A unified theory of zonal flows and corrugations in drift wave turbulence

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Motivation

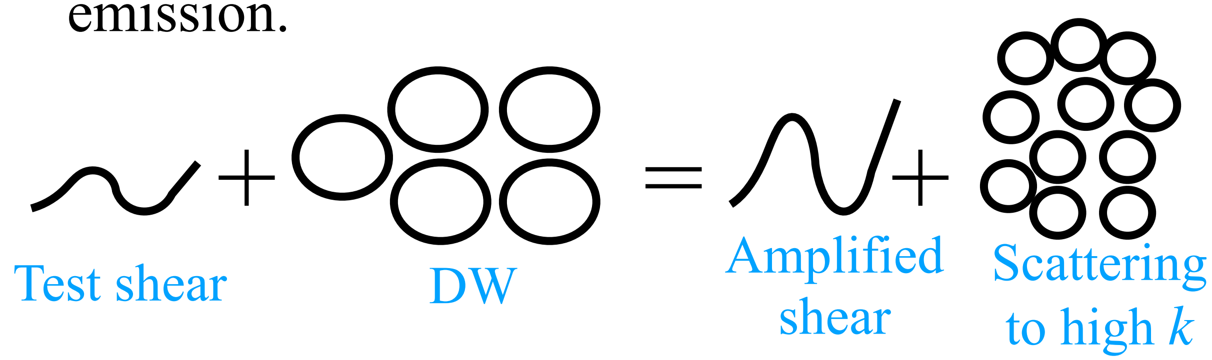
- Almost all theoretical models of zonal flow generation divide cleanly into:

1. Calculation of zonal flow dielectric or screening response, with occasional mention of wavy component beat noise [RH 1998, HR 1999]

$$\frac{\partial}{\partial t}\left\langle \left|\phi_q\right|^2 \right\rangle = \frac{2\tau_c \left\langle \left|S_q\right|^2 \right\rangle}{\left|\epsilon_{neo}\right|^2}$$

Emission from polarization interaction

2. Modulational stability calculations, which consider response of a pre-existing gas of drift waves to infinitesimal test shears or profile corrugations, but ignore noise emission.



- What happens when noise meets modulation? Langevin equation with -ve damping $\frac{\partial \phi_q}{\partial t} - \gamma_q \phi_q = noise$

- Unstable system + noise gets tricky.
  A unified theory of zonal modes is needed.

- Need spectral closures, which treat incoherent noise emission and coherent response on equal footing.

Motivation

- There are both zonal flows and density corrugations at the simplest level of description of DW-ZFT.

- Zonal flows result from the inverse cascade of kinetic energy - this is well known. What about the density corrugations?

- How are the zonal density and zonal flow correlated ? → staircase?

- What are the implications of zonal noise on the feedback loop dynamics?

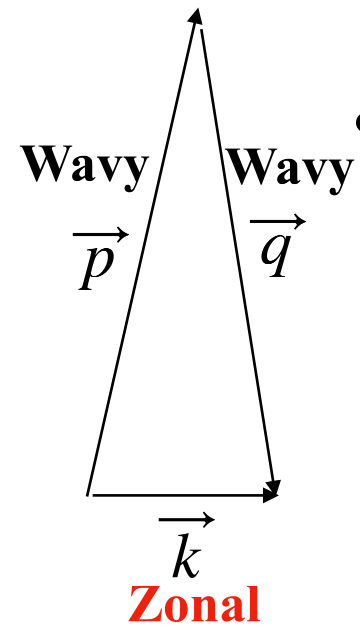
- How does zonal noise affect the dynamics of L-H transition?

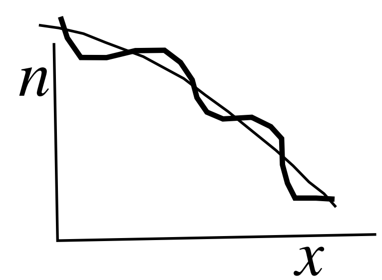
Spectral evolution of zonal intensity and density corrugation

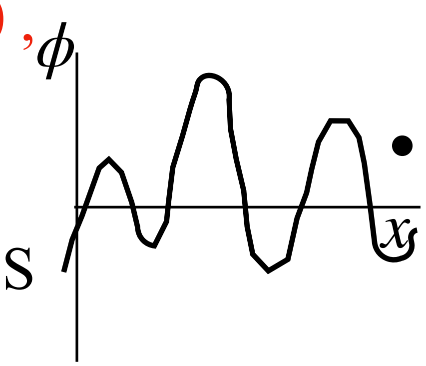
$$\left(\frac{\partial}{\partial t}+2\mu k_x^2\right)\left\langle\left|\phi_k\right|^2\right\rangle+2\eta_{1k}\left\langle\left|\phi_k\right|^2\right\rangle+\Re\left[2\eta_{2k}\left\langle n_k\phi_k^\star\right\rangle\right]=F_{\phi k}$$

$$\left(\frac{\partial}{\partial t}+2D_nk^2\right)\left\langle\left|n_k\right|^2\right\rangle+2\zeta_{1k}\left\langle\left|n_k\right|^2\right\rangle+\Re\left[2\zeta_{2k}\left\langle n_k^\star\phi_k\right\rangle\right]=F_{nk}$$

- $\eta_{1k}\propto k_x^2$ and -ve for $\frac{\partial I_q}{\partial q_x}<0\rightarrow$ transfer to large scales by NEGATIVE VISCOSITY
- Modulational instability when $-\eta_{1k}>\mu k_x^2$ defines a critical spectral slope !
- Zonal growth is maximum when $\alpha_q\rightarrow\infty$ $\Longrightarrow$Non-adiabatic fluctuations inhibit transfer to large scales !
- $\eta_{2k}^{(r)}>0$ ALWAYS for $\frac{\partial I_q}{\partial q_x}<0$ $\Longrightarrow$ Forward transfer when $\Re\left\langle n_k\phi_k^\star\right\rangle<0$, backward transfer when $\Re\left\langle n_k\phi_k^\star\right\rangle>0$
- Noise, $F_{\phi k}=4\sum_q\Pi_q^2\Theta_{k,-q,q}^{(r)}$ where RS $\Pi_q=q_yq_xI_q$. ALWAYS +ve and of envelop scale !







- NL damping rate ζ_{1k}, cross-coefficient ζ_{2k} and advection noise F_{nk} ALL +ve and scale as $1/\alpha_q^2$.

  ➡Density cascade forward in k_x !

  ➡Corrugations ↓as α_q↑.

  ➡Corrugation is determined by noise vs diffusion balance.

- Important for the nonlinear dynamics underlying staircases. Forward cascade in k-space is supporting the idea of (inhomogeneous) mixing in real space.

Spectral evolution of zonal cross-correlation

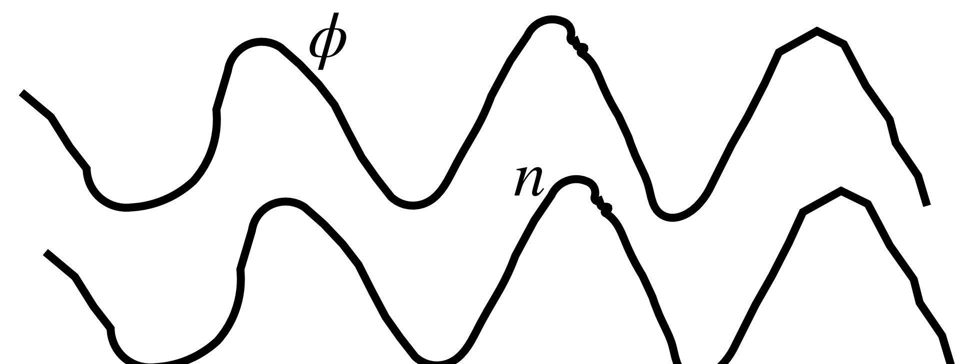
From zonal vorticity and zonal density equation one can obtain **NEW!**

$$\frac{\partial}{\partial t}\left\langle \bar{n}\,\nabla_x^2\bar{\phi}\right\rangle - \left(\mu + D_n\right)\left\langle \nabla_x^2\bar{n}\,\nabla_x^2\bar{\phi}\right\rangle = \left\langle \Gamma_{nx}\nabla_x^3\bar{\phi}\right\rangle + \left\langle \nabla_x\Pi_{xy}\nabla_x\bar{n}\right\rangle$$

- $\Longrightarrow$Zonal correlations are determined by correlation of fluxes and profiles. Zonal correlations are relevant to spatial structure of profile.

- Significant for layering or staircase structure - potential and density are aligned in staircase!

Q: When do zonal density and zonal potential align?



From spectral closure calculations, in steady state

$$\Re\left\langle n_k\phi_k^{\star}\right\rangle = \frac{2\eta_{2k}^{(r)}\left\langle \left|n_k\right|^2\right\rangle + 2\zeta_{2k}^{(r)}\left\langle \left|\phi_k\right|^2\right\rangle}{-\left(\mu + D_n\right)k_x^2 - 2\xi_{1k}^{(r)}} = \begin{cases} +ve \quad when \; -\left(\mu + D_n\right)k_x^2 - 2\xi_{1k}^{(r)} > 0 \\ -ve \quad when \; -\left(\mu + D_n\right)k_x^2 - 2\xi_{1k}^{(r)} < 0 \end{cases}$$

Where $\xi_{1k}^{(r)} = \eta_{1k} + \zeta_{1k}$= non-lin zonal damping rate + non-lin corrugation damping rate

- $\Longrightarrow$Zonal density and potential are correlated (anti-correlated) when the modulational growth of zonal flow is more (less) than modulational damping of corrugations.

Summary of zonal flow and corrugations interaction

(a) Zonal flow - Vorticity equation - Polarization charge flux		
Process	**Impact**	**Key physics**
Polarization noise	Seeds zonal flow	Polarization flux correlation, +ve definite
Zonal flow response (comparable to noise)	Drives zonal shear using DW energy	Non-local inverse transfer in k_x , -ve viscosity
Zonal shear straining of small scale	Regulates waves via straining	Stochastic refraction straining waves, induced diffusion to high k_x
(b) Density corrugations - Density equation - Particle flux		
Density advection beat noise	Seeds density corrugation	Advection beats due to non-adiabatic electrons.
Density corrugations response	Damps and regulates density corrugations	Non-local forward transfer in k_x +ve diffusivity, turbulent mixing weak for $\alpha >> 1$
Zonal shear straining of small scale	Regulates waves via straining	Stochastic refraction straining waves, induced diffusion to high k_x
(c) Zonal cross-correlation - Vorticity and density transport processes		
ZCC response	Sets corrugation - shear layer correlation; staircase states	Growth of zonal intensity must exceed the modulational damping rate of corrugation

Feedback loop with nonlinear zonal noise

How does zonal noise affect the feedback loops?

Turbulence energy ε evolves as

Induced diffusion

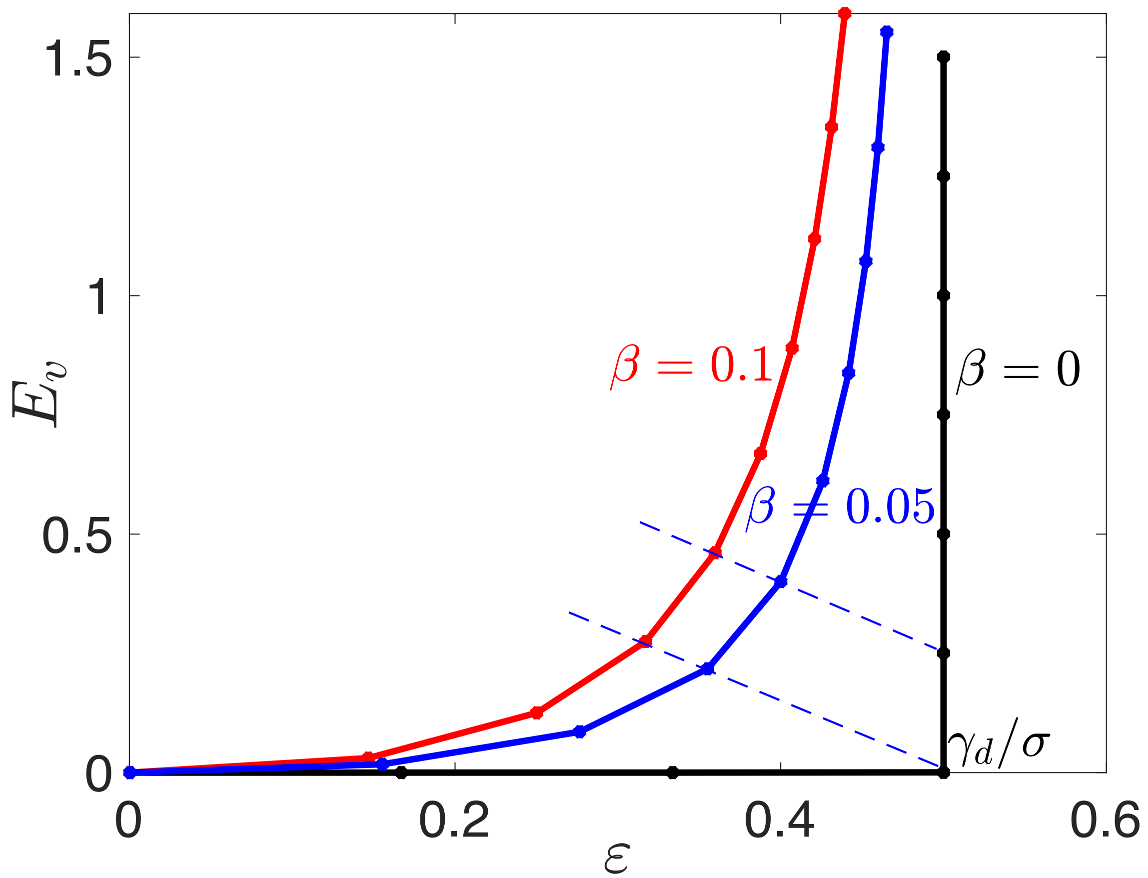
$$\frac{\partial \varepsilon}{\partial t} = \gamma\varepsilon - \sigma E_v \varepsilon - \eta\varepsilon^2$$

Nonlinear damping

Zonal flow energy E_v evolves as

Modulational growth

$$\frac{\partial E_v}{\partial t} = \sigma\varepsilon E_v - \gamma_d E_v + \beta\varepsilon^2$$



<u>Without noise:</u>

- Threshold in growth rate $\gamma > \eta\gamma_d/\sigma$ for appearance of stable zonal flows. Turbulence energy increases as γ/η below the threshold, until it locks at γ_d/σ, at the threshold.

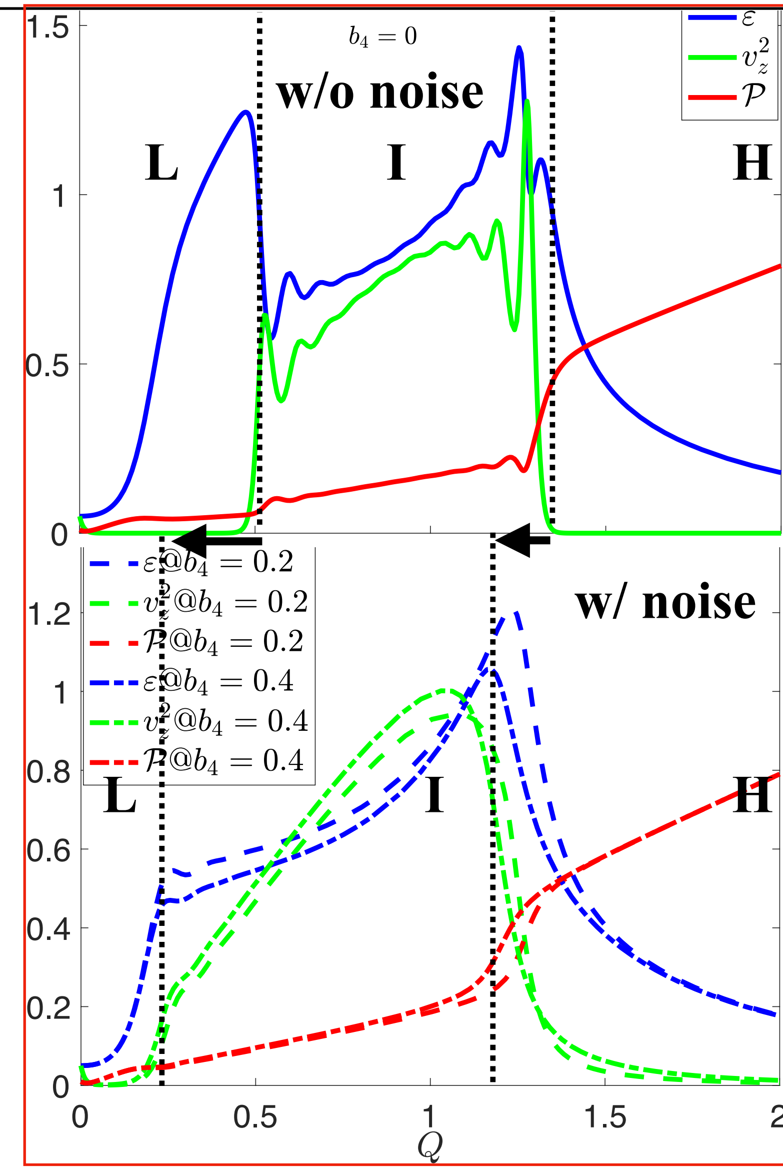
<u>With noise:</u>

- Both zonal flow and turbulence co-exist at any growth rate: - No threshold in growth rate for zonal flow excitation.

- Zonal flow energy is related to turbulence energy as $E_v = \beta\varepsilon^2/\left(\gamma_d - \sigma\varepsilon\right)$.

- Turbulence energy never hits the modulational instability threshold, absent noise!

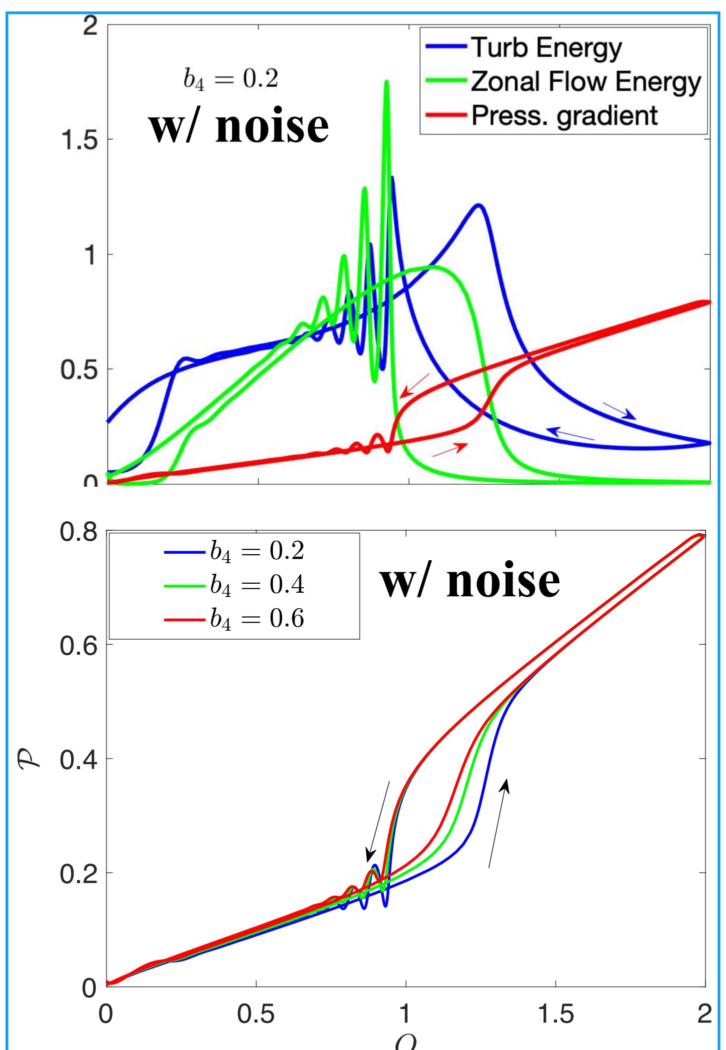
Noise effect on L - H transition and L-H-L hysteresis



<u>With Noise:</u> KD 03 + Noise

- Significant zonal flows appear much below the modulational instability threshold. No ZF threshold in Q! Zonal flows exist at all Q!

- Turbulence level is reduced, no overshoot, zonal flow enhanced.



- Amplitude of I-Phase oscillations reduced.

- H mode power threshold reduced.

- The I-phase in the back transition is more oscillatory than that in the forward transition.

- Hysteresis with noise is robust w.r.t the variations in the initial conditions and the power retreat point in the H mode.

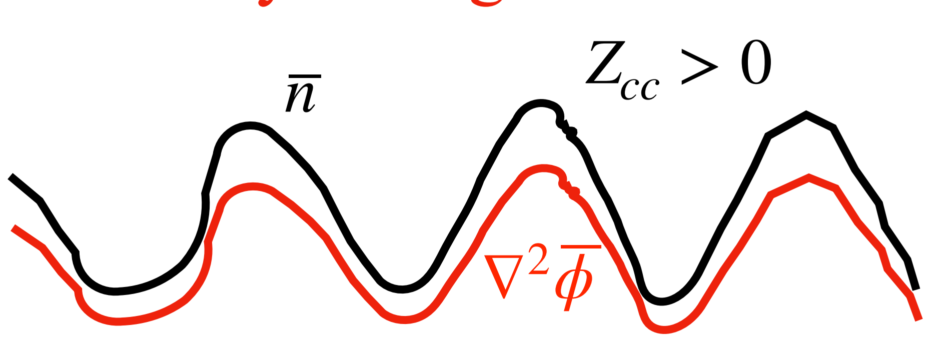
- The area enclosed by the hysteresis curve decreases with noise.

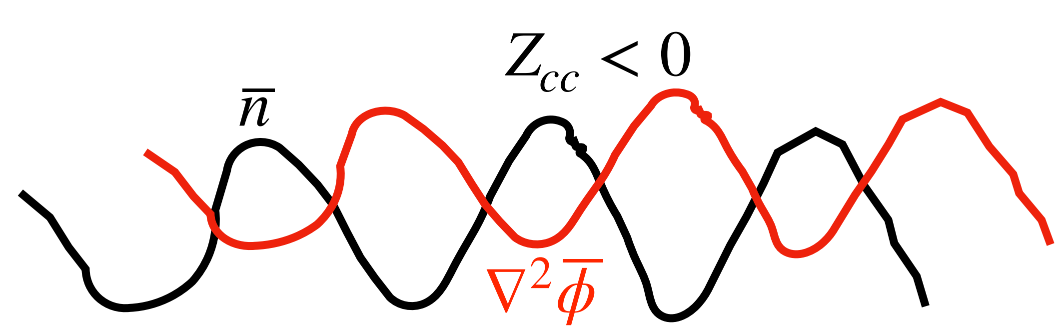
Conclusions I

We presented a unified theory of zonal mode dynamics. Derived a unified set of spectral equations, encompassing nonlinear response, polarization and advection beat noise.

New theoretical results:

- Vorticity flux correlations drive zonal flow noise. Likewise, density flux correlations drive corrugation noise.
- While effective viscosity for zonal flows can go negative, the zonal diffusivity remains positive for $\alpha > 1$. Bi-directional transfer- KE energy to large scale and internal energy to small scales.
- $Z_{cc} \equiv \langle \overline{n} \nabla^2 \overline{\phi} \rangle$ determine the phasing of density corrugations and shear layers. Z_{cc}>0 when modulational growth of zonal shear exceeds the damping of density corrugations.





Conclusions II

Implications:

- Polarization beat noise and modulational effects are comparable intrinsically (both driven by Reynolds stress!).

    - Expands the range of zonal flow activity relative to that predicted by modulational instability calculations.

    - Increases branching ratio of zonal flow energy to turbulence energy.

- Interaction of zonal noise and modulation has significant effect on feedback processes and thus the global characteristics of DW-ZFT.

    - Regarding the L-H transition: Noise eliminates the threshold for zonal flow excitation, and so expands the predicted range of the intermediate phase, drastically reduces the turbulence overshoot.

    - Answers: if zonal flows are the L-H trigger, then what triggers the trigger? → Polarization beat noise triggers the trigger!

    - The energy transfer to zonal flow is accelerated which lowers the threshold for L-H transition.

For experimentalists (Analog+Digital)

- Test the spectral transfer mechanism for corrugations $\rightarrow$ Bicoherence, etc.

- The zonal cross-correlation has not been measured and its relation to staircase structure has not been tested. Do so !

- The improved L-H transition model presented in this paper is testable. In particular, the weak overshoot, expanded domain of zonal mode activity, absence of a modulation instability and the level of residual H mode turbulence are all more consistent with experimental results than the results of earlier reduced models. Quantitative study?

N B: Well known that zonal flows appear before the I-phase. $\Longrightarrow$ Noise !

Future directions

- Deeper understanding of zonal flow generation :

  - Does shearing occurs in an intermittent and bursty avalanche - like feedback events? PDFs?

  - Does a critical spectral slope self-organize from these interactions?

- Understanding interaction of corrugations with avalanches:

  - Corrugations in state of high Z_{cc} sustained as localized transport barriers, staircases etc. localized by accompanying shear flow?

  - Corrugations in state of low Z_{cc} likely to overturn, and drive avalanches, as in running sandpile?

  - Relevant for TEM turbulence. Does the density gradient state consist of standing corrugations , running avalanches or mixtures thereof ?

- Theory should better understand the effect of noise on staircase, which have been considered only in context of Mean Field theory.

- Relation between Z_{cc} and the staircase structure: Does the physics of Z_{cc} set the relative positions of corrugations and shear layer? Is there a single Z_{cc} for staircase state ? Or a band ?