A Mean Field Model of the L→H Transition in a Stochastic Magnetic Field

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Motivation and Background

Mean Field Model (a work in progress)

- What determines $\langle v'_E \rangle$?
  
  **Key novelty:** $\langle J_r \rangle$ induced by magnetic perturbation
  
  $\rightarrow \langle V_\theta \rangle$ and $\langle V_\phi \rangle$ evolution
  
  $\rightarrow$ Particle and heat flux
  
  Turbulence intensity

- Modified 1D “Predator-Prey” model for $\langle E_r \rangle'$

Lessons learned, so far

Conclusion and Open Issues
Stochastic magnetic field

- The phenomenon of **chaos of magnetic field lines** is known as magnetic stochasticity (or magnetic chaos)

- In early studies, magnetic stochasticity is thought to be **bad for confinement** due to the enhanced radial transport of particles and energy along the chaotic field lines.

- However, at end of 1970s, magnetic stochasticity can be used to control the transport of energy and particles. → **stochasticity as a positive**: ergodic divertor concept
Most of today’s tokamaks need to use RMP (resonant magnetic perturbation) before $L \rightarrow H$ transition, in order to mitigate/suppress large ELMs, including first

RMPs are thought to produce stochastic layer near separatrix

The stochastic B field observed to influence

- threshold and power
- $V_{E \times B}$ and flows
- fluctuations

Stochastic magnetic field induced by RMP
Motivation (why?)

- RMPs increase $P_{th}$ of L$\rightarrow$H transition (experiments on DIII-D, MAST, AUG and KSTAR).

Here, stochastic B-field is thought to be caused by resonant $\delta B$.

Need model to elucidate the physical mechanism.

So, how stochastic B-field affects $\langle \nu_E' \rangle$?
\( \langle E_r \rangle \) structure: reversal by RMPs

\( E_r \) reversal or ‘bifurcation’: \( E_r \) well reduced or inverted to \( E_r \) hill with increasing RMP. Edge \( E_r \) shear layer sits in stochastic field region.

Clear Change in \( E'_r \) due to increase of RMP field
Reduced toroidal/poloidal flow by RMPs

L. Schmitz et al NF 2019

RMP increased toroidal (co)rotation and shear at separatrix

Clear effect on flows, and would further affect

Kriete, PoP 2020

RMP reduces mean turbulence poloidal velocity

\[ E_r = \frac{1}{enB} \frac{\partial}{\partial r} \left( P_i + v_\phi B_\theta - v_\theta B_\phi \right). \]
Fluctuations point: RMPs degrade energy transfer to mean flows prior to L-H transition

Energy balance between turbulence and mean flow\(^{1,2}\)

**Mean flow energy**: \( E_\bar{v} = \frac{1}{2} n_0 m_i \langle \tilde{v}_\theta \rangle^2 \)

**Turbulent flow energy**: \( E_\tilde{v} = \frac{1}{2} n_0 m_i (\langle \tilde{v}_r^2 \rangle + \langle \tilde{v}_\theta^2 \rangle) \)

**Thermal free energy**: \( E_\tilde{n} = \frac{1}{2} n_0 T_e \langle \tilde{n}_e / n_0 \rangle^2 \)

- \( E_\bar{v} \) **increases** prior to L-H in axisymmetric case
- \( E_\bar{v} \) **decreases** due to RMPs, but total turbulent energy only changes slightly
- Power transfer to zonal flow decreases

**Mean flow energy** \( E_\bar{v} \)

- **no** MPs
- **non-res.** MPs
- **resonant** MPs

**Total turbulent energy** \( E_\tilde{n} + E_\tilde{v} \)

Need more power to supply sufficient energy to the mean flow
Reduced Reynold stress/force by RMP

Y. Xu, 2007 NF

\[ \langle \frac{\partial \tilde{V}_y}{\partial \tilde{V}_y} \rangle \times 10^6 \text{ m}^2/\text{s}^2 \]

![Graph showing the effect of RMP on Reynold stress/force](image)

- Increase \( I_{\text{DED}} \) reduces Reynold force.
- RMP reduces Reynold force in L-mode, thus increasing power required for H-mode.

Effects of RMP on transition? ↔ Model!

Kriete, PoP 2020

![Graph showing the effect of RMP on Reynold stress/force](image)
Model – $\langle E_r \rangle$ as critical

- Goal: ascertain change due to RMP mean field model for $\langle E_r \rangle$ and $\langle \nu'_E \rangle$

  - Radial force balance equation: $\langle E_r \rangle$ is determined by

    $$\langle E_r \rangle = \frac{\langle \nabla P_i \rangle}{ne} - \langle V_\theta \rangle B_\phi + \langle V_\phi \rangle B_\theta$$

    **Diamagnetic drift**       **Poloidal rotation**       **Toroidal rotation**

    $\Gamma_e, Q_i$                $\langle J_r \rangle \leftrightarrow \langle \tilde{b}_r \tilde{b}_\theta \rangle$

  - Direct effect of $|\tilde{b}_r|^2$ on turbulence discussed later. Take turbulence as electrostatic in L-mode.

  - Prescribe stochastic B-field
\[ \langle J_r \rangle \text{ is key to } L \rightarrow H \text{ trigger mechanism} \]

- **L→H transition:**
  - \[ V'_E \text{ shearing feedback (mainly } \nabla P) \]
  - “trigger” due to \[ \langle J_r \rangle \]

- **Several candidates for \( \langle J_r \rangle \)**
  - Radial flux of polarization charge \( \rightarrow \) turbulence intensity gradient scale
    \[ \rho_s^2 \langle \tilde{v}_r \nabla^2 \tilde{\phi} \rangle \rightarrow \frac{\partial}{\partial r} \langle \tilde{V}_\theta \tilde{V}_r \rangle : \text{Reynolds force} \]
  
  **Critical Reynolds work criterion** [Rich literatures: Diamond, Tynan et al]

  - Neoclassical polarization \( \rightarrow \rho_{\theta i} \text{ scale} \) [S-I. Itoh, et al PRL, 60 (1988) 2276]

  - Orbit loss \( \rightarrow \rho_{\theta i}, \ldots \) [K.C.Shaing, PoF B: Plasma Physics 4 (1992) 171]

  - **New player here:** \( \langle J_r \rangle = \frac{\langle \tilde{j}_{\|} \tilde{B}_r \rangle}{B} \)

  stochastic field intensity profile scale, how enters?
Common element: $\langle J_r \rangle$ induced by stochastic B-field

- Ambipolarity breaking due to stochastic field $\Rightarrow \langle J_r \rangle$

$$\langle J_r \rangle = \langle \vec{J} \cdot \hat{e}_r \rangle = \frac{\langle \vec{J}_\parallel \vec{B}_r \rangle}{B} \quad \langle J_\parallel \rangle = \langle J_{\parallel,e} \rangle + \langle J_{\parallel,i} \rangle$$

- From Ampere law: $\vec{J}_\parallel = -\frac{c}{4\pi} \nabla^2 \vec{A}_\parallel$

**Stochastic field produces currents in plasmas**

$$\langle J_r \rangle = \frac{\langle \vec{J}_\parallel \vec{B}_r \rangle}{B} = -\frac{c}{4\pi B} \left[ \frac{\partial}{\partial y} \vec{A}_\parallel \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \vec{A}_\parallel \right]$$

$$= -\frac{c}{4\pi B} \frac{\partial}{\partial x} \left[ \left( \frac{\partial}{\partial x} \vec{A}_\parallel \right) \left( \frac{\partial}{\partial y} \vec{A}_\parallel \right) \right] = \frac{c}{4\pi B} \frac{\partial}{\partial x} \langle \vec{B}_x \vec{B}_y \rangle$$

$$= \frac{cB}{4\pi} \frac{\partial}{\partial x} \langle \vec{b}_x \vec{b}_y \rangle \Rightarrow \frac{cB_0}{4\pi} \frac{\partial}{\partial r} \langle \vec{b}_r \vec{b}_\theta \rangle$$

Note: $\langle J_r \rangle$ tracks momentum, not heat transport.
Phase in Maxwell stress

- Maxwell stress \( \langle \vec{B}_r \vec{B}_\theta \rangle = \sum_k |\vec{A}_k|^2 \langle k_x k_y \rangle \)

\( \vec{A}_k \) tilted by developing \( E \times B \) flow

\[
\frac{\partial A_x}{\partial t} + V \cdot \nabla A = \eta J \\
\frac{\partial A_y}{\partial t} + V'_E \frac{\partial A}{\partial y} + \vec{V} \cdot \nabla A = \eta J
\]

\[
k_x = k_x^{(0)} - k_y V'_E \tau_c
\]

\( \tau_c: \{ \text{shear fluctuations} \} \)

- Hence, \( \langle \vec{B}_r \vec{B}_\theta \rangle = - \sum_k |\vec{A}_k|^2 \langle k_y^2 V'_E \tau'_c \rangle \)

- Hence, Reynolds and Maxwell cross-phase closely linked.

F-19
30min
Chang-Chun Chen
University of California, San Diego
On How Decoherence of Vorticity Flux by Stochastic Magnetic Fields Quenches Zonal Flow Generation
Stochastic B-field affects $\langle V_\theta \rangle$

- For $V_\theta$:
  Poloidal momentum balance

  \[
  \frac{\partial \langle V_\theta \rangle}{\partial t} = -\mu (\langle V_\theta \rangle - V_{\theta,\text{neo}}) - \frac{\partial}{\partial r} \left( \langle \tilde{V}_\theta \tilde{V}_r \rangle - \frac{1}{4\pi \rho} \langle \tilde{B}_r \tilde{B}_\theta \rangle \right)
  \]

- For SS:  $\langle V_\theta \rangle = V_{\theta,\text{neo}} - \frac{1}{\mu} \frac{\partial}{\partial r} \left( \langle \tilde{V}_\theta \tilde{V}_r \rangle - \frac{1}{4\pi \rho} \langle \tilde{B}_r \tilde{B}_\theta \rangle \right)$

  \[
  = V_{\theta,\text{neo}} - \frac{1}{\mu} \frac{\partial}{\partial r} \left( \frac{1}{B^2} \tau_c V'_E \frac{l}{1 + \alpha V'^2_E} - \frac{B^2}{4\pi \rho} \tau'_c V'_E |\tilde{b}_r|^2 \right)
  \]

  with

  \[
  \mu = \mu_0 (1 + \frac{\nu_{C_X}}{\nu_{ii}}) \nu_{ii} q^2 R^2 \quad V_{\theta,\text{neo}} \approx -1.17 \frac{\partial T_i}{\partial r} \quad \tau'_c = \left( \frac{k^2 V'^2_E D_T}{3} \right)^{-1/3}
  \]

- $V'_E$ phasing via tilt tends to align turbulence and stochastic B-field, which counteracts the spin-up of $\langle V_\theta \rangle$.

- $\frac{\partial}{\partial r} |\tilde{b}_r|^2$ not only $|\tilde{b}_r|^2$, plays a role.
Stochastic B-field affects $\langle V_\phi \rangle$

- For $V_\phi$: 
  \[
  \frac{\partial \langle V_\phi \rangle}{\partial t} + \nabla \cdot \langle \vec{V}_r \vec{V}_\phi \rangle = \frac{1}{\rho_c} \langle J_r \rangle B_\theta + S_M
  \]
  \[
  \langle \vec{V}_r \vec{V}_\phi \rangle = -\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle, \quad \chi_\phi = \chi_T = \frac{\rho_s^2 c_s}{L_T}, \quad S_M = S_a \exp\left(-\frac{r^2}{2L_{M,dep}^2}\right)
  \]

  Only consider diffusive term.

- G.B.

  Momentum source tracks heat (from core).

  \[
  \Rightarrow \frac{\partial \langle V_\phi \rangle}{\partial t} = \frac{\partial}{\partial r} \left( \chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle \right) + \frac{1}{4\pi \rho} \frac{B_\theta}{B} \frac{\partial}{\partial r} \langle \vec{B}_r \vec{B}_\theta \rangle + S_M
  \]

- For SS:
  \[
  \frac{\partial}{\partial r} \left( \chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle \right) = -\frac{V_{Ti}^2 B_\theta}{\beta B} \frac{\partial}{\partial r} \langle \vec{b}_r \vec{b}_\theta \rangle - S_M
  \]

  Stochasticity edge toroidal velocity, shear

  \[
  \frac{\partial}{\partial r} \langle V_\phi \rangle |_{r_{sep}} = -\frac{1}{\chi_\phi} \int_0^{r_{sep}} S_M dr - \frac{V_{Ti}^2 B_\theta}{\beta \chi_\phi B} \langle \vec{b}_r \vec{b}_\theta \rangle |_{r_{sep}}
  \]

  Integrated external torque

  With $\langle \vec{b}_r \vec{b}_\theta \rangle = k_T^2 V_E' \tau_c' |\vec{b}_r|^2$

  ✓ Force through radial current across separatrix.

  ✓ Shear affected by stochasticity.
Stochastic B-field affects electron density flux

- For electron density:
  \[
  \frac{\partial n_e}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r\Gamma_e) = S_p
  \]

  with \( \Gamma_e = -(D_{\text{neo}} + D_T) \frac{\partial n}{\partial r} + \Gamma_{e,\text{stoch}} \) \( \Gamma_{e,\text{stoch}} \) \( D_{\text{neo}} = (m_e/m_i)^{1/2} \chi_{i,\text{neo}} \)
  \( D_T \sim bD_{\text{GB}} \) with \( b<1 \)

  \[
  S_p = \Gamma_a \frac{a - r + d_a}{L_{\text{dep}}^2} \exp\left(-\frac{(a + d_a - r)^2}{2L_{\text{dep}}^2}\right)
  \]

- The stochastic field can induce particle flux (\( n_e = n_i \)):
  \[
  \Gamma_{e,\text{stoch}} = \frac{c}{4\pi e} \langle \tilde{b}_r \nabla_\perp^2 \tilde{A}_\parallel \rangle + n \langle \tilde{V}_{\parallel,i} \tilde{b}_r \rangle
  \]

  with
  \[
  \langle \tilde{b}_r \tilde{f}_\parallel \rangle \quad \langle \tilde{b}_r \tilde{f}_{\parallel,i} \rangle
  \]

  \[
  \checkmark \frac{c}{4\pi e} \langle \tilde{b}_r \nabla_\perp^2 \tilde{A}_\parallel \rangle = -\frac{c}{4\pi e} \frac{\partial}{\partial r} \langle \tilde{b}_r \tilde{b}_\theta \rangle = -n \frac{D_B}{\beta} \frac{\partial}{\partial r} \langle \tilde{b}_r \tilde{b}_\theta \rangle
  \]

  \[
  \checkmark n \langle \tilde{V}_{\parallel,i} \tilde{b}_r \rangle : \text{parallel ion flow along tilted field lines}
  \]
Stochastic B-field affects electron density flux

- The stochastic field can induce particle flux:

\[ \Gamma_{stoch} = \frac{c}{4\pi e} \langle \tilde{b}_r \nabla^2_\perp \tilde{A}_\parallel \rangle + n \langle \tilde{V}_{\parallel,i} \tilde{b}_r \rangle \]

- And from \( \nabla_\parallel V_\parallel \cong 0 \), \( \tilde{v}_{\parallel,i} \sim -\tilde{b}_r \frac{\partial \langle V_{\parallel,i} \rangle}{\partial r} / (ik_\parallel) \)

\[ \langle \tilde{b}_r \tilde{V}_{\parallel,i} \rangle = -D_{M, \text{eff}} \frac{\partial \langle V_{\parallel,i} \rangle}{\partial r} \]

\( \rightarrow \) Modify \( l_{ac} \) due to \( E \times B \) shear

\[ l_{ac, \text{eff}} = \frac{l_{ac} l_{V'_E}}{l_{ac} + l_{V'_E}}, \quad l_{ac} = qR, \quad l_{V'_E} = \frac{C_s}{|k_\theta \Delta V'_E|} \sim \frac{C_s}{|V'_E|} \]

\( V_{\text{pinch}} \) is not diffusive, induced by stochastic B field

\( \checkmark \) Flow gradient drives particle flux.
Ion heat flux with stochastic field

- For ion temperature:
  \[
  n \frac{\partial T_i}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r Q_i \right) = S_H
  \]

  with
  \[
  Q_i = -n \left( \chi_{i,\text{neo}} + \chi_{i,T} \right) \nabla T_i + Q_{i,\text{stoch}}
  \]

  \[
  S_H = Q_a \exp \left( -\frac{r^2}{2L_{h,\text{dep}}} \right)
  \]

- The stochastic field affects ion heat flux:
  \[
  Q_{i,\text{stoch}} = \int V_\parallel \langle \tilde{B}_r \delta f \rangle (V_\parallel^2 + V_\perp^2) = -\frac{\partial \langle T_i \rangle}{\partial r} \sqrt{\chi_{i,\parallel i} \chi_{i,\perp i} \langle \tilde{b}_r^2 \rangle} l_{ac} k_\perp
  \]
  \[
  = -\nu_{th,i} D_M, \text{eff} \frac{\partial \langle T_i \rangle}{\partial r} \approx -D_{B,i} D_M \sqrt{k_\perp^2} \frac{\partial \langle T_i \rangle}{\partial r}
  \]

- Potentially important as threshold power is directly related to heat flux.
- Power uptake determines turbulence and Reynolds force.
Turbulence intensity

For turbulence intensity:

\[
\frac{\partial I}{\partial t} = \frac{\gamma_L}{(1+\alpha V'_E)^\sigma} I - \beta I^2 \rightarrow I = \frac{\gamma_L}{(1+\alpha V'_E)^\sigma} \frac{1}{\beta}
\]

\(\gamma_L\) — growth rate, \(\beta\) — nonlinear decay rate, \(V'_E\) — \(E \times B\) shear rate

\[
\frac{\gamma_L}{(1+\alpha V'_E)^\sigma} \sim \beta \quad \alpha = \frac{1}{\alpha_0(c_s/(qR))^2}, \quad \alpha_0\text{ and } \sigma\text{ are adjustable.}
\]

\[
\gamma_L = \gamma_{L0} \left(\frac{C_s}{R}\right) \sqrt{\frac{R}{L_T} - \left(\frac{R}{L_T}\right)_{\text{crit}}} \sim \gamma_{L0} \left(\frac{C_s}{R}\right), \quad \gamma_{L0} \approx 0.01
\]

threshold
So, mean field “Predator-Prey” model

- RFB equation: \[ \langle V_E \rangle' = \frac{1}{eB} \frac{\partial}{\partial r} \langle \nabla P_i / n \rangle - \frac{\partial}{\partial r} \langle V_\theta \rangle + \frac{B_\theta}{B} \frac{\partial}{\partial r} \langle V_\phi \rangle \]

- Five field model:
  1. Turbulence intensity
     \[ \frac{\partial}{\partial t} I = \frac{v_L}{(1 + \alpha V_E'^2)} I - \beta I^2 \]
  2. Ion temperature
     \[ n \frac{\partial T_i}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r Q_i) = S_H \]
  3. Electron density
     \[ \frac{\partial n_e}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_e) = S_p \]
  4. Poloidal flow
     \[ \langle V_\theta \rangle = V_{\theta,neo} + \frac{1}{\mu} \frac{\partial}{\partial r} \left( \frac{1}{B^2} \tau_c V_E' \frac{I}{1 + \alpha V_E'^2} - \frac{B^2}{4\pi \rho} \tau_c' V_E' \frac{|\tilde{b}_r|^2}{B^2} \right) \]
  5. Toroidal flow
     \[ \frac{\partial}{\partial r} \langle V_\phi \rangle |_{r_{sep}} = - \frac{1}{\chi_\phi} \int_0^{r_{sep}} S_M dr - \frac{V_{Ti}^2}{\beta \chi_\phi} \frac{B_\theta}{B} \langle \tilde{b}_r \tilde{b}_\theta \rangle |_{r_{sep}} \]

\[ \langle J_r \rangle \] directly related to \( \Gamma_e \) and Maxwell stress in flows
The sources

\[ S_H = Q_a \exp\left(-\frac{r^2}{2L_{h,dep}^2}\right) \]  \( L_{h,dep} = 0.15a \)

Heat source

\[ S_p = \Gamma_a \frac{a - r + d_a}{L_{dep}^2} \exp\left(-\frac{(a + d_a - r)^2}{2L_{dep}^2}\right) \]  \( L_{dep} = 0.1a \)

Particle source

\[ S_M = S_a \exp\left(-\frac{r^2}{2L_{M,dep}^2}\right) \]  \( L_{M,dep} = 0.15a \)

Momentum

- Now momentum source tracks heat, but with different coefficients
- Try to change the direction of rotation

\[ |\tilde{b}_r| = |\tilde{b}_0| \frac{1}{(L_{dep} - r)^3} \]  \( L_{dep} = 1.05a \)

Perturbed magnetic field

- Adjust \( |\tilde{b}_0| \), now use \( 10^{-4} \sim 10^{-3} \), may broaden the width of stochastic layer by changing profile

Stochastic layer

Heat/momentum source

Particle source
Lessons learned

- Novel \( \langle J_r \rangle \) enters due to Stochastic field

- Generates Maxwell stress/force, which competes with Reynolds stress/force

- Effects on
  - flow (poloidal and toroidal velocity)
  - phase
  - transport (particle and thermal)

- Reynolds and Maxwell phases linked by shearing

\[
\langle \tilde{b}_r \tilde{b}_\theta \rangle = k_y^2 V'_E \tau'_c |\tilde{b}_r|^2
\]
Ongoing: toward to 1D solution

Reduced model as before, the model with:

- $|\tilde{b}_r|^2$
  Amplitude; scale
- Reduced RS in $\langle V_\theta \rangle$
  Quenching the spin-up of $V_\theta$
- Intrinsic torque in $\langle V_\phi \rangle$
- Effect of $|\tilde{b}_r|^2$ on $Q_i, \Gamma_e$

Goals:

- Output the time evolution of $I, V_\phi, V'_E, n, T_i$ for $L \to H$ physics modified by stochastic B-field.

$Q_{i,crit}(|\tilde{b}_r|^2, \frac{\partial}{\partial r} |\tilde{b}_r|^2 \ldots \ldots)$
Conclusions

- Ambipolarity breaking ⇒ $\langle \tilde{b}_r \tilde{b}_\theta \rangle$, contribute to $\langle J_r \rangle$
- Both amplitude and profile of $|b_r|^2$ matter.
- $V'_E$ phasing ⇒ stochastic $\langle \tilde{b}_r \tilde{b}_\theta \rangle$ opposes turbulence $\langle \tilde{V}_r \tilde{V}_\theta \rangle$
  phase linked
- Intrinsic toroidal torque, such that reversal or spin-up of edge $\langle V_\phi \rangle$ occur with RMP, $\langle \tilde{b}_r \tilde{b}_\theta \rangle$ enters edge $\langle V_\phi \rangle$
- $|b_r|^2$ can modify $T_i$ and $n_e$ profiles
- ......
Open issues

- Direct effect of stochasticity on turbulence ($l_K$ v.s. $l_\parallel$ v.s. $l_{mf_p}$)
- Collisionality scaling
- Cost of $P_{th}$ to obtain H-mode with $E_r > 0$ ($E_r$ hill) ⇒ Trade-off for particle control?
- RMP → island + stochastic region
- ……
Thank you very much!

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