On How Decoherence of Vorticity Flux by Stochastic Magnetic Fields Quenches Zonal Flow Generation

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Outline

- Introduction

  Resonant Magnetic Perturbation plays an important role in momentum transport in edge plasma evolution.

- Model & Calculation

- Results

  a. Suppression of PV diffusivity and the shear-eddy tilting feedback loop.

  b. Power threshold increment for L-H transition.

  c. Intrinsic Rotation in presence of stochastic fields.

- Conclusions

- Future Work: Mixing length in presence of stochastic fields.

Why we study stochastic fields in fusion device?

Shear Flow

↓

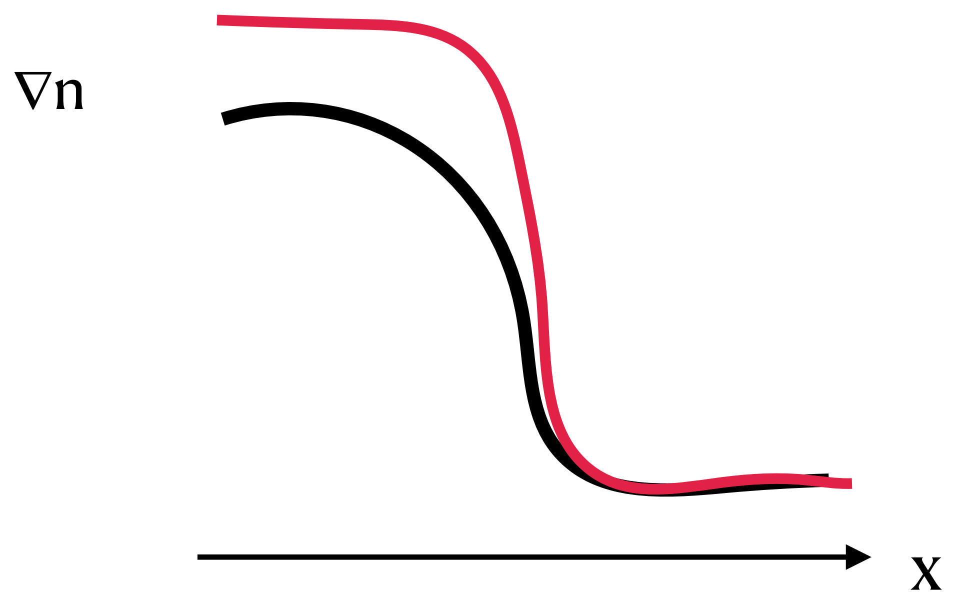
Quench Turbulence

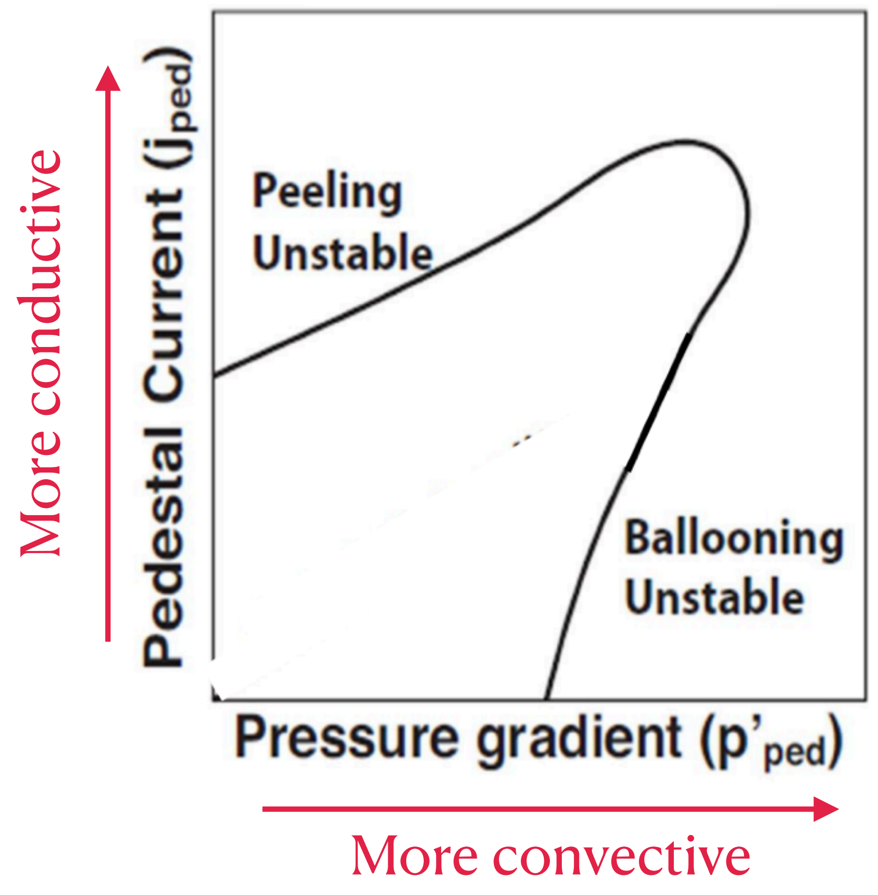
↓

Pedestal Formation

↓

Edge-Localized Mode (ELM)





Edge-Localized Mode (ELM)

↓

Edge gradient ↑

↓

Peeling-ballooning mode

↓

ELM Burst

Suppress (by inducing magnetic perturbation)

Boundary Control: Resonant Magnetic Perturbation (RMP)

- ELMs are quasi-periodic relaxation events occurring at edge pedestal in H-mode plasma.
- ELMs can damage wall components of a fusion device.

Stochastic field effect is important for boundary control

Shear Flow

Edge-Localized Mode (ELM)

ELM Burst

Boundary Control: Resonant Magnetic Perturbation (RMP)

Suppress (by inducing magnetic perturbation)

Stochasticity: Overlapping of Magnetic islands.

Trade off: RMPs controls gradients and mitigates ELM, but raise the **power threshold**.

Key Questions:

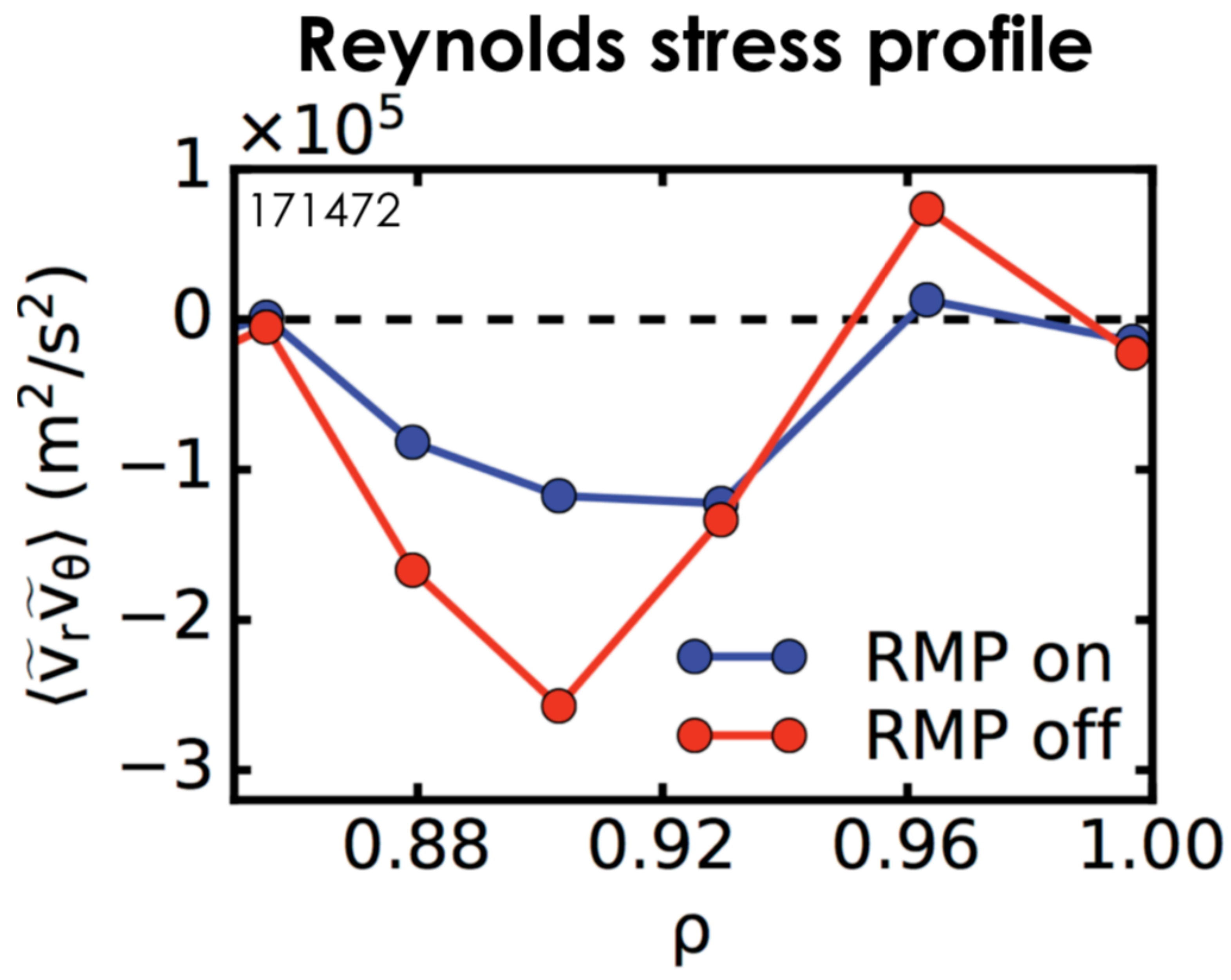
How RMPs influence the Reynolds stress and hence suppress the zonal flow?

How stochastic fields increase the power threshold of L-H transition?

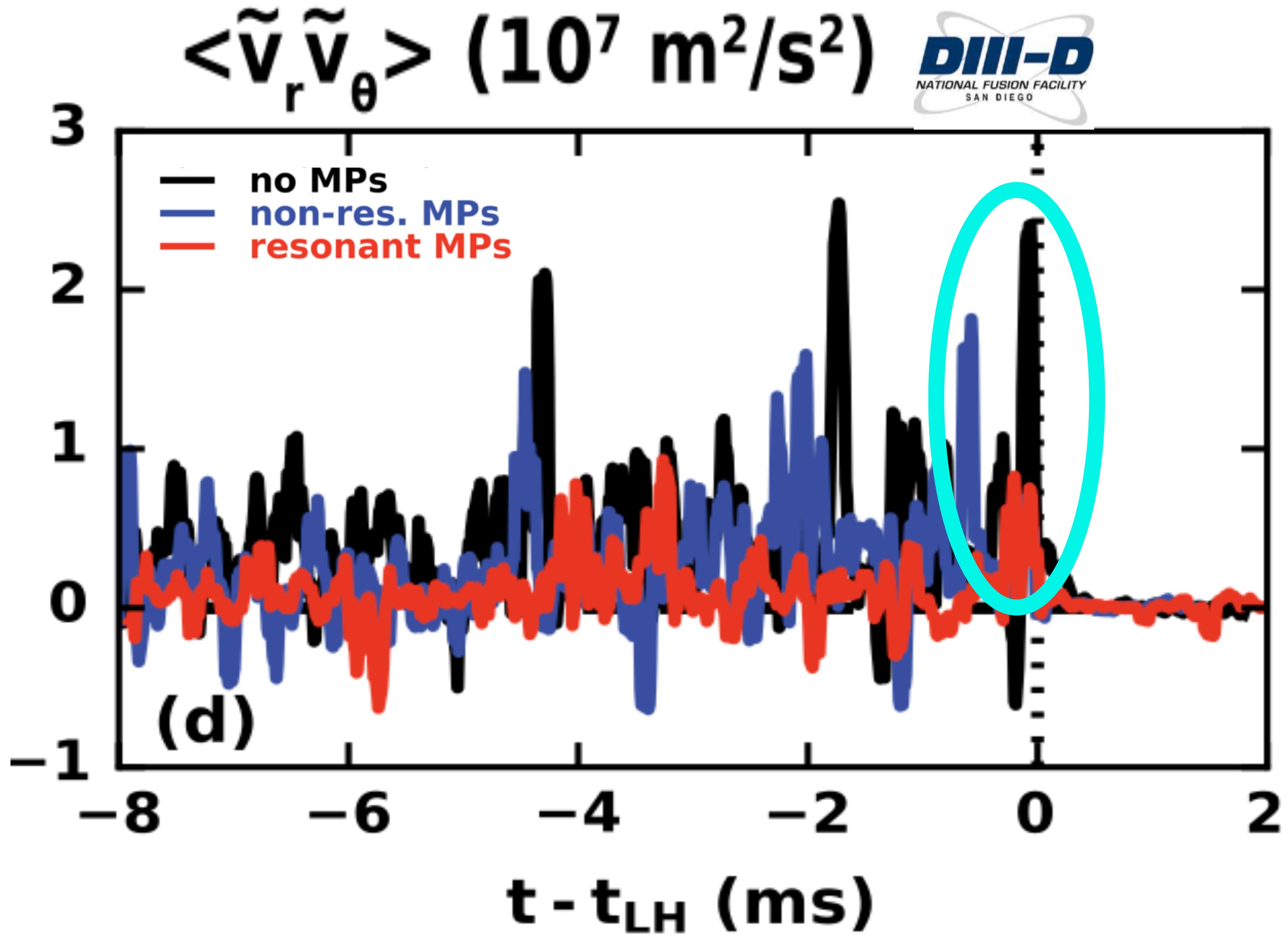
We examine the physics of stochastic fields interaction with zonal flow near the edge.

(Chen et al.,PoP **28**, 042301 (2021))

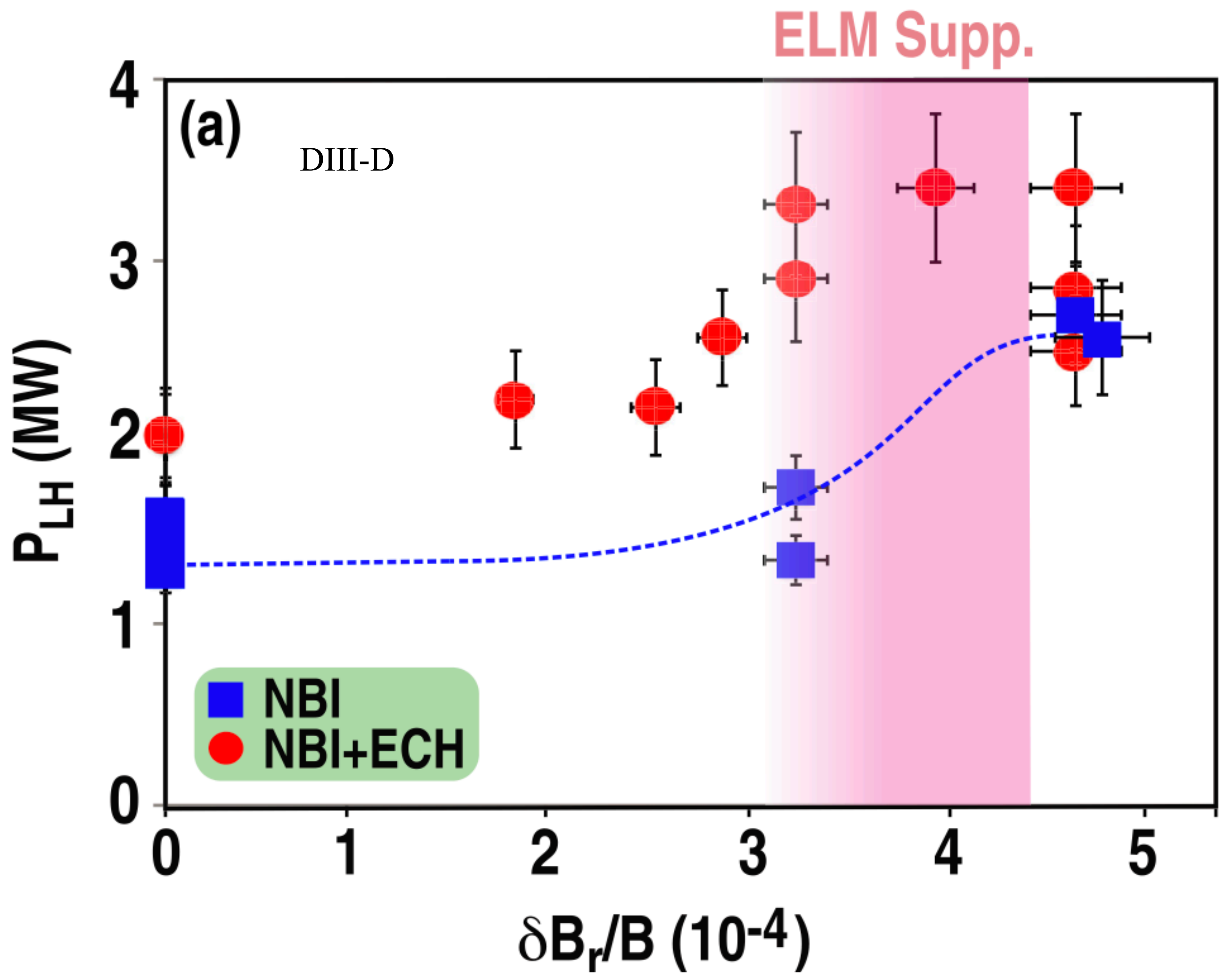
Experimental Results with RMP for L-H Transition — fluctuations



(D. Kriete et al, PoP **27** 062507 (2020))



(D. Kriete et al, PoP **27** 062507 (2020))

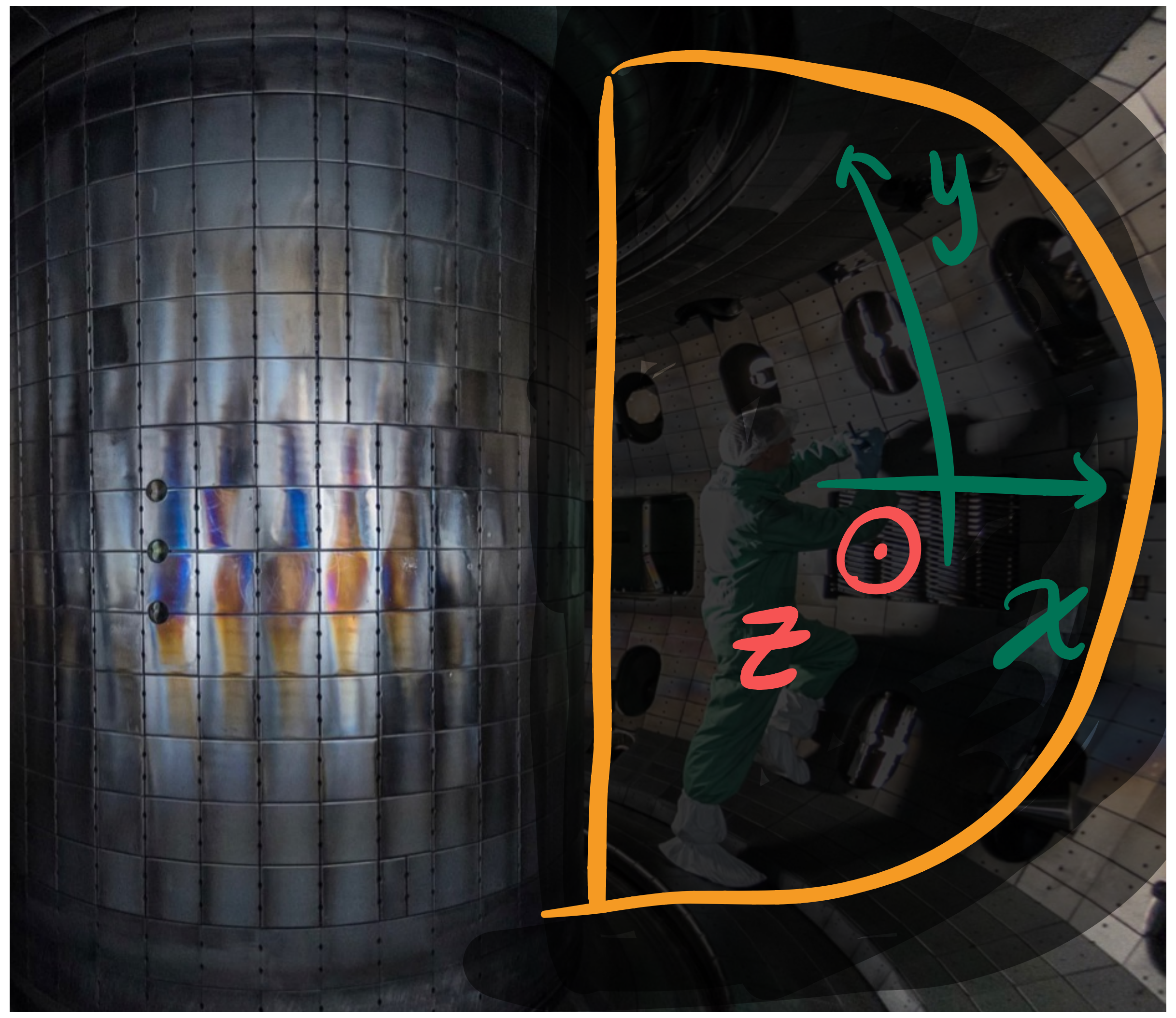


(L. Schmitz et al, NF **59** 126010 (2019))

DIII-D Experimental results: RMPs lower the Reynolds stress and increase the power threshold of L-H transition.

Model

(Chen et al.,PoP **28**, 042301 (2021))



1. Cartesian coordinate: strong mean field B_0 is in z direction (3D).
2. Rechester & Rosenbluth (1978): waves, instabilities, and transport are studied in the presence of **external excited, static, stochastic fields.**
3. $\underline{k} \cdot \underline{B} = 0$ (or $k_{\|} = 0$) **resonant at rational surface in third direction —**

   $\omega \to \omega \pm v_A k_z$, **and** Kubo number: $Ku_{mag} = \dfrac{l_{ac} |\widetilde{\mathbf{B}}|}{\Delta_{\perp} B_0}$).
4. Four-field equations —
   (a) Potential vorticity equation—vorticity $-\nabla^2 \psi \equiv \zeta$
   (b) Induction equation — $\mathbf{A}, \mathbf{J}$
   (c) Pressure equation — $\mathbf{p}$
   (d) Parallel flow equation — $\mathbf{u_z}$

Well beyond HM model

We use mean field approximation:

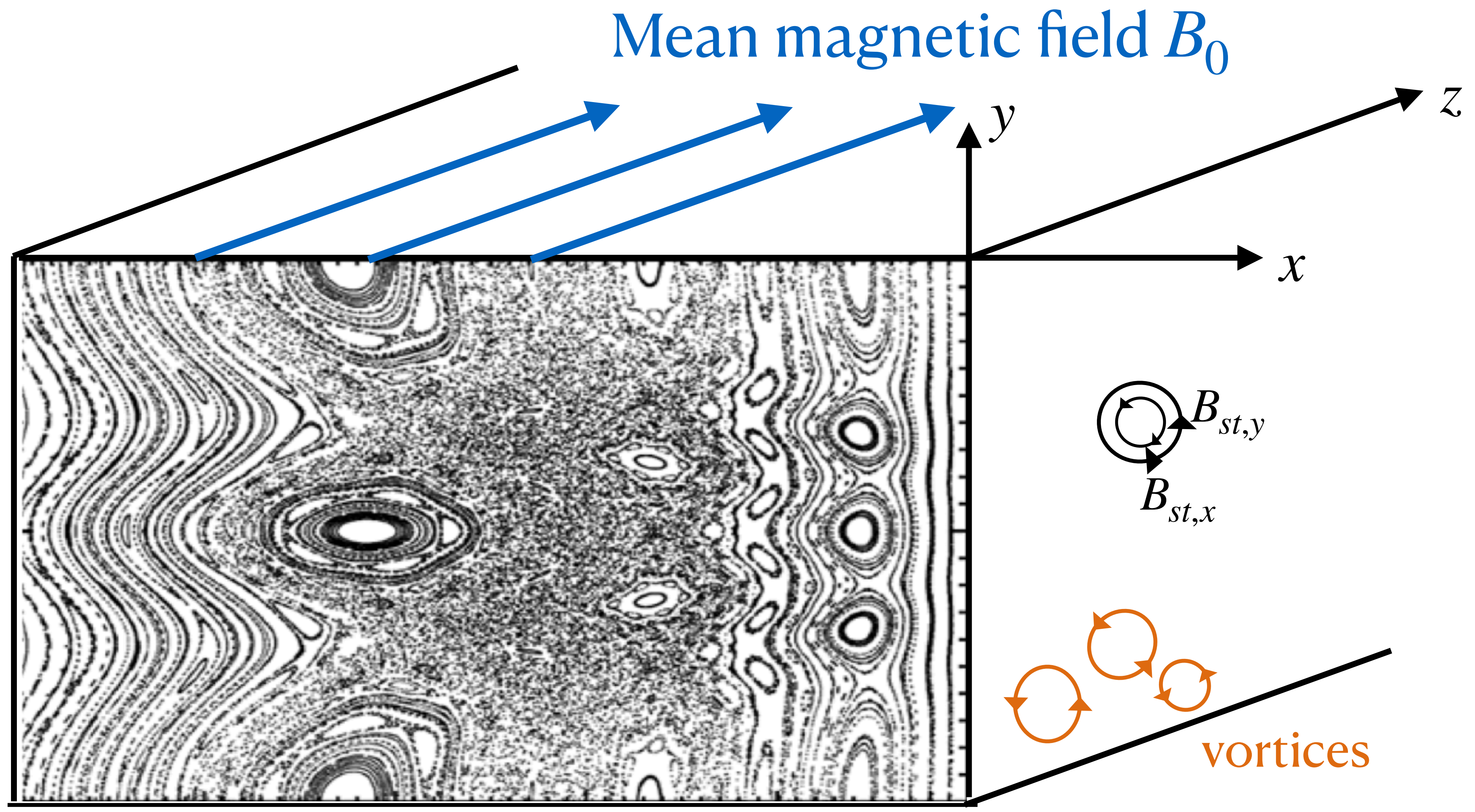
$$\zeta = \langle \zeta \rangle + \boxed{\tilde{\zeta},}$$ Perturbations produced by turbulences

$$where\ \langle\ \rangle = \frac{1}{L}\int dx \frac{1}{T}\int dt \qquad \langle \zeta \rangle = \frac{\partial v_{E\times B}}{\partial x} \quad (E \times B \text{ shear})$$

ensemble average over the zonal scales

We define rms of normalized stochastic field $b \equiv \sqrt{\overline{(B_{st}/B_0)^2}}$

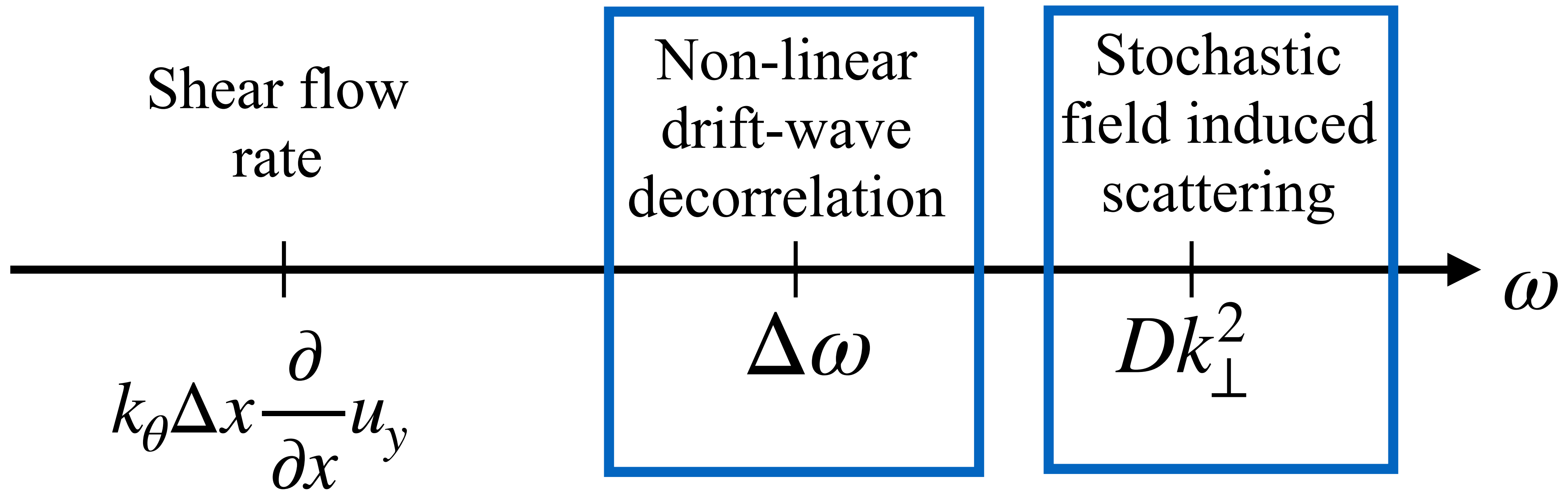
(See also works done by M. Leconte et al.)



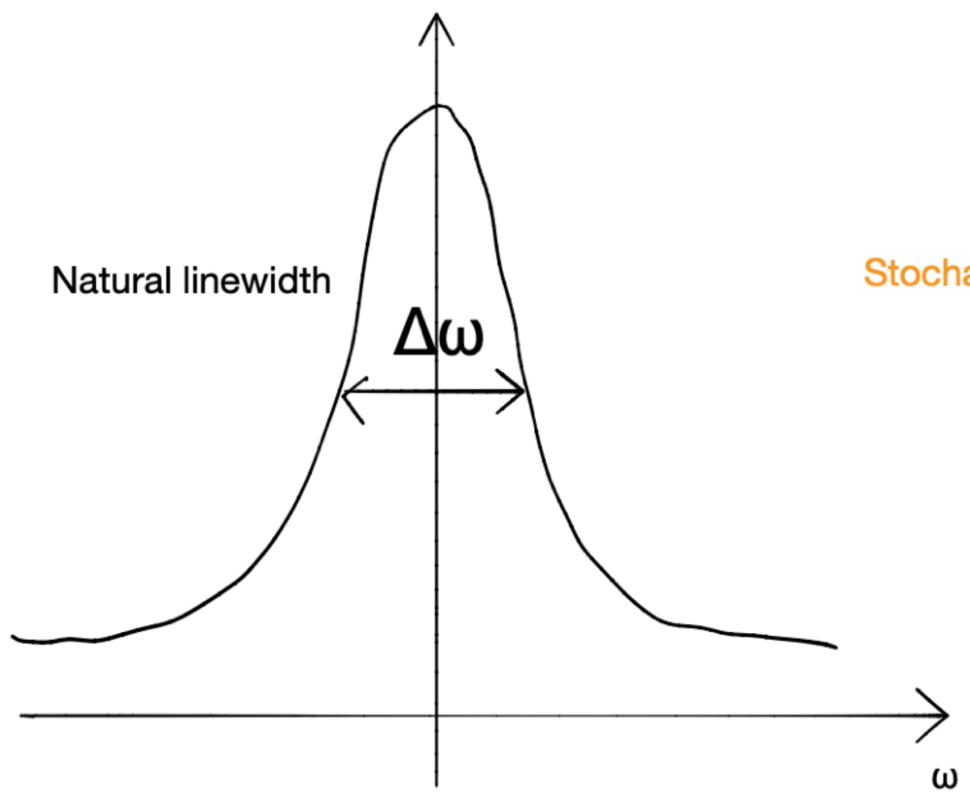
Magnetic islands overlapping forms stochastic

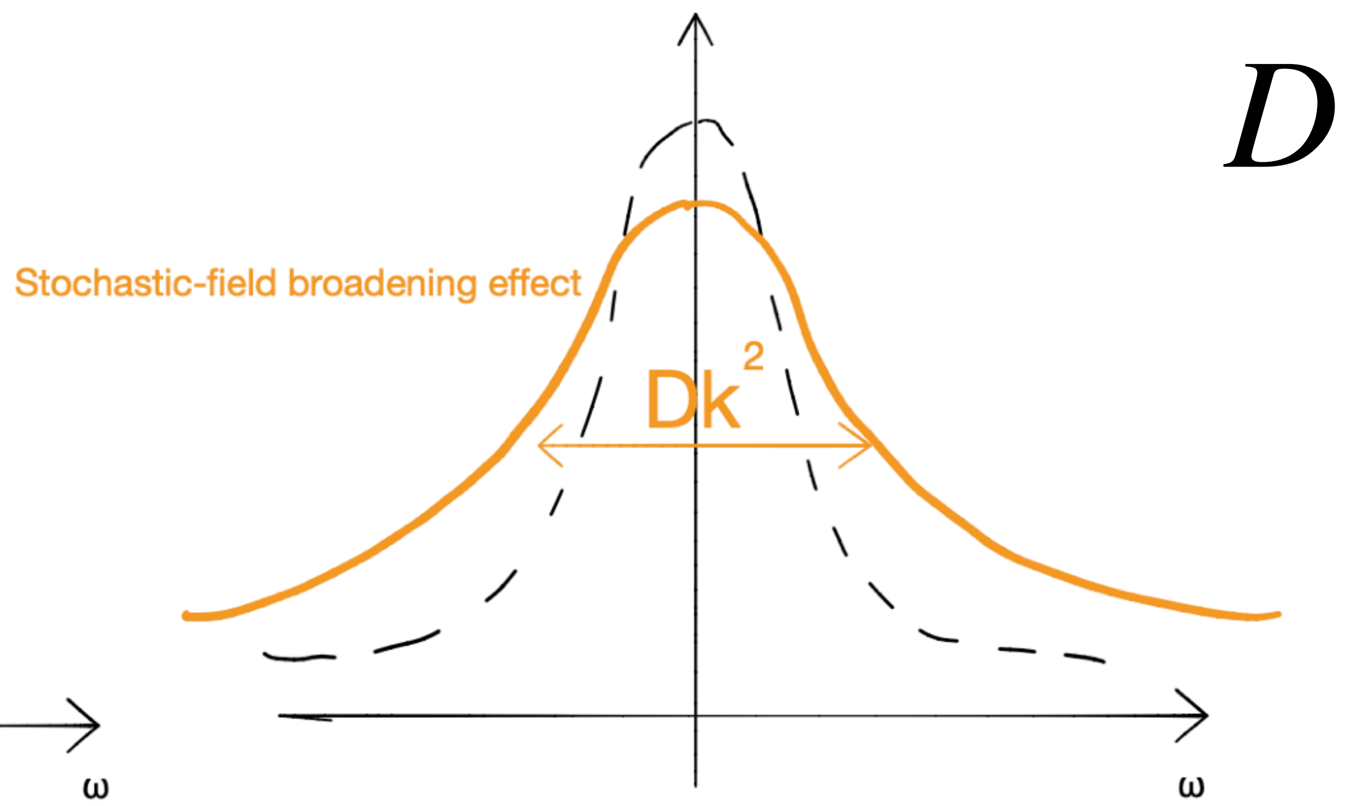
When does stochastic field effect becomes significant?

We consider timescales: (Chen et al.,PoP **28**, 042301 (2021))



Stochastic field decoherence beats the self-decoherence.





$$D \equiv v_A D_M = v_A \sum_k \pi\delta(k_z) b_k^2 \propto \boxed{B_{st}^2}$$

(Independent of B_0)

Magnetic diffusivity

Auto-correlation length l_{ac} ($\propto v_A$)

Perturbations propagate ultimately **in** $\perp$ (along stochastic fields)

→ characteristic velocity (v_A) emerges from the calculation of $\underline{\nabla} \cdot \underline{J} = 0$

Why v_A ?

Derivation of Magnetic Diffusivity

Vorticity equation: $(\frac{\partial}{\partial t} - \mathbf{u} \cdot \nabla)\nabla^2\phi - v_A(\cdot \nabla_\parallel + b_{st,\perp} \cdot \nabla_\perp)J_\parallel = 0$

$$\begin{cases} 0^{\text{th}} \text{ order} : v_A \frac{\partial}{\partial z} J_{0,z} = 0 \\ 1^{\text{st}} \text{ order} : (\frac{\partial}{\partial t} - \langle u_y \rangle \frac{\partial}{\partial y})\nabla^2 \widetilde{\phi} - v_A(\nabla_\parallel + b_{st,\perp} \cdot \nabla_\perp)\widetilde{J}_\parallel = 0 \end{cases}$$

$$\boxed{\text{Curly bracket : } \{\ \} = \int_{-\infty}^{+\infty} d\tau}$$

$$\{\frac{i}{-b_{st,\perp}k_\perp}\} = \int_{-\infty}^{+\infty} d\tau \{e^{ib_{st,\perp}k_\perp \int_0^\tau d\tau'}\} = \int_{-\infty}^{+\infty} d\tau e^{-k_i D_{M,ij} k_j \tau}$$

$$\int_0^{+\infty} d\tau = \int_0^{+\infty} \frac{dl}{|v_A|}$$

dl is along magnetic fields

Characteristic velocity of $b_{st,\perp}$ (parallel wave packet transit timescale)

$$D \equiv v_A D_M = v_A \sum_k \frac{B_{st,k}^2}{B_0^2} \pi \delta(k_z) \propto \cancel{v_A} \frac{1}{\cancel{v_A^2}} \cancel{v_A} B_{st}^2$$

Diffusivity D is independent of B_0 .

Dimensionless Parameters

How `stochastic' is magnetic field?

Alfvénic Dispersion

$v_A/L_{\|}$

(excited by drift-Alfvénic coupling)

v.s

Stochastic broadening

$Dk_{\perp}^2$

Ku_{mag} **(Magnetic Kubo number)**

$$\equiv \frac{\textbf{stochastic field scattering length}}{\textbf{perpendicular magnetic fluctuation size}} = \frac{l_{ac}b}{\Delta_{\text{eddy}}} \lesssim 1,$$

(for a b given)

Two dimensionless Parameters: $Dk_{\perp}^2 > \Delta\omega$

$$\begin{cases} l_{ac} \simeq Rq \\ \epsilon \equiv L_n/R \sim 10^{-2} \\ \beta \simeq 10^{-2\sim-3} \\ \rho_* \equiv \frac{\rho_s}{L_n} \simeq 10^{-2\sim-3} \end{cases}$$

1.

$$b^2 \equiv \left(\frac{\delta B_r}{B_0}\right)^2 > \sqrt{\beta}\rho_*^2\frac{\epsilon}{q} \sim 10^{-8}$$

Criterion for stochastic fields effect important to L-H transition.

2.

Broadening parameter

$$\alpha \equiv \frac{b^2}{\rho_*^2\sqrt{\beta}}\frac{q}{\epsilon} > 1$$

$\alpha = 1$:
stochastic broadening = natural linewidth

Decoherence of eddy tilting feedback

Snell's law:

$$\frac{d}{dt}k_x = -\frac{\partial(\omega_0 + u_y k_y)}{\partial x} = -k_y\frac{\partial u_y}{\partial x}$$

Gives an non-zero $\langle k_x k_y \rangle$

$\rightarrow \langle \tilde{u}_x \tilde{u}_y \rangle \propto \langle k_x k_y \rangle$

shear flow

Self-feedback loop:

The $E \times B$ shear generates the $\langle k_x k_y \rangle$ correlation and hence support the non-zero Reynolds stress.

$$\langle \tilde{u}_x \tilde{u}_y \rangle \simeq \sum_k \frac{|\tilde{\phi}_k|^2}{B_0^2}\left(k_y^2 \frac{\partial u_y}{\partial x}\tau_c\right)$$

The Reynold stress modifies the shear via momentum transport.

Shear flow reinforce the self-tilting.

Dispersion relation of drift-Alfvén coupling

$$\omega^2 - \omega_D \omega - k_\parallel^2 v_A^2 = 0$$

+

Stochastic Fields Effect

$$k_\parallel = k_\parallel^{(0)} + \underline{b}_\perp \cdot \underline{k}_\perp$$

$$\omega = \omega_D + \delta\omega$$

$$(\omega_D + \delta\omega)^2 - \omega_D(\omega_D + \delta\omega) - (k_\parallel + \underline{b} \cdot \underline{k}_\perp)^2 v_A^2 = 0$$

eigen-frequency shift

$$\delta\omega \simeq \frac{v_A^2}{\omega_D}\left(2k_\parallel \underline{b} \cdot \underline{k}_\perp + (\underline{b} \cdot \underline{k}_\perp)^2\right)$$

$$\omega_D \text{ (drift wave turbulence frequency)} \equiv \frac{k_y \rho_s C_s}{L_n}$$

Decoherence of eddy tilting feedback

Expectation frequency:

$$\left\langle \delta\omega \simeq \frac{v_A^2}{\omega_0}(2k_\parallel \underline{b}\cdot \underline{k}_\perp + (\underline{b}\cdot\underline{k}_\perp)^2) \right\rangle$$

Ensemble average of eigen-frequency shift

$$\langle\delta\omega\rangle \simeq \frac{v_A^2}{\omega_0}\langle(\underline{b}\cdot\underline{k}_\perp)^2\rangle = \frac{1}{2}\frac{v_A^2}{\omega_0}b^2k_\perp^2$$

$$\omega = \omega_D + \delta\omega$$

$$\langle\omega\rangle \simeq \omega_D + \frac{1}{2}\frac{v_A^2}{\omega_D}b^2k_\perp^2$$

Snell's law:

$$\frac{d}{dt}k_x = -\frac{\partial\omega_k}{\partial x}$$

$$= -k_y\frac{\partial u_y}{\partial x} - \frac{1}{2}\frac{v_A^2k_\perp^2}{\omega_D}\frac{\partial b^2}{\partial x}$$

Due to the Ensemble average eigen-frequency shift

Self-feedback loop is broken by b^2:

$$\langle\tilde{u}_x\tilde{u}_y\rangle \simeq \sum_k \frac{|\tilde{\phi}_k|^2}{B_0^2}\left(k_y^2\frac{\partial u_y}{\partial x}\tau_c + \frac{1}{2}k_y\frac{v_A^2k_\perp^2}{\omega_D}\frac{\partial b^2}{\partial x}\tau_c\right)$$

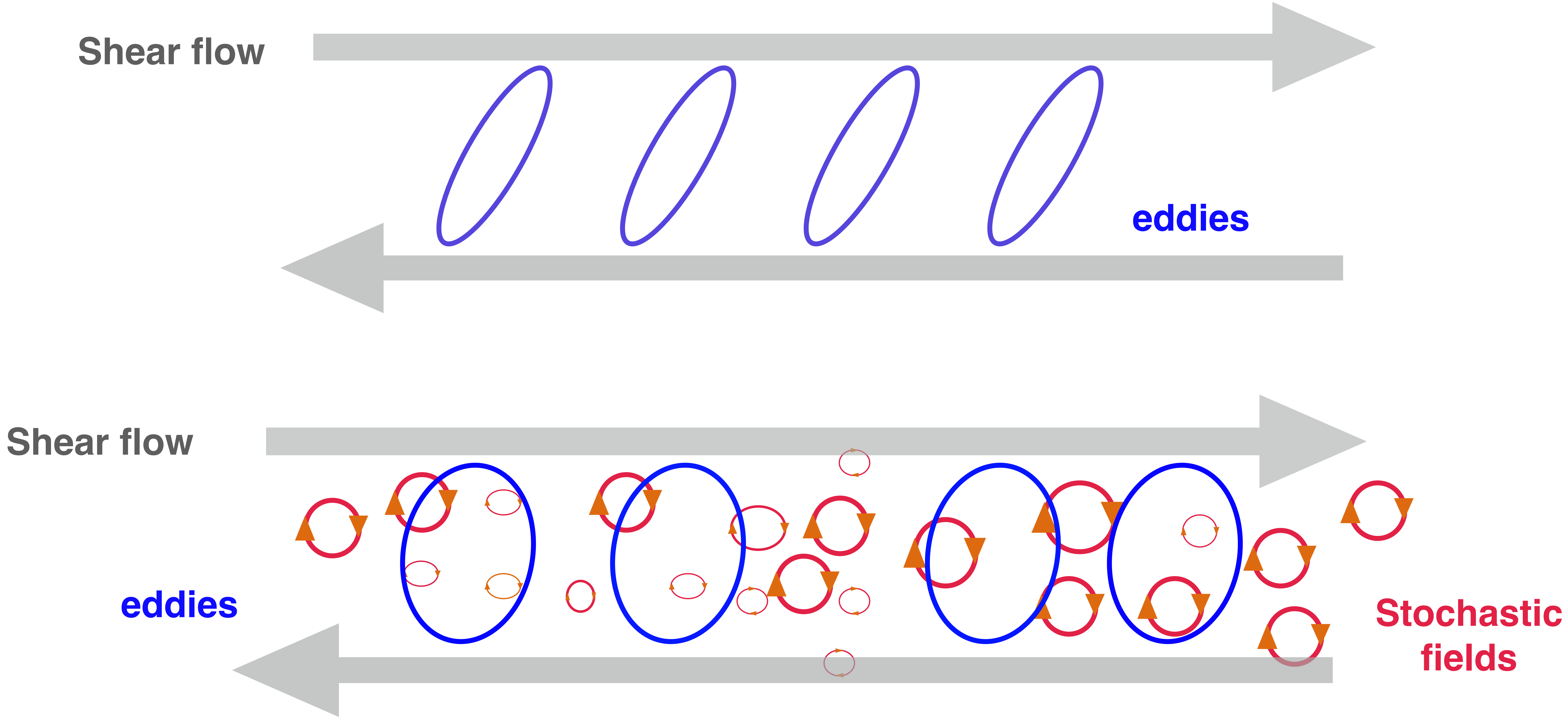
Stochastic dephasing

Stochastic fields (random ensemble of elastic loops) act as elastic loops and resist the tilting of eddies.

→change the cross-phase btw $\tilde{u}_x$ and $\tilde{u}_y$.

(Chen et al.,PoP **28**, 042301 (2021))

Decoherence of eddy tilting feedback



Stochastic fields interfere with shear-tilting feedback loop.

Results—Suppression of PV diffusivity

The ensemble average Reynolds force $\frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle$:

$$\text{PV flux} = \langle \tilde{u}_x \tilde{\zeta} \rangle = \frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle = -\underbrace{\boxed{D_{PV}}}_{\text{PV diffusivity}} \frac{\partial}{\partial x}\langle \zeta \rangle + \underbrace{\boxed{F_{res}}}_{\text{Residual Stress}} \underbrace{\kappa}_{\text{Curvature}} \frac{\partial}{\partial x}\langle p \rangle$$

Suppressed by stochastic fields

Taylor Identity: $\langle \tilde{u}_x \tilde{\zeta} \rangle = \frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle$

$$\langle \zeta \rangle = \frac{\partial v_{E\times B}}{\partial x} \quad (E \times B \text{ shear})$$

$$\boxed{D_{PV}} = \sum_{k\omega} |\tilde{u}_{x,k\omega}|^2 \frac{\boxed{v_A} b^2 l_{ac} k^2}{\bar{\omega}^2 + \left(v_A b^2 l_{ac} k^2\right)^2}$$

PV transport will be suppressed by stochastic fields via decoherence.

$$F_{res} \simeq \sum_{k\omega} \frac{-2k_y}{\bar{\omega}\rho} D_{PV,k\omega}$$

$$\bar{\omega} \equiv \omega - \langle u_y \rangle k_y$$

$$\text{Zonal flow acceleration} = \frac{\partial}{\partial t}\langle u_y \rangle = \boxed{D_{PV}} \frac{\partial}{\partial x}\langle \zeta \rangle - \boxed{F_{res}} \kappa \frac{\partial}{\partial x}\langle p \rangle$$

Zonal flow acceleration is slowed down by the stochastic field.

This stochastic dephasing is insensitive to turbulent modes, e.g. ITG, TEM,…etc.

Macroscopic Impact

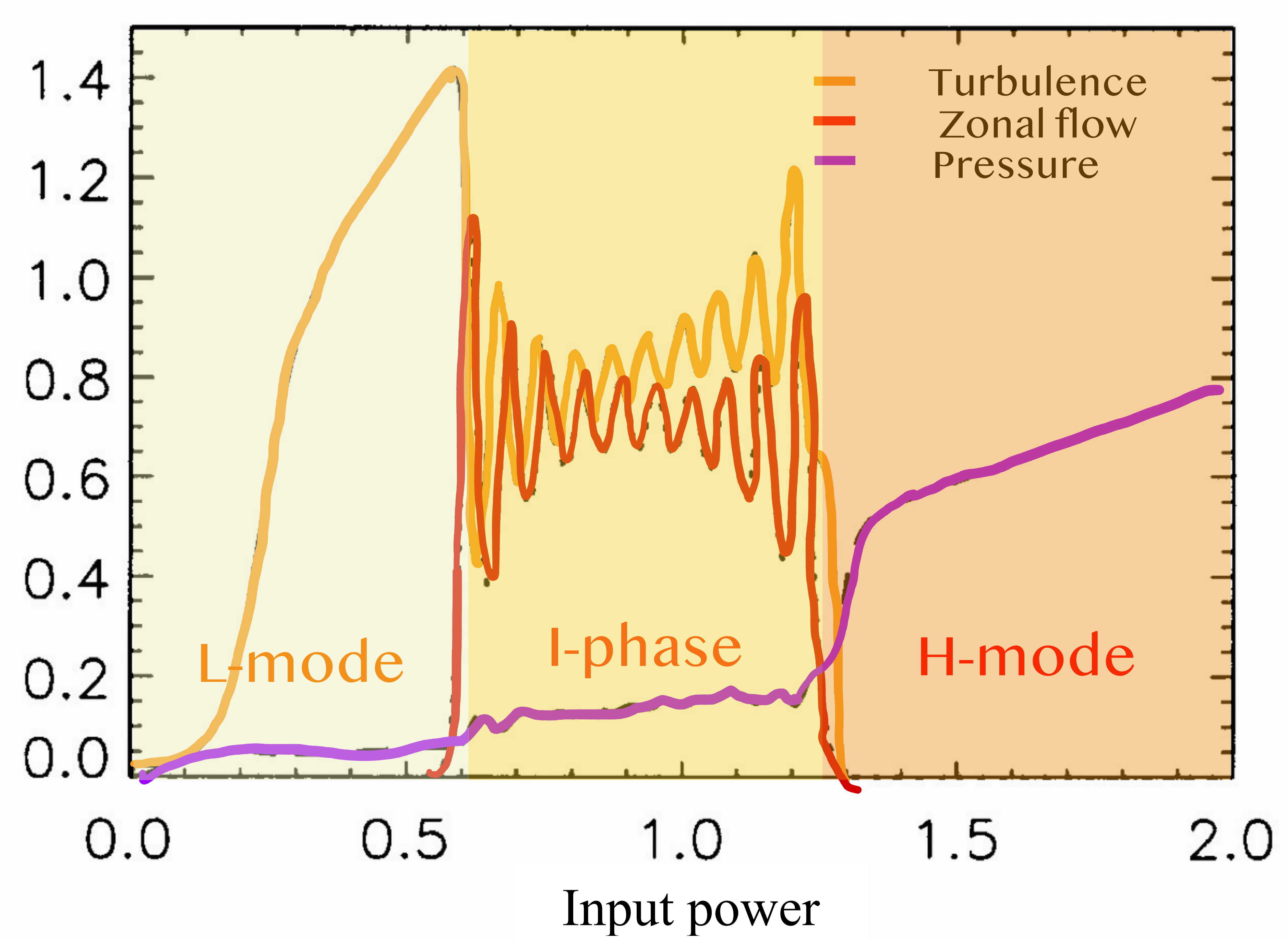
Results — Increment of P_{LH}

Stochastic field stress dephasing effect requires: $\Delta\omega \leq k_{\perp}^{2}D$ (where $D = D_M v_A$).

This gives Broadening parameter (α): $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} > 1$

α quantifies the strength of stochastic dephasing.

$$\begin{cases} l_{ac} \simeq Rq \\ \epsilon \equiv L_n/R \sim 10^{-2} \\ \beta \equiv \frac{P_{thermal}}{P_{mag}} \simeq 10^{-2\sim-3} \\ \rho_* \equiv \frac{\text{gyro-radius}}{\text{density scale length}} \\ \quad \equiv \frac{\rho_s}{L_n} \simeq 10^{-2\sim-3} \\ q(\text{stafety factor}) \equiv \frac{rB_t}{RB_p} \end{cases}$$



Kim-Diamond Model

(Kim & Diamond, PoP **10**, 1698 (2003))

This reduce model for the L-H transition is useful for testing trends in power threshold increment induced by stochastic fields.

Predator: zonal flow
prey: turbulence

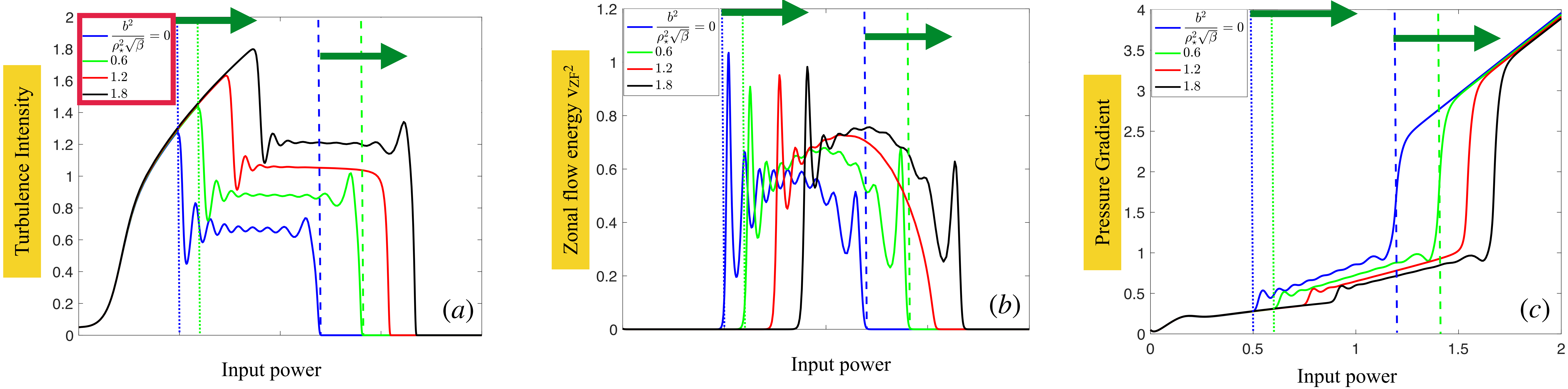
We expect stochastic fields to raise L-H transition thresholds.

Macroscopic Impact

Results — Increment of P_{LH}

$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} = 0.0, 0.2, 0.4, 0.6, 0.8...., 2.0$$

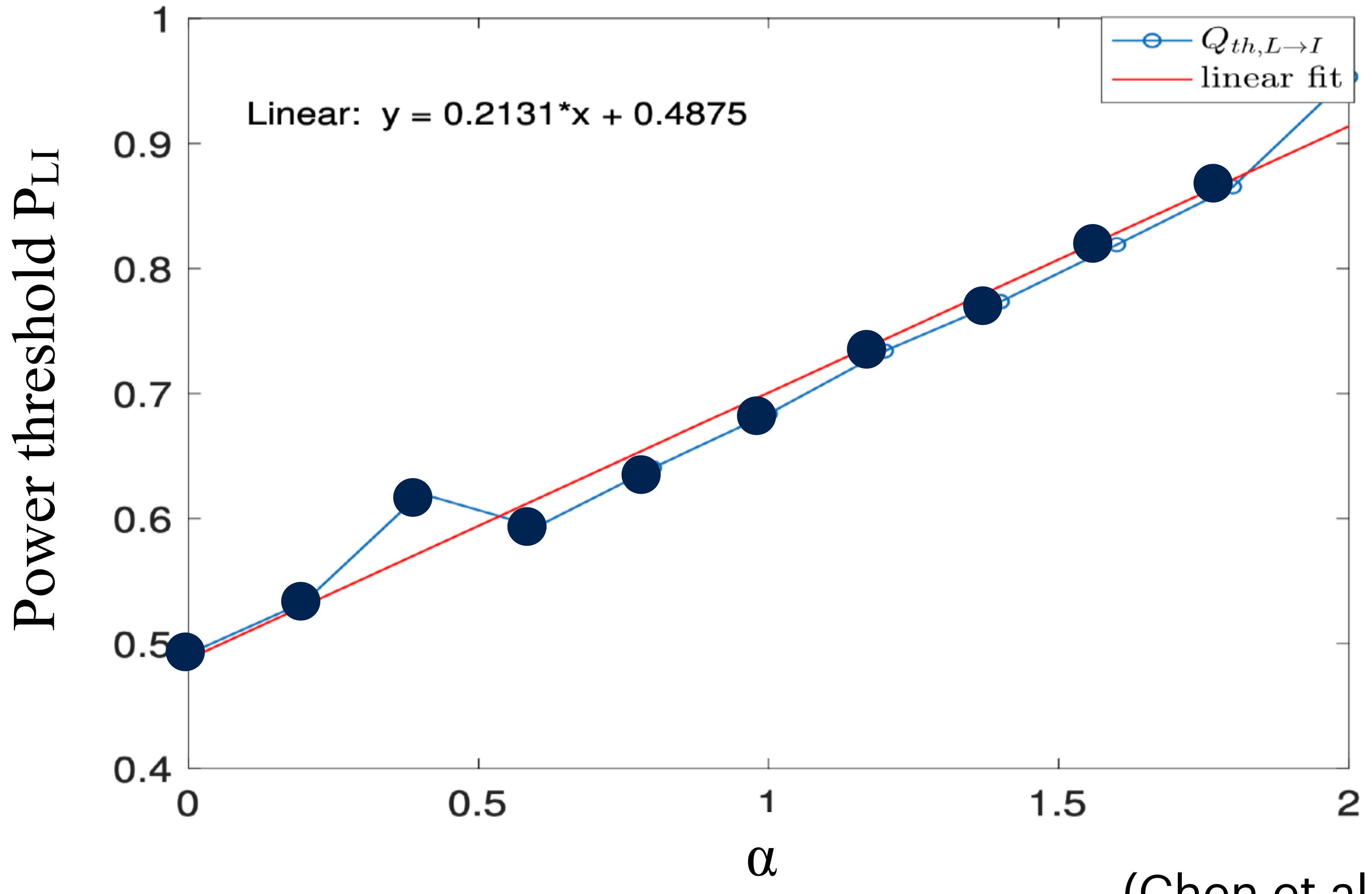
$\alpha \neq 0$

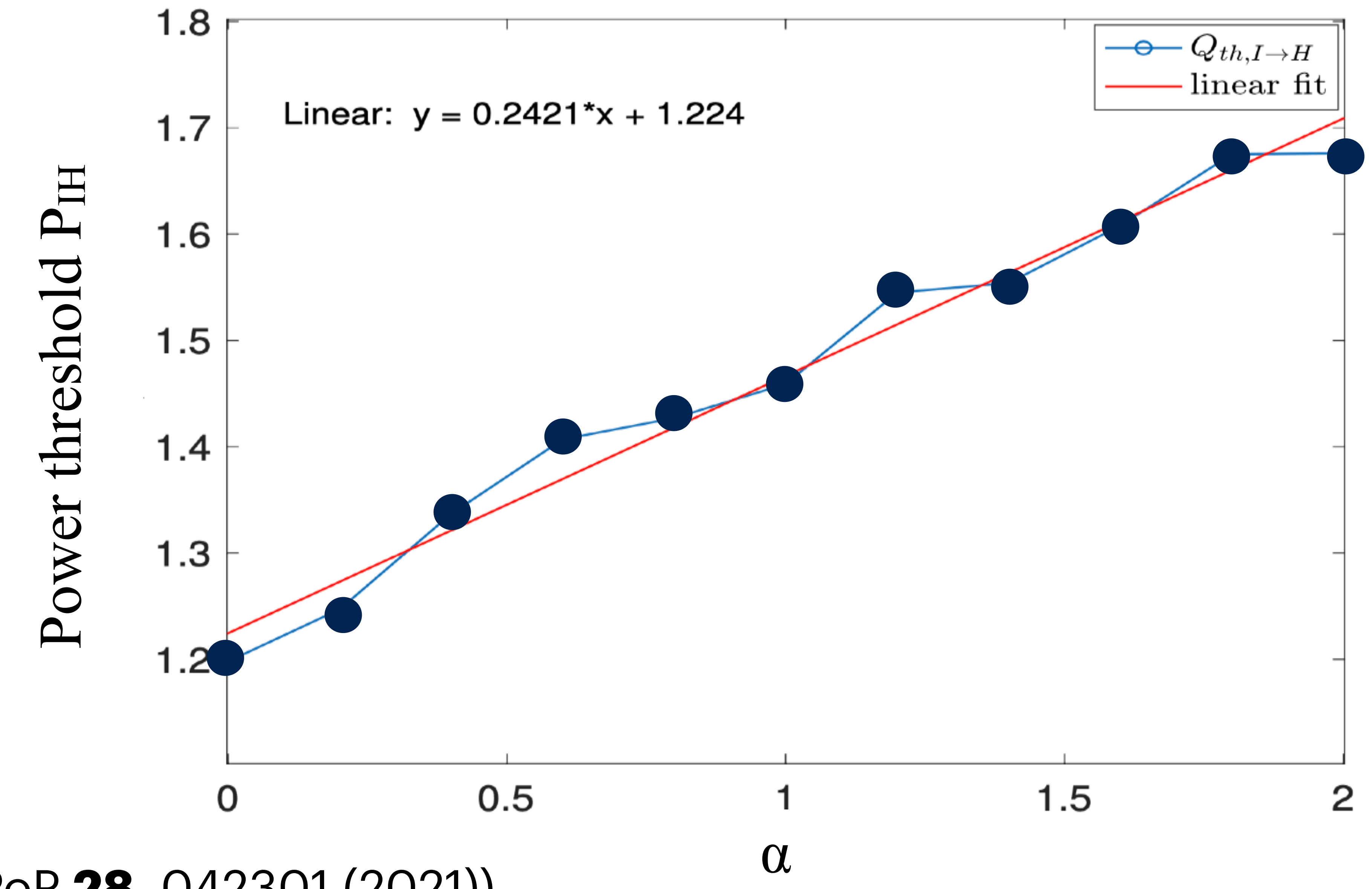


The threshold increase due to stochastic dephasing effect is seen in turbulence intensity, zonal flow, and pressure gradient.

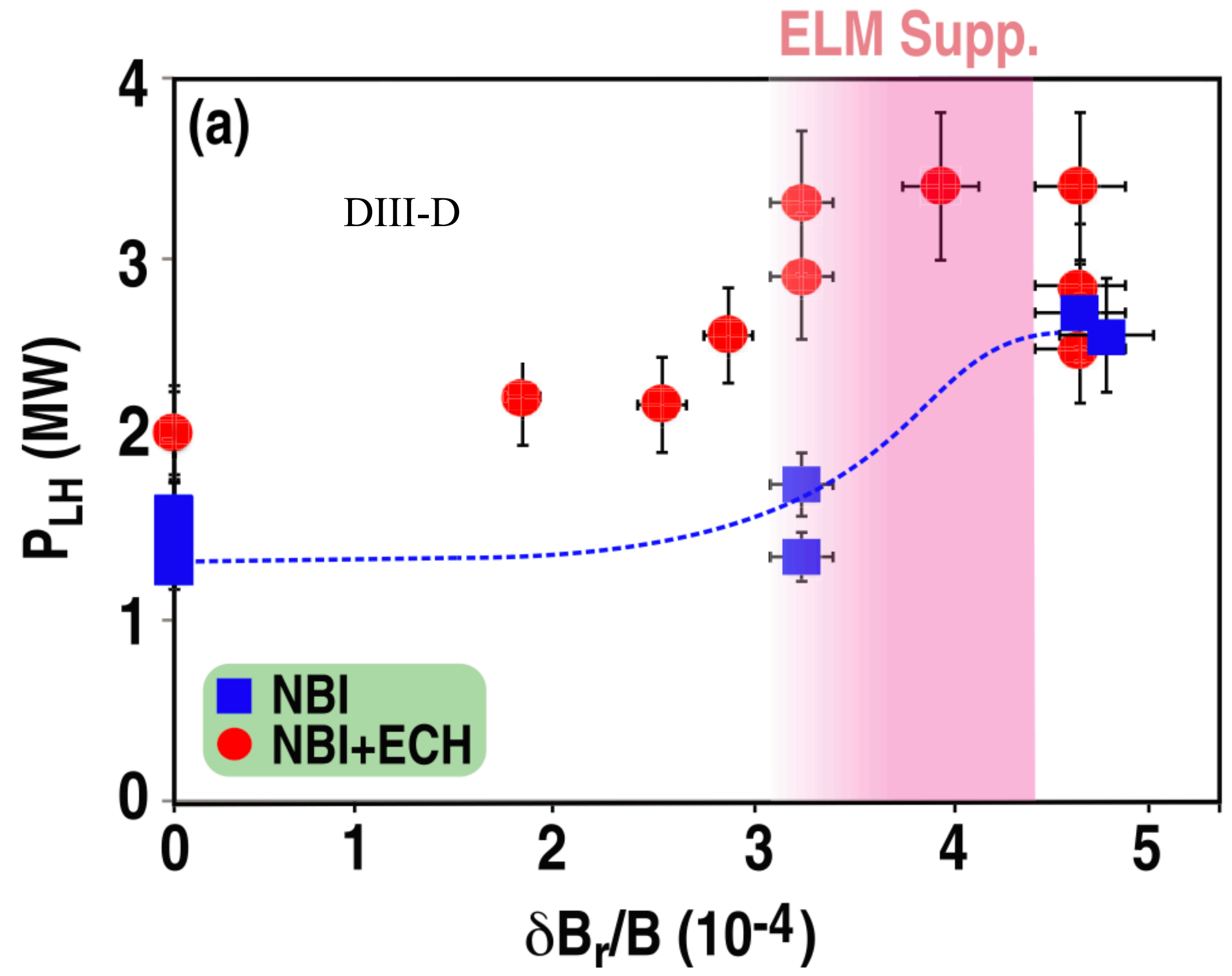
(Chen et al.,PoP **28**, 042301 (2021))

Results — Increment of P_{LH}





(Chen et al.,PoP **28**, 042301 (2021))



(L. Schmitz et al, NF **59** 126010 (2019))

Broadening parameter

$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$$

α quantifies the strength of stochastic dephasing.

The threshold increase linearly, in proportional to α. This is due to stochastic dephasing effect.

Intrinsic Rotation and Kinetic Stress

From parallel acceleration:

$$\frac{\partial}{\partial t}u_z + (\mathbf{u}\cdot\nabla)u_z = -\frac{1}{\rho}\frac{\partial}{\partial z}p$$

+

Stochastic Fields Effect

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z}^{(0)} + \underline{b}\cdot\underline{\nabla}_{\perp}$$

$$\frac{\partial}{\partial t}\langle u_z\rangle + \frac{\partial}{\partial x}\langle\tilde{u}_x\tilde{u}_z\rangle = -\frac{1}{\rho}\frac{\partial}{\partial x}\langle b\tilde{p}\rangle$$

Toroidal Reynolds Stress

Kinetic Stress

$$\langle\tilde{u}_x\tilde{u}_z\rangle = -\nu_{turb}\frac{\partial}{\partial x}\langle u_z\rangle + F_{z,res}\frac{\partial}{\partial x}\langle p\rangle$$

Turbulent viscosity

Toroidal Residual Stress

Influence intrinsic rotation

$$\nu_{turb} = \sum_{k\omega}|\tilde{u}_{x,k\omega}|^2\frac{2C_sb^2l_{ac}k^2}{\omega_{sh}^2 + (2C_sb^2l_{ac}k^2)^2}$$

Pat Diamond's talk 15:15 pm

- The sound speed is the relevant speed (acoustic dynamics).
  Stochastic fields effect is weaker ($C_sD_M < v_AD_M$).
- $F_{z,res} \sim \sum_{k\omega}\frac{-k_z}{\omega_{sh}\rho}\nu_{turb,k\omega}$.

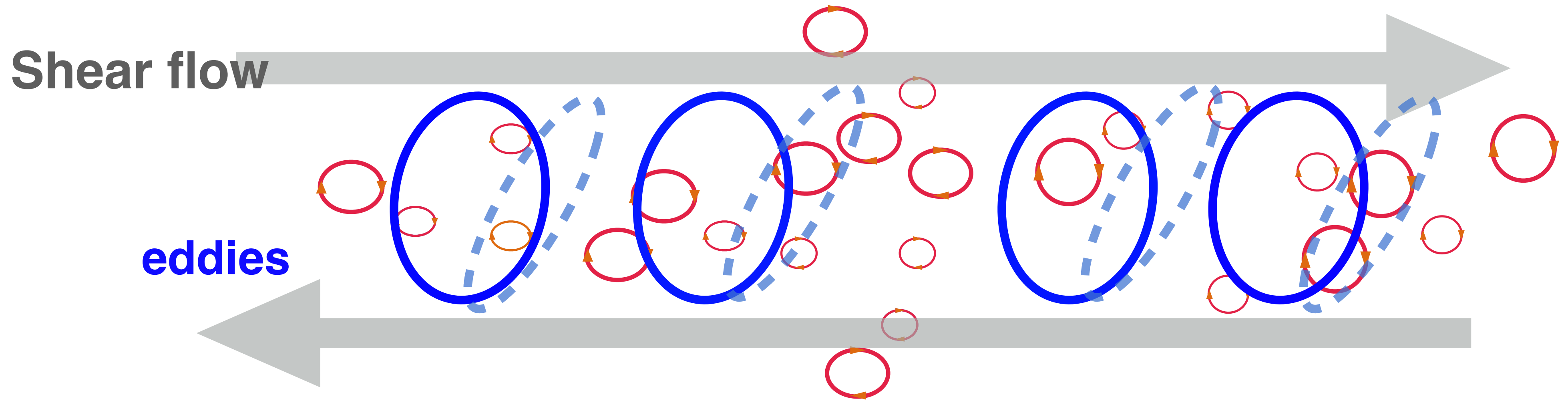
$F_{z,res}$ Requires symmetry breaking $\langle k_zk_y\rangle \neq 0$

(Chen et al.,PoP **28**, 042301 (2021))

Stochastic fields reduce the toroidal stress and hence slow down the intrinsic rotation.

Conclusions

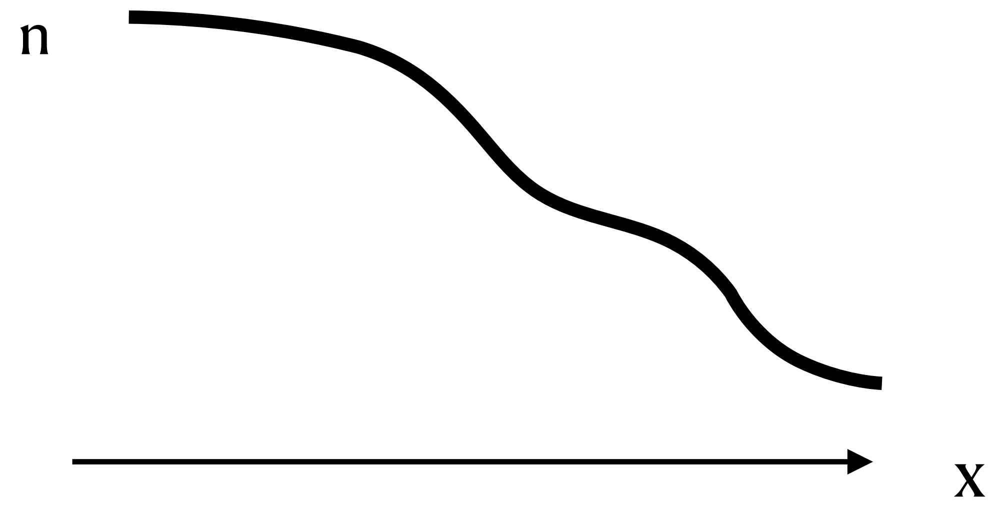
- **Dephasing effect** caused by stochastic fields quenches poloidal Reynolds stress (e.g. $\Delta\omega < Dk_\perp^2$). Here, $D = v_A D_M$.



- b^2 shift L-H threshold to higher power, in proportional to $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$.
- Stochastic fields have weaker effect on reducing toroidal Reynolds stress, since $C_s D_M < v_A D_M$. Need to revisit symmetry breaking $\langle k_y k_z \rangle \neq 0$ calculation (for $F_{z,res}$) in stochastic magnetic field.

Future Works

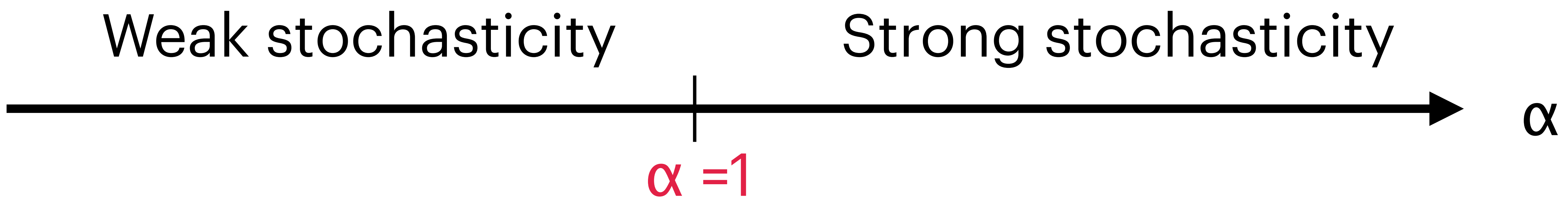
- We study the scale corrugation of staircases in presence of stochastic fields.
- Detailed calculations for symmetry breaking of toroidal residual stress.



Takeaways for Experimentalists

- Reynolds stress suppression due to stochastic dephasing→ generation of zonal flow is suppressed.
  Zonal intensity stays the same but damping occurs due to the stochastic dephasing.

- Stochastic fields broadening effect can be parameterized by α.

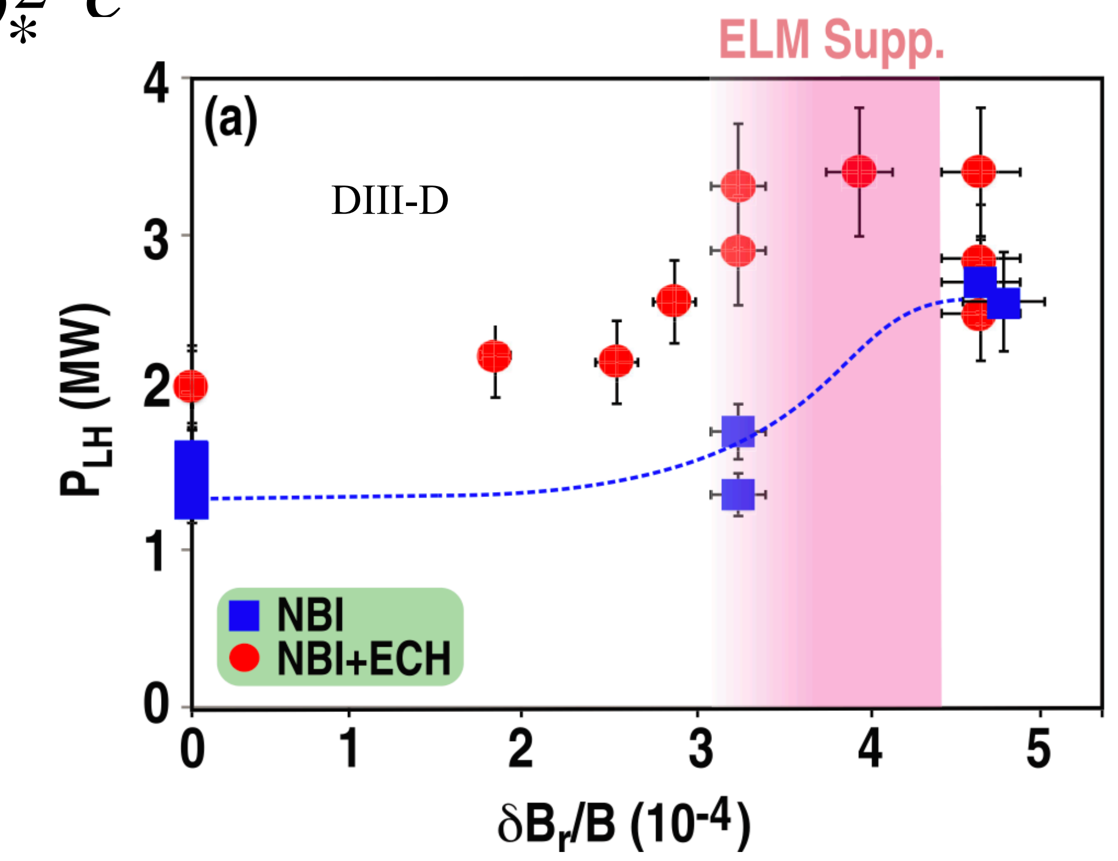


- b^2 shift L-H threshold to higher power, in proportional to $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$.

$$\alpha \propto \frac{1}{\rho_*^2}$$

ρ_* is small → $\alpha \uparrow$ (pessimistic)

- Our results predicts the power threshold of L-H transition increases linearly as stochastic magnetic field intensity increases.



(L. Schmitz et al, NF **59** 126010 (2019))

Thank you!

Fate of Spatial structure of zonal flow?

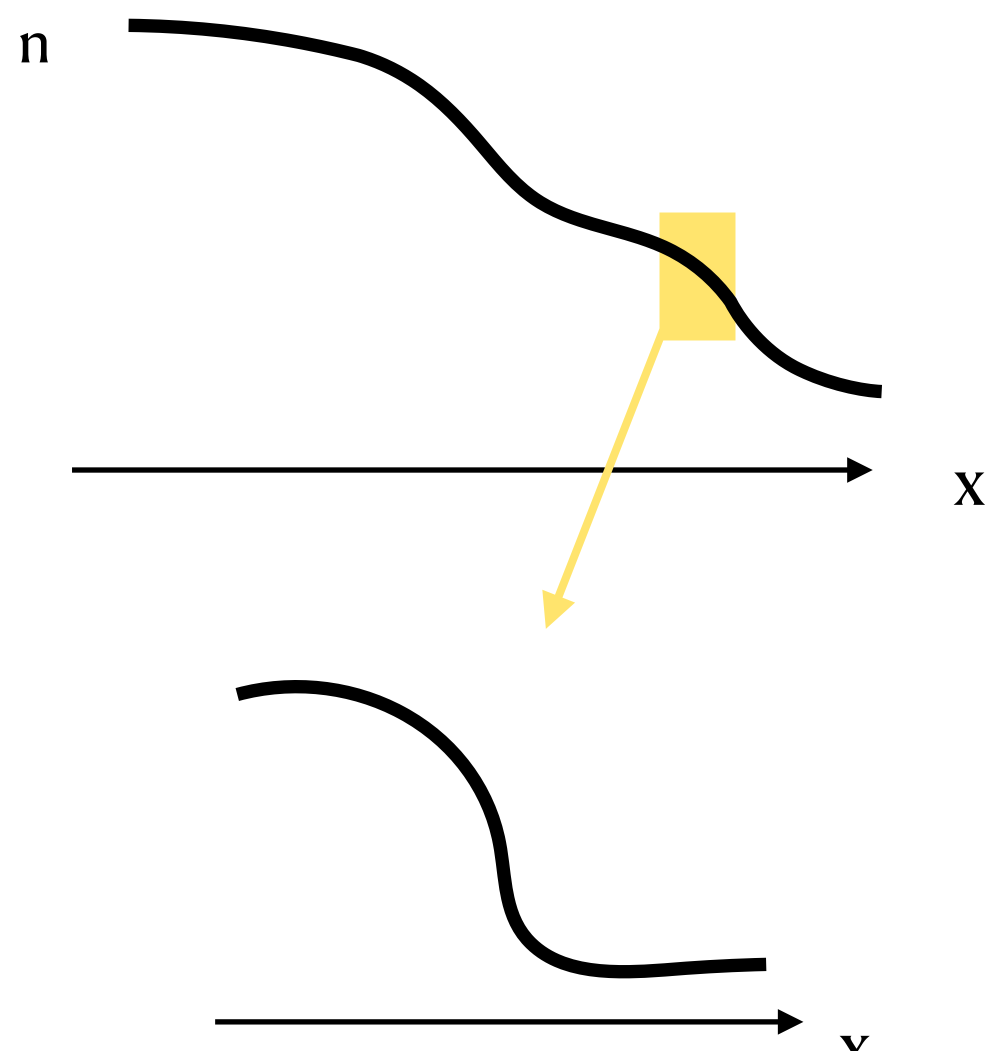
Shear Flow

↓

Edge-Localized Mode (ELM)

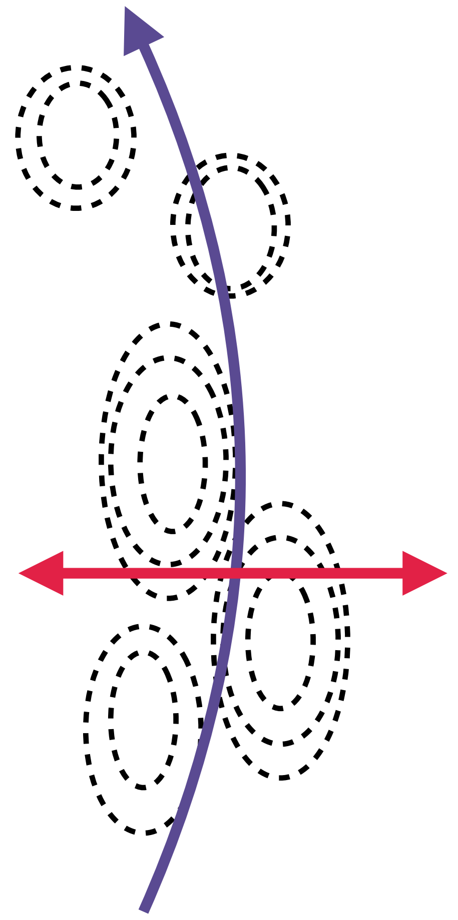
↓

Edge gradient ↑



Density corrugation

Poloidal zonal



Zonal flow width

Zonal flow width is related to corrugation length.

We are interested in zonal flow width in presence of stochastic fields.

Backup Slides

Layering Structure—Mixing Length Model

A mixing length model for layering:

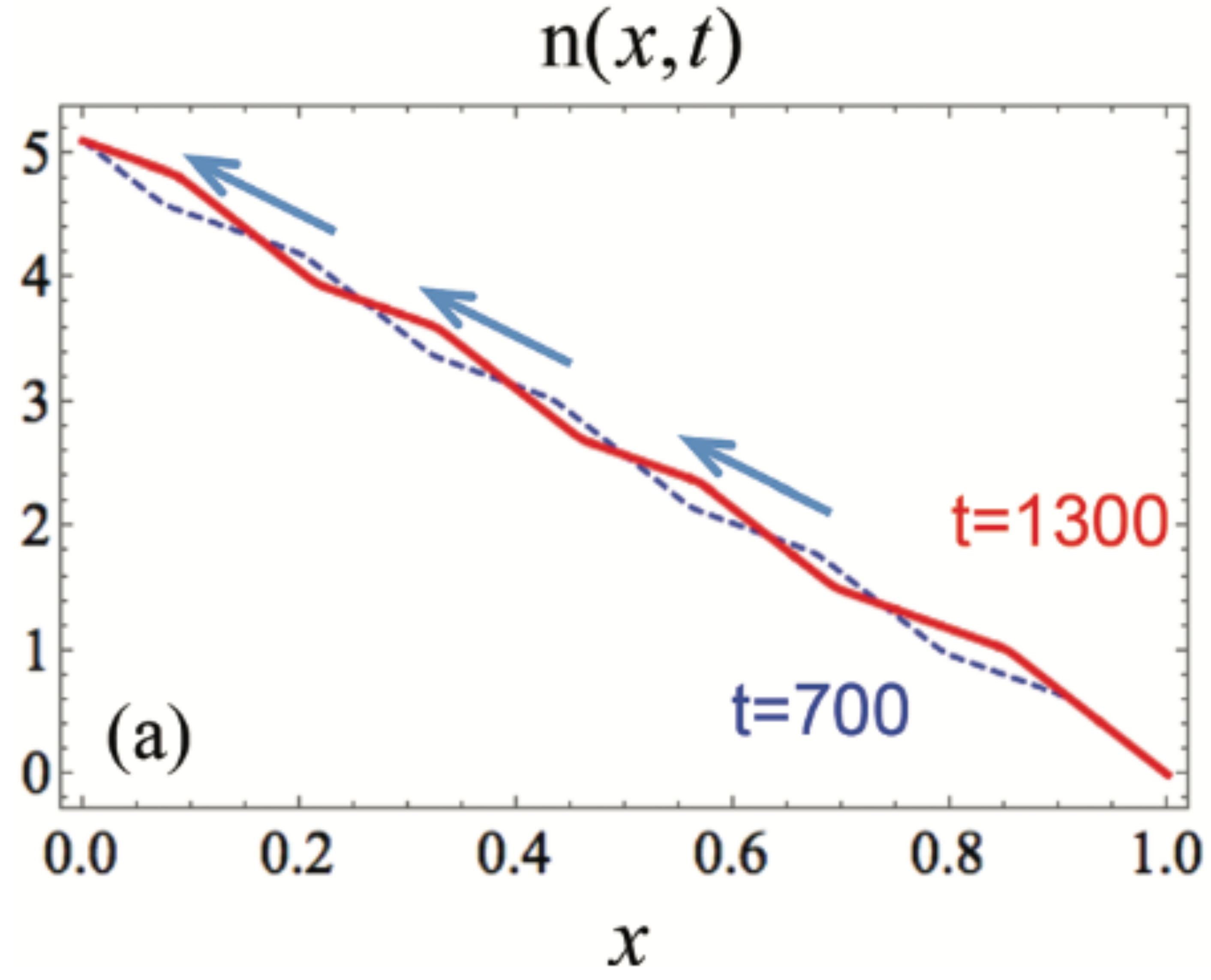
- Reduce evolution equations (based on H-W model).
- Energy and Potential entropy (PE) conserved.

Density: $\frac{\partial}{\partial t}\langle n\rangle = \underbrace{\frac{\partial}{\partial x}\left(D_n \frac{\partial \langle n\rangle}{\partial x}\right)}_{\text{turb. particle diffusion}} + D_c \frac{\partial^2}{\partial x^2}\langle n\rangle$

Potential Vorticity: $\frac{\partial}{\partial t}\langle \zeta\rangle = \frac{\partial}{\partial x}\left(\underbrace{(D_n - \chi)}_{\text{residual stress}} \frac{\partial \langle n\rangle}{\partial x}\right) + \underbrace{\chi \frac{\partial^2}{\partial x^2}\langle \zeta\rangle}_{\text{turb. Viscous diffusion}} + \mu_c \frac{\partial^2}{\partial x^2}\langle \zeta\rangle$

Turbulent potential Enstrophy: $\frac{\partial}{\partial t}\epsilon = \underbrace{\frac{\partial}{\partial x}\left(D_\epsilon \frac{\partial \epsilon}{\partial x}\right)}_{\text{PE diffusion}} + \underbrace{\chi[\frac{\partial (n-\zeta)}{\partial x}]^2}_{\text{mean-turb PE Coupling}} - \underbrace{\epsilon_c^{-1/2}\epsilon^{3/2}}_{\text{PE Dissipation}} + P$

n : density
ζ : potential vorticity
ϵ : turbulent PE $\quad \epsilon \equiv (\delta n - \delta\zeta)^2/2$
D_n : turbulent particle diffusivity
χ : turbulent vorticity
P : production



Ashourvan & Diamond, PoP **24,** 012305 (2017)

Density corrugation forms staircase-like structure.

Backup Slides

Scale Selection

The mixing length (l_{mix}) depends on **two scales:**

- Driving scale: l_0
- Rhines scale: $l_{RH} = \dfrac{\sqrt{\epsilon}}{|\partial_x q|}$

$$\Rightarrow \quad \text{mixing scale: } l_{mix} = \frac{l_0}{(1 + l_0^2(\partial_x q)^2/\epsilon)^{\kappa/2}} = \boxed{\frac{l_0}{(1 + l_0^2/l_{RH}^2)^{\kappa/2}}}$$

$$\begin{cases} \text{Strong mixing } (l_{RH} > l_0): & l_{mix} \simeq l_0 \text{ (Weak mean PV gradient)} \\ \text{Weak mixing } (l_0 > l_{RH}): & l_{mix} \simeq l_0^{1-\kappa} l_{RH}^{\kappa} \text{ (Strong PV gradient)} \end{cases}$$

l_{mix} (hybrid length scale) sets the scale of zonal flow.

What is the effect of stochastic fields on staircases?

Main effect of diffusivity D_n and χ

For α_{DW} (a measurement of the resistive diffusion rate in the parallel direction) > 1 in H-W regime:

Density diffusivity:

$$D_n \simeq \frac{l_{mix}^2 \epsilon}{\alpha_{DW}}$$

Resistive diffusion rate:

$$\alpha_{DW} = \frac{k_{\parallel}^2 v_{the}^2}{\nu}$$

+

Stochastic Fields Effect

$$k_{\parallel} = \underline{k} \cdot \underline{\hat{b}_0} \simeq \frac{1}{Rq} + \underline{b}_{\perp} \cdot \underline{k}_{\perp} \simeq \frac{1}{Rq} + \frac{b_{\perp}}{l_{mix}}$$

$\rightarrow$

$$D_n \simeq \frac{l_{mix}^2 \epsilon \nu / v_{the}^2}{(\frac{1}{Rq})^2 + (\frac{b}{l_{mix}})^2}$$

Same for χ (or D_{PV} in this case).

Competition btw $\frac{1}{Rq}$ v.s. $\frac{b_{\perp}}{l_{mix}}$ gives $Ku_{mag} = bRq/l_{mix}$ $\rightarrow$ $Ku_{mag} = Ku_{mag}(l_{mix})$

The mixing length is not likely affected by b^2.

A change of scale selection or staircase corrugation requires $Ku_{mag} \geq 1$.

Conclusions

- The mixing length is not likely affected by b^2. To change mixing length, we need $Ku_{mag} \geq 1$.