Competition of Mean Perpendicular and Parallel Flows in a Linear Device

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Background

- **Intrinsic** axial flows observed in linear device (CSDX)
- Linear device studies suggest **dynamical** competition between mean perpendicular and parallel flows

- **Dynamical**: $V_\perp$ and $V_\parallel$ exchange energy with the background turbulence, and each other.
  - Energy balance between $V_\perp$ and $V_\parallel$
  - **Tradeoff** between $V_\perp$ and $V_\parallel
Experimental observations: $V'_\perp$ and $V'_\parallel$

- $V'_\parallel$ scaling with $\nabla n_0$
  - Analogy to Rice-type scaling:
    $\Delta V'_\parallel \propto \nabla T$ [Rice et al, PRL, 2011]

- $V'_\perp$ scaling with $\nabla n_0$
  - Tradeoff between $V'_\perp$ and $V'_\parallel$
  - $V'_\perp$ saturation by $V'_\parallel$
Measurements: Parallel Reynolds Stress $\langle \tilde{v}_r\tilde{v}_\parallel \rangle$

$|\text{Reynolds force}| \gg |\text{axial pressure gradient}|$

$\Rightarrow V_\parallel \text{ driven by turbulence} \Rightarrow V'_\parallel \sim \nabla n_0$

$\Pi^{Res}_\parallel \text{ scaling with } \nabla n_0$

\[\sigma_{VT} = 2.93 \pm 1.09\]
Outline of the Rest

• Introduction
  – Key questions and why
  – Current status of model

• Exploration of coupling
  – Study turbulent energy branching between $V_\parallel$ and $V_\perp$
  – Reynolds power ratio $P_\parallel^R / P_\perp^R$ decreases as $V_\perp$ increases
    → tradeoff between $V_\perp$ and $V_\parallel$
  – $P_\parallel^R / P_\perp^R$ maximum occurs when $|\nabla V_\parallel|$ is below the PSFI (parallel shear flow instability) threshold
    → saturation of intrinsic $V_\parallel$

• Are shear suppression “rules” correct?
  – Revisiting the resonance effect
  – Wave-flow resonance suppresses instability
  – $V_\perp'$ weakens resonance → enhances instability
  – Implication for zonal flow dynamics
Key Questions and Why

• What’s the coupling between mean perpendicular and parallel flows ($V_\perp$ and $V_\parallel$)?
  – How do they interact? → How do they compete for energy from the background turbulence?
  – How does $V_\parallel$ affect the production and saturation of intrinsic $V_\perp$?

• Why should we care?
  – Relevant to L-H transition
    • Both $V'_\perp$ and $V_\parallel$ increase, during transition.
    • The coupling of the two is relevant to transition threshold and dynamics.
  – Linear device (CSDX) studies suggest competition between $V_\perp$ and $V_\parallel$
Why linear device?

• Relevance: zero magnetic shear $\leftrightarrow$ Enhanced-confinement states (H-mode) favor low magnetic shear.

• Self-generated, sheared $V_\perp$ (zonal flow) observed, which regulates the drift wave turbulence.

• *Intrinsic* $V_\parallel$ observed: driven by drift wave turbulence ($\nabla n_0$) via turbulent Reynolds work, i.e. $-\partial_r \langle \tilde{v}_r \tilde{v}_\parallel \rangle V_\parallel$.
  $\rightarrow$ New in linear device (zero magnetic shear). New mechanism for $V_\parallel$ generation proposed. [Li et al, PoP 2016 & 2017]

• Advantage of CSDX: *unique measurements of parallel Reynolds stress* $\langle \tilde{v}_r \tilde{v}_\parallel \rangle$ *and Reynolds power* $(-\partial_r \langle \tilde{v}_r \tilde{v}_\parallel \rangle V_\parallel)$
  $\rightarrow$ Not achieved in tokamak cores or other linear devices.
Current status of model

• Conventional wisdom of $V_\perp \to V_\parallel$ coupling:
  – $V_\perp'$ breaks the symmetry in $k_\parallel$, but requires finite magnetic shear
  – **Not applicable** in linear device (straight magnetic field)

• $V_\parallel \to V_\perp$ coupling:
  – 3D coupled drift-ion acoustic wave system [Wang et al, PPCF 2012]
  – Coupling between fluctuating PV and parallel compression $\langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle$
    breaks PV conservation
    → Sink/source for fluctuating potential enstrophy density
    → Zonal flow generation
  – Perpendicular flow dynamics:

\[
\frac{\partial}{\partial t} \left[ V_\perp - L_n \left( \frac{\tilde{q}^2}{2} \right) \right] \sim -v_i V_\perp + L_n \left[ \frac{\partial}{\partial r} \left( \tilde{v}_x \frac{\tilde{q}^2}{2} \right) \right] + \mu \langle (\nabla \tilde{q})^2 \rangle - \langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle
\]

  – Collisional damping
  – PV diffusion

\[
\langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle \sim - \sum_k \frac{\Delta \omega_k}{\omega_k^2} k_\parallel^2 |\phi_k|^2 < 0
\]
Section II: Exploration of $V_\perp$-$V_\parallel$ Coupling

- **Goal:** study *how extrinsic flows affect Reynolds powers*
  - generation of intrinsic flows
  - turbulent energy branching between intrinsic $V_\perp$ and $V_\parallel$
- **Analogy to biasing experiments**
- **Hasegawa-Wakatani drift wave**
  - near adiabatic electron:
    \[ \tilde{n} = (1 - i\delta)\phi, \delta \ll 1 \]
    \[ \frac{D}{Dt} \tilde{n} + \tilde{v}_r \frac{\nabla n_0}{n_0} + \nabla_\parallel \tilde{v}_|| = D_\parallel \nabla^2_\parallel (\tilde{n} - \tilde{\phi}), \]
    \[ \frac{D}{Dt} \nabla^2_\perp \tilde{\phi} + \tilde{v}_r V''_\perp = D_\parallel \nabla^2_\parallel (\tilde{n} - \tilde{\phi}), \]
    \[ \frac{D}{Dt} \tilde{v}_|| + \tilde{v}_r V'_\parallel = \nabla_\parallel \tilde{n}, \]

- **Prescribed flows vary in x direction:**
  \[ V_\perp = V_\perp^{max} \sin[q_x(x - L_x/2)]; V_\parallel = -V_\parallel^{max} \sin[q_x(x - L_x/2)] \]
- **Fourier decomposition in y, z directions:**
  \[ \tilde{f} = \sum_k f_k(x) e^{i(k_y y + k_\parallel z)} e^{-i(\omega_k + i\gamma_k)t}, \text{where } \tilde{f} = \tilde{n}, \tilde{v}_||, \tilde{\phi} \]
- **Solve for growth rate, frequency, and eigenmode function $\phi_k(x)$ for drift wave instability ($\nabla n_0$ driven) with prescribed $V_\perp$ and $V_\parallel$
Bottom Line: $\nabla n_0$ is the Primary Instability Drive

- Other potential drives:
  - $V_{\perp}''$ → Kelvin-Helmholtz instability
  - $\nabla V_{||}$ → Parallel shear flow instability

- KH is not important
  - $V_{\perp}''$ drive weaker than $\nabla n_0$ drive, i.e. $|k_y \rho_s^2 V_{\perp}''| \ll \omega_{*e}$
  - $V_{\perp}$ affects the drift wave instability via wave-flow resonance $\omega_k = k_y V_{\perp}$ (see Section III)

- **PSFI stable** in CSDX
$\nabla V_\parallel$ has little effect on drift wave instability

- Influence drift wave instability via frequency shift

$\gamma_k \sim \omega_{*e} - \omega_k \sim \frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2} \omega_{*e} + \frac{k_\theta k_\parallel \rho_s c_s V'_\parallel}{\omega_{*e}}$
Definition: Reynolds Power

- Mean flow evolution is powered by Reynolds power
  \[ \frac{1}{2} \frac{\partial |V_\parallel|^2}{\partial t} \sim - \frac{\partial}{\partial x} \langle \tilde{v}_x \tilde{v}_\parallel \rangle V_\parallel \]
  - Parallel Reynolds power of a single eigenmode
    \[ P_{\parallel}^R = \int_0^{L_x} dx \left[ - \frac{\partial}{\partial x} (\tilde{v}_x^* \tilde{v}_{\parallel,k}) \right] V_\parallel \]
  - Perpendicular Reynolds power of a single eigenmode
    \[ P_{\perp}^R = \int_0^{L_x} dx \left[ - \frac{\partial}{\partial x} (\tilde{v}_x^* \tilde{v}_{\perp,k}) \right] V_{\perp} \]
- Effects of extrinsic \( V_\parallel \) and \( V_{\perp} \) on the ratio \( P_{\parallel}^R / P_{\perp}^R \) are studied
Coupling of $V_\perp$ and $V_\parallel$ ↔ Ratio of Reynolds Powers

- Ratio $P_\parallel^R / P_\perp^R$ decreases with $V_\perp$
  - Energy branching of $V_\parallel$ reduced
  - $V_\perp$ reduces generation of $V_\parallel$
  - *Competition* between $V_\perp$ and $V_\parallel$

- Increase $V_\parallel$ → $P_\parallel^R / P_\perp^R$ turnover
  - *before* $\nabla V_\parallel$ hits PSFI threshold
  - Max energy branching of $V_\parallel$ below PSFI threshold
  - $V_\parallel$ saturates *below* PSFI threshold

Reduced model developed to study the coupling → See poster 43 on Thursday afternoon
Section III: Revisiting Shearing Effects

• **Are conventional shear suppression “rules” correct?**
  • Aim to test well known (mis)conceptions about shearing effects on stability
  • Conventional wisdoms:
    – $E \times B$ flow shear suppresses instability $\iff$ Is it correct?
    – Wave-flow resonance effect is often overlooked, though was mentioned in past works.
  • Findings:
    – Explore linear instability, using **fixed extrinsic flows**
    – Wave-flow resonance stabilizes drift wave instability
    – Perpendicular flow shear weakens the resonance, and thus **destabilizes** the instability
  • Implications for zonal flow generation and saturation:
    – Revisit predator-prey model with resonance effects
      $\Rightarrow$ Mechanism for **collisionless** zonal flow damping (without involving tertiary instability, such as KH)
Wave-flow resonance

- Resonance: $\omega_k - k_y V_\perp - k_\parallel V_\parallel$
  $|k_\parallel|/k_y << 1 \rightarrow$ Resonance dominated by $\omega_k - k_y V_\perp$

- Hasegawa-Wakatani drift wave model, with extrinsic $V_\perp$
  \[
  \frac{D}{Dt} \tilde{n} + \tilde{v}_r \frac{\nabla n_0}{n_0} = D_\parallel \nabla^2_\parallel (\tilde{n} - \tilde{\phi}),
  \]
  \[
  \frac{D}{Dt} \nabla^2_{\perp} \tilde{\phi} + \tilde{v}_r V''_\perp = D_\parallel \nabla^2_\parallel (\tilde{n} - \tilde{\phi})
  \]

- **KH drive negligible** $\rightarrow \nabla n_0$ driven instability
  - Near adiabatic electron: $\tilde{n} = (1 - i\delta)\phi$, $\delta << 1$
    - $\delta = (\omega_{*e} - \omega_k + k_y V_\perp)/k_\parallel^2 D_\parallel^2 = v_{ei}(\omega_{*e} - \omega_k + k_y V_\perp)/k_\parallel^2 v_{The}^2$
  - In the limit of strong resonance, i.e. $\gamma_k << \omega_k - k_y V_\perp << \omega_{*e}$,
    $\delta \rightarrow v_{ei}\omega_{*e}/k_\parallel^2 v_{The}^2$

- Resonance affects the eigenmode scale $\rightarrow$ Influence instability
Resonance and Instability Related to Mode Scale

• Eigenmode equation with resonant effect:

\[
(\omega_k - k_y V_\perp + i\gamma_k) \rho_s^2 \partial_x^2 \phi = \left[ (1 + k_y^2 \rho_s^2 - i\delta)(\omega_k - k_y V_\perp + i\gamma_k) - \omega* \right] \phi
\]

• Mode scale defined as

\[ L_m^2 \rho_s^2 \equiv \rho_s^2 \int_0^{L_x} dx |\partial_x \phi|^2 \bigg/ \int_0^{L_x} dx |\phi|^2 \]

• Results:

\[
\omega_k - k_y V_\perp = \frac{\omega* (1 + k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2)}{(1 + k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2)^2 + \delta^2},
\]

\[
\gamma_k = \frac{\delta (\omega_k - k_y V_\perp)}{1 + k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2} = \frac{\delta \omega*}{(1 + k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2)^2 + \delta^2}.
\]

• In the limit of strong resonance

\[
\gamma_k \ll \omega_k - k_y V_\perp \ll \omega*\]

\[
\omega_k - k_y V_\perp \sim \omega* L_m^2 / \rho_s^2
\]

\[
\gamma_k \sim \delta (\omega_k - k_y V_\perp) L_m^2 \sim \delta \omega* L_m^4 / \rho_s^4
\]

• Eigenmode peaks \((L_m^{-2} \rho_s^2)\) increases) as resonance becomes stronger

• Resonance suppresses drift wave instability
Perpendicular flow shear **destabilizes** turbulence

- Mean perpendicular flow shear increases mode scale $L_m/\rho_s$
  - Weakens resonance
  - Enhances instability
- KH drive **negligible** compared to $\nabla n_0$

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**Mode Structure**

- max shear = $4v_d/L_x$
- max shear = $8v_d/L_x$
- max shear = $12v_d/L_x$
- max shear = $16v_d/L_x$

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**Resonance**

$\left( \omega - k_0 V_{max} \right) / \omega_{ce}$

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**Growth Rate**

$\gamma / \omega_{ce}$
Implications for Zonal Flow Dynamics

• Connection to collisionless damping of ZF

• Zonal flow evolution \( \rightarrow \) Mean enstrophy equation:

\[
\frac{\partial}{\partial t} \int dr \frac{\langle \rho \rangle^2}{2} = \int dr \langle \tilde{v}_r \tilde{\rho} \rangle \frac{d\langle \rho \rangle}{dr} - v_i \int dr \langle \rho \rangle^2 + \ldots
\]

• Vorticity \( \rho \equiv \nabla_\perp^2 \phi \) flux: \( \langle \tilde{v}_r \tilde{\rho} \rangle = -D_\rho \frac{d\langle \rho \rangle}{dr} + \Gamma^\text{Res}_\rho \)

Conserves enstrophy between mean flow and fluctuations

\[
\frac{\partial}{\partial t} \int dr \frac{\langle \rho \rangle^2}{2} = - \int dr D_\rho \left( \frac{d\langle \rho \rangle}{dr} \right)^2 + \int dr \Gamma^\text{Res}_\rho \frac{d\langle \rho \rangle}{dr} - v_i \int dr \langle \rho \rangle^2 + \ldots
\]

• \( v_i \to 0 \rightarrow \) Dimits shift regime \( \rightarrow \) Resonance gives collisionless damping

• Collisionless damping by turbulent viscosity: \( d\langle \rho \rangle/dr \sim \Gamma^\text{Res}_\rho / D_\rho \)

• Resonance sets \( D_\rho \rightarrow \) ZF damping

\[
\Gamma^\text{Res}_\rho = k_y c_s^2 |\phi_k|^2 \left[ \frac{\gamma_k \omega^*_e + \alpha_n (\omega^*_e - \omega_k + k_y V_\perp)}{|\omega_k - k_y V_\perp + i \alpha_n|^2} - \frac{\gamma_k |\omega^*_e|}{|\omega_k - k_y V_\perp|^2} \right] \quad D_\rho = k_y^2 c_s^2 |\phi_k|^2 \frac{\gamma_k}{|\omega_k - k_y V_\perp|^2}
\]
Collisionless ZF damping by vorticity flux resonance

- Resonance replaces need for KH:

\[ \gamma_k = \text{linear instability } (\gamma_L) + \text{resonance absorption } \gamma_R \sim \gamma_R \left( \omega_k - k_y V_\perp \right) \]

Analogy to ion-acoustic absorption during collapse of Langmuir waves

- Resonance induces collisionless damping through \( D_\rho \)

- Revisit predator-prey model with resonance effect

→ Mechanism for collisionless damping, without KH
Summary

• Experimental observations suggest competition between mean $V_\perp$ and $V_\parallel$

• Reynolds power ratio $P_\parallel^R / P_\perp^R$ changes with prescribed extrinsic mean flows
  – $P_\parallel^R / P_\perp^R$ decreases with $V_\perp \rightarrow$ tradeoff between $V_\perp$ and $V_\parallel$
  – $P_\parallel^R / P_\perp^R$ maximum occurs before $\nabla V_\parallel$ hits PSFI threshold

• Testing misconceptions of shearing effects on stability
  – Wave-flow resonance suppresses instability
  – $V'_\perp$ weakens resonance $\rightarrow$ $V'_\perp$ enhances instability $\rightarrow$
  – Resonance produces turbulent viscosity
    $\rightarrow$ collisionless damping of ZF, without involving KH
  – Suggest revisit predator-prey model with resonance effects
    $\rightarrow$ mechanism for collisionless ZF damping, without tertiary instability