Studies of Turbulence-driven FLOWs:

a) $V_\perp$, $V_\parallel$ Competition in a Tube

b) Revisiting Zonal Flow Saturation

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Outline

• Background: turbulence driven $V_{\perp}$ and $V_{\parallel}$ observed in CSDX
• Questions: How do they interact? How do they saturate?
• Increment study of $V_{\perp}$, $V_{\parallel}$ competition
  → Analogous to perturbation experiments
• Zonal flow saturation by wave-flow resonance
  • Wave-flow resonance effects on linear stability
    → *Flow shear enhances instability via resonance*
  • Collisionless ZF saturation by resonance
    → *Derives* mesoscopic ZF scale, i.e. $L_{ZF} \sim \sqrt{\rho_s L_n}$
    → Extended predator-prey model, compared to old model
Background

- **Intrinsic** axial and azimuthal flows observed in linear device (CSDX)
  - Increase B → scans mean flows-both $V_{\perp}$ and $V_{\parallel}$
- **Dynamical** competition between perpendicular and parallel flows
- $V_{\perp}$ and $V_{\parallel}$ exchange energy with the turbulence, and each other.
  → Study energy apportionment between $V_{\perp}$ and $V_{\parallel}$
  → Tradeoff between $V_{\perp}$ and $V_{\parallel}$
Key Questions

• What’s the coupling between mean perpendicular and parallel flows ($V_\perp$ and $V_\parallel$)?
  – How do they compete for energy from turbulence?

• How/Why do flows saturate, especially in collisionless regime?

• Why?
  – Linear device (CSDX) studies suggest apportionment of turbulence energy between $V_\perp$ and $V_\parallel$
  – Relevant to L-H transition
    • Both $V'_\perp$ and $V'_\parallel$ increase, during transition.
    • The coupling of the two is relevant to transition threshold and dynamics.
Current status of coupling model

- Conventional wisdom of $V_\perp \rightarrow V_\parallel$ coupling:
  - $V'_\perp$ breaks the symmetry in $k_\parallel$, but requires finite magnetic shear
  - *Not applicable* in linear device (straight magnetic field)
- $V_\parallel \rightarrow V_\perp$ coupling via parallel compression:
  - 3D coupled drift-ion acoustic wave system [Wang et al, PPCF 2012]
  - Coupling between fluctuating PV and parallel compression $\langle \tilde{q} V_\parallel \tilde{v}_\parallel \rangle$
    - *breaks PV conservation*
    - Sink/source for fluctuating potential enstrophy density
    - Zonal flow generation
V⊥ and V∥ competition

- Increment study of V⊥ and V∥ effects on Reynolds powers
  - *Turbulent energy branching* between V∥ and V⊥
  - Reynolds power ratio $P^R_∥/P^R_\perp$ decreases as $V_\perp$ increases
    - tradeoff between $V_\perp$ and $V∥$
  - $P^R_∥/P^R_\perp$ maximum occurs when $|\nabla V∥|$ is below the PSFI (parallel shear flow instability) threshold
    - saturation of intrinsic $V∥$
Exploration of $V_\perp$-$V_\parallel$ Coupling

- **Goal:** *How do extrinsic flows affect powers?*
  - **Turbulent energy branching** between intrinsic $V_\perp$ and $V_\parallel$
  - How does $V_\perp$ affect intrinsic $V_\parallel$ generation?

- Analogous to perturbation experiments, i.e. fix one flow and increase the other through external momentum source

- **Collisional drift wave**
  - near adiabatic electron:
    \[
    \tilde{n} = (1 - i\delta)\phi, \delta \ll 1
    \]

- **Slab geometry**
  \[
  \frac{D}{Dt} \tilde{n} + \tilde{v}_r \frac{\nabla n_0}{n_0} + \nabla_\parallel \tilde{v}_\parallel = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi}),
  \]
  \[
  \frac{D}{Dt} \nabla_\perp^2 \tilde{\phi} + \tilde{v}_r V_\perp'' = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi}),
  \]
  \[
  \frac{D}{Dt} \tilde{v}_\parallel + \tilde{v}_r V_\parallel' = \nabla_\parallel \tilde{n},
  \]
\( \nabla n_0 \) is the Primary Instability Drive

- Other potential drives:
  - \( V''_\perp \) → Kelvin-Helmholtz instability
  - \( \nabla V_\parallel \) → Parallel shear flow instability

- KH is not important
  - \( V''_\perp \) drive weaker than \( \nabla n_0 \) drive, i.e. \( |k_y \rho_s^2 V''_\perp| \ll \omega_e \)

- \( \nabla V_\parallel \) in CSDX is well below the PSFI linear threshold
  \( \rightarrow \text{PSFI stable in CSDX} \)
Coupling of $V_\perp$ and $V_\parallel$ ↔ Ratio of Reynolds Powers

- Ratio $P_\parallel^R / P_\perp^R$ decreases with $V_\perp$
  → Energy branching of $V_\parallel$ reduced
  → $V_\perp$ reduces generation of $V_\parallel$
  → Suggest *competition* between $V_\perp$ and $V_\parallel$

- Increase $V_\parallel$ → $P_\parallel^R / P_\perp^R$ turnover *before* $\nabla V_\parallel$ hits PSFI threshold
  → Max energy branching of $V_\parallel$ below PSFI threshold
  → Suggest $V_\parallel$ saturates *below* PSFI threshold

![Graph 1](image1.png)

![Graph 2](image2.png)
Partial Summary 1

- CSDX experiments suggest **energy apportionment** between mean $V_\perp$ and $V_\parallel$
- Increment study on Reynolds power ratio $P_{\parallel}^R / P_{\perp}^R$
  - Analogous to perturbation study
  - $P_{\parallel}^R / P_{\perp}^R$ decreases with $V_\perp \rightarrow$ tradeoff between $V_\perp$ and $V_\parallel$
  - $P_{\parallel}^R / P_{\perp}^R$ maximum occurs **before** $\nabla V_\parallel$ hits PSFI threshold
Collisionless zonal flow saturation

• Wave-flow resonance prominent in linear device (CSDX)
  – Enters turbulence regulation, both linearly and nonlinearly
  – Flow shear is not the exclusive control parameter

• Resonance suppresses linear instability by wave absorption
  – Are shear suppression “rules” correct?
  – $V_\perp'$ weakens resonance $\Rightarrow$ flow shear enhances instability via resonance
Collisionless zonal flow saturation (cont’d)

• Collisionless Zonal flow saturation by resonant PV mixing
  – Model of resonant PV mixing
  – Resonant diffusion of vorticity saturates zonal flow in collisionless regime
  – Incorporated in an extended predator-prey model
  – Drift wave mixes PV at zonal flow shear below that for KH/tertiary excitation
Wave-flow resonance effect on linear stability

- Resonance: $\omega_k - k_y V_\perp - k_\parallel V_\parallel$
  $|k_\parallel|/k_y \ll 1 \rightarrow$ Resonance set by $\omega_k - k_y V_\perp$

- Hasegawa-Wakatani drift wave model, with extrinsic $V_\perp$

\[
\left( \frac{d}{dt} + \tilde{v}_E \cdot \nabla \right) \tilde{n} + \tilde{v}_x \frac{\nabla n_0}{n_0} = D_\parallel \nabla^2 (\tilde{n} - \tilde{\phi}) + D_c \nabla^2 \tilde{n},
\]

\[
\left( \frac{d}{dt} + \tilde{v}_E \cdot \nabla \right) \tilde{\rho} + \tilde{v}_x \langle \rho \rangle' = D_\parallel \nabla^2 (\tilde{n} - \tilde{\phi}) + \chi_c \nabla^2 \tilde{\rho},
\]

- KH drive negligible, i.e. $|k_y \rho_s^2 \langle v_y \rangle''| \ll \omega_e \rightarrow$ Drift wave instability dominant
  - Near adiabatic electron: $\tilde{n} = (1 - i\delta)\phi$, $\delta \ll 1$
  - $\delta = (\omega_e - \omega_k + k_y V_\perp)/k_\parallel^2 D_\parallel^2 = \nu_e (\omega_e - \omega_k + k_y V_\perp)/k_\parallel^2 v_{Th,e}^2$

- Resonance reduces the eigenmode scale $\rightarrow$ Suppresses instability

(Width of eigenmode)
Perpendicular flow shear **destabilizes** turbulence

- Mean perpendicular flow shear increases mode scale $L_m/\rho_s$
  - Weakens resonance
  - Enhances instability
- KH drive **negligible** compared to $\nabla n_0$

### Mode Structure

- Blue: max shear = $4v_d/L_x$
- Orange: max shear = $8v_d/L_x$
- Yellow: max shear = $12v_d/L_x$
- Purple: max shear = $16v_d/L_x$

### Resonance

### Growth Rate
Zonal Flow Saturation: Motivation

• Why?
  – Crucial to understand Dimits state physics
    → Collisionless zonal flow saturation, i.e. collisional damping → 0

• Tertiary instability does not work
  – Severely damped by magnetic shear
  – Observed mean flow shear is always below the threshold for tertiary instability excitation
Nonlinear Model: **Resonant PV Mixing**

- **Density:**
  \[
  \frac{\partial}{\partial t} \langle n \rangle = \frac{\partial}{\partial x} D_{n,\text{turb}} \frac{\partial}{\partial x} \langle n \rangle + D_c \nabla^2 \langle n \rangle,
  \]

- **Vorticity:**
  \[
  \frac{\partial}{\partial t} \langle \rho \rangle = \frac{\partial}{\partial x} \left[ (D_{n,\text{turb}} - D_q^{\text{res}}) \frac{\partial}{\partial x} \langle n \rangle + D_q^{\text{res}} \frac{\partial}{\partial x} \langle \rho \rangle \right] - \mu_c \langle \rho \rangle - \mu_{NL} \langle \rho \rangle + \chi_c \nabla^2 \langle \rho \rangle,
  \]

- **PE:**
  \[
  \frac{\partial}{\partial t} \Omega = D_{\Omega} \frac{\partial}{\partial x} \Omega + D_q^{\text{res}} \left[ \frac{\partial}{\partial x} \left( \langle n \rangle - \langle \rho \rangle \right) \right]^2 - \varepsilon_c \Omega^{3/2} + \gamma_L \Omega.
  \]

PE = Potential Enstrophy, i.e. \( \Omega \equiv \langle \tilde{\rho}^2 \rangle \)

- \( \mu_{NL} = \mu_{NL} \langle v_y \rangle \): nonlinear damping rate driven by tertiary mode

  **Irrelevant** to most cases we have encountered

- \( D_c, \mu_c, \chi_c \): collisional particle diffusivity, flow damping, vorticity diffusivity \( \Rightarrow \) vanishing in collisionless regime
**Resonant PV diffusion**

- PV flux $\rightarrow$ turbulent PV diffusion:
  \[
  \langle \tilde{v}_x \tilde{q} \rangle = -D_{q,turb} \frac{\partial}{\partial x} \langle q \rangle 
  \]
  \[\downarrow\]
  \[D_{q,turb} = \text{Resonant} + \text{Non-resonant}\]

- Resonant PV diffusivity:
  \[
  D_{q,\text{res}} = \sum_k |\tilde{v}_x|^2 \pi \delta(\omega_k - k_y V_\perp) \sim \sum_k \tau_{c,k} k_y \rho_s^2 c_x^2 |\phi_k|^2
  \]
  \[
  \tau_{c,k} \sim \left[ |v_{g,y} - v_{ph,y}| \Delta k_y + v_{g,x} \Delta k_x \right]^{-1}
  \]

- Non-resonant PV diffusivity:
  \[
  D_{q,\text{non-res}} = \sum_{\omega_k \neq k_y \langle v_y \rangle} k_y^2 \rho_s^2 c_s^2 |\phi_k|^2 \frac{|\gamma_k|}{|\omega_k - k_y \langle v_y \rangle|^2} \sim \sum_{\omega_k \neq k_y \langle v_y \rangle} \frac{k_y^2 \rho_s^2 c_s^2}{k^2 D ||} \frac{k_y \rho_s^2 + L_m^2 \rho_s^2}{1 + k_y^2 \rho_s^2 + L_m^2 \rho_s^2} |\phi_k|^2
  \]

Resonant diffusivity exceeds non-resonant part:
\[
D_{q}^{\text{non}} / D_{q}^{\text{non-res}} \sim \tau_{c,k} k^2 v_{The}^2 / \nu_{ei} \gg 1
\]
Collisionless saturation by resonant diffusion of vorticity

- Zonal flow evolution $\leftrightarrow$ Mean enstrophy equation:

$$\frac{\partial}{\partial t} \int dr \frac{\langle \rho \rangle^2}{2} = \int dr \langle \tilde{\nu}_r \tilde{\rho} \rangle \frac{d\langle \rho \rangle}{dr} - v_i \int dr \langle \rho \rangle^2 + \cdots$$

Vorticity ($\rho \equiv \nabla^2 \phi$) flux:

$$\langle \tilde{\nu}_r \tilde{\rho} \rangle = -D_q^{res} \frac{d\langle \rho \rangle}{dr} + \Gamma^{Res}_{\rho}$$

Conserves enstrophy between mean flow and fluctuations

$$\frac{\partial}{\partial t} \int dr \frac{\langle \rho \rangle^2}{2} = - \int dr D_q^{res} \left( \frac{d\langle \rho \rangle}{dr} \right)^2 + \int dr \Gamma^{Res}_{\rho} \frac{d\langle \rho \rangle}{dr} - v_i \int dr \langle \rho \rangle^2 + \cdots$$

- $v_i \rightarrow 0 \rightarrow$ Dimits shift regime $\rightarrow$ Resonant diffusion saturates ZF

- Collisionless damping by turbulent viscosity: $d\langle \rho \rangle/dr \sim \Gamma^{Res}_{\rho} / D_q^{res}$
  - Resonant vorticity diffusivity $D_q^{res} \rightarrow$ ZF saturation
Mesoscopic stationary zonal flow

- Balance vorticity flux: $\langle \tilde{v}_x \tilde{\rho} \rangle = -D_{q}^{\text{res}} \frac{d\langle \rho \rangle}{dx} + \Gamma_{\rho}^{\text{Res}} = 0$

  $\rightarrow \langle v_y \rangle'' = d\langle \rho \rangle/dx \sim \Gamma_{\rho}^{\text{Res}} / D_{q}^{\text{res}}$

- Vorticity flux driven by $\nabla n$: $\Gamma_{\rho}^{\text{Res}} = (D_{n,\text{turb}} - D_{q}^{\text{res}}) \frac{\partial}{\partial x} \langle n \rangle$

- Resonant PV diffusivity:\n  
  $D_{q}^{\text{res}} = \sum_k \tau_{c,k} k_y^2 \rho_s^2 c_x^2 |\phi_k|^2$ with $\tau_{c,k} \sim [(v_{g,y} - v_{p,h,y}) \Delta k_y + v_{g,x} \Delta k_x]^{-1}$

- Stationary flow:\n  
  $\langle v_y \rangle'' = \langle \rho \rangle' = \left(1 - \frac{D_{n,\text{turb}}}{D_{q}^{\text{res}}} \right) \frac{\partial \langle n \rangle}{\partial x} \sim -\frac{c_s}{\rho_s L_n} \left(1 - \frac{1}{\tau_{c,k} D_{||} k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2} \right)$

  $\rightarrow$ Zonal flow scale: $L_{ZF} \sim \sqrt{\rho_s L_n} \rightarrow \rho_s \ll L_{ZF} \ll L_n$

\[ \text{This} \ \text{derives} \ \text{the} \ \text{standard} \ \text{ordering}, \ \text{which} \ \text{is} \ \text{just} \ \text{invoked}, \ \text{in} \ \text{ad} \ \text{hoc} \ \text{way.} \]

$L_m$: radial mode scale of drift wave eigenmode, regulated by resonance.
Extended Predator-Prey Model

- **Mean flow energy:**

\[
\frac{L_{ZF}^2}{2} \frac{dV^{''2}}{dt} = \alpha_1 |V^{''}| E - \alpha_2 V^{''2} E - \gamma_{NL} V^{''2} - \mu_c V^{''2}. 
\]

- **Turbulence energy (PE):**

\[
\frac{dE}{dt} = -\alpha_1 |V^{''}| E + \alpha_2 V^{''2} E - \varepsilon_c E^{3/2} + \gamma_L E.
\]

\(L_{ZF}:\) zonal flow profile scale, \(\rho_s \ll L_{ZF} \ll L_n\)

- Production by residual vorticity flux
- Resonant diffusion of vorticity
- Collisional Damping
- Nonlinear damping by tertiary modes
- Forward cascade of PE
- Linear instability
Turbulence and flow states

- Compare by regime:

<table>
<thead>
<tr>
<th>Regime</th>
<th>Collisionless</th>
<th>Weak Collisional</th>
<th>Strong Collisional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collisional Damping Strength</td>
<td>$\mu_c \ll \alpha_2 E$</td>
<td>$\alpha_2 E \ll \mu_c \ll 4\gamma_L \alpha_1^2/\varepsilon_c^2$</td>
<td>$\mu_c \gg 4\gamma_L \alpha_1^2/\varepsilon_c^2$</td>
</tr>
<tr>
<td>Flow State</td>
<td>$\alpha_1/\alpha_2$</td>
<td>$\alpha_1 \gamma_L^2/\mu_c \varepsilon_c^2$</td>
<td>$\gamma_L/\alpha_1$</td>
</tr>
<tr>
<td>Turbulence Energy</td>
<td>$\gamma_L^2/\varepsilon_c^2$</td>
<td>$\gamma_L^2/\varepsilon_c^2$</td>
<td>$\gamma_L \mu_c/\alpha_1^2$</td>
</tr>
</tbody>
</table>

- Collisionless = collisional damping/viscosity $\to 0$

- Collisionless saturation compared to usual collisional damping:
  - Turbulence energy determined by linear stability and small scale dissipation
    - Different from usual models, where turbulence energy $\sim$ flow damping
  - Flow state basically independent of stability drive
    - There can be flows in nearly marginal turbulence
Analogy to Landau Damping Absorption in Langmuir Turbulence

<table>
<thead>
<tr>
<th></th>
<th>Langmuir Turbulence Collapse</th>
<th>Collisionless ZF Saturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary player</td>
<td>Plasmon-Langmuir wave</td>
<td>Drift wave turbulence</td>
</tr>
<tr>
<td>Secondary player</td>
<td>Ion- acoustic wave (caviton)</td>
<td>Zonal flow</td>
</tr>
<tr>
<td>Free energy source</td>
<td>Langmuir turbulence driver</td>
<td>$\nabla n, \nabla T$ drives</td>
</tr>
<tr>
<td>Final State</td>
<td>(Nearly) empty cavity</td>
<td>Saturated zonal flow and residual turbulence</td>
</tr>
<tr>
<td>Resonance</td>
<td>Landau damping</td>
<td>Resonant wave absorption</td>
</tr>
<tr>
<td>Other damping effects</td>
<td>Ion-acoustic radiation</td>
<td>Kelvin-Helmholtz relaxation</td>
</tr>
</tbody>
</table>

- Landau damping: flattens PDF (negative slope) in phase space
- Resonant PV mixing: homogenizes mean PV in real space
Partial Summary 2

• Resonance effects on linear stability
  – Wave-flow resonance suppresses instability
  – $V'_\perp$ weakens resonance $\Rightarrow V'_\perp$ enhances instability via resonance

• Resonant diffusion of vorticity saturates zonal flow in collisionless regime
  – Resonant PV mixing $\Leftrightarrow$ resonant diffusion of PV
  – Model shows that stationary zonal flow scale is mesoscopic, i.e. $\rho_s \ll L_{ZF} \ll L_n$, since $L_{ZF} \sim \sqrt{\rho_s L_n}$
  – Extended predator-prey model
    $\Rightarrow$ turbulence energy $\sim \gamma_L^2 / \epsilon_C^2$ not $\sim \gamma_L$
  – Flow independent of turbulence level/drive
    $\Rightarrow$ flow in marginal turbulence