Learning a model for mean-field turbulence dynamics

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Introduction: can a computer do plasma physics?

- Calculating turbulent fluxes is an important challenge for the plasma physicist
- Introduce new **machine learning technique** for studying fluxes
- Can the algorithm pick up physics we missed?

**Figure** A computer studies tokamak physics
Introduction: this talk

- Apply new method to resistive DW turbulence via 2D Hasegawa-Wakatani system
- Reproduce analytic result for particle flux, including often-overlooked term induced by ZF
- Discuss implications of ZF term, future directions

Figure: Snapshot of vorticity field from simulation of 2D HW
Background
Drift-wave turbulence

- Drift-wave turbulence features complex interaction between mean profile, ZF, and turbulence
- Dynamics controlled by cross-correlations between fluctuating quantities (turbulent fluxes). E.g. particle flux $\Gamma = \langle \tilde{n}\tilde{v}_x \rangle$ and Reynolds stress $\Pi = \langle \tilde{v}_x\tilde{v}_y \rangle$
- Difficult to calculate! Not many approaches beyond quasilinear theory

Figure Feedback loop illustrating interaction of mean fields in DW turbulence
Machine learning approach
Basic formalism: pushing MFT to the limit

- Seek maps $f_q$ which send local mean fields to local fluxes, i.e. $f_q : (n, \partial_x n, \ldots, \phi, \partial_x \phi, \zeta, \partial_x \zeta, \ldots, \varepsilon, \partial_x \varepsilon, \ldots) \mapsto \langle \tilde{q}\tilde{v}_x \rangle$. Here $\varepsilon = (\tilde{n} - \tilde{\zeta})^2$ is turb. PE, $\zeta = \partial_x^2 \phi$ is vorticity.

- Idea: rather than attempt direct calculation or fitting parameters, use supervised learning to train a neural network on numerical simulations.

- Essentially nonlinear, model-free regression. Could capture physics missed by human?

**Figure** Schematic of machine learning method.
Detailed methods

- As proof of concept, learn particle flux $\Gamma$ from simulations of 2D HW (at fixed $\alpha = 2$). Advantages: fast simulations, captures full feedback loop, yet simple to treat analytically
- Constrain problem using symmetries of HW:
  1. Invariance under uniform shifts $n \to n + n_0$ and $\phi \to \phi + \phi_0$ eliminates dependence on $n, \phi$
  2. Invariance under boosts in $y$
     \[
     \begin{align*}
     \phi & \to \phi + v_0 x \\
     y & \to y - v_0 t
     \end{align*}
     \]
     eliminates dependence on ZF speed $\partial_x \phi$
  3. Reflection symmetries $x \to -x, y \to -y$ and $\phi \to -\phi, n \to -n, x \to -x$ and $\phi \to -\phi, n \to -n, x \to -x$
     which are enforced by duplicating and transforming data
Results
Particle flux learned by NN

NN learns a model roughly of the form

$$\Gamma = -D_n \varepsilon \partial_x n - D_\zeta \varepsilon \partial_x \zeta$$

Usual QL flux plus an “off-diagonal” term driven by vorticity! (no clear dependence on other quantities)

Figure Curves (at fixed $\varepsilon = 10$, $\zeta = 1$, $\partial_x \varepsilon = 0$, and various $\partial_x n$) of $\Gamma$ vs vorticity gradient. Appears to be simple linear combination of $\partial_x n$ term and $\partial_x \zeta$ term
Derivation of off-diagonal term

Careful analytic treatment in adiabatic limit reproduces off-diagonal term. Need include frequency shift due to ZF!

\[ \omega_k = \frac{k_y}{1 + k^2} (\partial_x n - \partial_x \zeta) + O(\alpha^{-2}) \]

\[ \gamma_k = \frac{k_y^2}{\alpha (1 + k^2)^3} (\partial_x n - \partial_x \zeta) (k^2 \partial_x n + \partial_x \zeta) + O(\alpha^{-2}) \]

\[ \Gamma = \text{Re} \sum_k -i k_y \tilde{n}_k \tilde{\phi}_k^* \]

\[ = \sum_k -k_y^2 \partial_x n (\gamma_k + \alpha) + \alpha k_y \omega_{r,k} \frac{|\tilde{\phi}_k|^2}{\omega_{r,k}^2 + (\gamma_k + \alpha)^2} \]

\[ = \frac{1}{\alpha} \sum_k -\frac{k_y^2}{1 + k^2} (k^2 \partial_x n + \partial_x \zeta) |\tilde{\phi}_k|^2 + O(\alpha^{-2}) \]
Comparison to QLT (diagonal term)

Compare NN result to QLT result using spectrum centered at most unstable $k$ for $\partial_x \zeta = 0$

$$\varepsilon_k = \frac{\langle \varepsilon \rangle}{4\pi \Delta k_x \Delta k_y} e^{-k_x^2/2\Delta k_x^2} \left( e^{-(k_y-\sqrt{2})^2/2\Delta k_y^2} + e^{-(k_y+\sqrt{2})^2/2\Delta k_y^2} \right)$$

**Figure Curves (at fixed $\zeta = 1$, $\partial_x \zeta = \partial_x \varepsilon = 0$, and various $\varepsilon$) of $\Gamma$ vs density gradient from NN**

**Figure Corresponding curves from QLT with $\Delta k_x = \Delta k_y = 1.5$**
Comparison to QLT (off-diagonal term)

**Figure** Curves (at fixed $\zeta = 1$, $\partial_x n = \partial_x \varepsilon = 0$, and various $\varepsilon$) of $\Gamma$ vs vorticity gradient from NN

**Figure** Corresponding curves from QLT with $\Delta k_x = \Delta k_y = 1.5$
Discussion
Implications of off-diagonal term

- Off-diagonal often dismissed, but coupling same order of magnitude ($\sim 0.5$) as that of usual $\partial_x n$ term. Machine picks it out very clearly!

- Consequence: ZF can induce staircase pattern on profile. If $V_y = V_0 \sin(qx)$, $\partial_x \zeta$ term will contribute

  \[ \partial_t \langle n \rangle \sim -\frac{k_y^2 q^3 V_0 \langle \varepsilon \rangle}{\alpha(1 + k^2)^3} \cos(qx) \]

- Alternate mechanism independent of usual shear suppression, bistability. Could explain DIII-D pedestal staircase (Ashourvan et al. PRL 2019)?

Figure Cartoon indicating how ZF can induce profile staircase via pinch
Conclusions and future work

- Machine learning technique teaches us that shear-induced off-diagonal flux is **significant** effect.
- Eventually, ML method may be applicable to more complex systems that resist analytic treatment.
- In progress:
  1. Develop more sophisticated ML methods, e.g. spatially nonlocal model. Goal: Reynolds stress
  2. Complete analytic study of effects of ZF frequency shift
- How important is off-diagonal flux relative to other staircasing mechanisms?
Hasegawa-Wakatani system

- Simplest realistic model for drift-wave turbulence which captures full feedback loop

\[
\partial_t n + \mathbf{v}_E \cdot \nabla_\perp n = \alpha (\tilde{\phi} - \tilde{n}) + \text{dissipation}
\]

\[
\partial_t \zeta + \mathbf{v}_E \cdot \nabla_\perp \zeta = \alpha (\tilde{\phi} - \tilde{n}) + \text{dissipation}
\]

with \( \mathbf{v}_E = \hat{z} \times \nabla_\perp \phi \), \( \zeta = \nabla_\perp^2 \phi \) and \( \alpha = \eta k_\parallel^2 \) the adiabatic operator (representing parallel electron response)

- Averaging over symmetry directions (\( \langle \cdots \rangle \)) yields

\[
\partial_t \langle n \rangle + \partial_x \Gamma = \text{dissipation}
\]

\[
\partial_t \langle \zeta \rangle - \partial_x^2 \Pi = \text{dissipation}
\]

where \( \Gamma = \langle \tilde{n} \tilde{v}_x \rangle \) and \( \Pi = \langle \tilde{v}_x \tilde{v}_y \rangle \)

- How to calculate \( \Gamma, \Pi \)?
Figure Error bar estimates for NN results for $\partial_x \zeta = 0$

Figure Error bar estimates for NN results for $\partial_x n = 0$