Physics of Scale Selection for Mesoscale Patterns in Drift Wave Turbulence

Special Focus: Physics of ExB Staircase（楼梯）

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Outline

- **Basics** of nonlinear patterns
  - **Pattern formation** **(scale, structure ?)** in magnetic confinement plasma↔**Scale selection**

    Especially, zonal flow → staircase
  - Formation mechanism and **feedback loops**

    → Shearing and Rhines mechanisms, transport bifurcation, ……
- **Recent model study** of zonal pattern formation / sustainment
  - ✓ Parameter scans
  - ✓ Feedback loop studies
  - ✓ Turbulence spreading
  - ✓ Initial condition sensitivity
  - ✓ Boundary condition sensitivity…….
- **Conclusions and plan**

Scales Determine Transport

- Several spatial scales enter transport
  - ✓ Drift wave, micro-scale ρ
  - ✓ Mean flow, macro-scale $L_n \sim a$
  - ✓ Zonal flow, meso-scale ($\sqrt{\rho L_n}$?)

• ρ: Larmor radius
• a: minor radius
• $L_n = -n/\nabla n$: density scale length

- Goal→**Predicting turbulence and transport in saturated states**

$$D = D_{\text{Bohm}} \left(\frac{\rho}{a} \right)^{\alpha}, 0 < \alpha < 1. \quad \begin{cases} \alpha = 0, \ Bohm\ scaling\ (Bad) \\ \alpha = 1, \ Gyro-Bohm\ scaling\ (Good) \end{cases}$$

→Turbulent diffusivity D scaling with ρ/a is a key question in fusion!

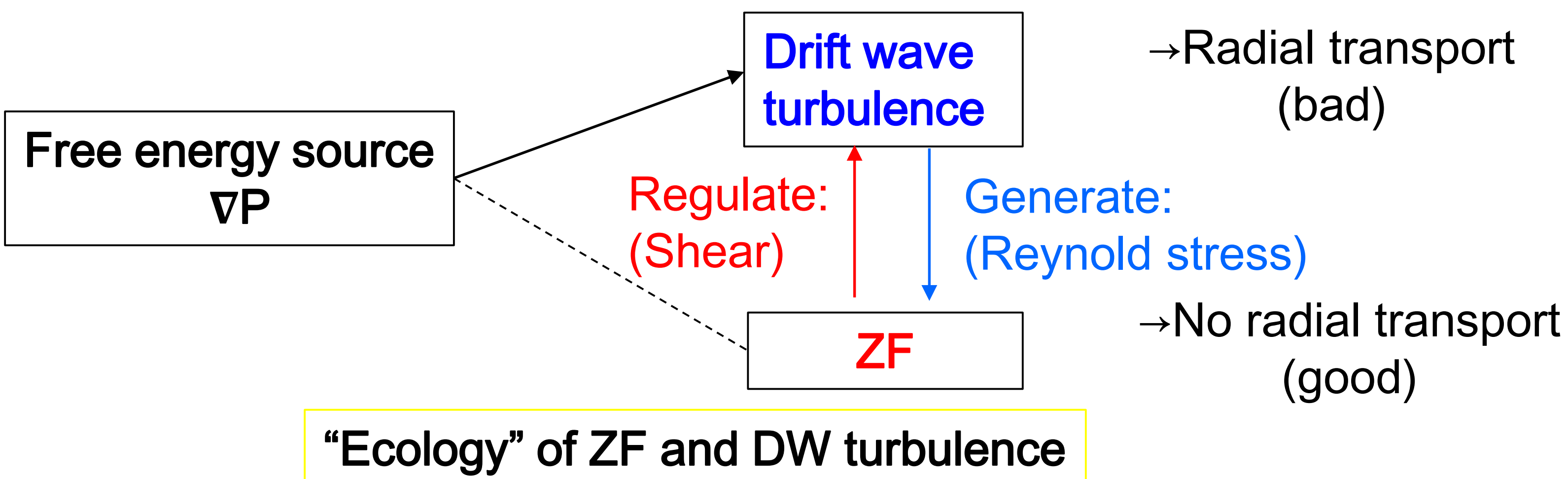
→Value of α is directly linked to **meso-scale selection**

- The ecology of **feedback loops** must enter the scaling of **spatial structure**

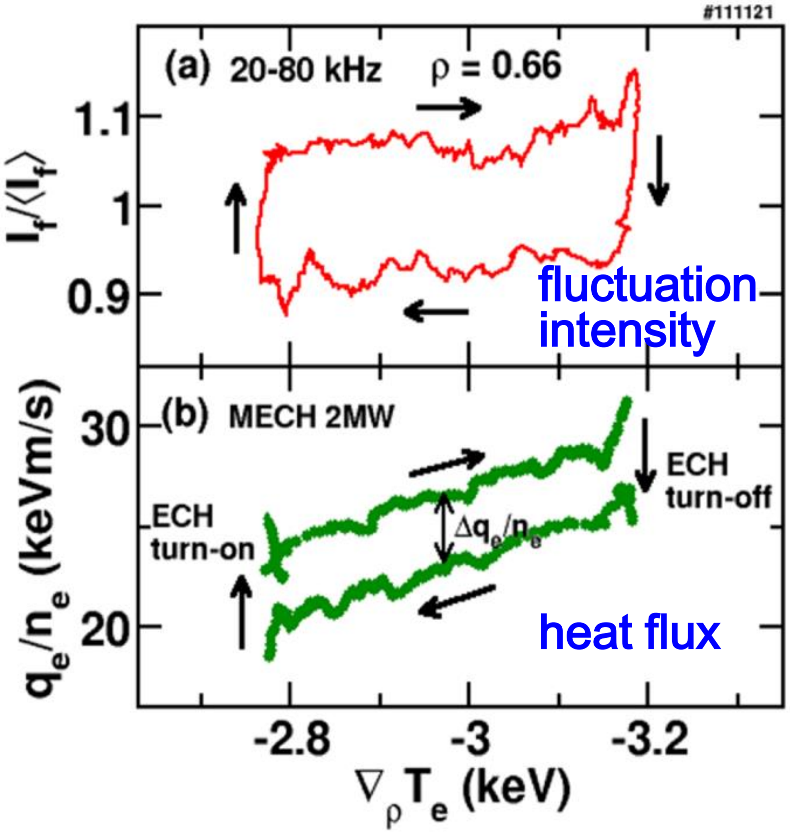
$$|\delta\varphi| \sim l_{mix} \sim \frac{1}{1+\sigma(v'_{E\times B})^2}$$

Physics of Scale Selection

- **Formation of patterns** in self-organizing, non-equilibrium and nonlinear systems is widely observed.
  - ✓ Layered Stratification
  - ✓ Potential Vorticity (PV) and ExB Staircase
  - ✓ Transport Barrier
  - ✓ Zonal Flow (ZF) …….
- ZF:
  →minimal inertia and Landau damping ➡ benign repository for free energy
  →regulate turbulent transport, trigger L-H and ITBs
- Closing the feedback loop when predators meet the prey



Important Element: GLOBAL Bistability



[S. Inagaki, NF 2013]

LHD, **L-mode**

- Experimental observation of **hysteresis** reflected by the non-linear ∇T_e dependence of fluctuation intensity (a) and heat flux (b).

  →self-organized **global hysteresis**

- Hysteresis strongly suggests **bistability** in the fluctuation intensity
- Here, not due to local transport barrier

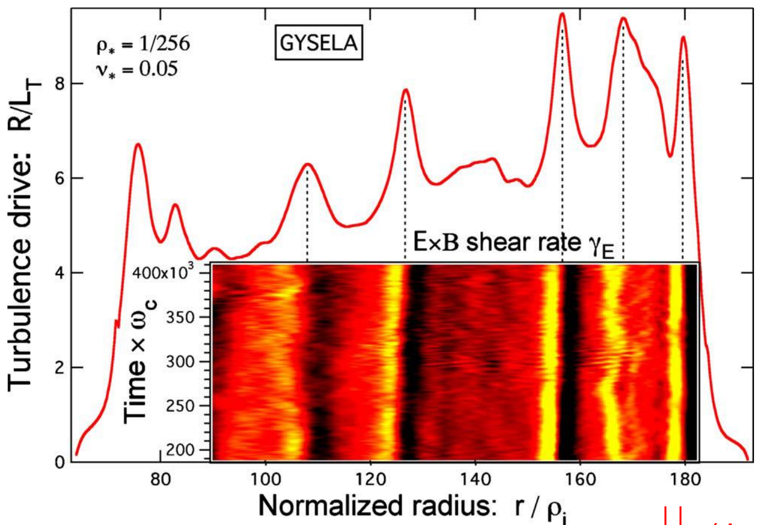
A Hint: ExB Staircase

- ✓ In ion temperature gradient (ITG) turbulence, quasi-regular (spatial) and long-lived (temporal) E×B flows coexisting with temperature corrugations are observed numerically + experimentally

→ Meso-scale E×B staircase (楼梯), by analogy to PV ($n - u$) staircases and atmospheric jets

(density → n, vorticity → u)

Dritschel, McIntyre, Journal of the Atmospheric Sciences, 2008



(Steps) (Jumps)

[G. Dif-pradalier, *et al*, Physical Rev. E **82**, 025401 (2010)]
[G. Dif-pradalier, *et al*, , Phys. Rev. Lett. **114**, 085004 (2015).]

"A natural and dynamic means for the simultaneous existence of these **two antagonistic trends**"

- ✓ Clear scale selection
- ✓ Clear link between ZF scale ↔ avalanche scale

$\delta^{\text{flow}} \sim 10\, \rho_s$ (Jumps)

$\Delta^{\text{stat}} \sim 40\, \rho_s$ (Steps)

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Simulation ↔ Reduced Models

- Works: simulations in ITG turbulence (GYSELA, GKNET)

G. Hornung et al NF 2017
G. Dif-Pradalier et al NF 2017
W. Wang, Y. Kishimoto et al NF 2018

Leave many questions:

→Conditions for existence of the E×B staircase?

→Explore dimensionless parameter dependence?

→Vary boundary condition (B. C.) and initial condition?

→Robustness in parameter space?

→Impact on transport and confinement?

→Relation to the barrier? …….

- Especially, **which feedback is critical** for the formation and sustainment of ExB staircase?

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Reduced models: Why and What ?

- Reduced models: self-consistently relate variations in **mean plasma fields** to **fluctuation intensity** !

  lower computational cost, flexibility, simplicity, variety,
  supply essential understanding……

- Hasegawa–Wakatani (H–W) system
  - ✓ Simple **generic** system describing **collisional** drift wave turbulence, which conserves energy and **P**otential **E**nstrophy↔ **PE** $\equiv \frac{1}{2}(n-u)^2$

    n → density, u → vorticity
  - ✓ **Inhomogeneous PV mixing and symmetry** generate ZF from turbulence via transport

- Flexible reduced model to understand the physics of meso-scale patterns derived from H-W equation
  - ✓ Inhomogeneous effect of the mean field (density, PV……)
  - ✓ **Mixing length: key element** → What is it ?

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Mixing Length→Rhines Scale

- Mixing length is a nonlinear hybrid of two length scales
  - A constant **excitation** scale l_0
  - A **dynamic** length scale l_d
    a function of the **system gradient**

$$l_{mix}^{\kappa} = \frac{l_0^2}{1 + l_0^2 / l_d^2}$$

- Not the conventional mode scale B B Kadomtsev 《Plasma turbulence》, 1965

- Inhomogeneous PV mixing process selects the mixing scale l_{mix} to generate pattern.

  →Feedback loop strongly depends on l_{mix}

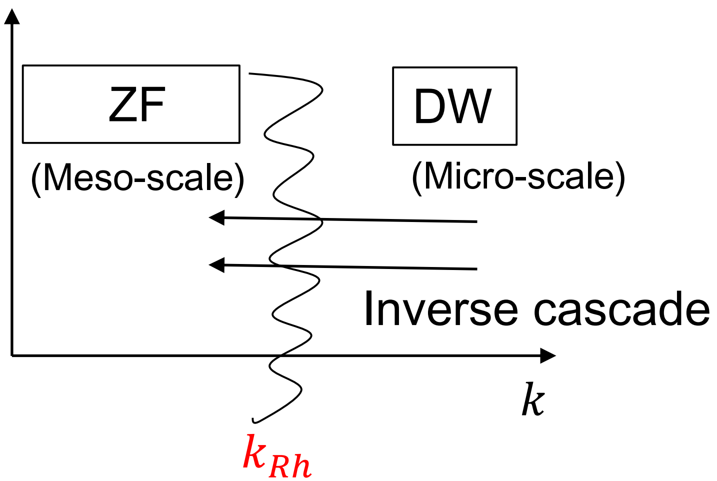
- l_d=l_{Rh}↔ **Rhines scale**

What is Rhines Scale ?

- Scale where **decorrelation rate** crosses **mismatch frequency** (due to dissipation)

(wave coupling mismatch)

$$k_{Rh}\tilde{v} = \omega_{MM} \Rightarrow l_d = l_{Rh} = \frac{\sqrt{\varepsilon}}{\left|\partial_x (n-u)\right|}$$



- ε : **fluctuation** PE
- $\partial_x(n-u)$: **mean** PV gradient

$$l_{mix} = \frac{l_0}{\left(1 + l_0^2\left[\partial_x(n-u)\right]^2 / \varepsilon\right)^{\kappa/2}}$$

- $l < l_{Rh} \rightarrow$ short memory, turbulence is eddy-like → strong mixing
- $l > l_{Rh} \rightarrow$ long memory, turbulence is wave-like → weak mixing

Reduced Model from H-W Equation

- Evolution of mean density n, vorticity u and turbulent potential enstrophy ε

$$\partial_t n = \partial_x D_n \partial_x n + D_c \partial_x^2 n, \quad (1)$$

$$\partial_t u = \partial_x (D_n - \chi) \partial_x n + \chi \partial_x^2 u + \mu_c \partial_x^2 u, \quad (2)$$

$$\partial_t \varepsilon = \partial_x D_\varepsilon \partial_x \varepsilon + \chi [\partial_x (n-u)]^2 - \varepsilon_c^{-1} \varepsilon^{3/2} + \mathscr{P}. \quad (3)$$

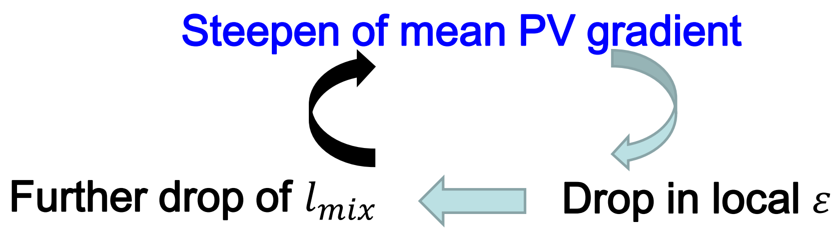
Adiabatic limit

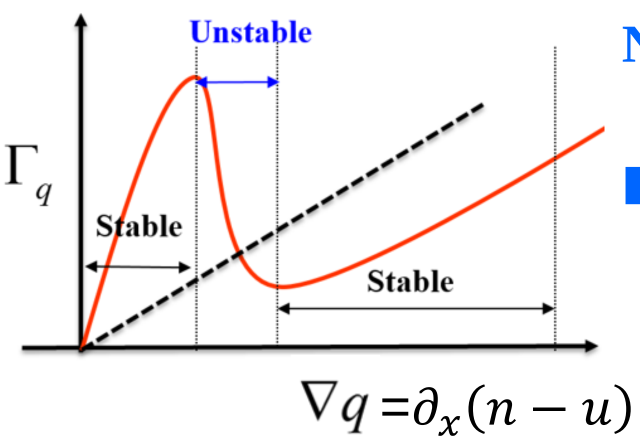
A. Ashourvan and P. H. Diamond, PoP 2017, PRE 2016

- $D_n \approx l^2 \frac{\varepsilon}{\alpha} \quad \chi(x) = c_\chi l^2 \frac{\varepsilon}{\alpha} \quad D_\varepsilon(x) \cong \beta l^2 \varepsilon^{1/2}$

$$l_{mix} = \frac{l_0}{\left(1 + l_0^2 \left[\partial_x (n-u) \right]^2 / \varepsilon \right)^{\kappa/2}}$$

- PV = q $= n - u$



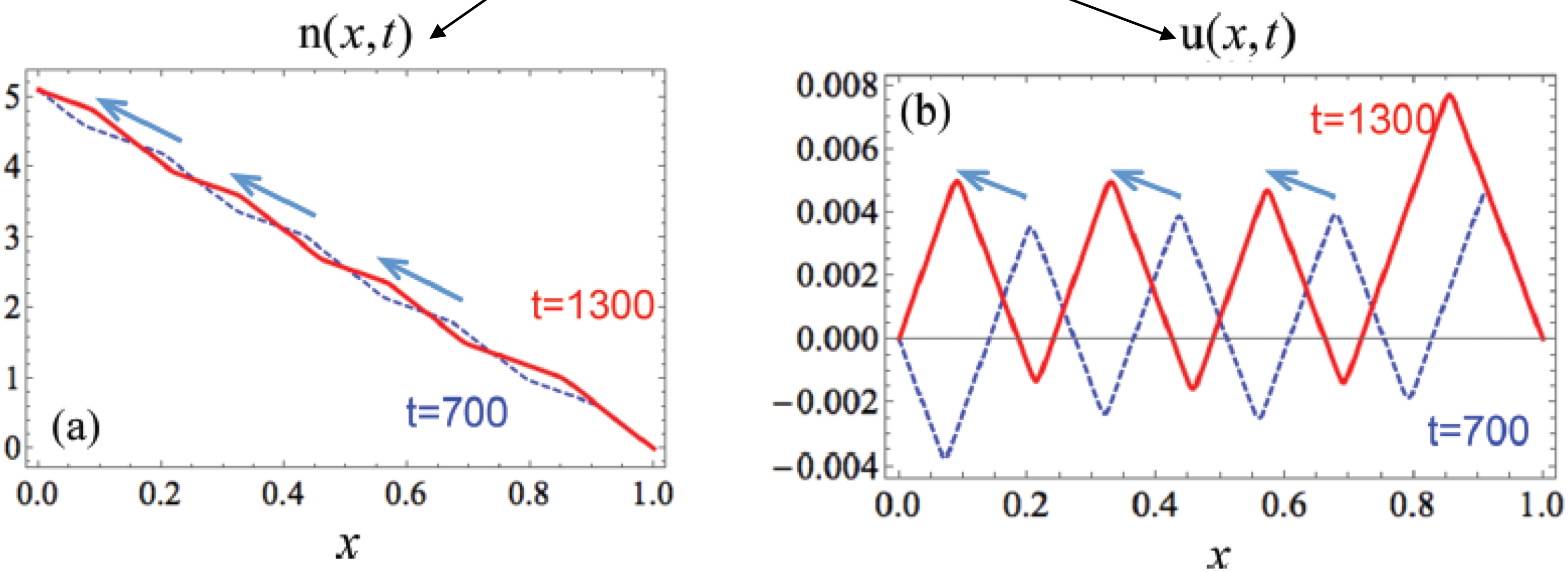


Negative diffusion region---bistable mixing

✓ **The positive feedback loop**
→Drives the pattern
→Leads to nonlinear feature formation in the mean profile.

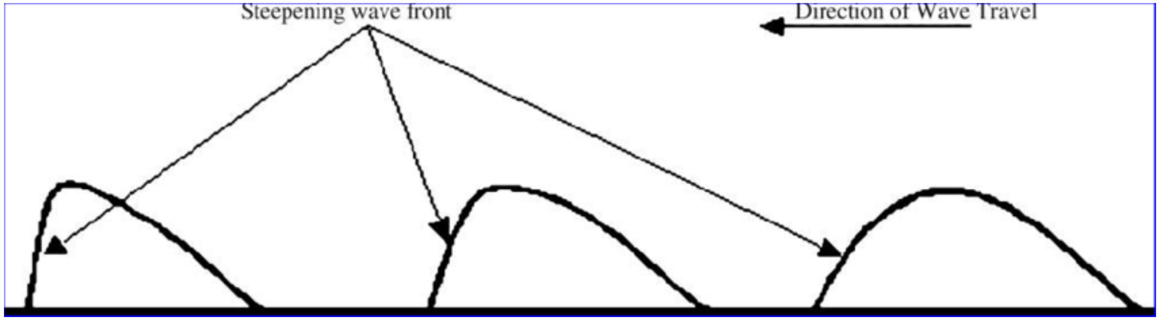
Reduced Model→Pattern Formation

- Formation of **density staircase** and **vorticity corrugation** from modulation



→ **"Staircase"** pattern : consequence of modulational bistability feedback

- Similar to shock wave, which forms due to **self-steepening** of ordinary waves



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Key questions---Apply Reduced Models

- What are the **dimensionless parameters** and how does the pattern structure respond to parameter scan?
- What is the **principal feedback loop** physics?
- How does the pattern respond to **turbulence spreading** and/or avalanching?
- Does the pattern have **memory** of initial condition?
- What is the effect of **mean $\boldsymbol{E} \times \boldsymbol{B}$ shear** on pattern? (Macro$\leftrightarrow$ Meso scale)

…..

Parameter sensitivity studies

- Dimensionless parameters

  - $Re \sim \frac{\sqrt{\varepsilon} l^2}{\mu_c}$
  - $\alpha = \frac{k_\parallel^2 v_{th}^2}{\omega \nu_e}$ >1, fixed
  - $Pr \sim \frac{\mu_c}{D_c}$ (μ_c: Collisional viscosity; D_c: Collisional diffusivity)
  - $\frac{Drive}{dissipation} \sim \frac{\varepsilon_c \gamma_\varepsilon}{\sqrt{\varepsilon}}$

- Re and Pr scans:

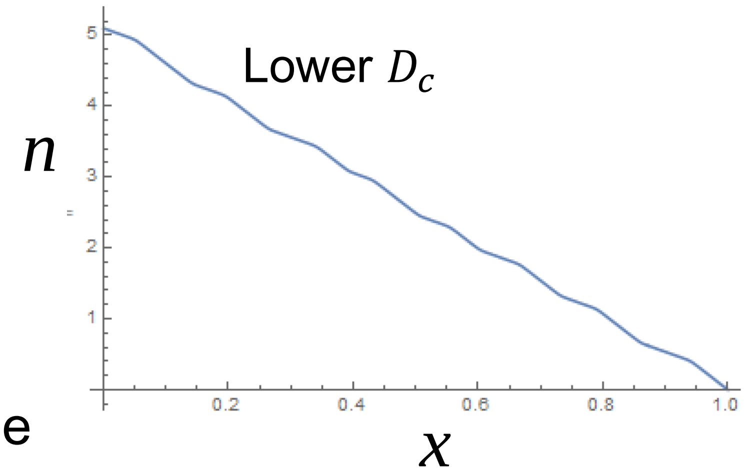
  FOM : Number of steps (N_s) in density profile

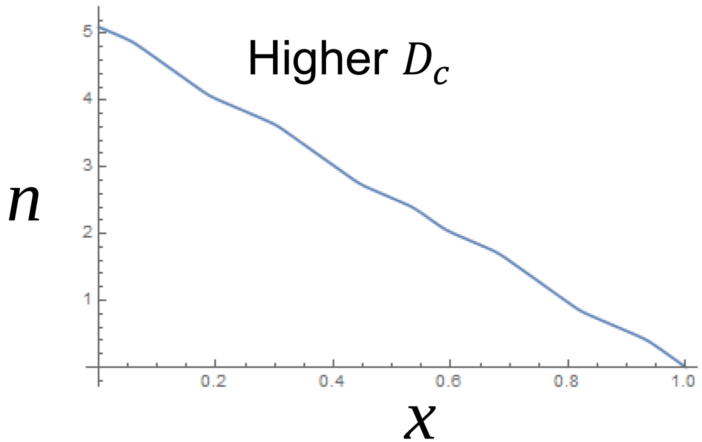
  $\leftrightarrow$ structure complexity

  ✓ Both $\mu_c \nearrow$ and $D_c \nearrow$ (remove energy) ➡ $N_s \swarrow$ ➡ Damp the staircase

  ✓ $Pr \swarrow$ ➡ $N_s \swarrow$

  ✓ $\gamma_\varepsilon \nearrow$ ➡ $N_s \nearrow$





What is the Feedback Mechanism?

- In the case of mean perpendicular shears, mixing length is presented

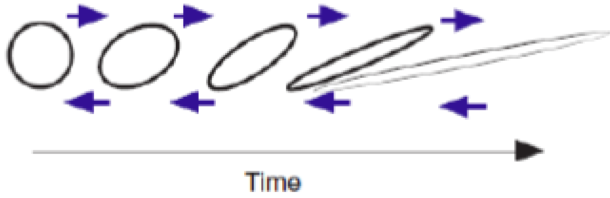
$$l_{mix}^2 = \frac{l_0^2}{\left[1+\left(\bar{v}'_{\boldsymbol{E}\times\boldsymbol{B}}\right)^2 \tau_c^2\right]^{\kappa}}$$

- l_0: the mixing scale without $v_{\boldsymbol{E}\times\boldsymbol{B}}$
- τ_c: fluctuation correlation time
- $\bar{v}'_{\boldsymbol{E}\times\boldsymbol{B}}$: perpendicular shear rate

- The correlation time is given

$$\tau_c = \left(u^2 \varepsilon / l_0^2\right)^{-1/4}$$

- ε: turbulent PE
- $u = \bar{v}'_{\boldsymbol{E}\times\boldsymbol{B}}$



$$l_{mix} = \frac{l_0}{\left[1+\frac{|u|}{l_0\sqrt{\varepsilon}}\right]^{\kappa/2}}$$

↔ Feedback loop works through shearing dependent mixing length

- Rhines scale: $$l_{mix} = \frac{l_0}{\left(1+l_0^2\left[\partial_x\left(n-u\right)\right]^2/\varepsilon\right)^{\kappa/2}}$$

∇n ← → $\partial_x u$

A. Ashourvan and P. H. Diamond, PoP 2017, PRE 2016

- Multiple candidate, which mechanism is key?

Outcome (parameters as before)

- If **only $E \times B$ shearing** feedback ($u = \bar{v}'_{E\times B}$)

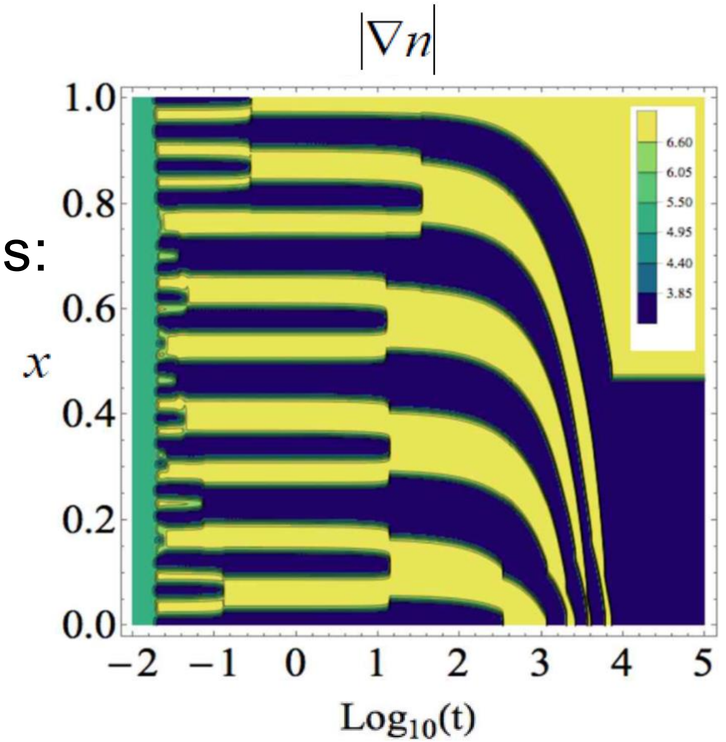
  → **No** layer staircase structure forms !

- If **Rhines scale** feedback (∇n and $\partial_x u$)

  →**Recover** staircase pattern, three stages:

  - Microscale instabilities→Non-linear (NL) mesoscale structure
  - → Merger
  - → Migration

  → Check $\partial_x u$!



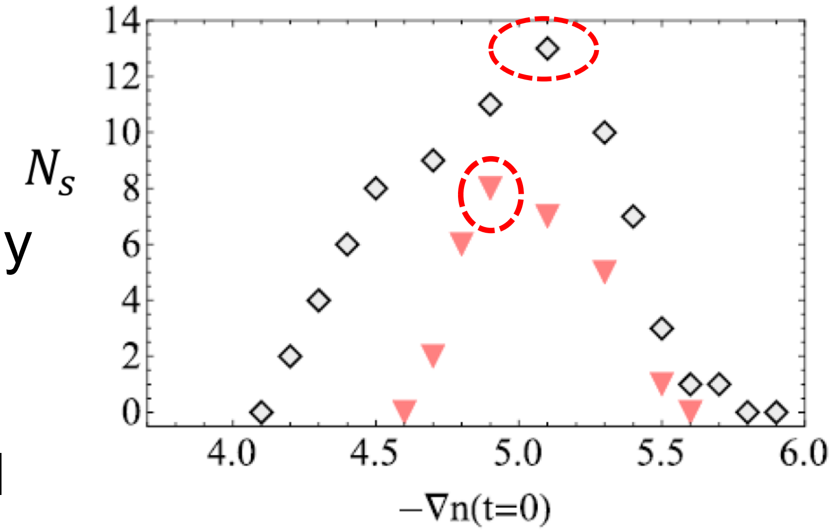
- If **turn off** $\partial_x u$ → **Recover** pattern

↔**Experimental, Jiang Min**

➡ Feedback through NL **gradient drive** of mixing **is key loop ！**

Sensitivity to (Initial) Gradient Drive

- Scan initial gradient drive $\nabla n(t=0)$



  - Initial rise ↔ increased free energy to staircase structure → $N_s \nearrow$
  - Further rise → $N_s \swarrow$ ↔ effects of diffusive dissipation limiting small scales

➡
- ✓ Free energy and dissipation define pattern structure
- ✓ **Minimal step scale** exists

- Flux Drive? What defines the selected scale? (Ongoing)

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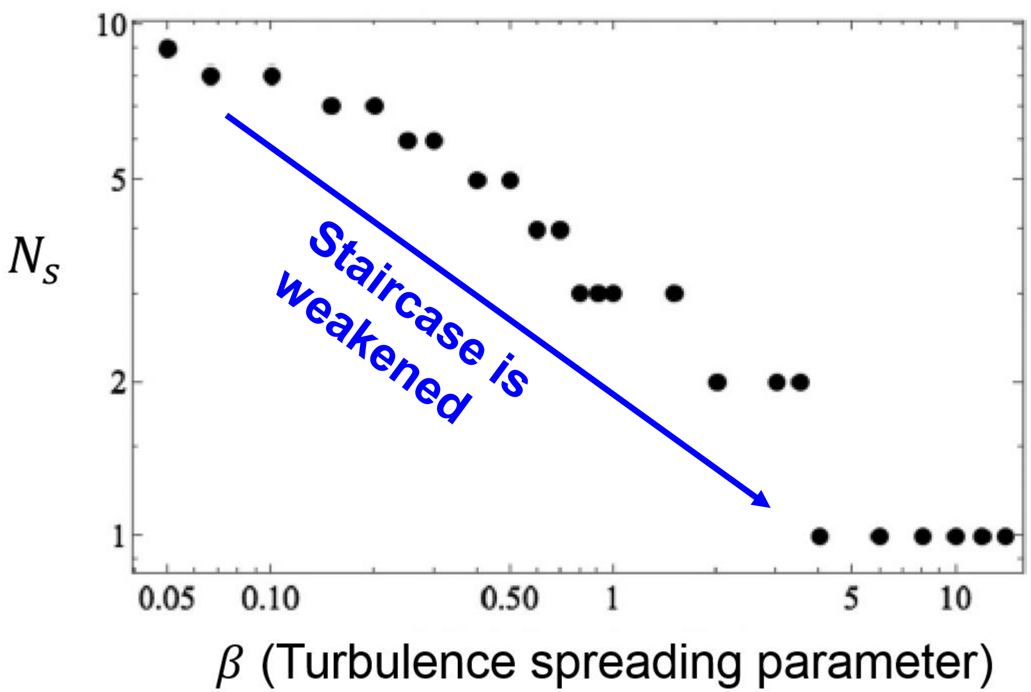
Response to turbulence spreading

- Evolution of fluctuation potential enstrophy (PE)
  - β measures the effect of turbulence spreading of **PE ~ fluctuation intensitity**
  - β represents complex mode interaction physics



✓ Moderate increase in β **weakens** the staircase

➡

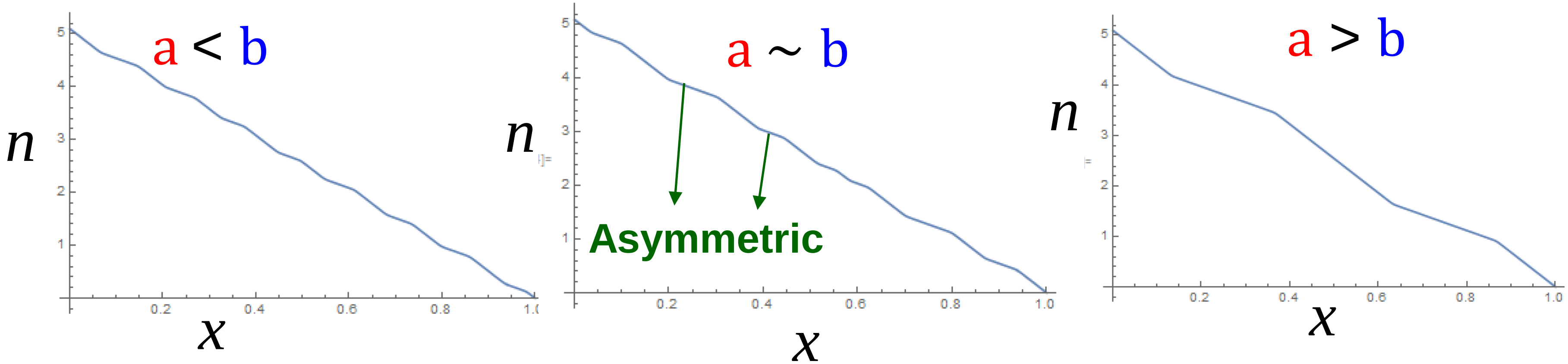
✓ Pattern is **sensitive** to turbulence spreading

Interaction of Zonal and Mean Shear (I)

- Initialize: $u(t=0) = \nabla_\perp^2 \varphi = a\sin(n\pi x) + b$

initial zonal shear (a) — mean shear (b)

I. Fixed mean shear b, vary zonal shear a



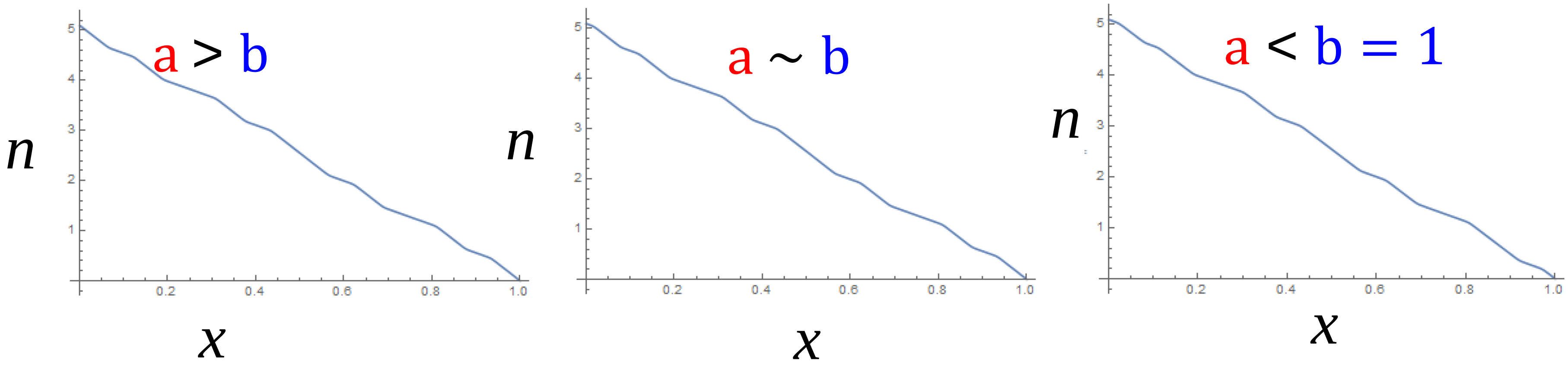
- ✓ Asymmetric pattern can form when zonal shear is comparable with mean shear
- ✓ Stronger zonal shear (a > b) will destroy pattern

Interaction of Zonal and Mean Shear (II)

- Initialize: $u(t=0) = \nabla_\perp^2 \varphi = a\sin(n\pi x) + b$

initial zonal shear (a) — mean shear (b)

II. Fixed zonal shear a, vary mean shear b



✓ b ≤1, increasing mean shear leaves pattern **unchanged**

✓ Stronger mean shear (b >1) destroys the pattern

⇨ **Pattern retains memory of initialization**. Both zonal and mean shear are relevant !

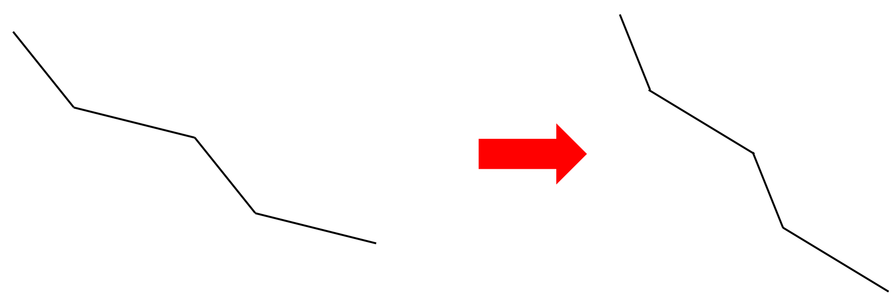
Further Results: Pattern Threshold

In shearing feedback loop:

- Lower the damping value adopted in Rhines feedback loop
  - → Recover the pattern
  - →But, sustainment of the pattern is difficult

- Increase **(a)** the $\nabla n(t=0)$ and **(b)** the initial period of zonal shear
  - →N_s: first increase and then decrease !

- Increasing the **zonal shear** causes the jumps to become much **steeper** !



Summary of results

Existence of density staircase and vorticity corrugation in H-W system

- Pattern responds to the **plasma parameters**
  - Increase flow **viscosity** (μ_c), particle diffusion **damping** (D_c):
    $\rightarrow$ **damp** pattern.
    $\rightarrow$ Moreover, D_c is more effective
  - Increase production $\rightarrow$ selects minimal step scale
- **Feedback loop** physics

  Shearing vs Rhines → NL ∇n dependence of l_{mix} is key loop!

  Surprise ! : shearing feedback is **ineffective**
- Moderate turbulence spreading **weakens** the Pattern
- Pattern **retains** the memory of initial scale
- Both **mean** and zonal $E \times B$ shear act on zonal pattern

Conclusions

- Origin of staircase is simple: quasi-periodic zonal pattern in u, n formed by **self-sharpening of modulation**
  - Principal feedback is through gradient nonlinearity of mixing and transport (**here, ∇n determines**)
  - Pattern **scale** emerges as **minimum** set by ∇n, dissipation, period of zonal shear……
- Reduced models are **useful** complements to large scale simulation !

Ongoing and Future Work

- **Understanding**
  - Why shearing feedback is not effective?
  - Requirement of driving gradient nonlinearity to sustain pattern?
  - Other feedback mechanism?
- **Flux driven** studies (Extend Ashourvan, Diamond studies)
  - Scaling of step size ?
  - Quantity the global hysteresis of state ?
  - Determined condition for ITB transition ? (global→local barrier ?)
- **Modify the model** to treat mean shear consistently, and to treat high density (i.e., **hydrodynamic**) regime ?……

Thank you very much for your attention!

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