Hysteresis as a Probe of Turbulent Bifurcation in Intrinsic Rotation Reversals on Alcator C-Mod

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Outline

• Why study hysteresis?
  What is rotation reversal hysteresis, and what can it tell us about turbulence?

• Experimental characterization of hysteresis on C-Mod
  What is experimentally observed in hysteresis, and can we infer any changes in turbulence?

• Understanding observations through quasilinear modelling
  Can we use reduced models to identify a candidate theory for the reversal, despite the complexity of plasma turbulence?
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Focus: Turbulent Transport of Heat and Momentum in Tokamaks

Two Major Questions:

• **Energy Confinement** – How does turbulence influence transport of injected power in the tokamak?

• **Toroidal Rotation** – When do large-scale coherent flows form from turbulence in tokamaks?
  
  • **Fusion energy context**: How is this interrelated with energy confinement?

https://fusion.gat.com/theory/Gyromovies
Introduction: A Tale of Two Transitions in Tokamak Turbulence

- **LOC/SOC transition** is a universally observed transition in confinement time found in L-mode plasmas

- **Intrinsic Rotation Reversal** is a spontaneous reorganization of toroidal rotation in plasmas with no external momentum input

- Rich phenomenology observed (impurity asymmetry, density profile peaking, cold pulse propagation, etc...)¹
  - How are these transitions linked? Can we build a qualitative understanding of the underlying turbulent state?

¹Rice NF 2013
Background: Understanding of Drift Wave Turbulence Necessary to Characterize LOC/SOC

- Gyrokinetic drift wave turbulence is responsible for most heat and particle transport in tokamaks
- Observed rotation profiles require a non-zero residual stress, driven by turbulence\(^1\)
- LOC/SOC and reversal thought to be linked to a transition in DW turbulence from TEM to ITG\(^2,3\)
- Studies based on linear stability alone have been inconclusive

\[
\Pi_{r\phi} = -\chi_{\phi}\langle v_{\phi}\rangle' + V\langle v_{\phi}\rangle + \pi^R_{r\phi}
\]

momentum flux = diffusion + pinch + residual

\(^1\)Diamond NF 2013
\(^2\)Diamond PoP 2008
\(^3\)Camenen PPCF 2017
Motivation: Hysteresis Experiments Provide Controlled Probe of Turbulent Transition!

- Reversals exhibit **hysteresis**, so the same mean plasma parameters manifest different (bifurcated) rotation states

- **This Work**\(^1\):
  1. The physics of the dominant linear instability alone is **not enough** to explain the link between transitions
  2. A candidate **subdominant mode transition** found through quasilinear modelling which is consistent with the observed transport

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\(^1\)Cao et al NF 2019
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Experiments show hysteresis is a reproducible phenomenon across multiple shots

5.4 T, 0.8 MA
Experiments show hysteresis is a reproducible phenomenon across multiple shots

- Hysteresis robust to perturbative LBO
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- Hysteresis robust to perturbative LBO
- Well-defined and well-separated rotation states $\Rightarrow$ distinct states of turbulence

$\leftarrow$ LOC

$\Rightarrow$ SOC
Nearly Indistinguishable Density and Temperature Profiles Can Lead to Different Rotation States

• Profiles are shown here for **LOC** (t=0.96 s) and **SOC** (t=0.6 s)
  - Electron profiles from same discharge (Thomson, GPC-ECE)
    - error rigorously estimated with GPR\(^1\)
  - Ion profiles from different but matched shots (HIREXSR XICS)

\(^1\)Chilenski NF 2017
Linear Gyrokinetic Simulations Show Mode Stability Unchanged across LOC/SOC Transition

• Linear CGYRO run for multiple times in the same shot in rotation reversal region
  • Matched profiles from LOC and SOC
  • ±1σ scan from SOC, shown in gray
• Ion-scale instabilities robustly remain ion-directed near transition – change in dominant linear instability not sufficient to explain LOC/SOC transition!
  • Consistent with previous work on Alcator C-Mod and AUG plasmas\textsuperscript{1,2,3}
  • Note: <2σ from TEM/ITG boundary
• Motivates a need to look at subdominant modes to diagnose turbulent state

\[ \omega_R \left[ \frac{c_s}{a} \right] \quad r/a = 0.575 \]
\[ \gamma \left[ \frac{c_s}{a} \right] \]

\textsuperscript{1}White PoP 2013
\textsuperscript{2}Sung PoP 2016
\textsuperscript{3}Erofeev NF 2017
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Separation of Linear and Nonlinear Physics: Quasilinear Transport Approximation (QLTA)

• In linear mode QLTA (mQLTA), flux is given by the sum over modes of a quasilinear weight (linear eigenmode structure) times a mode intensity (nonlinear saturation)

\[
\text{Flux} = \sum_k \text{weight} \cdot \text{intensity}
\]

\[
Q_{e,\text{turb}} = \sum_k W_{Qe,k} \left\langle \phi_k^2 \right\rangle
\]

• Note on Applicability: Weights used in mQLTA match weights from fully nonlinear simulation\(^1\); cross-phases match experiment\(^2,3\)

\(^1\)Waltz PoP 2009  
\(^2\)White PoP 2010  
\(^3\)Freethy PoP 2018
mQLTA: Experimental Fluxes Provide a Constraint on Nonlinear Mode Saturation Levels

$$\text{Flux} = \sum_{k} \text{weight} \cdot \text{intensity}$$

- **Turbulent Fluxes** inferred from power balance (TRANSP - NEO)
- **Quasilinear Weights** from Linear Gyrokinetic Simulation (CGYRO)
  - *Example*: The only non-zero weight on high-\(k\) modes is electron heat flux, so they only contribute to electron heat flux
- Can’t directly invert equation to solve for mode intensities
Reduced Family Model Allows Understanding Mixed Mode Picture

- Not interested in detailed shape of spectrum, only general trends
- Construct a reduced model that lumps related modes into ‘families’:
  \[
  \text{Flux} = \sum_{\text{families}} \text{weight} \cdot \text{intensity}
  \]
- Greatly reduces the number of unknowns, and allows for the experimental fluxes to be used meaningfully as constraints
Reduced Family Model Allows Understanding Mixed Mode Picture

\[ \text{Flux} = \sum \text{weight} \cdot \text{intensity} \]

**ITG (a,b)**  
Ion-scale, ion-directed modes; separated into low-k (a) and mid-k (b) based on particle transport:  
(a) Net outward particle transport  
(b) Roughly balanced particle transport

**TEM**  
Intermediate-scale \( k_y \rho_s \gtrsim 1 \), electron or hybrid-direction modes; marginal modes, with inward particle pinch

**ETG**  
Electron-scale electron-directed modes; exhausts mostly electron heat flux, ions adiabatic
Novel Candidate Subdominant Mode Transition Identified, Consistent with Observed Transport

- No core fueling, so $\Gamma_e = 0$
- To satisfy particle flux constraint:

<table>
<thead>
<tr>
<th>LOC-like</th>
<th>SOC-like</th>
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<td>Active Mode Families</td>
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<td>TEM</td>
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<td>Particle Flux Balance</td>
<td>ITGa balances</td>
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![Graph showing different scales and balances](image)

**APS-DPP 2019**
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• Away from transition, continuity implies different transport dependencies

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Mechanism leading to Transition and Bistability still Unknown

Experimental Constraints on Possibilities:

1. We’re on a stability boundary, so small profile changes lead to big changes in turbulence
   • Robustness to perturbative LBO seems to rule this out

2. Mean rotation profiles contribute to change in turbulence through ExB shearing
   • Reminiscent of L-H transition, next slide will discuss...

3. Change in meso-scale or micro-scale turbulent nonlinear interactions (e.g. entering/leaving weak turbulence regime)
   • QL may fail for TEMs$^{1,2}$, and shear flows affect cross-phase$^3$. How to identify key physics, and what happens to QL estimates?

$^1$Xiao PRL 2009
$^2$Kosuga PoP 2011
$^3$Terry PRL 2001
Change in mean ExB shear across transition possibly non-negligible?

- Mean ExB shearing rate calculated from force balance
  \[ \gamma_E = \frac{r \, d\omega_0}{q \, dr} \]

- CGYRO scans show shear is not enough to linearly stabilize observed ion modes, but could play a role in saturation of more marginal modes
  - Not conclusive based on linear arguments alone! Need to consider nonlinear interactions\(^1\)

- Possibly amenable to reduced dynamical modelling through predator-prey (similar to L-H\(^2,3\))

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\(^1\)Waltz et al PoP 1998  
\(^2\)Diamond et al PRL 1994  
\(^3\)Kim et al PRL 2003
Summary and Paths Forward

• Experiments show changes in toroidal rotation and turbulent residual stress despite nearly identical density and temperature profiles
  • A change in dominant linear instability alone is not sufficient to explain the LOC/SOC transition

• Quasilinear modelling identifies a candidate subdominant ITG/TEM transition which is consistent with the observed transport
  • Reminiscent of a “population collapse” or quenching of marginal TEM turbulent intensity

• Provides a stepping stone for future inquiry:
  • Hints towards nonlinear physics required to adapt reduced dynamical models
  • Hysteresis experiments in nonlinear global simulation to probe residual stress physics
  • Characterize turbulent fluctuations in hysteresis experiments on other machines
References

Extra Slides
What Evidence Exists for ITG/TEM Picture of LOC/SOC Transition?

• Modelling Consistent with TEM in deep LOC, ITG in deep SOC
  • Romanelli (analytic w/ mixing length), Erofeev, Grierson (Predictive Transport Modeling w/ TGLF)
  • Rodriguez-Fernandez (role of TEMs vs ITGs in determining cold pulse dynamics)

• Residual Stress physics and Theoretical Suspicions
  • Wang, Grierson (GTS papers), Hornsby (GKW)
  • Camenen, Diamond (wave group velocity in relation with momentum)

• Experimental Signatures
  • JET and Tore Supra Citrin (QC-TEMs go away at LOC/SOC; “TEMs being unstable is not enough”)
1.1 MA Ohmic – **different** toroidal rotation, **same** local profiles, **different** PCI spectra
PCI Spectra provide circumstantial evidence of turbulence change in hysteresis experiments

• Circumstantial evidence suggests a sudden change in turbulence, in direction of LOC => SOC, before the reversal finishes
  • Reminder: Bistability robust to perturbative laser blow-off injection

Observed $\pm k_R$ Fluctuation Asymmetry

$$\frac{(P + k_R - P - k_R)}{\bar{P}}$$

High: $300 \text{ kHz} < f < 700 \text{ kHz}$
Low: $50 \text{ kHz} < f < 200 \text{ kHz}$
What Exactly are the intermediate-k Modes?

Appear in the crossover between TEM and ITG branches of dispersion, when the overlap decreases, almost certainly the remnants of TEM+ITG; unstable TEM also have inward particle pinch
Nonlinear Heat Flux Spectra Possibly Consistent with mQLTA Prediction

Heat flux spectra at r/a=0.8

Peaking factor 1.5

Peaking factor 2

Ion heat flux spectrum more peaked in SOC!
Ways the Quasilinear Approximation Could Fail, and Resulting Physics

Possibilities include...

1. Wave-particle nonlinearity (Kubo number)
   • ITG Kubo # good, TEM Kubo # > 1

2. DW-DW interactions
   • TEM dispersion weak at ion-scales

3. DW-Flow shear interaction (note difference between trapped and passing particle response)
   • Hahm-Burrell Shearing parameter
Calculating Gradients and Uncertainties has “Coastline Paradox” Problem

• Coastline measurement paradox involves the measurement of arclength, which depends on derivative

\[ \int ds \approx \sum \sqrt{\Delta x^2 + \Delta y^2} \]

• Problem is ill-posed; answer tends to diverge as length of ruler gets smaller e.g. detail increases as length-scale used to view increases, typically with \( \alpha < 0 \) [Richardson]

\[ L(G) = MG^\alpha \]

• Note relationship with anomalous dissipation in turbulence, appearance of singularities in limit of zero dissipation

• This is an issue intrinsic to the derivative operator (related to sensitivity to high frequency components), not a quirk of the measurement! There is no such thing as a gradient measurement without a length scale.

• Importance of different length scales imply different physics
What about momentum?

• Don’t expect many intrinsic stress sources to be captured by local linear runs

• [Grierson PRL 2017] Turbulent momentum diffusion balances with intrinsic stress generation
Momentum Transport Predicted is Primarily Diffusive
Flow Shear Affects Growth Rates of $k_y \rho_s \sim 1$

Marginal Modes

- Performed scans reducing mean ExB and Mach flow shear (color scale on plot to the right)
- Marginal modes with large enough $k_y \rho_s$ strongly affected by flow shear, while ion modes relatively unchanged
- Provides possible mechanism for bifurcation (increasing flow shear -> changed mode population -> increasing flow shear)
Hysteresis is observed at multiple currents, and survives perturbation from LBO.

Low-power ICRF heating ramps also lead to hysteresis, with similar
Fluctuation Measurements Change Despite Nearly Identical Profiles

- Power spectra of complex signal from 88 GHz channel of C-Mod midplane O-Mode reflectometer shown for LOC and SOC
- Reflectometry is sensitive to density perturbations, $k_\perp$ up to 10 cm$^{-1}$ [Lin PPCF 2001]
- Open question if change in power spectrum actually due to change in turbulence, not just Doppler shift