Bistable Dynamics of Turbulence Intensity in a Corrugated Temperature Profile

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Outline

- Motivations
- Mesoscale temperature profile corrugation and nonlinear drive
- Bistable spreading of the turbulence intensity:

  —subcritical excitation

  —propagation

Motivations

How turbulent fluctuations penetrates stable domains?

$\longrightarrow$ Anomalous transport

$\longrightarrow$ Collapse of H-mode

Most previous works treat turbulence spreading as a Fisher front.

Conventional wisdom: Fisher front with a nonlinear diffusivity

Generic structure of Fisher spreading equation:

$$A \equiv -\partial_x T$$

$$\frac{\partial}{\partial t} I = \gamma_0 \left(\langle A \rangle - A_c \right) I - \gamma_{nl} I^2 + D_1 \frac{\partial}{\partial x} \left(I \frac{\partial}{\partial x} I \right)$$

linear excitation **nonlinear propagation**

Nontrivial solution requires:

$$\langle A \rangle > A_c \Rightarrow I \propto \langle A \rangle - A_c$$

Suffering from two serious drawbacks:

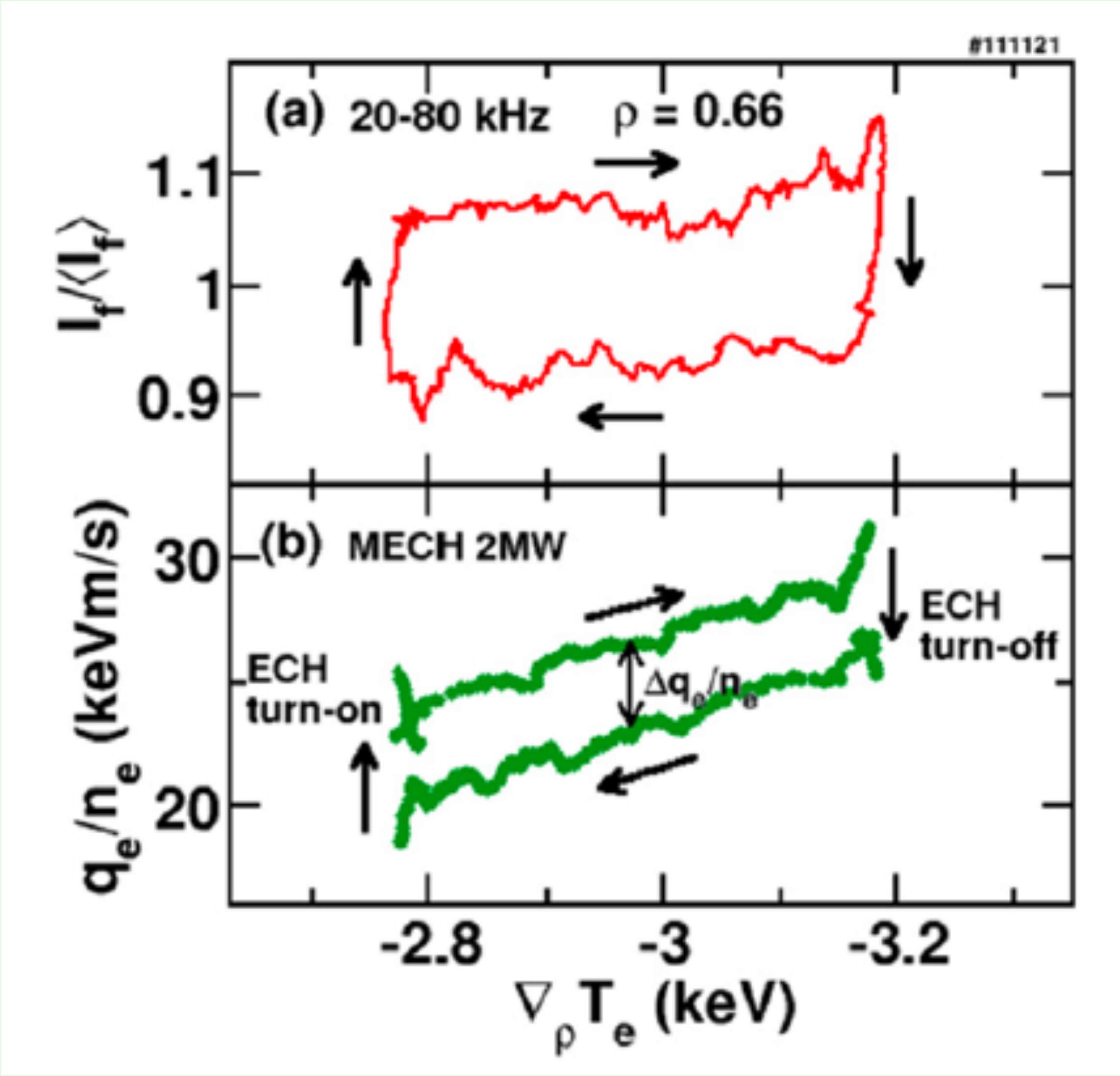
***insufficient near marginal state**

****can be strongly damped in subcritical region**

Motivations

The turbulence intensity is ***unistable*** in the Fisher model. However:

A hysteretic relation between turbulence intensity and temperature gradient also observed:



Inagaki2013

$\Longrightarrow$ An indication of bistability of the turbulence intensity!

We missed something here??

$$\frac{\partial}{\partial t} I = \gamma_0 \left(\langle A \rangle - A_c + \boxed{??} \right) I - \gamma_{nl} I^2 + D_1 \frac{\partial}{\partial x} \left(I \frac{\partial}{\partial x} I \right)$$

In this talk, we propose the missed piece is the nonlinear drive induced by the **corrugation of the temperature field**.

The key for *nonlinear* turbulence excitation:

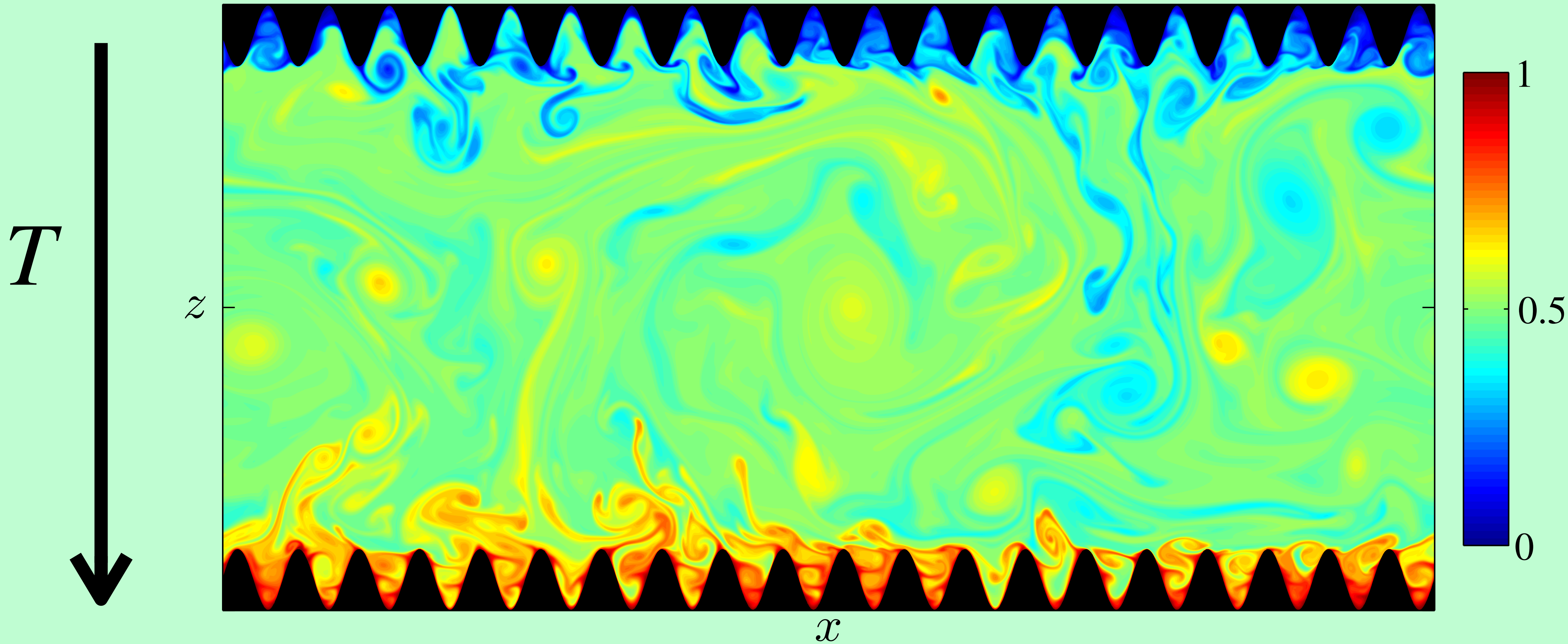
temperature corrugation by inhomogeneous turbulent mixing.

$$\Uparrow$$

Inevitable consequence of potential enstrophy conservation

A consistent treatment of
multi-scale, **multi-field** couplings is required…

An example from Rayleigh-Bernard convection



Toppaladodi et al 2017

Roughened temperature profile enhances turbulent heat flux.

How mesoscale fields impact evolution of turbulence intensity?

Drive: $\left(\nabla T\right)_{meso}$

Dissipation: $\left\langle V\right\rangle'_{ZF} \xrightarrow{\text{local force balance}} \left\langle V\right\rangle'_{ZF} \propto \left(\nabla^2 T\right)_{meso}$

$\Rightarrow$ Generally, drive&dissipation act in different regions.

How the turbulence intensity is **excited and spreads i**n the presence of a corrugated temperature profile?

The Model: bistable turbulence Intensity

The basic structure of I's evolution is

$$\frac{\partial}{\partial t} I = \gamma_0 \left(\langle A \rangle - A_c + \Theta(\widetilde{A}_m)\widetilde{A}_m \right) I - \gamma_{nl} I^2 + D_1 \frac{\partial}{\partial x}\left(I \frac{\partial}{\partial x} I \right)$$

nonlinear drive

For a mean field approximation, $\Theta(\tilde{A}_m)\tilde{A}_m = \left\langle \Theta(\tilde{A}_m)\tilde{A}_m \right\rangle + \widetilde{\Theta(\tilde{A}_m)\tilde{A}_m} \simeq \left\langle \Theta(\tilde{A}_m)\tilde{A}_m \right\rangle$

$\Rightarrow$

$$\frac{\partial}{\partial t} I = \gamma_0 \left(\langle A \rangle + \langle \Theta(\widetilde{A}_m)\widetilde{A}_m \rangle - A_c \right) I - \gamma_{nl} I^2 + D_1 \frac{\partial}{\partial x}\left(I \frac{\partial}{\partial x} I \right)$$

Relation between

$$\langle \Theta(\widetilde{A}_m)\widetilde{A}_m \rangle \text{ and } \boldsymbol{I}\,?$$

Strength of Mesoscopic ∇T Fluctuations

$$\frac{\partial}{\partial t}T + \nabla \cdot Q_T = \chi_{neo}\frac{\partial^2}{\partial x^2}T + S\delta(x) \qquad \textbf{(*)}$$

$$T = \langle T \rangle + \tilde{T} = \langle T \rangle + \tilde{T}_m + \tilde{T}_s,\ Q_T = \tilde{v}T,\ \tilde{T}_m : \text{meso scale};\ \tilde{T}_s : \text{micro scale}$$

Define two types of average:

$\langle .. \rangle_s$ – micro timescale; $\langle .. \rangle_m$ – meso timescale $\quad \Rightarrow \langle \langle T \rangle_s \rangle_m = \langle T \rangle_m \equiv \langle T \rangle$

Multiplying T on both sides of $(*)$ and carrying out a double average $\langle \langle .. \rangle_s \rangle_m$ yields

Entropy balance of the turbulence:

$$A_0 \langle Q_T \rangle_m + \langle \langle \tilde{A}_m \tilde{v} \tilde{T} \rangle_s \rangle_m \simeq \chi_{neo} \langle \tilde{A}_m^2 \rangle_m$$

entropy dissipation due to neoclassical diffusion

entropy production due to turbulent mixing of the mean temperature field

triple coupling between micro and meso scales → The essential process for subcritical turbulence excitation

A closure on the triple coupling: ‘negative’ thermal conductivity

$$\left\langle \tilde{A}_m \tilde{v} \tilde{T} \right\rangle_s = \tilde{A}_m \underline{\left\langle \tilde{v} \tilde{T} \right\rangle_s}$$

up gradient heat flux on mesoscale $\left\langle \tilde{v} \tilde{T} \right\rangle_s = \chi_m \tilde{A}_m = -\left| \chi_m \right| \tilde{A}_m$

$\chi_m < 0$ the negative diffusivity.

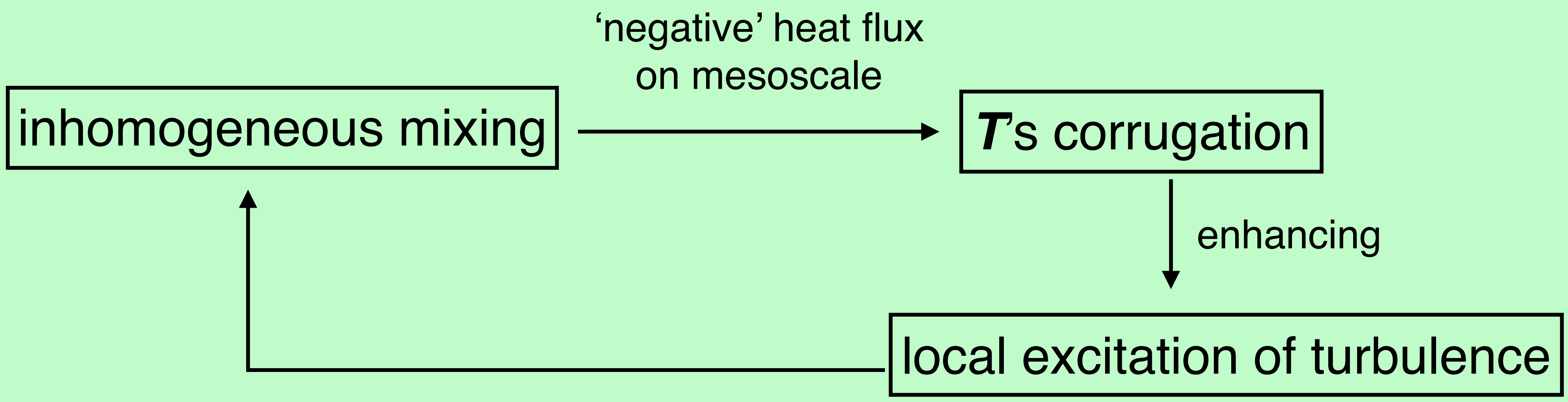
The underlying physics: roll-over of Q_m vs $\partial_x T_m$ duo to ZF shear.

$\Longrightarrow$ Zeldovich relation in multi-cale coupling system:

$$\langle \tilde{A}_m^2 \rangle = \frac{D_0 \langle A \rangle^2}{\chi_{neo} + |\chi_m|} I$$

T’s profile gets corrugated by the inhomogeneous turbulent mixing.

The closed loop



Bistable spreading of the turbulence intensity: subcritical excitation

I's evolution with subcritical drive(**Fitzhugh-Nagumo type, not Fisher!**)

$$\frac{\partial}{\partial t}I \;=\; \gamma_0\left(\langle A\rangle - A_C\right)I + \sqrt{\frac{\gamma_0^2 D_0 \langle A\rangle^2}{\chi_{neo}+|\chi_m|}}I^{3/2} - \gamma_{nl}I^2 \;+\; D_1\frac{\partial}{\partial x}\left(I\frac{\partial}{\partial x}I\right)$$

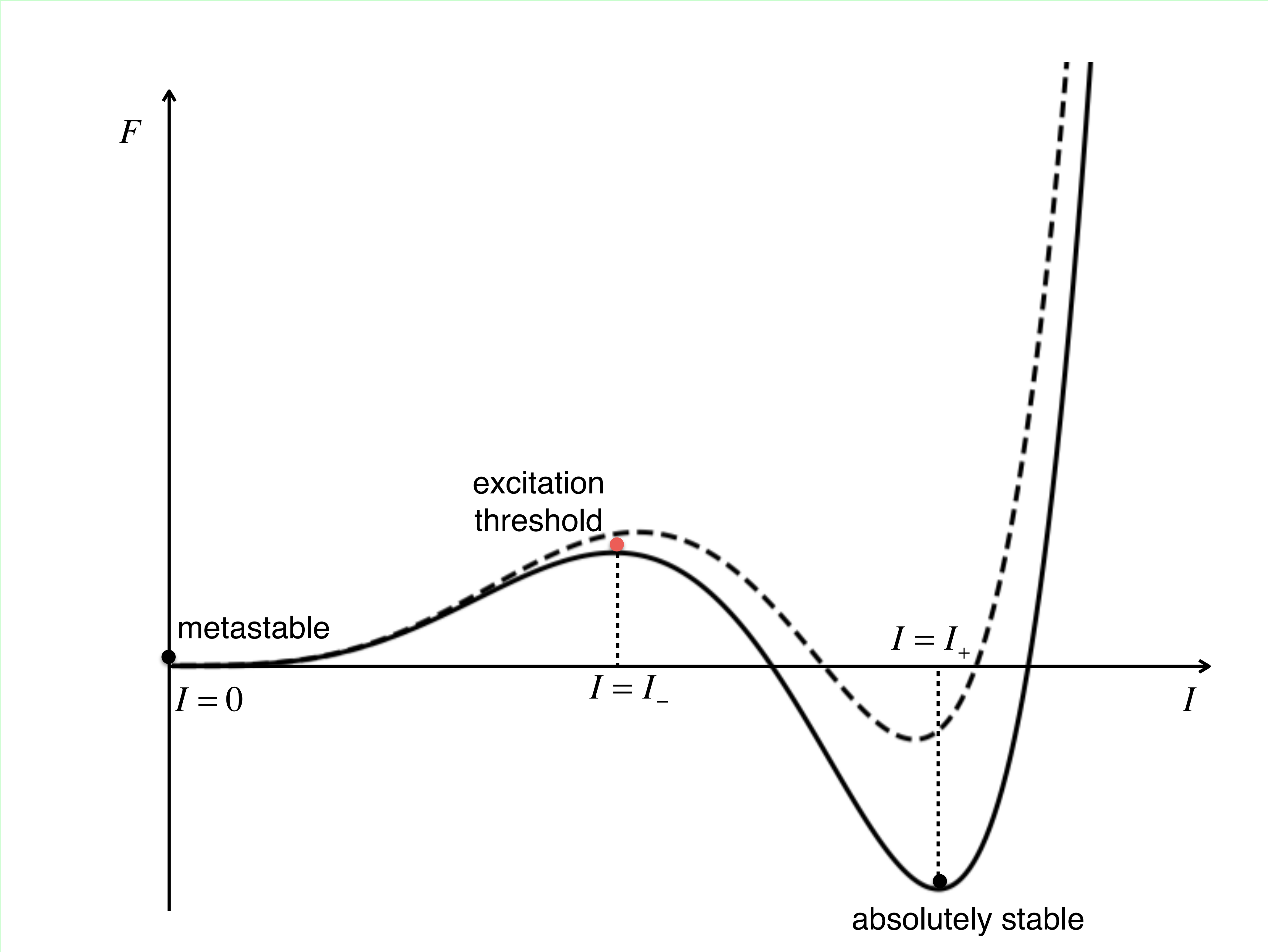
On the subcritical excitation

$$\frac{\partial}{\partial t}I = -\frac{\delta F(I)}{\delta I}$$

$$F(I) = -\frac{1}{2}\gamma_0\left(\langle A\rangle - A_C\right)I^2 - \frac{2}{5}\sqrt{\frac{\gamma_0^2 D_0\langle A\rangle^2}{\chi_{neo}+|\chi_m|}}I^{5/2} + \frac{1}{3}\gamma_{nl}I^3$$

Two stable solutions: $I = 0 \quad \text{and} \quad I = I_+$

One unstable solution: $I = I_-$



F
excitation threshold
metastable
$I = I_+$
$I = 0$
$I = I_-$
I
absolutely stable

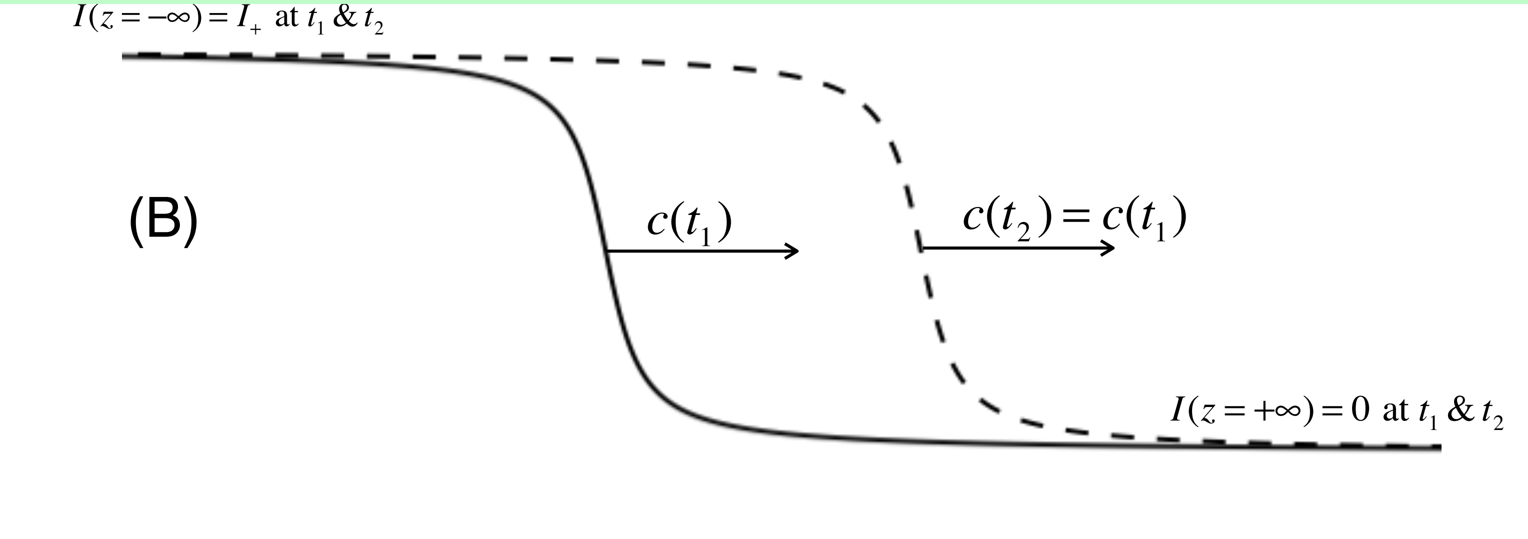
How the bistable spreading happens?

Looking for wave-like solution: $I(x,t) = I(z)$ with $z = x - c^{*}t$

$$-c^{*}\frac{d}{dz}I = -\frac{\delta F}{\delta I} + D_1 I \frac{d^2}{dz^2}I + D_1\left(\frac{d}{dz}I\right)^2$$

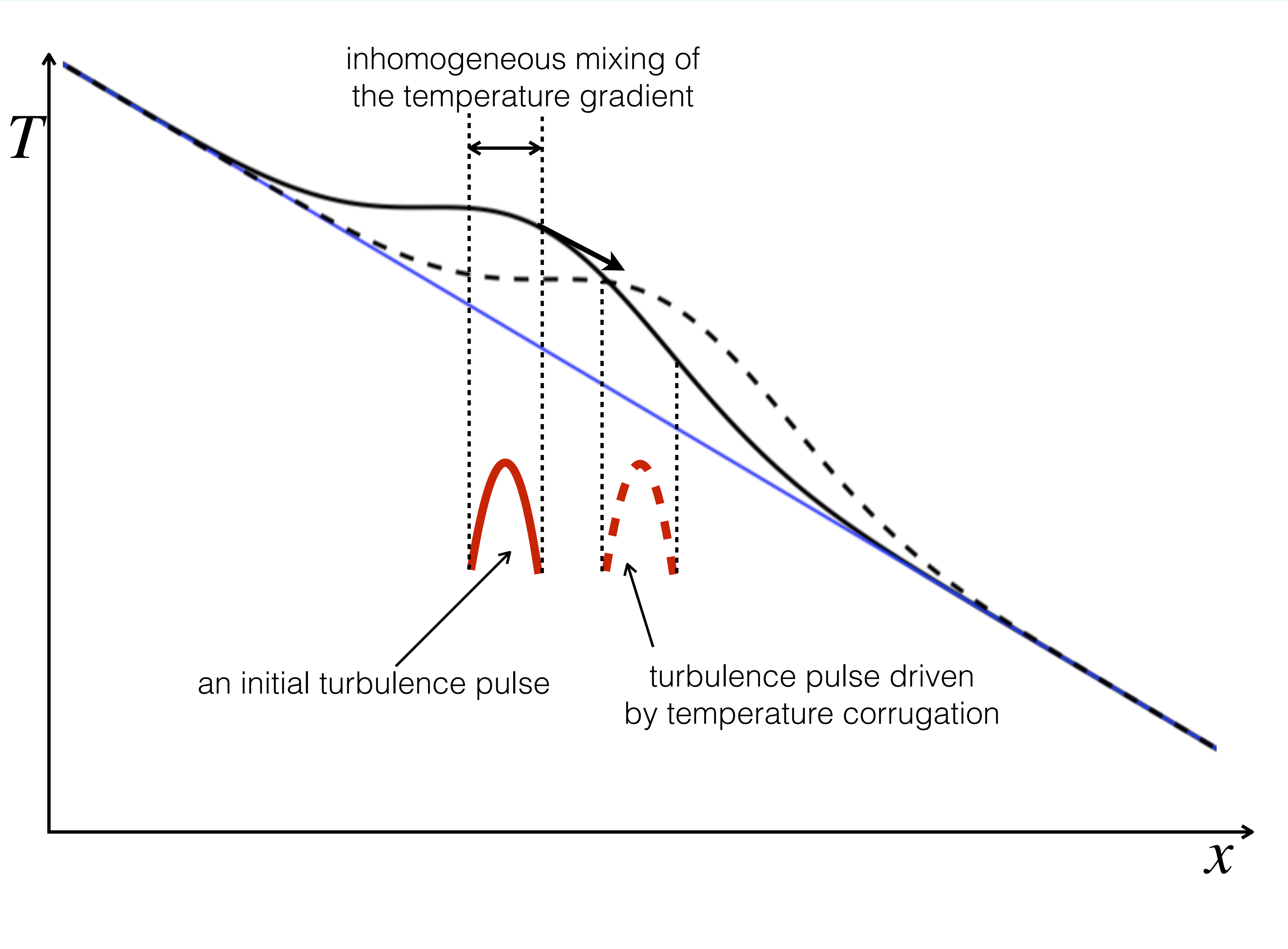
Propagation speed of the bistable front:

$$c^{*} = \frac{F(+\infty) - F(-\infty)}{\int_{-\infty}^{+\infty} I'^2 dz} + \frac{-\frac{D_1}{2}\int_{-\infty}^{+\infty} I'^3 dz}{\int_{-\infty}^{+\infty} I'^2 dz}$$



$: C^{*} > 0$

How the bistable spreading happens?



Summary&future work

Mesoscale temperature profile corrugations provide a natural way for subcritical turbulence excitation, and the following spreading.

Next:

Temperature profile evolution needs to be treated in a more consistent way;

Stability problem of the front, i.e., can the front be splitter by any external/internal noise?