., $\omega_k/\omega_{di} \simeq \mathbf{b}\times\nabla \ln n_0\cdot\mathbf{k}/\mathbf{b}\times\nabla \ln B\cdot\mathbf{k} \simeq R_0/L_n$. Substituting this relationship into the preceding equation, we can obtain

$$\begin{aligned}
Q_i^{Res} &= -2\sqrt{\pi}\tau \exp\left(-\frac{3}{4}\frac{R_0}{L_n}\right)\left(\frac{3}{4}\right)^{7/2}\left(\frac{R_0}{L_n}\right)^{3/2}\left(\frac{R_0}{L_n} - 2\right) R_0 \\
&\quad \times \sum_k |k_\theta| c_s \rho_s \left\langle |\tilde{\Phi}_k|^2 \right\rangle n_0 \nabla T_{i0}. \qquad \text{(A5)}
\end{aligned}$$

We can see that the contribution from resonant ions to the ion energy flux is a diffusive component.

Next, we calculate the contribution from nonresonant ions to the energy flux,

$$\begin{aligned}
Q_i^{NR} &= \frac{1}{3}\left\langle \delta v_r^* \int d^3v \left(mv_\parallel^2 + 2\mu B\right) \delta f_i^{NR} \right\rangle \\
&= \left\langle \sum_k k_\theta c_s \rho_s |\tilde{\Phi}_k|^2 \tau \frac{4}{3\sqrt{\pi}} n_0 T_{i0} \int_{-\infty}^{+\infty} d\hat{v}_\parallel \int_0^{+\infty} d\hat{v}_\perp \hat{v}_\perp (\hat{v}_\parallel^2 + \hat{v}_\perp^2) \exp[-(\hat{v}_\parallel^2 + \hat{v}_\perp^2)] \frac{|\triangle\omega_k|}{\omega_k^2} \right. \\
&\quad \left. \times \left\{ \omega_{di}(2\hat{v}_\parallel^2 + \hat{v}_\perp^2) - \omega_{*i}\left[1 + \eta_i\left(\hat{v}_\parallel^2 + \hat{v}_\perp^2 - \frac{3}{2}\right)\right]\right\} \right\rangle \\
&= \left\langle \sum_k k_\theta c_s \rho_s |\tilde{\Phi}_k|^2 \tau n_0 T_{i0} \frac{|\triangle\omega_k|}{\omega_k^2} \left[\frac{10}{3}\omega_{di} - \omega_{*i}(1 + \eta_i)\right] \right\rangle \\
&\simeq -\sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \right\rangle \left(\frac{10}{3}\frac{1}{R_0} n_0 T_{i0} + \nabla n_0 T_{i0} + n_0 \nabla T_{i0}\right). \qquad \text{(A6)}
\end{aligned}$$

The ion energy flux coming from non-resonant ions consists of convective components driven by the inhomogeneous magnetic field and the density gradient and a diffusive component.

Combining both contributions from resonant and nonresonant ions, we can obtain the total ion energy flux

$$
\begin{aligned}
Q_i \;=\; & -\left[\sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \right\rangle \right. \\
& \left. +2\left(\frac{3}{4}\right)^{7/2} \sqrt{\pi}\tau R_0 \left(\frac{R_0}{L_n}\right)^{3/2} \left(\frac{R_0}{L_n} - 2\right) \exp\left(-\frac{3}{4}\frac{R_0}{L_n}\right) \sum_k |k_\theta| c_s \rho_s \left\langle |\tilde{\Phi}_k|^2 \right\rangle \right] n_0 \nabla T_{i0} \\
& + \left(\frac{1}{L_n} - \frac{10}{3}\frac{1}{R_0}\right) \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \right\rangle n_0 T_{i0}. \qquad \text{(A7)}
\end{aligned}
$$

## APPENDIX B: ENERGY FLUX FOR TRAPPED ELECTRONS

From eqs. (16) and (17), the resonant and non-resonant nonadiabatic electron distribution functions are

$$
h_{ek}^{Res} \;=\; i\left\{\omega_k - \omega_{*e}\left[1 + \eta_e\left(\hat{\hat{E}} - \frac{3}{2}\right)\right]\right\} \pi\delta\left(\omega_k - \omega_{*e}\frac{L_n G}{R_0}\hat{\hat{E}}\right) \tilde{\Phi}_k F_{e0}, \qquad \text{(B1)}
$$

$$
h_{ek}^{NR} \;=\; i\left\{\omega_k - \omega_{*e}\left[1 + \eta_e\left(\hat{\hat{E}} - \frac{3}{2}\right)\right]\right\} \frac{|\triangle\omega_k|}{\omega_k^2} \left(1 + 2\frac{\omega_{*e}}{\omega_k}\frac{L_n G}{R_0}\hat{\hat{E}}\right) \tilde{\Phi}_k F_{e0}. \qquad \text{(B2)}
$$

The sign of trapped electron precession drift frequency is the same as that of electron diamagnetic frequency. Therefore, there is no resonant contribution to the trapped electron energy flux for ITG turbulence. However, for TEM turbulence, the trapped electron energy flux contains both resonant and non-resonant contributions.

The contribution from the trapped electron precession drift resonance to the trapped electron energy flux is

$$
\begin{aligned}
Q_e^{Res} \;=\; & \left\langle \delta v_r^* \int^{tr} d^3v E h^{Res} \right\rangle \\
=\; & -\left\langle \sum_k k_\theta c_s \rho_s |\tilde{\Phi}_k|^2 \sqrt{2\epsilon/\pi}\; n_0 T_{e0} \int_{\kappa_0}^1 d\kappa^2 \frac{1}{\sqrt{\kappa^2 - \sin^2\frac{\theta}{2}}} \int_0^\infty d\hat{\hat{E}} \hat{\hat{E}}^{3/2} \right. \\
& \left. \times \left\{\omega_k - \omega_{*e}\left[1 + \eta_e\left(\hat{\hat{E}} - \frac{3}{2}\right)\right]\right\} \exp\left(-\hat{\hat{E}}\right) \pi\delta\left(\omega_k - \omega_{*e}\frac{L_n G}{R_0}\hat{\hat{E}}\right) \right\rangle \\
=\; & -\left\langle \sum_k k_\theta c_s \rho_s |\tilde{\Phi}_k|^2 2\sqrt{\pi} f_t n_0 T_{e0} \exp\left(-\frac{\omega_k}{\omega_{*e}}\frac{R_0}{L_n G}\right) \left(\frac{R_0}{L_n G}\right)^{5/2} \left(\frac{\omega_k}{\omega_{*e}}\right)^{3/2} \frac{\omega_k}{|\omega_{*e}|} \right. \\
& \left. \times \left\{1 - \frac{\omega_{*e}}{\omega_k}\left[1 + \eta_e\left(\frac{\omega_k}{\omega_{*e}}\frac{R_0}{L_n G} - \frac{3}{2}\right)\right]\right\} \right\rangle, \qquad \text{(B3)}
\end{aligned}
$$

where $f_t = \sqrt{2\epsilon}\int_{\kappa_0}^1 d\kappa \left(\kappa/\sqrt{\kappa^2 - \sin^2(\theta/2)}\right)$ is the fraction of trapped electrons. Here, we consider the contribution from resonant trapped electrons, so we approximate the turbulence frequency $\omega_k$ by the electron diamagnetic drift frequency $\omega_{*e}$ for simplicity. Then the preceding equation can be written as

$$Q_e^{Res} = -2\sqrt{\pi} f_t \exp\left(-\frac{R_0}{L_n G}\right) \frac{R_0}{G} \left(\frac{R_0}{L_n G}\right)^{3/2} \left(\frac{R_0}{L_n G} - \frac{3}{2}\right) \sum_k |k_\theta| c_s \rho_s \left\langle |\tilde{\Phi}_k|^2 \right\rangle n_0 \nabla T_{e0}. \tag{B4}$$

This resonant part of trapped electron energy flux is a diffusive component.

The non-resonant trapped electrons' contribution to the trapped electron energy flux is

$$\begin{aligned} Q_e^{NR} &= \langle \delta v_r^* \delta P_{e,tr}^{NR} \rangle = \left\langle \delta v_r^* \int^{tr} d^3 v E h^{NR} \right\rangle \\ &= -\left\langle \sum_k k_\theta c_s \rho_s |\tilde{\Phi}_k|^2 \frac{2}{\sqrt{\pi}} f_t n_0 T_{e0} \int_0^\infty d\hat{\hat{E}} \hat{\hat{E}}^{3/2} \exp\left(-\hat{\hat{E}}\right) \right. \\ &\quad \left. \times \left\{ \omega_k - \omega_{*e} \left[ 1 + \eta_e \left( \hat{\hat{E}} - \frac{3}{2} \right) \right] \right\} \frac{|\triangle \omega_k|}{\omega_k^2} \left( 1 + 2 \frac{\omega_{*e}}{\omega_k} \frac{L_n G}{R_0} \hat{\hat{E}} \right) \right\rangle \\ &= -\frac{3}{2} f_t \sum_k k_\theta c_s \rho_s \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle \omega_k|}{\omega_k} \right\rangle n_0 T_{e0} \\ &\quad -\frac{3}{2} f_t \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle \omega_k|}{\omega_k^2} \right\rangle \left( n_0 \nabla T_{e0} + T_{e0} \nabla n_0 + 5 \frac{G}{R_0} n_0 T_{e0} \right). \end{aligned} \tag{B5}$$

Here, the last term comes from the next order term in the expansion of the inverse of the trapped electron propagator. The corrections to $T_{e0}\nabla n_0$ and $n_0 \nabla T_{e0}$ related terms coming from the next order are neglected. This non-resonant part of trapped electron energy flux contains a diffusive component and convective components driven by the density gradient and the inhomogeneous magnetic field.

Combining the contributions from both resonant and nonresonant trapped electrons' contribution, the total trapped electron energy flux is

$$\begin{aligned} Q_e &= -\left[ \frac{3}{2} f_t \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle \omega_k|}{\omega_k^2} \right\rangle \right. \\ &\quad \left. + 2 f_t \sqrt{\pi} \frac{R_0}{G} \left( \frac{R_0}{L_n G} \right)^{3/2} \left( \frac{R_0}{L_n G} - \frac{3}{2} \right) \exp\left( -\frac{R_0}{L_n G} \right) \sum_k |k_\theta| c_s \rho_s \left\langle |\tilde{\Phi}_k|^2 \right\rangle \right] n_0 \nabla T_{e0} \\ &\quad + \frac{3}{2} \frac{1}{R_0} f_t \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle \omega_k|}{\omega_k^2} \left( 1 - \frac{\omega_k}{\omega_{*e}} \right) \left( \frac{R_0}{L_n} - \frac{5G}{1 - \omega_k / \omega_{*e}} \right) \right\rangle n_0 T_{e0}. \end{aligned} \tag{B6}$$

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TABLE I: Various cases for the condition $\omega_{di} > k_\parallel v_\parallel$. $\rho_i$ is the ion gyroradius, $q$ is the safety factor, $qR_0\triangle\theta$ and $\triangle x$ are the widths of the mode structure along the magnetic field line and in the x directions, respectively.

| | |
|---|---|
| Ballooning mode | $k_\theta \rho_i \gg 1/q$ |
| Extended toroidal mode | $k_\theta \rho_i \gg 1/(q\triangle\theta)$ |
| Slab mode | $\rho_i/\triangle x \gg \hat{s}/q$ |


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TABLE II: Pieces of ion and trapped electron energy fluxes. The factor $k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \right\rangle$ is omitted in the $\nabla B$ driven and $\nabla n$ driven terms. The second term in $\chi_i$ is present for ITG turbulence but not for TEM turbulence. The second term in $\chi_e$ is present for TEM turbulence but not for ITG turbulence.

| | $\nabla B$ driven pinch velocity | $\nabla n$ driven convective velocity | $\chi$ |
|---|---|---|---|
| Ion | $-\frac{10}{3}\frac{1}{R_0}$ | $\frac{1}{L_n}$ | $\sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \right\rangle$ |
| | inward | outward for | $+2\left(\frac{3}{4}\right)^{7/2}\sqrt{\pi}\tau R_0 \left(\frac{R_0}{L_n}\right)^{3/2}\left(\frac{R_0}{L_n}-2\right)$ |
| | | $\nabla n_0 < 0$ | $\times \exp\left(-\frac{3}{4}\frac{R_0}{L_n}\right)\sum_k |k_\theta| c_s \rho_s \left\langle |\tilde{\Phi}_k|^2 \right\rangle$ |
| Electron | $-\frac{15}{2} f_t \frac{G}{R_0}$ | $\frac{3}{2} f_t \frac{1}{L_n}\left(1-\frac{\omega_k}{\omega_{*e}}\right)$ | $\frac{3}{2} f_t \sum_k k_\theta^2 c_s^2 \rho_s^2 \left\langle |\tilde{\Phi}_k|^2 \frac{|\triangle\omega_k|}{\omega_k^2} \right\rangle$ |
| | inward for | outward for | $+2 f_t \sqrt{\pi}\frac{R_0}{G}\left(\frac{R_0}{L_n G}\right)^{3/2}\left(\frac{R_0}{L_n G}-\frac{3}{2}\right)$ |
| | $\hat{s} \gtrsim -0.89$ | $\nabla n_0 < 0$ and $\omega_k < \omega_{*e}$ | $\times \exp\left(-\frac{R_0}{L_n G}\right)\sum_k |k_\theta| c_s \rho_s \left\langle |\tilde{\Phi}_k|^2 \right\rangle$ |