Subcritical turbulence spreading and avalanche birth

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In magnetic fusion plasma, turbulence driven by linear instability

However, turbulence is still found to be present in linearly stable regions

Explanation: turbulence can spread

Basic example of nonlocality

Figure: Experiment [Nazikian et al., 2005] clearly showing fluctuations in stable zone
Turbulence spreading: old news?
Challenge the conventional wisdom on spreading (supercritical Fisher model)
Suggest a new model based on subcritical turbulence, which testably differs from old story
Will see that new model also serves as basic framework for avalanching

Figure: Conventional wisdom on turbulence spreading
For the impatient: preview of results

- New model accounts for robust penetration of turbulence into stable regions via **ballistic propagation**, whereas old model features weak, evanescent penetration $\ell \sim \Delta_c$
- New model features threshold for propagation of a puff of turbulence, akin to an avalanche
- Power law threshold for puff size vs. intensity

Penetration into stable zone in Fisher model (left) and new model (right)
Outline

1. Background: turbulence spreading

2. Fisher model

3. Bistable model

4. Avalanche threshold

5. Conclusions
Background: turbulence spreading
What the Fick?: turbulence spreading

- Turbulence can radially self-propagate via **nonlinear coupling**. Intensity profile gradient $\rightarrow$ intensity flux
- Can penetrate linearly stable zones
- Decouples flux-gradient relation: local turbulence intensity now depends on global properties of the profiles
- Spells doom for local Fickian transport models i.e. $Q \propto \partial_x T$

**Figure:** Mesoscale gradient in intensity envelope generates turbulence flux
Figure: Spatiotemporal evolution of flux-surface-averaged turbulence intensity in toroidal GK simulation. Linearly unstable region is $0.42 < r < 0.76$; profiles are fixed. From [Wang et al., 2006]
Fisher model
Conventional wisdom: Fisher model

- Conventional wisdom [Gürcan and Diamond, 2005, Hahm et al., 2004, Naulin et al., 2005] for spreading is Fisher-type equation for turbulence intensity:

\[
\frac{\partial t}{t} I = \gamma_0 I - \gamma_{nl} I^2 + \frac{\partial_x (D_0 I \partial_x I)}{
\text{local lin.}
\text{growth/decay}
\text{local nonlin.}
\text{coupling to}
\text{dissipation}
\text{nonlin. diffusion of turb. energy}}
\]

- When \( \gamma_0 > 0 \), uniform fixed points are "laminar" \( I = 0 \) and "saturated turbulence" \( I = \frac{\gamma_0}{\gamma_{nl}} \)

- Dynamics characterized by traveling fronts connecting roots, with speed \( c = \sqrt{\frac{D_0 \gamma_0^2}{2\gamma_{nl}}} \)
Figure: Evolution of traveling turbulence front in Fisher model. From [Gürcan and Diamond, 2006]
Consider spreading of turbulence from linearly unstable to linearly stable zone.

Simple model: $\gamma_0 > 0$ for $x < 0$, $\gamma_0 < 0$ for $x > 0$.

Allow turbulent front to form in lefthand region and propagate.

Penetration is **weak**: forms stationary, exponentially-decaying profile with $\lambda \sim \sqrt{D_0/\gamma_{nl}} \sim \Delta_c$. Puny!

**Figure**: A front of turbulence crosses into stable zone and penetrates a finite depth.
Does Fisher-type spreading make sense?

- No
- Fisher model purports to describe spreading of a patch of turbulence in linearly unstable zone
- Begs the question: why didn’t noise already excite the whole system to turbulence?
- Only relevant if $\gamma_0 \ll c/\Delta x$ i.e. $\Delta x^2 \gamma_{nl} \ll D_0$
- Otherwise, physical fronts separating laminar/turbulent domains generally require bistability à la [Pomeau, 1986]
Bistability

Figure: Free energy of unistable system, corresponding to Fisher

Figure: Free energy of bistable system
Bistable model
A new model is born

Heinonen and Diamond 2019: propose phenomenological model of form

\[ \frac{\partial I}{\partial t} = \gamma_1 I + \gamma_2 I^2 - \gamma_3 I^3 + \partial_x (D_0 I \partial_x I) \]

- local lin. growth/decay
- nonlin. instability
- nonlin. coupling to dissipation
- nonlin. diffusion of turb. energy

Simplest extension of Fisher-like model with bistability

New physics: nonlinear turbulence drive \( \propto I^2 \). Can sustain sufficiently large fluctuations even when linearly damped

**Bistable** in weak damping regime

Estimate \( \gamma_1 \sim \epsilon \omega_* \), \( \gamma_{2,3} \sim \omega_* \), \( D_0 \sim \chi_{GB} \) (drift-wave/Gyro-Bohm scaling)
Evidence for bistability/subcriticality

- [Inagaki et al., 2013]: experiments demonstrate hysteresis between fluctuation intensity and driving gradient (no TB present). Suggests bistable S-curve relation?

- Turbulence subcritical in presence of strong perpendicular flow shear [Barnes et al., 2011] or in the presence of magnetic shear [Drake et al., 1995]

- Profile corrugations [Guo and Diamond, 2017] and phase space structures [Lesur and Diamond, 2013] can drive nonlinear instability

**Figure:** Hysteresis between intensity and gradient, flux and gradient
Bistable regime

- Qualitatively similar to Fisher EXCEPT in bistable/weak damping case.
- Can then transform to Zel’dovich/Nagumo equation
  \[ \partial_t I = f(I) + \partial_x (D I \partial_x I) \]
  \[ f(I) \equiv \gamma I (I - \alpha)(1 - I) \]
- Unlike Fisher, traveling fronts admitted (even though damped)!
  \[ c \sim \sqrt{D\gamma} \text{ (depends on } \alpha), \ell \sim \sqrt{D/\gamma} \]
  \[ \alpha \equiv I_- / I_+, \quad \gamma \equiv I_+^2 \gamma_3, \quad D \equiv I_+ D_0, \quad I_{\pm} \equiv \left( \gamma_2 \pm \sqrt{\gamma_2^2 - 4|\gamma_1|\gamma_3} \right)/2\gamma_3 \]

Figure: Reaction function has stable nodes at \( I = 0, 1 \) and unstable node at \( I = \alpha \)
Penetration into stable zone: new model

- Take $\gamma_1 = \gamma_g > 0$ for $x < 0$, $\gamma_1 = -\gamma_d < 0$ for $x > 0$
- In contrast to Fisher, a new front with reduced speed/amplitude forms in second region if weakly damped

$$\left( \gamma_d < \frac{15\gamma_2^2}{64\gamma_3} \right)$$
- Hence: can have **ballistic propagation into stable zone**!
- Much stronger penetration than possible in Fisher—resolves issue of feeble, evanescent penetration
Figure: Spreading into stable zone in GK simulation with magnetic shear [Yi et al., 2014]. Evidence of ballistic propagation? More careful study needed!
Avalanche threshold
Avalanches

- Bursty, intermittent transport events associated with SOC
- Accounts for a large percentage of total flux
- Initially localized fluctuation cascades through neighboring regions via gradient coupling, simultaneous firing of many cells
- What does this have to do with spreading?

**Figure**: Cartoon depicting generic avalanche process via overturning of fluctuation into neighboring cells
Spreading vs. avalanching

- Fast, mesoscopic turb front propagation
- Interaction of a small scale (DW, cell) with a mesoscale (envelope, avalanche)
- Turbulence intrinsic to avalanching → drives spreading
- Unified model?

Figure: Spreading and avalanching both result from coupling of small scale $k$ with mesoscale $q$ ($q \ll k$)
Depiction of avalanching

**Figure:** Pressure (left) and potential (right) contours for simulations of resistive drift interchange turbulence [Carreras et al., 1996]. Diagonal lines → propagating transport events
Local threshold behavior

- In contrast to Fisher, sufficiently large localized puff of turbulence will grow into front and spread. Suggestive of an avalanche triggered by sufficiently strong initial seed
- How to determine threshold?

Two puffs differing only in spatial size are initialized; one grows and spreads, other collapses
Obviously puff amplitude must exceed $l_0 = \alpha$ or else
$\gamma_{\text{eff}} = (l - \alpha)(1 - l) < 0$

Consider “cap” of puff (part exceeding $l = \alpha$)

Competition between diffusion of turbulence out of cap and total nonlinear growth in cap

Sets threshold lengthscale $\sqrt{D/\gamma}$

**Figure:** “Cap” of initial data. There is a competition between nonlinear growth and turbulence diffusion here.
Avalanche threshold II

- Analytic result: puff grows if
  \[ L > L_{\text{min}} \sim (I_0 - \alpha)^{-1/2} \]

- Near linear marginality, threshold is weak:
  \[ l_\perp \sim \frac{\gamma_1}{\gamma_2} \ll 1, \quad L_{\text{min}} \sim \left( \frac{\chi_G B}{\omega_*} \right)^{1/2} \sim \Delta_c \]

- Thus, avalanche could be triggered by noise. Another possibility: corrugation

Figure: Numerical result for threshold at \( \alpha = 0.3 \) for three types of initial data (Gaussian \( I_1 \), Lorentzian \( I_2 \), parabola \( I_3 \)), compared with analytical estimate
Conclusions
## Fisher vs. new model

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<th>new model</th>
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<td>above lin. marginal</td>
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<td>ballistic or evanescent</td>
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Bistability in the wild: testing the model

Two key tests:

- To investigate avalanches: perturb plasma locally, observe spatiotemporal response à la [Van Compernolle et al., 2015]. Need distinguish from linear mode response!

- Can we see ballistic penetration of stable region in numerical experiments? More careful study à la [Yi et al., 2014]

Figure: Cartoon (poloidal cross section) depicting basic setup for avalanching experiment observing response to local pulse.
[Inagaki et al., 2013] is interesting but not the last word. We suggest:

- More basic experiments exploring $\tilde{n}/n$ vs $\nabla T$ hysteresis
- Better resolution of dependence of fluctuation intensity on the input power
- More careful study of relaxation after ECH is turned off
- More information on fluctuation field (e.g. spatial correlations)
- Simultaneous measurement of zonal flow pattern
Spreading in context

- How does spreading affect profiles in a real system?
- Spreading will be most important when profiles force sharp $\nabla I$
- Basic example: NML. Spreading reduces turbulence intensity, leading to increased pedestal height/width — spreading can be “good” for confinement
- More details: see Rameswar Singh’s talk, NO4.2 “When does turbulence spreading matter?”

**Figure:** Intensity and pressure profiles; $\sigma =$spreading strength
Conclusions

- Update to Fisher model that allows for physical fronts separating laminar/turbulent domains and robust penetration of stable regions
- Supported by substantial evidence for subcritical turbulence
- Provides simple framework for understanding avalanching: local exceedance of nonlinear instability threshold by turbulent puffs
- Key testable predictions: ballistic spreading into weakly linearly damped regions, power-law threshold for spreading of puffs
- Need more experiments in the vein of Inagaki to study bistability
<table>
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<tr>
<th>References I</th>
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Isn’t this just quasilinear theory?!  

- Quasilinear theory describes spreading of active region in phase space  
- Related concept but there are key differences  
- TS: active region remains **fixed**  
- Real/phase space distinction important. We can compute propagation speeds  
- QL spreading more similar to avalanching (gradient propagation). Realistic model should incorporate both effects
Cousin models

- Compare to bistable models for subcritical transition to fluid turbulence [Barkley et al., 2015, Pomeau, 2015].
- Compare to [Gil and Sornette, 1996] model for sandpile avalanches

\[ \partial_t S = \gamma \left( |\partial_x h| / g_c - 1 \right) S + \beta S^2 - S^3 + \partial_x (D_S S \partial_x S) \]
\[ \partial_t h = \partial_x (D_h S \partial_x h). \]

- \( S \leftrightarrow I, \ h \leftrightarrow p \)
- Weak gradient coupling limit \( D_h \ll D_S \Rightarrow \) our model
- Strong gradient coupling limit: \( S \) slaved to \( h \). \( \partial_x h \propto S^{-1} \Rightarrow \) linear term is \( c - \gamma S \), where \( c \) is a constant which depends on BCs. Bistable again!