

On How Edge Shear Layer Collapse Defines Greenwald Limit $n_g \sim I_p$

[Rameswar Singh and P H Diamond, Nucl. Fusion **61**(2021) 076009]

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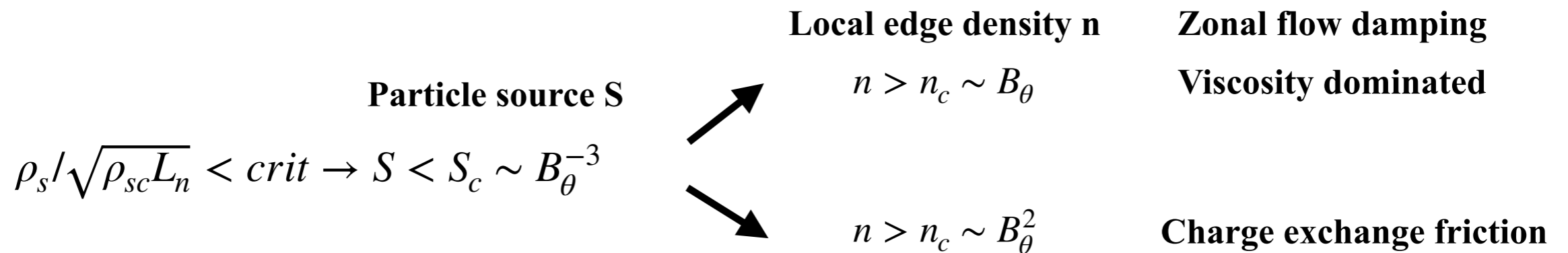
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Summary

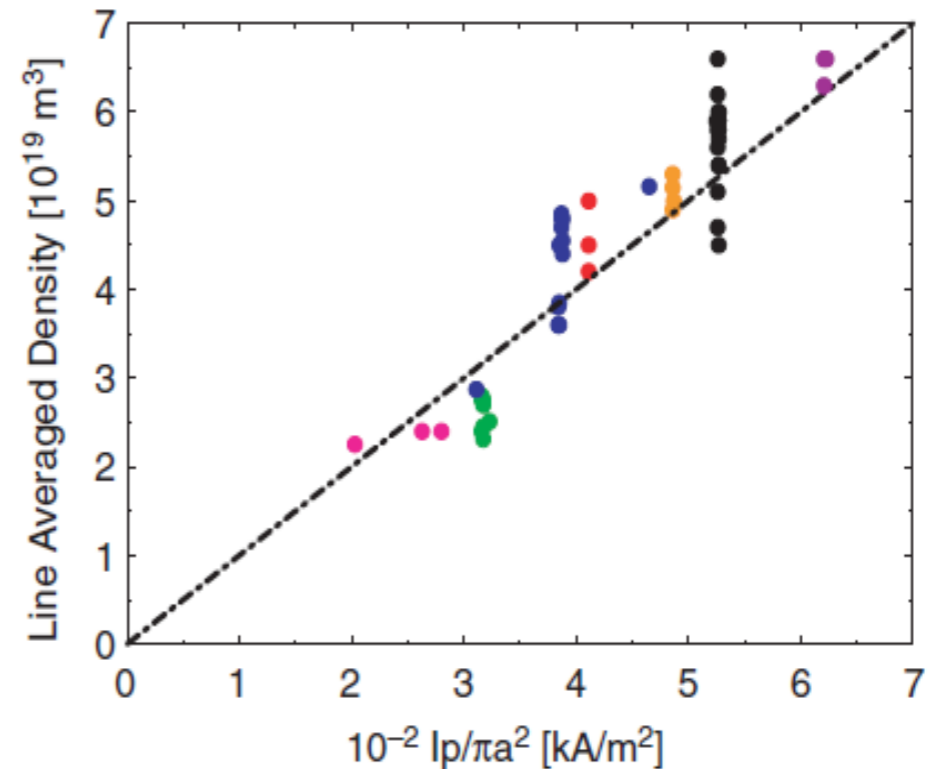
- Edge shear layer collapse triggers the sequence of events leading to Greenwald density limit phenomenology.
- A theory of edge shear layer collapse for $n \rightarrow n_g$ in the adiabatic regime.
- Neoclassical zonal flow screening is key to the origin of current scaling.
- Neoclassical screening + drift wave - zonal flow dynamics \longrightarrow a novel predator - prey model.
- *The threshold condition for edge shear layer collapse $\rho_s l \sqrt{\rho_{sc} L_n} < crit$. Here, ρ_s is ion sound radius, ρ_{sc} is screening length and L_n is density scale length. *Smaller ρ_{sc} i.e., higher B_θ expands the regime of zonal flow persistence.**



- Shear layer collapse aggravates radiative cooling effects! Radiative cooling is secondary (i.e., a consequence of) to the transport bifurcation.

Density limit basics

- Discharge terminates when line integrated density exceeds a critical value $\bar{n}_g = I_p / \pi a^2$.
- Why care? Fusion power $\propto n^2$.
- A fundamental limit on performance.
- Not a dimensionless number
▶ more physics involved.
- Still begging the origin of current I_p scaling!?
- Also manifested in RFPs.
- What about Stellarators?



From discharge termination studies in Alcator-C, Greenwald PPCF 2002

Often associated with macroscopic phenomena

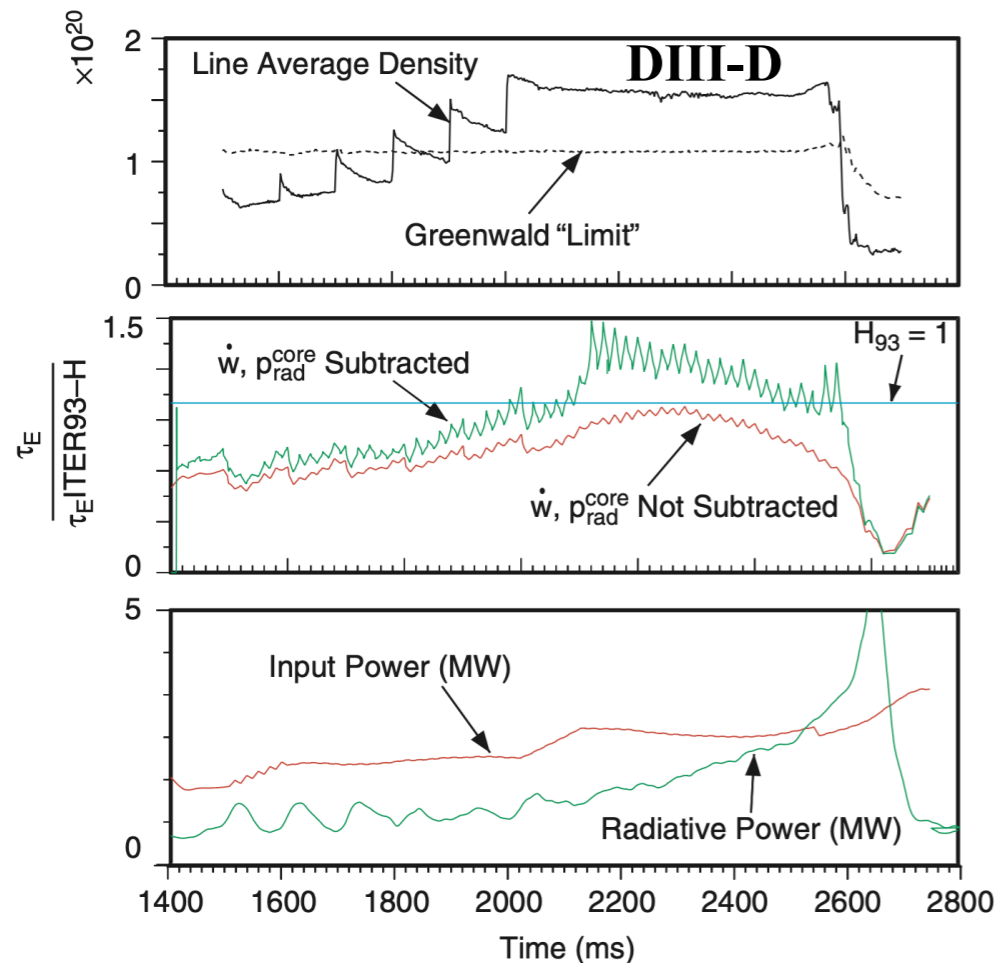
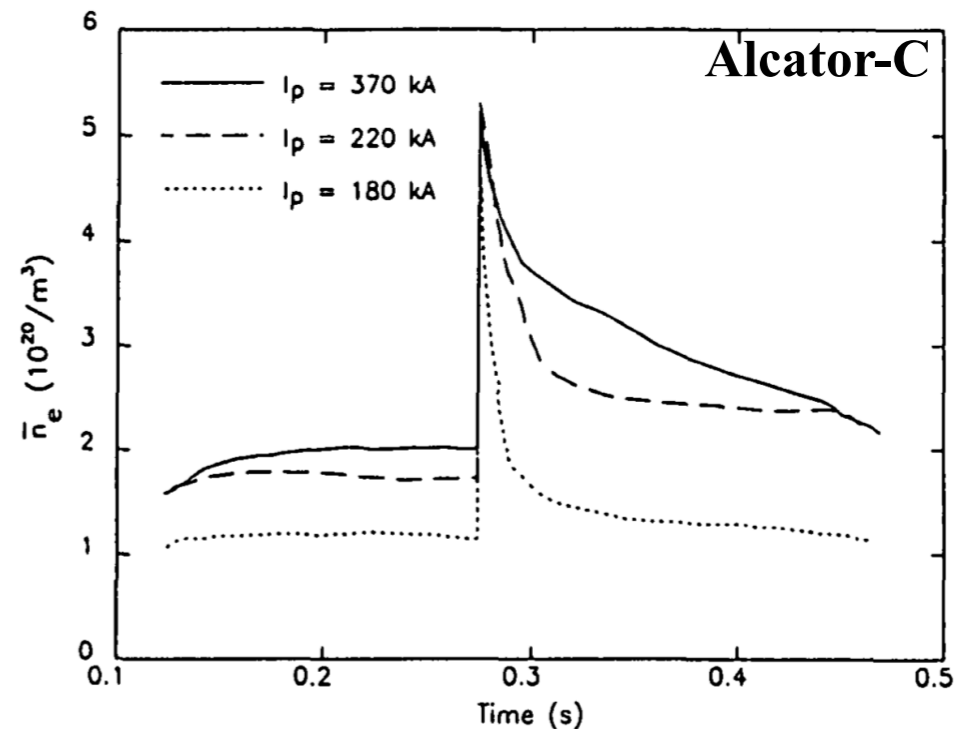
- Global thermal collapse, Radiative condensation / MARFEs.
- Poloidal detachment, Divertor detachment, MHD activity -radiation driven islands.

What's beyond macroscopic ?

Connections with transport physics

Role of particle transport?

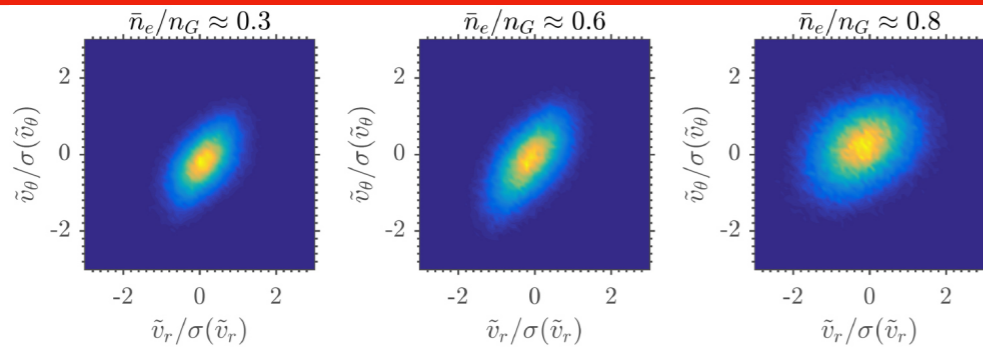
- A SOFT limit: Shallow pellet injection in plasma with $\bar{n} = n_g$ triggered transient particle increased relaxation to n_g by transport rather than by disruption! [Greenwald NF 1988]
- Shallow pellet injection avoids excessive edge cooling— No MARFEs, disruptions!



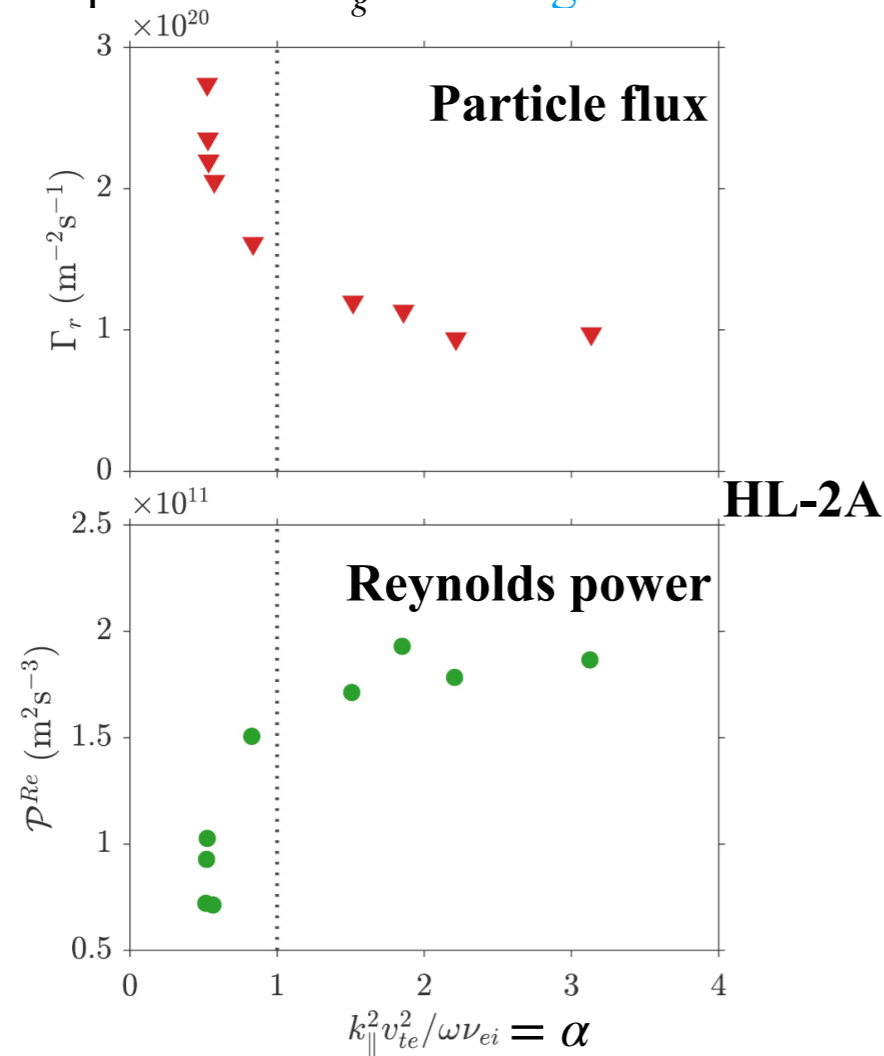
- Pellet in DIII-D beat n_g by peaked density profiles \longleftrightarrow enhanced core confinement. Accumulation of impurities \longrightarrow increase in radiation \longrightarrow disruption. [Mahdavi *et al* 1997]
- Disruption ensuing as a secondary consequence of strong edge cooling due to gas fueling/ radiative cooling.

Recent experiments

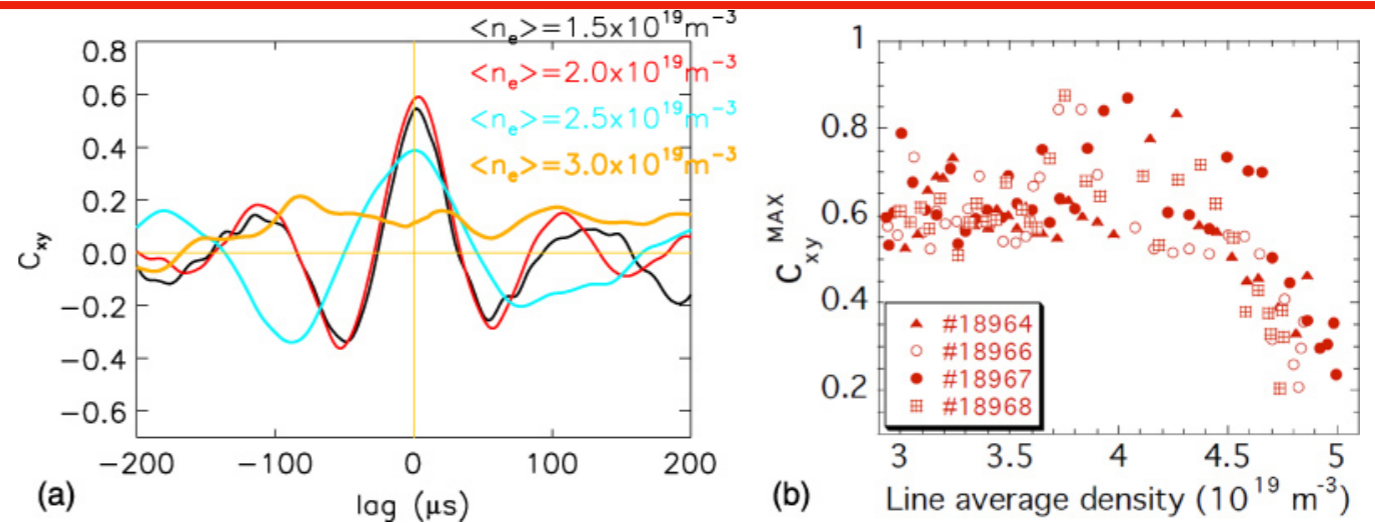
Is the density limit related to edge shear layer decay?



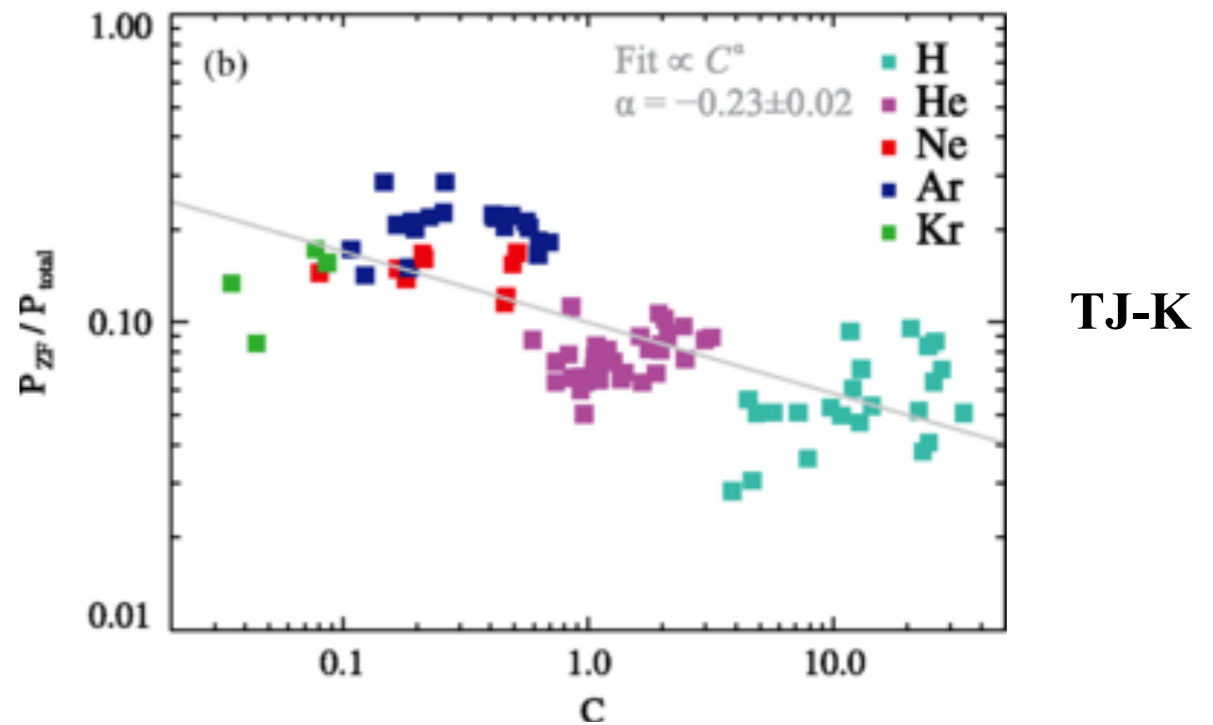
Radial and poloidal velocity correlations drops as $n \rightarrow n_g$ [Hong et al NF 2018](#)



Reynolds power $P_{Re} = -\langle v_\theta \rangle \partial_r \langle \tilde{v}_r \tilde{v}_\theta \rangle$ and particle flux \uparrow as α drops below 1.



Long range correlations (LRC) decrease as the line averaged density increases in both TEXTOR and TJ-II. **LRC** \leftrightarrow **ZF strength** [Y. Xu et al NF 2011](#)



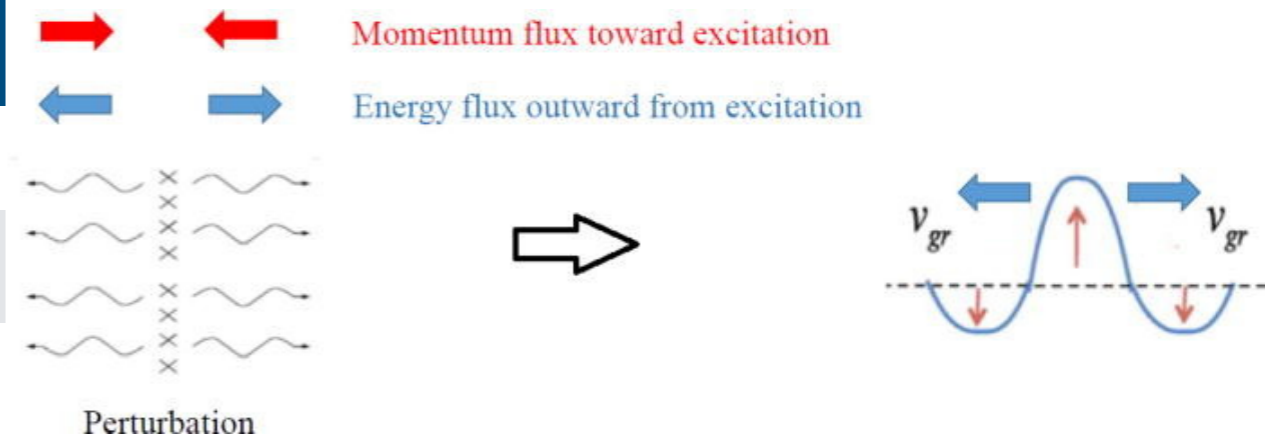
Zonal flow contribution to the total turbulent spectrum decreases with collisionality. [Schmid et al PRL 2017](#)

...and the most recent experiments by T Long et al @ J-TEXT (this meeting)

Shear layer collapse with hydrodynamic electrons: HDM theory and its limitations

- Clearly, shear layer collapse \longrightarrow increased turbulence and transport as $\bar{n} \rightarrow n_g$! Edge shear layer exist in all devices tokamaks, RFPs, Stellarators.
- Plasma response for Hasegawa - Wakatani :- HDM Theory [[Hajjar, Diamond, Malkov 2018](#)]

| Response | Adiabatic | Hydro |
|---|--------------------|----------------------|
| Particle flux Γ_n | $\sim \alpha^{-1}$ | $\sim \alpha^{-1/2}$ |
| Turbulent viscosity χ_y | $\sim \alpha^{-1}$ | $\sim \alpha^{-1/2}$ |
| Residual vorticity flux Π^{res} | $\sim \alpha^{-1}$ | $\sim \alpha^{1/2}$ |
| Vorticity gradient $\nabla_r^3 \bar{\phi} = \frac{\Pi^{res}}{\chi}$ | $\sim \alpha^0$ | $\sim \alpha^1$ |



In hydro-regime the link of wave energy flux to Reynolds stress is broken !

- $\Gamma_n, \chi \uparrow$ and $\Pi^{res}, \nabla_{\perp}^2 \bar{\phi} \downarrow$ as the electron response passes from adiabatic to hydrodynamic regime.
- Weak zonal flow production for $\alpha \ll 1 \rightarrow$ weak regulation of turbulence and enhancement of particle transport and turbulence.

Limitations (Key Questions)

- Connection of shear layer collapse scenario with Greenwald scaling $\bar{n}_g \sim I_p$?
- Shear layer collapse in hydro regime only. Not relevant for **hot** tokamaks. What of collapse in adiabatic ($\alpha > 1$) regime?
- Dimensionless parameter ?

Origin of current scaling

- Key physics: zonal flow drive is “screened” by neoclassical dielectric [Rosenbluth - Hinton

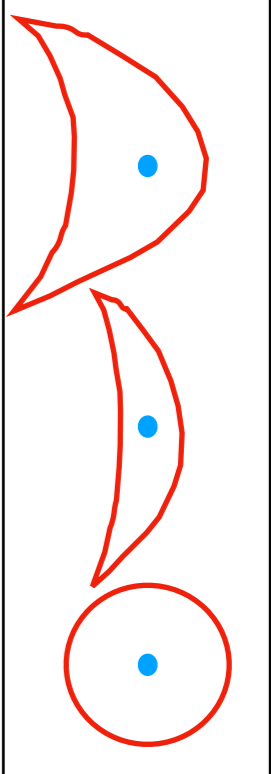
$$1998]. \frac{\partial}{\partial t} \langle |\phi_k|^2 \rangle = \frac{2\tau_c \langle |S_k|^2 \rangle}{|\epsilon(q)|^2}; \quad \epsilon = \epsilon_{cl} + \epsilon_{neo} = \frac{\omega_{pi}^2}{\omega_{ci}^2} \left\{ 1 + \frac{q^2}{\epsilon^2} \right\} k_r^2 \rho_i^2$$

← Emission from polarization interaction
← Neoclassical response
← Zonal wave #

banana regime

- Poloidal gyro-radius ρ_θ emerges as screening length! **Effective ZF inertia** \downarrow as $I_p \uparrow \rightarrow$ ZF strength increases with I_p , for fixed drive.
- But edge region is most likely in Plateau regime. [T Long et al NF 2019] \rightarrow Need revisit R-H screening calculations.

| Collisionality regimes | Screening length ρ_{sc} | Residual level $\frac{\phi_k(\infty)}{\phi_k(0)}$ | B_θ -dependence |
|------------------------|---|---|------------------------|
| Banana | $= \sqrt{\rho_s^2 + \rho_\theta^2} \approx \rho_\theta$ | $\approx \left(\frac{B_\theta}{B_T} \right)^2$ | Favorable |
| Plateau | $= \sqrt{\rho_s^2 + \mathcal{L} \rho_\theta^2} \approx \mathcal{L}^{1/2} \rho_\theta$ | $\approx \frac{1}{\mathcal{L}} \left(\frac{B_\theta}{B_T} \right)^2$ | Favorable |
| P-S | $= \rho_s$ | $= 1$ | None |



Here $\mathcal{L} = 1 - \frac{4}{3\pi} (2\epsilon)^{3/2} < 1$. Favorable I_p scaling persist in plateau regime. Robust trend!

No I_p scaling in P-S regime. Effective inertia minimum in P-S

Feedback loop with nonlinear zonal noise

Neoclassical screening + drift wave - zonal flow dynamics \longrightarrow a novel predator - prey model.

Turbulence energy E_t evolves as

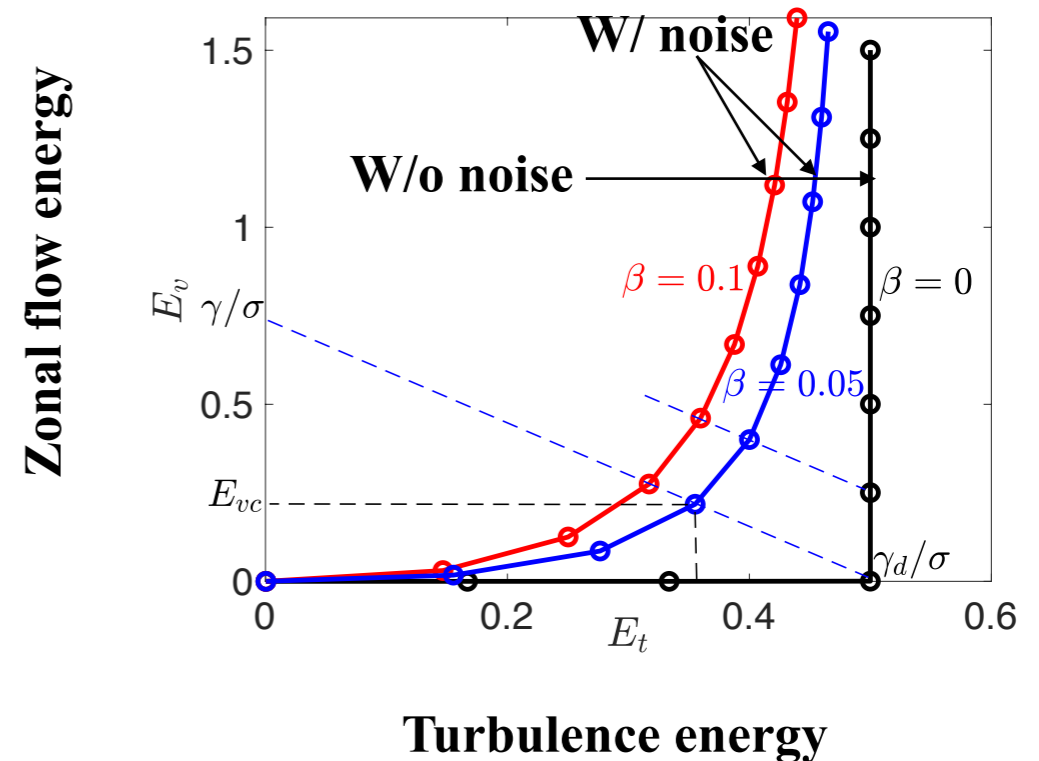
$$\frac{\partial E_t}{\partial t} = \gamma E_t - \underbrace{\sigma E_v E_t}_{\text{Induced diffusion}} - \underbrace{\eta E_t^2}_{\text{Nonlinear damping}}$$

Zonal flow energy E_v evolves as

$$\frac{\partial E_v}{\partial t} = \underbrace{\sigma E_t E_v}_{\text{Modulational growth}} - \gamma_d E_v + \underbrace{\beta E_t^2}_{\text{Zonal noise}}$$

Notice, $\sigma \sim \varepsilon^{-1} \sim B_\theta^2 \sim I_p^2$ and $\beta \sim \varepsilon^{-2} \sim B_\theta^4 \sim I_p^4$

$\implies I_p$ jacks up modulational growth and zonal noise \rightarrow stronger feedback on turbulence.



With noise:

[Singh and Diamond PPCF 2021]

- Both zonal flow and turbulence co-exist at any growth rate: - No threshold in growth rate for zonal flow excitation.
- Zonal flow energy is related to turbulence energy as $E_v = \beta E_t^2 / (\gamma_d - \sigma E_t) \uparrow$ with I_p .
- Turbulence energy never hits the 'old' modulational instability threshold!
- Turbulence energy \downarrow and zonal flow energy \uparrow :- Noise feeds energy into zonal flow!

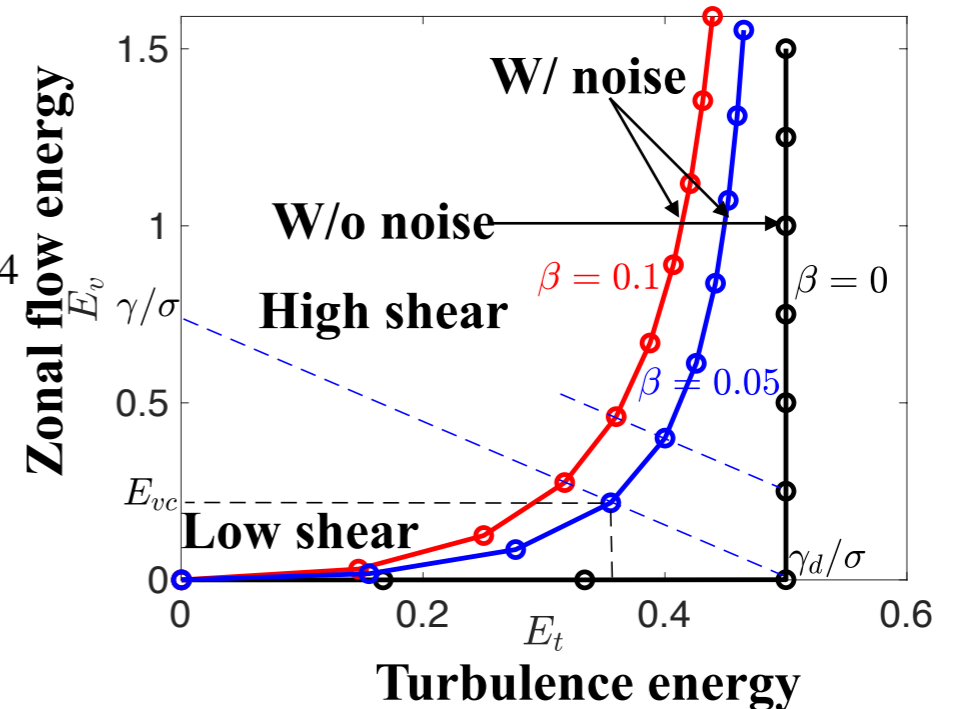
Shear layer collapse in adiabatic regime

- Criterion for zonal flow collapse with noise tracks that for collapse of zonal flow without noise i.e.,

$$E_v < E_{vc} \rightarrow E_{v0} < 0 \implies \gamma < \eta \frac{\gamma_d}{\sigma}$$

$$\implies \boxed{\frac{\rho_s}{\sqrt{\rho_{sc} L_n}}} < \left[\frac{\eta}{\Omega_i} \frac{\gamma_d}{2k_x^2 \rho_s^2 \Theta \Omega_i^2} \frac{\hat{\alpha}}{q_{\perp}^2 \rho_s^2} \frac{(1 + q_{\perp}^2 \rho_s^2)^3}{q_y^2 \rho_s^2} \right]^{1/4}$$

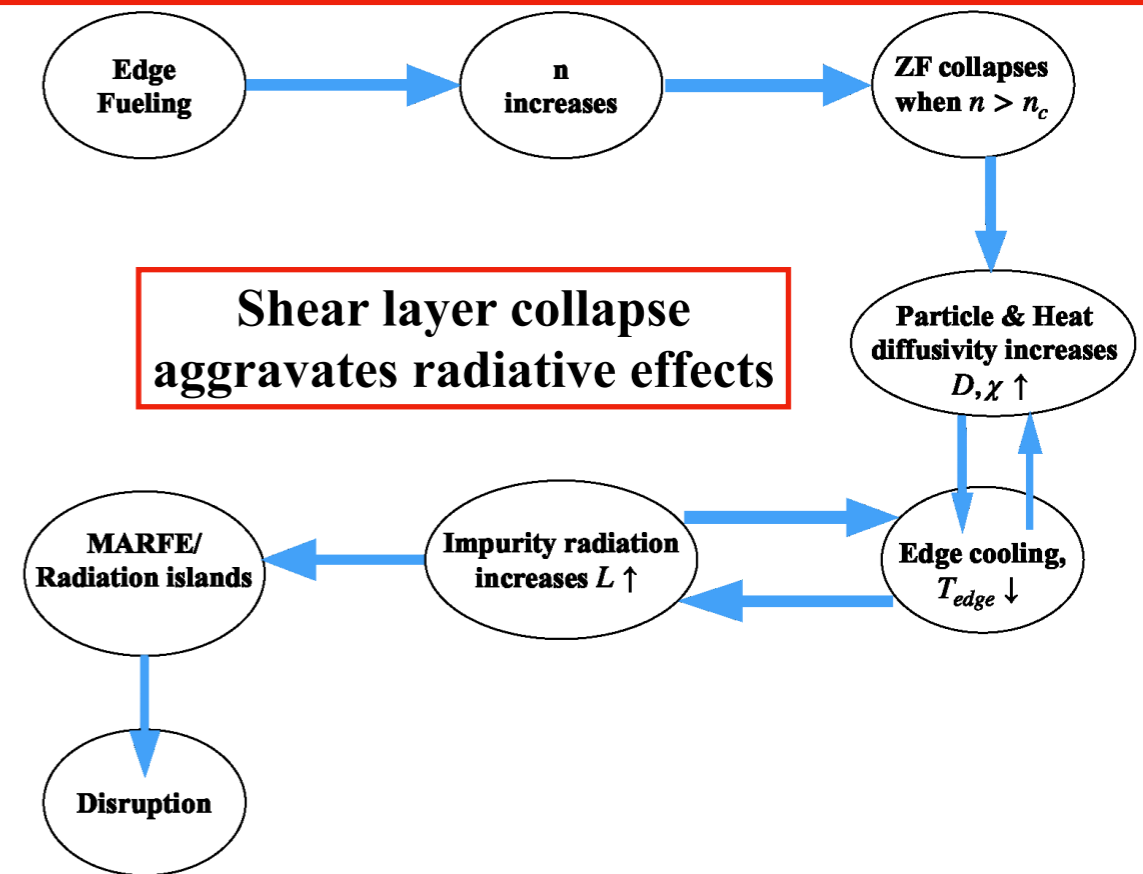
- Note that smaller ρ_{sc} i.e., higher B_{θ} enlarges the regime of zonal flow persistence.



- Particle diffusivity $D \sim \rho_{sc}^2 / \rho_s^2 \sim B_{\theta}^{-2} \sim I_p^{-2}$. [Consistent with T Long's expt. (this meeting)]
- Zonal flow collapse criterion in terms of particle source S : $S < S_{crit} \sim \rho_{sc}^3 / \rho_s^3 \sim B_{\theta}^{-3} \sim I_p^{-3}$. Critical particle source required to hold shear layer decreases with plasma current.
- In the viscosity dominated regime, zonal flows collapse when the local density $n > n_{crit} \sim \rho_s / \rho_{sc} \sim B_{\theta} \sim I_p$.
- In the charge exchange friction dominated regime, $n_{crit} \sim \rho_s^2 / \rho_{sc}^2 \sim B_{\theta}^2 \sim I_p^2$ (NB: stronger I_p dependence!)

Conclusions

- Density limit is linked to **shear layer collapse** which is (in part) controlled by neoclassical screening response.
- Neoclassical zonal flow screening is key to the emergence of the current scaling $n_g \sim I_p$.
- Radiative cooling is secondary (i.e., a consequence of) to the transport bifurcation.
- $\rho_s / \sqrt{\rho_{sc} L_n}$ emerges as the key dimensionless ratio which underpins the density limit. Zonal flows collapse when $\rho_s / \sqrt{\rho_{sc} L_n} < crit$. Smaller screening length ρ_{sc} i.e., higher B_θ expands the regime of zonal flow persistence.
- Using particle balance, the zonal flows collapse when the particle source $S < S_{crit} \sim B_\theta^{-3} \sim I_p^{-3}$. \implies Particle source required to hold the shear layer decreases with increasing current.
- In terms of local edge density, zonal flows collapse when $n > n_{crit}$. In a viscosity dominated regime $n_{crit} \sim B_\theta \sim I_p$ and in charge exchange friction dominated regime $n_{crit} \sim B_\theta^2 \sim I_p^2$.

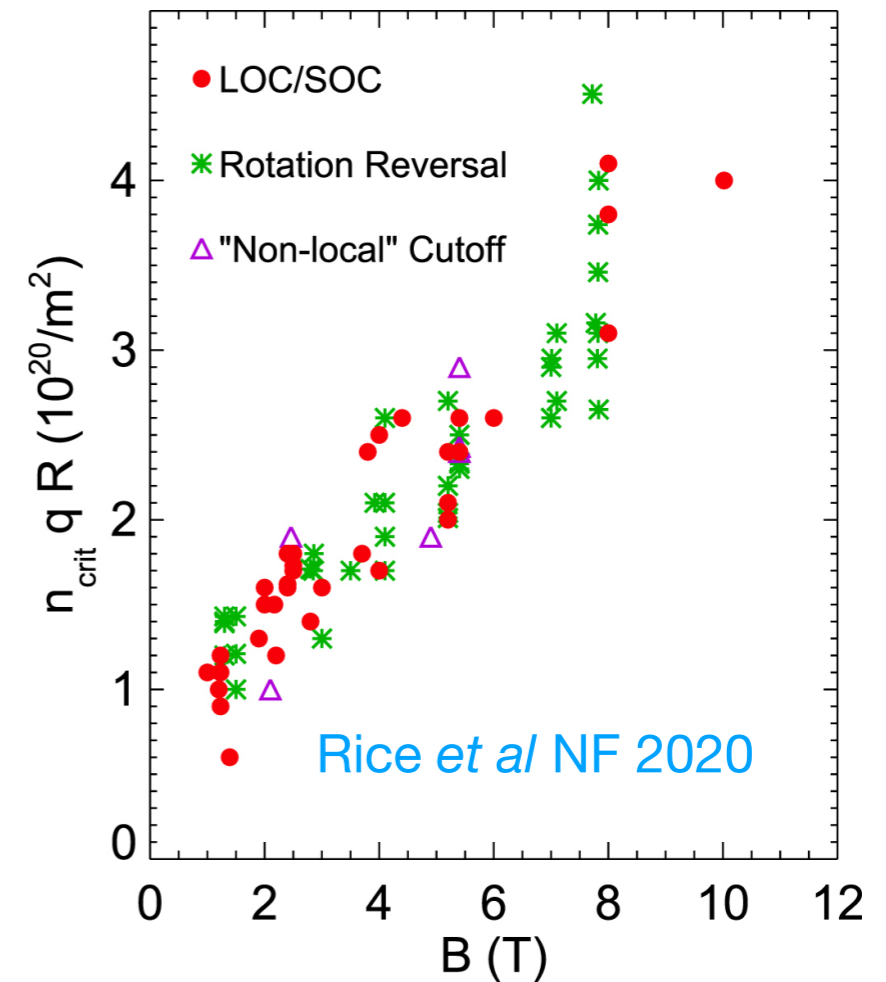


Implications for other devices

- Density limit is linked to shear layer collapse which is (in part) controlled by neoclassical screening response.
- In stellarators, the principal correction to classical screening is due to helically trapped particles.
- This has no obvious length scale other than ρ_i the zonal flow screening is classical. Thus the “effective inertia” for zonal flows in stellarators is lower than that for tokamaks.

Connection with Ohmic confinement phenomenology and L-H threshold power minimum

- Ohmic phenomenology (i.e., LOC-SOC, rotation reversal etc.) may be unified by the scaling relation $n_{crit}qR = B_T$. This is equivalent to $n/n_G = \mathcal{O}(1/2)$.
- Greenwald limit phenomenology-i.e., radiative cooling, Marfes, disruption etc., - set in for $n/n_G = \mathcal{O}(1)$
- Why the similarity to the Greenwald scalings, but for phenomena which occur at lower density?
- These occur for $n/n_G < 1$ \longrightarrow a precursor to the more violent phenomena associated with the density limit.
- $n_{crit}qR = B_T$ appears to be related to the minimum in the power threshold for L \rightarrow H transition.
- This may be due to the onset of pre-transition shear layer decay for $n/n_G < 1$.
- This in turn weakens the 'seed' shear which initiates the L \rightarrow H transition, and thus necessitates an increase in the power required for the transition.



Suggestions for experiments

- Shear layer collapse study in L mode with P_{aux} . (Note $\alpha \sim T^2/n$)
- Scale of shear layer collapse and its relation with screening length.
- Pellet/SMBI experiment of Greenwald with relevant fluctuations measurements. Relate the density relaxation time to the predictions based on transport dynamics.
- Is the critical edge density with RMP lower than without? Is this because zonal shears are already weakened by the RMP?
- Can edge biasing sustain $n > n_g$ by driving the edge shear layer, externally?
- Is the shear layer collapse transport bifurcation hysteretic or not?

Future directions in theory

- Interplay of B_θ scaling via ρ_{sc} with B_θ scaling from $k_{\parallel} = 1/qR$ (i.e., from Landau damping!?)
- H-mode density limit: H-L back transition by mean ExB shear collapse? Mechanism of mean shear collapse at high density?
- How flux surface shaping effects the shear layer collapse criterion? Can negative triangularity sustain $n > n_g$?