Competition of Mean Perpendicular and Parallel Flows in a Linear Device

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Background

• **Intrinsic** axial and azimuthal flows observed in linear device (CSDX)
• Increase B $\rightarrow$ scan mean flows-$V_\perp$ and $V_\parallel$
• **Dynamical** competition between mean perpendicular and parallel flows
• [See George Tynan’s talk earlier]

– **Dynamical**: $V_\perp$ and $V_\parallel$ exchange energy with the background turbulence, and each other.
  $\rightarrow$ Energy balance between $V_\perp$ and $V_\parallel$
  $\rightarrow$ **Tradeoff** between $V_\perp$ and $V_\parallel$
Key Questions and Why

• What’s the coupling between mean perpendicular and parallel flows ($V_{\perp}$ and $V_{\parallel}$)?
  – How do they interact?
    → How do they compete for energy from turbulence?
  – Can we have a reduced model of the coupling between $V_{\perp}$ and $V_{\parallel}$?

• Why should we care?
  – Linear device (CSDX) studies suggest apportionment of turbulence energy between $V_{\perp}$ and $V_{\parallel}$
  – Relevant to L-H transition
    • Both $V'_{\perp}$ and $V_{\parallel}$ increase, during transition.
    • The coupling of the two is relevant to transition threshold and dynamics.
Outline of the Rest

• Current status of model

• Exploration of $V_\perp$ and $V_\parallel$ competition
  – *Turbulent energy branching* between $V_\parallel$ and $V_\perp$
  – Reynolds power ratio $P_\parallel^R/P_\perp^R$ decreases as $V_\perp$ increases
    $\rightarrow$ tradeoff between $V_\perp$ and $V_\parallel$
  – $P_\parallel^R/P_\perp^R$ maximum occurs when $|\nabla V_\parallel|$ is below the PSFI (parallel shear flow instability) threshold $\rightarrow$ saturation of intrinsic $V_\parallel$

• Wave-flow resonance effects
  – Are shear suppression “rules” always correct?
  – $V'_\perp$ weakens resonance
    $\rightarrow$ *flow shear enhances instability*
  – Implication for zonal flow dynamics
Current status of model

- Conventional wisdom of $V_\perp \rightarrow V_\parallel$ coupling:
  - $V_\perp'$ breaks the symmetry in $k_\parallel$, but requires finite magnetic shear
  - *Not applicable* in linear device (straight magnetic field)

- $V_\parallel \rightarrow V_\perp$ coupling via parallel compression:
  - 3D coupled drift-ion acoustic wave system [Wang et al, PPCF 2012]
  - Coupling between fluctuating PV and parallel compression $\langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle$
    - *breaks PV conservation*
    - Sink/source for fluctuating potential enstrophy density
    - Zonal flow generation
Section II: Exploration of $V_\perp$-$V_\parallel$ Coupling

• Goal: study *how extrinsic flows affect Reynolds powers*
  
  $\rightarrow$ generation of intrinsic flows

  $\rightarrow$ *turbulent energy branching* between intrinsic $V_\perp$ and $V_\parallel$

• Analogous to increment study

• **Hasegawa-Wakatani** drift wave
  
  $\rightarrow$ near adiabatic electron:

  $\tilde{n} = (1 - i\delta)\phi$, $\delta \ll 1$

• Slab geometry

\[
\frac{D}{Dt} \tilde{n} + \tilde{v}_r \frac{\nabla n_0}{n_0} + \nabla_\parallel \tilde{v}_\parallel = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi}),
\]

\[
\frac{D}{Dt} \nabla^2_\perp \tilde{\phi} + \tilde{v}_r V''_\perp = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi}),
\]

\[
\frac{D}{Dt} \tilde{v}_\parallel + \tilde{v}_r V'_\parallel = \nabla_\parallel \tilde{n},
\]
Bottom Line: $\nabla n_0$ is the Primary Instability Drive

- Other potential drives:
  - $V_{\perp}'' \to$ Kelvin-Helmholtz instability
  - $\nabla V_{\parallel} \to$ Parallel shear flow instability

- KH is not important
  - $V_{\perp}''$ drive weaker than $\nabla n_0$ drive, i.e. $|k_y \rho_s^2 V_{\perp}''| \ll \omega_e$

- **PSFI stable** in CSDX
Definition: Reynolds Power

- Mean flow evolution is driven by Reynolds power
  \[ \frac{1}{2} \frac{\partial |V_\parallel|^2}{\partial t} \sim - \frac{\partial}{\partial x} \langle \tilde{v}_x \tilde{v}_\parallel \rangle V_\parallel \]
  - Parallel Reynolds power of a single eigenmode
    \[ P_\parallel^R = \int_0^{L_x} dx \left[ - \frac{\partial}{\partial x} (\tilde{v}_{x,k}^* \tilde{v}_{\parallel,k}) \right] V_\parallel \]
  - Perpendicular Reynolds power of a single eigenmode
    \[ P_\perp^R = \int_0^{L_x} dx \left[ - \frac{\partial}{\partial x} (\tilde{v}_{x,k}^* \tilde{v}_{\perp,k}) \right] V_\perp \]
- Effects of extrinsic $V_\parallel$ and $V_\perp$ on the ratio $P_\parallel^R / P_\perp^R$ are studied
Coupling of $V_\perp$ and $V_\parallel$ ↔ Ratio of Reynolds Powers

- Ratio $P_\parallel^R / P_\perp^R$ decreases with $V_\perp$
  → Energy branching of $V_\parallel$ reduced
  → $V_\perp$ reduces generation of $V_\parallel$
  → *Competition* between $V_\perp$ and $V_\parallel$

- Increase $V_\parallel$ → $P_\parallel^R / P_\perp^R$ turnover
  *before* $\nabla V_\parallel$ hits PSFI threshold
  → Max energy branching of $V_\parallel$ below PSFI threshold
  → $V_\parallel$ saturates *below* PSFI threshold

![Graph 1](image1.png)

![Graph 2](image2.png)
Section III: Revisiting Wave-Flow Resonance

[Li & Diamond, manuscript in preparation]

• **Are conventional shear suppression “rules” always correct?**
  – $E \times B$ flow shear suppresses instability $\leftrightarrow$ Is it correct with resonance?
  – Wave-flow resonance effect is often overlooked, though was mentioned in past works.

• **Findings:**
  – Wave-flow resonance stabilizes drift wave instability
  – Perpendicular flow shear weakens the resonance, and thus **destabilizes** the instability

• **Implications for zonal flow saturation:**
  – **Collisionless** zonal flow saturation (without involving tertiary instabilities, such as KH) set by resonance, $D_\rho \sim (\omega_k - k_y V_\perp)^{-2}$
Wave-flow resonance

- Resonance: $\omega_k - k_y V_\perp - k_\parallel V_\parallel$
  $|k_\parallel|/k_y \ll 1 \rightarrow$ Resonance dominated by $\omega_k - k_y V_\perp$

- Hasegawa-Wakatani drift wave model, with extrinsic $V_\perp$
  \[
  \frac{D}{Dt} \tilde{n} + \tilde{v}_r \frac{\nabla n_0}{n_0} = D_\parallel \nabla^2_\parallel (\tilde{n} - \tilde{\phi}),
  \]
  \[
  \frac{D}{Dt} \nabla^2_\perp \tilde{\phi} + \tilde{v}_r V_\perp'' = D_\parallel \nabla^2_\parallel (\tilde{n} - \tilde{\phi})
  \]

- **KH drive negligible** $\rightarrow$ Drift wave instability dominant
  - Near adiabatic electron: $\tilde{n} = (1 - i\delta)\phi$, $\delta \ll 1$
  - $\delta = (\omega* - \omega_k + k_y V_\perp)/k_\parallel^2 D_\parallel^2 = \nu_{ei}(\omega* - \omega_k + k_y V_\perp)/k_\parallel^2 v_{Te}^2$

- Resonance reduces the **eigenmode scale** $\rightarrow$ Suppresses instability
  (Width of eigenmode)
Perpendicular flow shear \textit{destabilizes} turbulence

- Mean perpendicular flow shear increases mode scale $L_m/\rho_s$
  \rightarrow Weakens resonance
  \rightarrow Enhances instability
- KH drive \textbf{negligible} compared to $\nabla n_0$
Implications for Zonal Flow Saturation

• Connection to collisionless saturation of ZF

• Zonal flow evolution $\Rightarrow$ Mean enstrophy equation:

$$\frac{\partial}{\partial t} \int dr \frac{\langle \rho \rangle^2}{2} = \int dr \langle \tilde{v}_r \tilde{\rho} \rangle \frac{d\langle \rho \rangle}{dr} - \nu_i \int dr \langle \rho \rangle^2 + \ldots$$

• Vorticity ($\rho \equiv \nabla^2 \phi$) flux: $\langle \tilde{v}_r \tilde{\rho} \rangle = -D_\rho \frac{d\langle \rho \rangle}{dr} + \Gamma^\text{Res}_\rho$ \hspace{1cm} Conserves enstrophy between mean flow and fluctuations

$$\frac{\partial}{\partial t} \int dr \frac{\langle \rho \rangle^2}{2} = - \int dr D_\rho \left( \frac{d\langle \rho \rangle}{dr} \right)^2 + \int dr \Gamma^\text{Res}_\rho \frac{d\langle \rho \rangle}{dr} - \nu_i \int dr \langle \rho \rangle^2 + \ldots$$

• $\nu_i \rightarrow 0 \Rightarrow$ Dimits shift regime $\Rightarrow$ Resonance saturates ZF, w/o KH

• Collisionless damping by turbulent viscosity: $d\langle \rho \rangle/dr \sim \Gamma^\text{Res}_\rho / D_\rho$

• Resonance sets $D_\rho \rightarrow$ ZF saturation

$$\Gamma^\text{Res}_\rho = \sum_k k_y c_s^2 |\phi_k|^2 \left[ \frac{\gamma_k \omega_e + \alpha_n (\omega_e - \omega_k + k_y V_\perp)}{|\omega_k - k_y V_\perp + i\alpha_n|^2} - \frac{|\gamma_k| \omega_e}{|\omega_k - k_y V_\perp|^2} \right], \quad D_\rho = \sum_k k_y^2 c_s^2 |\phi_k|^2 \frac{|\gamma_k|}{|\omega_k - k_y V_\perp|^2}$$
Summary

- CSDX experiments suggest energy apportionment between mean $V_\perp$ and $V_\parallel$
- Reynolds power ratio $P_\parallel^R / P_\perp^R$ changes in response to external flow increment
  - $P_\parallel^R / P_\perp^R$ decreases with $V_\perp$ → tradeoff between $V_\perp$ and $V_\parallel$
  - $P_\parallel^R / P_\perp^R$ maximum occurs before $\nabla V_\parallel$ hits PSFI threshold
- Testing misconceptions of shearing effects on stability
  - Wave-flow resonance suppresses instability
  - $V_\perp'$ weakens resonance $\rightarrow V_\perp'$ enhances instability
  - Resonance produces turbulent viscosity $\rightarrow$ collisionless saturation of ZF, without involving tertiary instabilities
Backup
Details on Acoustic Coupling

• $V_{||} \rightarrow V_{\perp}$ coupling via parallel compression:
  – 3D coupled drift-ion acoustic wave system [Wang et al, PPCF 2012]
  – Coupling between fluctuating PV and parallel compression $\langle \tilde{q} V_{||} \tilde{v}_{||} \rangle$
    breaks PV conservation
    $\rightarrow$ Sink/source for fluctuating potential enstrophy density
    $\rightarrow$ Zonal flow generation
  – Perpendicular flow dynamics:

\[
\frac{\partial}{\partial t} \left[ V_{\perp} - L_n \left( \frac{\tilde{q}^2}{2} \right) \right] \sim -v_i V_{\perp} + L_n \left[ \frac{\partial}{\partial r} \left( \tilde{v}_x \frac{\tilde{q}^2}{2} \right) \right] + \mu \langle (\nabla \tilde{q})^2 \rangle - \langle \tilde{q} V_{||} \tilde{v}_{||} \rangle
\]

PV diffusion

collisional damping

$\langle \tilde{q} V_{||} \tilde{v}_{||} \rangle \sim -\sum_k \frac{|\Delta \omega_k|}{\omega_k^2} k_{||}^2 |\phi_k|^2 < 0$
Stationary Zonal Flow Profile

- Turbulent viscosity set by resonance:

\[
D_\rho = \sum_k k_y^2 c_s^2 |\phi_k|^2 \frac{|\gamma_k|}{|\omega_k - k_y V_\perp|^2} \sim \sum_k \frac{k_y^2 \rho_s^2 c_s^2}{k_\parallel D_\parallel} \frac{k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2}{1 + k_y^2 \rho_s^2 \rho_s^2} |\phi_k|^2
\]

- Residual vorticity flux:

\[
\Gamma_\rho^{Res} = \sum_k k_y c_s^2 |\phi_k|^2 \left[ \frac{\gamma_k \omega_s + \alpha_n (\omega_{s} - \omega_k + k_y V_\perp)}{|\omega_k - k_y V_\perp + i \alpha_n|^2} \right] - \frac{|\gamma_k| \omega_{s}}{|\omega_k - k_y V_\perp|^2},
\]

- Reynolds force (i.e. net production) = 0

→ Stationary flow profile:

\[
\langle v_y \rangle'' = \langle \rho \rangle' = \frac{\Gamma_\rho^{Res}}{D_\rho} \sim -\frac{k_y^2 \rho_s^2 c_s^2}{(k_\parallel D_\parallel)^2} \frac{1}{L_n^3} \frac{1}{(1 + k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2)^2}.
\]
Resonance and Instability Related to Mode Scale

• Eigenmode equation with resonant effect:

\[ (\omega_k - k_y V_\perp + i \gamma_k) \rho_s^2 \phi_x^2 \phi = \left[ (1 + k_y^2 \rho_s^2 - i \delta)(\omega_k - k_y V_\perp + i \gamma_k) - \omega_e \right] \phi \]

• Mode scale:

\[ L_m^{-2} \rho_s^2 \equiv \rho_s^2 \int_0^{L_x} dx |\phi_x|^2 / \int_0^{L_x} dx |\phi|^2 \]

• Results:

\[ \left| \omega_k - k_y \langle v_y \rangle \right|_{\text{min}} \approx \frac{\omega_e}{1 + k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2} \]

\[ \gamma_k \approx \frac{\omega_e^2}{k_y^2 D_\parallel} \frac{k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2}{(1 + k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2)^3} \]

• Strong resonance:

\[ \gamma_k \ll \omega_k - k_y V_\perp \ll \omega_e \]

• Eigenmode peaks \((L_m^{-2} \rho_s^2)\) increases as resonance becomes stronger

• Resonance suppresses drift wave instability
### Analogy to Landau Damping Absorption

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Revisit predator-prey model

- Resonance induces collisionless saturation through $D_\rho$, apart from KH:
  \[
  \gamma_k = \text{linear instability } (\gamma_L) + \text{resonance absorption } \gamma_R \sim \gamma_R (\omega_k - k_y V_\perp)
  \]

  Analogy to ion-acoustic absorption during collapse of Langmuir waves

- Revisit predator-prey model with resonance effect
  → Mechanism for collisionless damping, without KH