On the Transport Physics of the Density Limit

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Collaborations:

- Theory
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 - Zhibin Guo (UCSD→PKU)
- Experiment
 - Rongjie Hong, G. Tynan, HL-2A Team (UCSD and SWIP)

Discussion: Martin Greenwald

Outline

- Basics of Density Limit → Mostly L-mode
 - General Trends
 - Some Indications of Transport as Fundamental
 - Modelling The Conventional Wisdom
- Recent Studies \rightarrow HL-2A (L-mode)
 - Edge Shear Layer Evolution as $\overline{n} \rightarrow \overline{n}_g$
 - Shear Layer $\leftarrow \rightarrow$ Electron Adiabaticity Connection
 - Synthesis
 - Confronting the Conventional Wisdom

A Theory of Shear Layer Collapse

- Thesis: For hydrodynamic electrons, drift wave turbulence cannot regulate itself via self-generated shear flows. Turbulence levels rise.
- A Simple Argument
- Collisional drift wave-zonal flow turbulence for $k_H^2 v_{T_e}^2 / \omega \gamma_e \ge 1$
- Scaling Comparison
- What of PV Mixing?
- Scenario for edge cooling

Implications and Directions

Some Thoughts on Density Limit in H-mode

Conclusion

Basics of Density Limits

Density Limits

- Not a review! Incomplete!
- Greenwald density limit:



Tokamak Operating Space

- Manifested on other devices (more later)
 - See especially <u>RFP</u>

- Global limit
- Simple dependence
- Begs origin of I_p scaling?!
- Most fueling via edge → edge
 transport critical to n
 imits



• Trends well established



- Often (but not always!) linked to:
 - MARFE (radiative condensation instability) $\leftarrow \rightarrow$ Impurity influx
 - MHD disruption
 - Divertor detachment
 - H→L Back-transition

- Argue:
 - 'Disruptive' scenarios <u>secondary</u> outcome, largely consequence of <u>edge</u>
 <u>cooling</u>, due fueling
 - \bar{n}_g reflects fundamental limit imposed by particle transport
- Some Evidence



- Density decays non-disruptively after pellet injection
- $\bar{n} \sim I_p$ asymptote
- Density limit enforced non-disruptively!

• More Evidence:



- Post pellet density decay rises with \bar{J}/\bar{n}

– Limit at:
$$\bar{J}/\bar{n} \sim 1$$

- Pellet in DIII-D beat \bar{n}_g
- Peaked profiles ← → enhanced core
 particle confinement ~ ITG turbulence
- Reduced particle transport → impurity accumulation

Looking at the Edge

• Edge Fueling ←→ edge transport crucial to density limit



- C-Mod SOL profiles
- As n ↑, high ⊥ transport region extends <u>inward</u>



- Scan of edge/SOL profiles, $\bar{n} \rightarrow \bar{n}_G$
- Large fluctuation activity develops in main plasma, inward from SOL, for $\bar{n} \rightarrow \bar{n}_G$

Tentative Conclusions

- Turbulence intensities
- ⊥ particle transport increases
- Pellet injection admits $\bar{n} > \bar{n}_g$, with non-disruptive

At edge, as $\overline{n} \rightarrow \overline{n}_{a}$

relaxation, as edge cooling avoided

Key Question:

 \rightarrow What physics is under-pinning of rise in

turbulence, transport as $\overline{n} \rightarrow \overline{n}_g$?

Conventional Wisdom (Rogers + Drake '98)

Reduced Fluid Simulation (no heat source)



- $\alpha_{MHD} = -Rq^2 d\beta/dr$
- $\leftrightarrow \nabla P \rightarrow$ ballooning drive

$$\alpha_d = \rho_S C_s t_0 / L_n L_0$$

$$t_0 = \frac{(RL_n 2)^{\frac{1}{2}}}{C_S}$$

- D+R on n-limit physics:
 - DWT → resistive ballooning turbulence
 - State of high $\nabla P, \beta$, cool electrons
 - Check: $\gamma > \omega_S, \omega_*$?

- $L_0 = 2\pi q \left(\frac{\gamma_e R \rho_s}{2\Omega_e}\right)^{1/2}$
- → Hybrid of drift frequency and adiabaticity

shear

So, Conventional Wisdom ->

- In density limit conditions, another linear instability resistive ballooning – emerges and dominates
- Transition mechanism/physics not addressed
- Is there more to this than convention?

Recent Studies on HL-2A

(Ronjie Hong, Tynan, P.D., HL-2A Team/NF2018)

➔ New twist: Edge Fluctuation Studies! (L-mode)

- Edge Langmuir probe array
- Curiously absent from \overline{n} limit literature

Basic Results

- OH, $I_p \sim 150 kA$, $B_T = 1.3T$, $q = 3.5 \rightarrow 4$
- $\bar{n} = 0.25 \rightarrow 0.9 \ \bar{n}_g$
- Profiles



• Fluctuation Properties



 $P_{Re} = -\langle V_{\theta} \rangle \partial_r \langle \tilde{V}_r \tilde{V}_{\theta} \rangle \rightarrow \text{energy gained by low-f flow}$ DROPS as $\bar{n} \rightarrow \bar{n}_g$

Further Studies of Stress and Flows



• Flow shearing rate <u>drops</u> as collisionality increases

 Reynolds power (to flow) drops as collisionality increases

cf: Schmid, et. al. 2017

Further Studies



- Joint pdf of \tilde{V}_r , \tilde{V}_{θ} for 3 densities
- $r r_{sep} = -1cm$
- Note:
 - Tilt lost, symmetry restored as $\bar{n} \rightarrow \bar{n}_g$
 - Consistent with drop in P_{Re}

Weakened production by Reynolds stress

Transport



- Γ_n rises as $\bar{n} \to \bar{n}_g$
- Density fluctuations rise dramatically.

The Key Parameter

- Electron adiabaticity emerges as the telling local parameter $\Rightarrow k_{\parallel}^2 V_{the}^2 / \omega \gamma$
- Drops from ~ 3 \rightarrow 0.5 during \bar{n} scan



• Reynolds work <u>plummets</u> as

 $k_{\parallel}^2 V_{the}^2 / \omega \gamma \ll 1$

- $P_{Re} \downarrow$ as shear layer weakens
- Turbulent particle flux rises as $P_{Re} \downarrow$

The Feedback Loop (per experimentalists)



- $k_{\parallel}^2 V_{the}^2 / \omega \gamma > 1$ to < 1
 - Weakens ZF (how?)
 - N.B. beyond damping?
 - Enhances turbulence
- Increased turbulent transport cools

edge



The Key Question

What is fate of ZF for hydrodynamic electrons

 $(k_{\parallel}^2 V_{the}^2 / \omega \gamma < 1)$? Underlying Physics?

• How reconcile with our understanding of drift wavezonal flow physics?

A Theory of Shear Layer Collapse

- (R. Hajjar, P.D., Malkov)
- <u>Thesis</u>: For hydrodynamic electrons, ZF <u>production</u> by drift wave turbulence drops
 - DWT cannot regulate itself by zonal flow shears
 - Turbulence, transport rise

N.B.

. . .

- Many simulation studies note weakening or outright disappearance of ZF in hydro. Regime
 - Numata, et. al. '07
 - Gamargo, et. al. '95
 - Ghantous & Gurcan, '15

 However, mechanism left un-addressed, as adiabatic electron regime of primary interest

Model: { Collisional Drift Wave Hasegawa-Wakatani

$$\frac{dn}{dt} = -\frac{v_{th}^2}{\nu_{ei}} \nabla_{\parallel}^2 (\phi - n) + D_0 \nabla^2 n$$

$$\frac{d\nabla^2 \phi}{dt} = -\frac{v_{th}^2}{\nu_{ei}} \nabla^2_{\parallel}(\phi - n) + \mu_0 \nabla^2(\nabla^2 \phi)$$

\rightarrow Simplest viable for edge

$$\alpha = \frac{k_{\parallel}^2 V_{th}^2}{\omega \gamma} \rightarrow \text{coupling parameter}$$

 \rightarrow Adiabaticity parameter

Fluctuations

$$\partial_t \tilde{n} + \tilde{v}_x \cdot \nabla \bar{n} = -\frac{v_{th}^2}{\nu_{ei}} \nabla^2_{\parallel} (\tilde{\phi} - \tilde{n}) - \{\tilde{\phi}, \tilde{n}\} + D_0 \nabla^2 \tilde{n}$$
$$\partial_t \nabla^2 \tilde{\phi} + \tilde{v}_x \cdot \nabla \overline{\nabla^2 \phi} = -\frac{v_{th}^2}{\nu_{ei}} \nabla^2_{\parallel} (\tilde{\phi} - \tilde{n}) - \{\tilde{\phi}, \nabla^2 \tilde{\phi}\} + \mu_0 \nabla^2 (\nabla^2 \tilde{\phi})$$

• Mean Fields:

$$\partial_t \bar{n} = -\partial_x \langle \tilde{V}_x \tilde{n} \rangle + D_0 \overline{\nabla}_x^2 \bar{n}$$
$$\partial_t \overline{\nabla}_x^2 \phi = -\partial_x \langle \tilde{V}_x \nabla^2 \phi \rangle + \mu_0 \nabla_x^2 \overline{\nabla}_x^2 \phi$$

A <u>Simple</u> Argument: <u>Wave Propagation</u> (Quasilinear)

• Fundamental dispersion character charges between $\alpha > 1$ and

 $\alpha < 1$, i.e.

• $\alpha > 1 \rightarrow$ traditional 'drift wave' scaling

$$\omega = \frac{\omega_*}{1 + k_\perp^2 \rho_s^2} + i \frac{\omega_{*e} k_\perp^2 \rho_s^2}{\alpha}, \qquad \alpha > 1$$

$$\bigwedge \text{ wave + inverse dissipation}$$

• $\alpha < 1 \rightarrow$ hydrodynamic 'convective cell' scaling

•
$$\omega = \left(\frac{|\omega_*|\widehat{\alpha}|}{2k_\perp^2 \rho_s^2}\right)^{1/2} (1+i), \qquad \widehat{\alpha} = \frac{k_\parallel^2 V_{th}^2}{\gamma}$$

Cell

Ubiquity of Zonal Flow?

- 'Standard argument': $ZF \rightarrow$ made of minimal
- My favorite: (GFD)
- "... the central result that a rapidly rotating flow, when stirred in a localized region, will converge angular momentum into the region" (Isaac Held, '01)

 $\begin{cases} Inertia \\ Damping \\ transport \\ transport \end{cases} (DI²H)$



Why?

• Direct proportionality of wave group velocity to Reynolds stress $\leftarrow \rightarrow$ spectral correlation $\langle k_x k_y \rangle$



Outgoing waves generate a flow convergence! → Shear layer spin-up

But for hydro limit:

•
$$\omega_r = \left[\frac{|\omega_{*e}|\hat{\alpha}|}{2k_\perp^2 \rho_s^2}\right]^{1/2}$$

•
$$V_{gr} = -\frac{2k_r \rho_s^2}{k_\perp^2 \rho_s^2} \omega_r \quad \overleftarrow{\leftarrow} \rightarrow \quad \langle \tilde{V}_r \tilde{V}_\theta \rangle = -\langle k_r k_\theta \rangle$$

- → Link between energy, momentum flux link <u>weakened</u> \downarrow
- → Eddy tilting ($\langle k_r k_\theta \rangle$) does not arise as consequence of causality
- → ZF generation <u>not</u> 'natural' outcome in hydro regime!

N.B. Issue is somewhat non-trivial in that:

- Symmetry breaking $\leftrightarrow \forall \nabla n$
- Mode coupling
- PV mixing
- \rightarrow All persist in hydrodynamic regime
- → Need look in depth

Reduced Model

- Utilize models for real space structure to address shear layer
 - e.g. { Balmforth, et. al. Ashourvan, P.D. → Outgrowth of staircase studies

See also: J. Li, P.D. '2018 (PoP)

- Exploit PV conservation:
 - $-q = \ln n \nabla^2 \phi \rightarrow \text{conserved PV}$

 $- \ \tilde{q} = \tilde{n} - \nabla^2 \tilde{\phi}$

So

• Natural description: $\langle n \rangle$, $\langle \nabla^2 \phi \rangle$, $\langle \tilde{q}^2 \rangle = \varepsilon$ $\varepsilon =$ fluctuation P.E.

Reduced Model, cont'd

$$\partial_t \varepsilon + \partial_x \Gamma_{\varepsilon} = -(\Gamma_n - \Pi)(\partial_x n - \partial_x u) - \varepsilon^{\frac{3}{2}} + P$$

Fluxes:

 $\Gamma_n \rightarrow \text{Partial flux } \langle \tilde{V}_x \tilde{n} \rangle$

<u>The Fluxes</u> – Physics Content

- Proceed by QLT
- $\Pi = -\chi_y \, \partial_x u + \Pi_{resid}$

- Diagonal, Residual $\leftarrow \rightarrow \nabla n$, via $\hat{\alpha}$ Shear relaxation <u>Production</u> \rightarrow key measure (K-H ignored)
- $\Gamma_n = -D_n \nabla n$
- Primary focus on scalings with $\underline{\alpha}$
- i.e. what changes as $\alpha > 1 \rightarrow \alpha < 1$

Basic Results

• Adiabatic ($\hat{\alpha} \gg |\omega|$)

$$n_{0}\Gamma_{n} = -\frac{\langle \delta v_{x}^{2} \rangle}{\hat{\alpha}} \frac{d\bar{n}}{dx} \simeq -\frac{\varepsilon l_{mix}^{2}}{\hat{\alpha}} \frac{d\bar{n}}{dx}$$

$$\Pi = -\frac{|\gamma_{m}| \langle \delta v_{x}^{2} \rangle}{|\omega|^{2}} \frac{d^{2}\bar{v}_{y}}{dx^{2}} - \frac{\omega_{ci} \langle \delta v_{x}^{2} \rangle}{\hat{\alpha}} \frac{d\bar{n}}{dx} \left(\frac{k_{\perp}^{2} \rho_{s}^{2}}{1 + k_{\perp}^{2} \rho_{s}^{2}}\right)$$

$$\simeq -\frac{\varepsilon l_{mix}^{2}}{\hat{\alpha}} \frac{d^{2}\bar{v}_{y}}{dx^{2}} - \frac{\omega_{ci} \varepsilon l_{mix}^{2}}{\hat{\alpha}} \frac{d\bar{n}}{dx}$$

• Reduction:

$$\Gamma_n \simeq -(\varepsilon l_{mix}^2/\hat{\alpha})\nabla \bar{n}$$
$$\chi_y \simeq \varepsilon l_{mix}^2/\hat{\alpha}$$
$$\Pi^{res} \simeq -(\omega_{ci}\varepsilon l_{mix}^2/\hat{\alpha})\nabla \bar{n}$$

<u>Results,</u> cont'd

• Hydrodynamic ($\hat{\alpha} \ll |\omega|$)

$$\begin{split} n_0 \Gamma_n &\simeq -\sqrt{\frac{k_\perp^2 \rho_s^2}{2k_\theta \rho_s c_s}} \sqrt{\frac{|d\bar{n}/dx|}{\hat{\alpha}}} \langle \delta v_x^2 \rangle \simeq -\frac{\varepsilon l_{mix}^2}{\sqrt{\hat{\alpha}|\omega^\star|}} \frac{d\bar{n}}{dx} \\ \Pi &= -\frac{|\gamma_m| \langle \delta v_x^2 \rangle}{|\omega|^2} \frac{d^2 \bar{v}_y}{dx^2} - \frac{\omega_{ci} \langle \delta v_x^2 \rangle}{k_\theta \rho_s c_s} \cdot \sqrt{\frac{k_\perp^2 \rho_s^2}{2}} \sqrt{\frac{\hat{\alpha}}{|\omega^\star|}} \\ &\simeq -\frac{\varepsilon l_{mix}^2}{\sqrt{\hat{\alpha}|\omega^\star|}} \frac{d^2 \bar{v}_y}{dx^2} - \frac{\omega_{ci} \varepsilon \sqrt{\hat{\alpha}} l_{mix}^2}{|\omega^\star|^{3/2}} \frac{d\bar{n}}{dx} \end{split}$$

• Reduction:

$$\Gamma_n \simeq -(\varepsilon l_{mix}^2 / \sqrt{\hat{\alpha} |\omega^*|}) \nabla \bar{n}$$
$$\chi_y \simeq \varepsilon l_{mix}^2 / \sqrt{\hat{\alpha} |\nabla \bar{n}|}$$
$$\Pi^{res} \simeq -(\omega_{ci} \varepsilon \sqrt{\hat{\alpha} l_{mix}^2} / |\omega^*|^{3/2}) \nabla \bar{n}$$

Shear Strength!?

• <u>Vorticity gradient</u> emerges as natural measure

of production vs. turbulent mixing



• Stationary vorticity flux:

$$\nabla u = \prod_{resid} / \chi_y$$

n.b.:
$$u' = (V'_y)'$$

 ∇u as FOM

• How characterize layer?

Shear Strength, cont'd



- Jump in flow shear over scale D equivalent to vorticity gradient on that scale
- Vorticity gradient characteristic of flow shear layer strength
- N.B. ∇u central measure of Rossby wave elasticity!

$$l_{Rh} \sim \left(\tilde{V} / \nabla u \right)^{1/2}$$

<u>Tabulation</u>: α scaling - <u>answer</u>

Plasma Response	$\begin{array}{c} \mathbf{Adiabatic} \\ \alpha \gg 1 \end{array}$	$\begin{array}{l} \mathbf{Hydrodynamic}\\ \alpha \ll 1 \end{array}$
Turbulent enstrophy $\sqrt{\varepsilon}$	$\sqrt{\varepsilon} \propto 1/\alpha$	$\sqrt{\varepsilon} \propto 1/\sqrt{\alpha}$
Particle Flux Γ	eq.(20a) $\Gamma \propto 1/\alpha$	eq.(24a) $\Gamma \propto 1/\sqrt{\alpha}$
Turbulent Viscosity χ_y	eq.(20b) $\chi_y \propto 1/\alpha$	eq.(24b) $\chi_y \propto 1/\sqrt{\alpha}$
Residual Stress Π^{res}	eq.(20c) $\Pi^{res} \propto -1/\alpha$	eq.(24c) $\Pi^{res} \propto -\sqrt{\alpha}$
$\frac{\Pi^{res}}{\chi_y} = (\omega_{ci} \nabla \bar{n}) \times$	$\left(\frac{\alpha}{ \omega^{\star} }\right)^{0}$	$\left(\frac{\alpha}{ \omega^{\star} }\right)$

- Note:
- $\alpha > 1, \nabla u \sim \alpha(0)$

$$\alpha < 1$$
, $\nabla u \sim \alpha$

- i.e. $\begin{cases} \chi_y \text{ rises} \\ \Pi_{resid} \text{ drops with } \alpha \end{cases}$
- Fluctuation Intensity rises
- Particle flux rises

Bottom Line

- Shear Layer, via production, collapses as $\alpha \downarrow < 1$
- Transport and fluctuations rise, as $\alpha \downarrow < 1$
- Edge $\alpha = k_{\parallel}^2 V_{the}^2 / \omega \gamma$ is <u>key local parameter</u>

What of 'PV Mixing' ?

- PV mixing persists in hydro regime
- Key: Unlike GFD/Adiabatic Regime,

PV mixed via several channels

• The Cartoons:



<u>PV, cont'd</u>

• H-W:

$$q = \ln n - \nabla^2 \phi$$

$$= \ln(n_0(x)) + \frac{|e|\hat{\phi}|}{T} + \tilde{h} - \rho_s^2 \nabla^2 \left(\frac{|e|\hat{\phi}|}{T}\right)$$

N.B. Boltzmann response does not contribute to net PV mixing

PV mixing

$$\Gamma_{q} = \langle \tilde{V}_{x}\tilde{h} \rangle - \rho_{s}^{2} \langle \tilde{V}_{x} \nabla_{x}^{2} \left(\frac{|e|\hat{\phi}}{T}\right) \rangle$$
Branching
ratio?!
PV flux Particle flux Vorticity flux,

Reynolds force

<u>PV, cont'd</u>

$$\Gamma_q = \langle \tilde{V}_{\chi} \tilde{h} \rangle - \rho_s^2 \ \langle \tilde{V}_{\chi} \ \nabla_{\chi}^2 \left(\frac{|e|\hat{\phi}}{T} \right) \rangle$$

- *α* > 1
 - Fields tightly coupled, $\sim \alpha$
 - Γ_n , Π_{resid} ~ $1/\alpha$
 - Both channels transport PV
 - ZF robust
- *α* < 1
 - Fields weakly coupled

$$\blacksquare$$
 – $\Gamma_n \sim 1/\sqrt{\alpha}$, $\Pi_{Resid} \sim \sqrt{\alpha}$

- <u>PV</u> transported via <u>particle flux</u>
- ZF dies

Edge Cooling Scenario



Inward turbulent spreading
 can increase resistivity and
 steepen *VJ*, resulting in MHD

N.B. For CDW, $Q \sim \Gamma_n$

Implications and Directions

Implications

- Density limit a 'back-transition' phenomenon
 - i.e. drift-ZF state \rightarrow convective cell, strong fluctuation turbulence
 - → scaling of collapse? (spatio-temporal)
 - → bifurcation? Trigger?, hysteresis?!
 - → control parameter $\leftarrow \rightarrow \alpha$ —
- Cooling front as secondary
 - → Extent penetration of turbulence spreading?
 - \rightarrow Strength, depth penetration \rightarrow operating regime

Directions

Experiment

- Test α criticality $\rightarrow \alpha \sim T_e^{\frac{5}{2}}/n$. Achieve $\bar{n}/\bar{n}_g > 1$ with $\alpha > 1$?
- $T \text{ vs } n \text{ trade-off at } \overline{n}_g$? Sustain $\overline{n} > \overline{n}_g$?!
- Hysteresis in *n* manifested? Space-time evolution of turbulence
- Localized edge shear layer response to SMBI, small pellets? Relaxation rate, persistence
- Established α vs $\overline{n}/\overline{n}_g$ connection
- Explore changes in bi-spectra <ZF|DW,DW> vs \bar{n}/\bar{n}_g (after Schmid, et. al.)
- Core-edge coupling?

Directions, cont'd

Theory / Model

- As usual, more 'stuff' in model...
- N.B. In HL-2A, $\alpha_{MHD} \uparrow 0.1 \rightarrow 0.3$

 $\alpha \downarrow 3 \rightarrow 0.5$

Onset of RBM dubious

- In particular:
 - Neutral penetration i.e. fueling source
 - \rightarrow CX damping of flows
 - Impurity \rightarrow build-up
 - $Q_{e,core}$ explicit

 $L \rightarrow H$ model of Miki et.al.

may be useful

Dynamical Modelling

- Feedback loop
- Macroscopics vs α
- Layer scale, expansion
- Heating vs fueling trade-off

•
$$\bar{n} / \bar{n}_g \leftrightarrow \alpha$$
 ?

Density Limit in H-mode

- SOL strongly turbulent; pedestal quiescent
- Shear layer at separatrix
- Turbulence penetration of pedestal (H→L
 BACK Transition) → needed for n

 Imit
 phenomena
- SOL turbulence set by:
 - Q
 - Fueling
 - Divertor conditions



n.b. SOL curvature unfavorable

Treat via **Box Model**

- Q_{\perp}, Q_{\parallel} regulate I_{SOL}
- Sufficient $I_{SOL} \rightarrow \text{ETB}$ penetration
- What are fueling, n_{SOL}, Q to trigger turbulence in flux and pedestal collapse. Barrier penetration is critical?
- Recent: H-mode density limit set by SOL ballooning?! (SOL P limit)





(ZBG, PD 2018)

Conclusions

- Density limit is consequence of particle transport processes
- L-mode density limit experiments:
 - Edge, turbulence-driven shear layer collapse
 - Local parameter $\alpha = k_{\parallel}^2 V_{th}^2 / \omega \gamma$
- Theory indicates:
 - Zonal flow production drops with α , $\alpha < 1$
 - Edge transport, turbulence ↑
 - → Self-regulation fails
- \bar{n} -limit in L-mode as transition from drift-zonal turb. \rightarrow strong drift turbulence