F-I31

On How Stochastic Magnetic Perturbations Influence Dynamics and Relaxation

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Outline

- Why this problem, <u>now</u>?
- Some History – mostly <u>Ancient</u>
- Formulating a “Simple” Problem !?
- Some Insight – inspired by a classic
- Two Scale Formulation and ‘Solution’
- What’s the <u>Physics</u>?

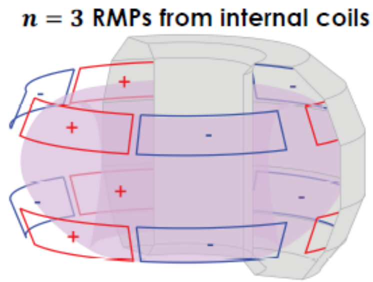
- What’s the Physics? cont’d
  - Nonlinear magnetic torque
  - $\langle E_{\parallel} \rangle$ along perturbed lines
  - ‘Screening’ and small scale $\tilde{\phi}$
  - Convective cells
- Where it stands – formulating a perturbation theory
- Conclusion: Lessons Learned
- Open Issues

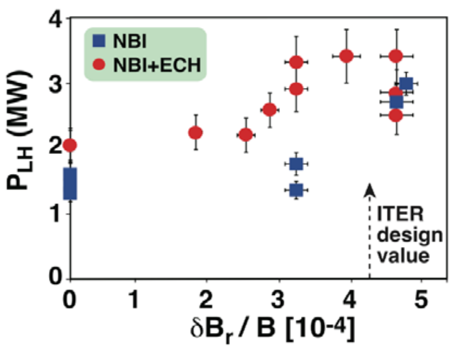
Why Now?

- Syntheses of good confinement and optimal power handing drive us to 3D, likely involving stochastic magnetic fields

i.e.

- RMP raises L→H transition threshold:
  - Stochastic layer ~ separatrix
  - Turbulence persists, though modified
  - Interaction with flows modified
    → decoherence, etc.
- Stellerator: can support stochastic regions





cf: C.C. Chen,
W.X. Guo, L. Schmitz

- Island Configurations (c.f. K. Ida)
  - ITBs, NTM, ...
  - Stochastic layers frequently present, near islands
- Disruptions
  - Thermal quench, etc. results from stochastic field
  - Confinement, including electron momentum (i.e. current quench), also of interest → disruption evolution.

➔ Fluctuations, Flows, $\langle E_r \rangle$ all modified ➔ Turbulence ?! structure

→ tensively studied

How is turbulence modified?

- How does a stochastic field modify the <u>instability process</u>?

  Question motivated by stochastic field transport (~ late 70’s)

Eg:

- <u>Classic of Ancient History</u>: Kaw, Valeo, Rutherford ‘79 et. seq.
  - Tearing, in braided magnetic field
  - ‘anomalous dissipation’ by $\left\{ \begin{array}{l} \text{electron viscosity} \\ \text{hyper-resistivity} \end{array} \right.$ ⇨ rescue resistive MHD

    i.e. $E_\parallel = \eta J_\parallel - \mu \nabla_\perp^2 J_\parallel$
  - lack of, or very simple, micro → macro connection

    Origin, content of μ ?

- Need to re-visit Simple Problem: → insight, guide simulation

- For $l_{mfp} < l_c < l_{macro}$:

$$\frac{\rho_0}{B_0^2} d_t \nabla_\perp^2 \phi = -\frac{1}{\eta} \nabla_\parallel^2 \phi - \frac{g}{B_0} \frac{\partial P}{\partial y} + \nu \nabla_\perp^2 \nabla_\perp^2 \phi$$

$$d_t P = -V_r \frac{dP_0}{dr} + \chi \, \nabla_\perp^2 P$$

- Key: $\boxed{\nabla_\parallel = \nabla_\parallel^{(0)} + \vec{b} \cdot \nabla_\perp}$ → parallel gradient along randomly tilted lines

  – Resistive Interchange – test wave → stability – the questions.....

  – $\vec{b}$ → stochastic variable → static $\langle \tilde{b}^2 \rangle_{k'}$ specified, $\vec{k}'$ s/t $|\vec{k}'| \gg |\vec{k}|$

  i.e. specify 2nd moment of Pdf($\tilde{b}$)

So

$$(\partial_t - \nu\nabla_\perp^2)\nabla_\perp^2\phi = -S\left(\nabla_\parallel^{(0)} + \vec{b}\cdot\nabla_\perp\right)\left(\nabla_\parallel^{(0)} + \vec{b}\cdot\nabla_\perp\right)\phi - \frac{g}{L_p}\partial_y P$$

→ Multiplicative Noise, Stochastic PDE

χ, ν tbd

$$\frac{1}{S} = \frac{\tau_A}{\tau_R}$$

$$\partial_t P - \chi\nabla_\perp^2 P = -\partial_y\hat{\phi} \qquad \nu, \chi : \text{TBD}$$

→ Resembles Schrodinger Eqn. with random potential (c.f. Kraichnan '61)

i.e. $-\frac{\hbar^2}{2m}\nabla^2\psi + U_0(x)\psi + \tilde{U}(x)\psi = E\psi$

Random Coupling Model

N.B.:

- Poses basic question → $\boxed{\text{Impact of 'random bending' ?}}$ → generic to instability processes
- Generic: $\nabla\cdot\vec{J} = 0$ → $\nabla_\parallel J_\parallel + \nabla_\perp J_\perp = 0$

  → Wandering Lines...

- Presume random magnetic perturbations ←→ Stochastic Lines   equivalence

  (dynamics +)   (static)

- Key Issue: Physics of 'Stochastic Bending'

$$\left(\nabla_{\parallel}^{(0)} + \vec{b} \cdot \nabla_{\perp}\right)\left(\nabla_{\parallel}^{(0)} + \vec{b} \cdot \nabla_{\perp}\right)\phi$$

- Insight from the Classics
  - Rechester and Rosenbluth '78: Test Particle Picture

    D_M ; l_{ac} , l_c , l_{mfp} - various orders etc.

  - Kadomtsev and Pogutse '78: $l_{mfp} < l_c$, Hydrodynamic

$\boxed{\nabla \cdot \vec{q} = 0}$ → aspect of self-consistency

$$\vec{q} = -\chi_{\parallel}\nabla_{\parallel}T\widehat{b} - \chi_{\perp}\nabla_{\perp}T$$

$$\vec{b} = \overrightarrow{b_0} + \overrightarrow{\tilde{b}}\ , \quad \nabla_{\parallel} = \nabla_{\parallel}^{(0)} + \vec{b} \cdot \nabla_{\perp}$$

$$\chi_{\parallel} \gg \chi_{\perp}$$

Requiring $\nabla \cdot \vec{q} = 0$ throughout prevents heat accumulation

K + P '78, cont'd

- Aim → mean heat flux
- For $K_u < 1$; (also triplet) ?
- $\langle q_r \rangle = -\chi_{\parallel} \left[\langle \tilde{b}_r^2 \rangle \frac{\partial \langle T \rangle}{\partial r} + \langle \tilde{b}_r \frac{\partial \tilde{T}}{\partial z} \rangle \right] - \chi_{\perp} \nabla_{\perp} \langle T \rangle$

  kinematic $\tilde{T}$ enters from $\widetilde{b \cdot \nabla T} \neq 0$

- For $\tilde{T}$: $\nabla \cdot \vec{q} = 0$ → $\tilde{T}_k = -\frac{\chi_{\parallel} i\, k_{\parallel}\, \tilde{b}_k\, \partial \langle T \rangle / \partial r}{\chi_{\parallel} k_{\parallel}^2 + \chi_{\perp} k_{\perp}^2}$ → cancellations
- N.B.: $\tilde{T}$ adjusts to satisfy $\nabla \cdot \vec{q} = 0$

  $\tilde{b}$ and $\nabla \cdot \vec{q} = 0$ necessitate $\tilde{T}$! $\tilde{T}$ not adjustable, arbitrarily

→ Significant departure from kinematics !

Why revisit K and P? – in this context??

- Structurally Similar...
- $\boxed{\nabla \cdot \vec{J} = 0 \text{ is constraint here}}$

→ system should prevent local charge accumulation! ←→ quasi-neutrality!

How?

- $\nabla_{\|} J_{\|} + \nabla_{\perp} \cdot J_{\perp} = 0$

parallel perp: polarization + P-S

- Given $\tilde{b}$, if $\nabla_{\|} \tilde{J}_{\|} \neq 0$ → $\nabla_{\perp} \cdot \tilde{J}_{\perp} \neq 0$ → $\tilde{\phi}$

Potential Fluctuations!

Stochastic fields → Stochastic cells

Stochastic Field must generate (micro) convective cells!

K and P, and Implications – Here

- Problem is multi-scale: $\bar{\phi}_k$ → test field (mean)

- $\phi = \bar{\phi}_k + \tilde{\phi}_{k'}$ $\quad |\vec{k}'| \gg |\vec{k}|$

$\tilde{b}_{k\prime}$

Potential fluctuations generated on small scale to maintain $\nabla \cdot \vec{J} = 0$

- Approach by method of averaging:

Focus on low order moments, initially.

$$T = \left(\nabla_{\parallel}^{(0)} + \vec{b} \cdot \nabla_{\perp}\right)\left(\nabla_{\parallel}^{(0)} + \vec{b} \cdot \nabla_{\perp}\right)\phi$$ → bending

➔

$$\bar{T} = \nabla_{\parallel}^{(0)2}\bar{\phi} + \nabla_{\perp} \cdot \langle \tilde{b}\tilde{b} \rangle \cdot \nabla_{\perp}\bar{\phi} + \langle \nabla_{\parallel}^{(0)}\tilde{b} \cdot \nabla_{\perp}\tilde{\phi} \rangle + \langle (\tilde{b} \cdot \nabla_{\perp})\nabla_{\parallel}^{(0)}\tilde{\phi} \rangle$$

'mean field' → $\vec{k}$

How get $\tilde{\phi}$?

K and P, and Implications, cont'd

dielectric operators

(enforce $\nabla \cdot \vec{J} = 0$ on k' scale)

- For $\tilde{\phi}$: $L_{k'}\,\tilde{\phi}_{k'} = \tilde{T}_{k'}$

- $\tilde{T} = \nabla_\perp \cdot \tilde{b}\nabla_\| \bar{\phi} + \nabla_\|^{(0)} \tilde{b} \cdot \nabla_\perp \bar{\phi}$

- $\tilde{\phi}$ is 'screened' response to $\tilde{b}_{k'}\bar{\phi}$

- $\tilde{b}_{k'}$ determines $\tilde{\phi}_{k'}$, via $\tilde{b}\,\bar{\phi}$ and response

- Of course, $\vec{k}$ scale, $\vec{k}'$ scale $\Big\}$ evolutions coupled

ultimately in $\bar{\phi}$ evolution

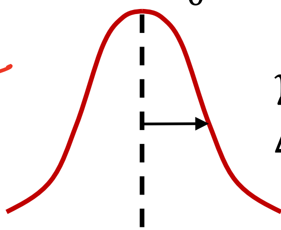
$\Rightarrow$ Feedback Loops....

Two Scale Formulation and 'Solution'

- The problem

Test Mode

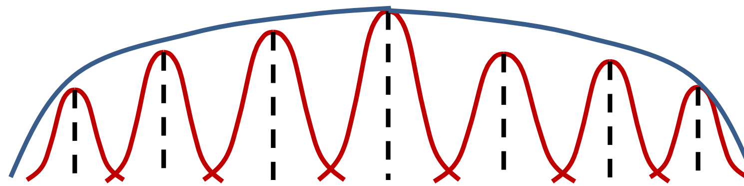
$\vec{k} \cdot \vec{B}_0 = 0$



$\gamma \sim S^{-1/3}\tau_A^{-1}$
$\Delta x \sim S^{-1/3}a$

+

Spectrum of prescribed,
static magnetic fluctuations

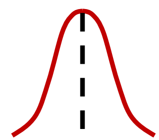


Densely Packed

low m

Resistive Interchange k

(single test mode)



$\vec{k}' \cdot \vec{B}_0 = 0$ { spectral scale specified

→ $|k'| \gg |k|$ ➔ multi-scale

→ Stability, intensity etc.

Many Questions...

$|b_{k'}|^2 = |b_0|^2 S(k_\theta)\Gamma(r - r_{k'})$

spatial form factor

$w_{k'} \ll \Delta x$

Toward a Solution

$\phi = \bar{\phi} + \tilde{\phi}$

$\bar{\phi} \rightarrow \vec{k}$ envelope

$\tilde{\phi}, \tilde{b}_{k'} \rightarrow \vec{k}'$

Envelope:

$$(\partial_t - \nu \nabla_\perp^2)\nabla_\perp^2 \bar{\phi} + \frac{S}{\tau_A}\, \partial_x \underbrace{|\tilde{b}_r|^2}_{①} \partial_x \bar{\phi} = -\frac{S}{\tau_A} \nabla_\parallel^{(0)2} \bar{\phi} - \frac{g}{L_P} \partial_y \bar{P} + \underbrace{\langle \nabla_\parallel^{(0)} \nabla_\perp \cdot (\tilde{b}\tilde{\phi}) \rangle}_{②} + \underbrace{\nabla_\perp \cdot \langle \tilde{b} \nabla_\parallel^{(0)} \tilde{\phi} \rangle}_{③}$$

$$(\partial_t - \chi \nabla_\perp^2)\bar{P} = -\bar{V}_r$$

Small Scale Fluctuation: Inversion $\rightarrow$ $\boxed{G(x, x'')}$

Greens Function

$$(\partial_t - \nu \nabla_\perp^2)\nabla_\perp^2 \tilde{\phi}_{k'} + \frac{S}{\tau_A} \nabla_\parallel^{(0)} \tilde{\phi}_{k'} + \frac{g}{L_P}\, \partial_y \tilde{P}_{k'} = -\frac{S}{\tau_A}\left[\nabla_\perp \cdot \left(\tilde{b}_{k'} \nabla_\parallel^{(0)} \bar{\phi}\right) + \nabla_\parallel^{(0)} \left(\tilde{b}_{k'} \nabla_\perp \bar{\phi}\right)\right]$$

$\tilde{P}$ equation

What's the Physics? - what does this mess mean?

N.B.: Clearly much more than hyper-resistivity....

①

$$\frac{S}{\tau_A}\,\partial_x\left|\tilde{b}_r\right|^2\partial_x\bar{\phi}$$

→ $\boxed{\text{magnetic vorticity damping}}$

→ 3rd order $\nabla_\parallel J_\parallel$

from

$$\sim\ \hat{b}\cdot\nabla_\perp\left(-\frac{1}{\eta}\hat{b}\cdot\nabla_\perp\right)\bar{\phi}$$

Re-express: $\frac{S}{\tau_A}\left|\frac{\tilde{B}_{rk'}}{B_0}\right|^2=\frac{V_A^2}{\eta}\frac{k_\theta'^2}{L_s^2}\,{w_I'}^4$

$w_I'\equiv$ island width

for stochastic field

(rms)

So: $\frac{S}{\tau_A}\,\partial_x\left|\tilde{b}\right|^2\partial_x\bar{\phi}\ \sim\ \frac{V_A^2}{\eta}\frac{k_\theta'^2}{L_s^2}\frac{{w_I'}^4}{(\Delta x)^2}\bar{\phi}$

estimate

$\bar{\phi}$ layer width

Magnetic Torque, cont'd

$(\nabla_\parallel J_\parallel)^{(1)} \sim \frac{V_A^2}{\eta}\frac{k_\theta^2}{L_s^2}(\Delta x)^2\,\bar{\phi}$ → bending term, linear

Key Question:

$(\nabla_\parallel J_\parallel)^{(3)} \sim \left(\nabla_\parallel J_\parallel^{(1)}\right)$ ←→ When will 3rd order magnetic torque balance first order

→ $\boxed{w_I' \sim \left[\frac{k_\theta^2}{k_\theta'^2}(\Delta x)^4\right]^{1/4}}$ → Width of small scale island needed.

→ Reminiscent of Rutherford '73; but with $k_\theta^2/k_\theta'^2$ factor, due multi-scale character

$(\nabla_\parallel J_\parallel)^{(2)} > \left(\nabla_\parallel J_\parallel^{(1)}\right)$ → magnetic torque supplants inertia in vorticity balance

unambiguously stabilizing basic vortex flow of mode

E-fields along Perturbed Lines ②, ③

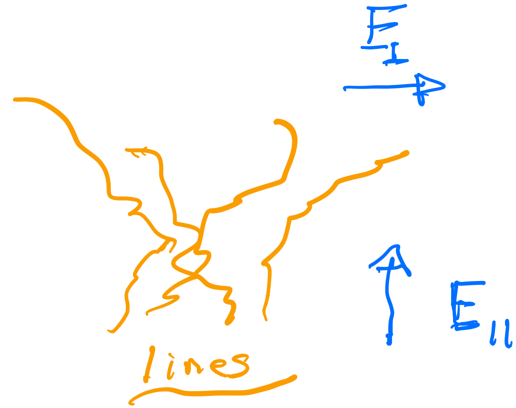
The rest...

Consider:

②

$$\left(\nabla_\parallel^{(0)} + \tilde{b}\cdot\nabla_\perp\right)\left(-\frac{1}{\eta}\left(\nabla_\parallel^{(0)} + \tilde{b}\cdot\nabla_\perp\right)\right)(\bar{\phi}+\tilde{\phi})$$

③



$$③ = \langle \nabla_\parallel^{(0)}(\nabla_\perp\cdot\tilde{b}\tilde{\phi})\rangle = \langle -\nabla_\parallel^{(0)}\left(\tilde{b}_\perp\cdot\vec{\tilde{E}}_\perp\right)\rangle$$

$$② = -\nabla_\perp\cdot\langle\tilde{b}\nabla_\parallel\tilde{\phi}\rangle = -\nabla_\perp\cdot\langle\tilde{b}_\perp\tilde{E}_\parallel\rangle$$

E field projections along wandering lines

$$\vec{\tilde{E}} = -\nabla_\perp\tilde{\phi} \quad , \qquad \tilde{\phi} \sim \tilde{b}\phi$$

$\nabla_\parallel J_\parallel \not\to 0$ at test wave resonant surface.

→ Nonlinear Bending + Resistivity → Dissipative Nonlinearity !

Screening, Small Scale $\tilde{\phi}$ and Convective Cells

How obtain $\tilde{\phi}$?

mean potential

$$L_{k+k'}\tilde{\phi}_{k+k'} = C\,\tilde{b}_k\,\bar{\phi}_k$$

→ ~ Langevin Eqn.

eigen mode operator with ν, χ

C.C.

noise/modulation (multiplicative)

$$\left(\partial_t V + \frac{\gamma}{m} V = \frac{\tilde{f}}{m}\right)$$

Convenient to take

k → slow interchange

k' → fast interchange

$|k| \ll |k'|$:

$$L_{k'}\tilde{\phi}_{k'} = C\,b_{k'}\bar{\phi}$$

$$\boxed{\tilde{\phi} = \int dr''\, G(r,r'')C b_{k'}\bar{\phi}}$$

→ obtains $\tilde{\phi}$ via Green's function

Screening, cont'd

How determine $\bar{\phi}$?

- Langevin Eqn. ←→ Fluctuation-Dissipation Theorem (?!)

$$\left|\tilde{\phi}_{k'}\right|^2 \approx \frac{|c|^2 \, |b_{k'}|^2 \left|\bar{\phi}\right|^2}{L_{-k'} L_{k'}}$$

$L_k \equiv$ operator

→ $k_\theta'^2 \gg k_\theta^2$

∴ fast interchange

~ stationarity → damped response → L must be $\boxed{\text{over-stable}}$

- ν, χ → turbulent diffusion from small scale electrostatic cells — $\underline{\nabla} \cdot \underline{\nabla} \nabla^2 \phi$ $\underline{\nabla} \cdot \underline{\nabla} \phi$

$$\nu, \chi \;\to \nu_T \qquad \nu_T \sim \left(g/L_p\right)^{1/2} k_\theta'^{\,-2} + \delta\nu_{\mathrm{T}}$$

saturated

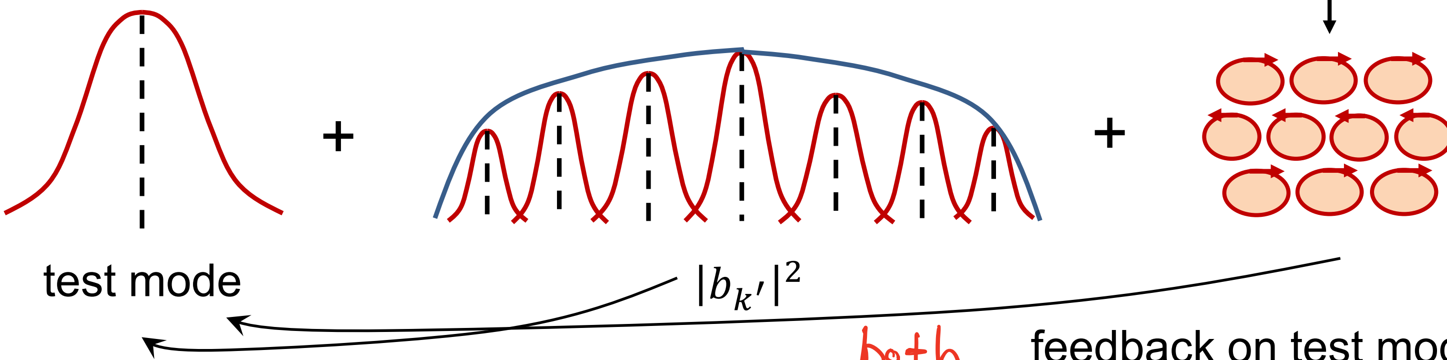
small increment add'l

$$\nu_T \approx \sum_{k'} |c_{k'}|^2 \langle \tilde{b}^2 \rangle_{k'} \left|\bar{\phi}\right|^2 \gamma_{k'}^{-1} \Big/ \left[k_\theta'^2 - \frac{g k_\theta'^{\,2}}{L_p \left(\nu_T k_\theta'^{\,2}\right)^2} \right]^2 \Rightarrow \underline{\delta\nu_T}$$

Screening, cont'd

- System really is :

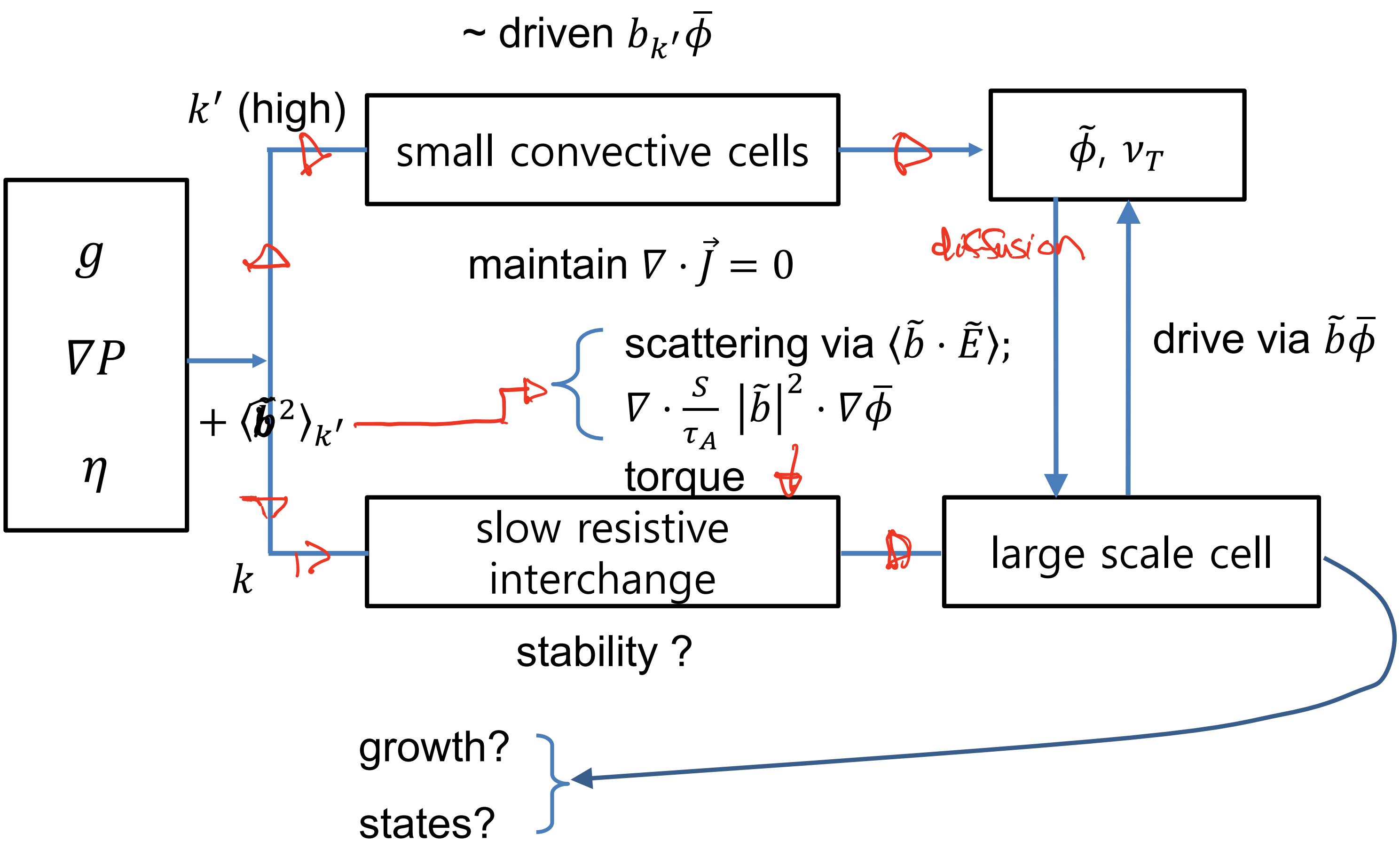
small scale cells driven



both feedback on test mode

- $\nu, \chi \;\rightarrow\;$ turbulent diffusion, due $|\tilde{\phi}|^2$
- Multi-scale interaction branches thru ES, Magnetic Scattering

The Big Picture:



Where Things Stand

- Integro-differential equation for $\bar{\phi}$ evolution in presence specified $|b_{k'}|^2$
- Technically complex...
- $(\nabla_{\parallel} J_{\parallel})^{(3)}$ magnetic torque is clear and novel effect, damping vorticity
- Can formulate perturbation theory $\gamma_k \rightarrow \gamma_k^{(0)} + \delta\gamma_k$, in terms quadratic form
- Crank ongoing ...

Conclusions – Lessons Learned, so far…

- Problem of instability in stochastic field is intrinsically multi-scale and dynamic: $\bar{\phi}$; $\tilde{\phi}$ and $\tilde{b}$
- To maintain $\nabla \cdot \vec{J} = 0$ for prescribed $\tilde{b}_{k'}$ + instability ➔ $\tilde{\phi}$ generated
- Physics: $\nabla_\perp \cdot J_\perp \neq 0$ to maintain $\nabla \cdot \vec{J} = 0$ ➔ Enter electrostatic micro-cells !
- Magnetic vorticity damping is generic to stochastic $\tilde{b}$ + turbulence
- Inertia → Inertia + $\frac{S}{\tau_A} \partial_x |b_r|^2 \partial_x \bar{\phi}$
- FOM : w_I' vs $\left[\left(k_\theta^2 / k_\theta'^{\,2}\right) (\Delta x)^4\right]^{1/4}$ For: $\nabla_{\parallel} J_{\parallel}^{(3)} \sim \nabla_{\parallel} J_{\parallel}^{(r)}$

Conclusions – Lessons Learned, so far...

- More generally, for turbulence $\tilde{\phi}$ in stochastic $\tilde{b}$; cannot treat as statistically independent i.e. $\boxed{\langle \tilde{b}\,\tilde{\phi}\,\rangle \neq 0}$
- small scale $\tilde{b}$ leaves 'footprint' on modes

Look Ahead:

- Complete analysis – bistability ?
- More effects ? → the usual...
- To resistive interchange turbulence... → flows ....
- Collisionless ➔ Alfven radiation into network of $\langle \tilde{b}^2 \rangle$

- Statistical analysis
  Pdf $(\tilde{b})$ → Distribution of Eigenvalues

(c.f. C-C Chen, this meeting)

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