Mean field model for turbulent transport in a stochastic magnetic field

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Outline

- Motivation and background
  - Why? → Interaction and co-existence of stochastic B field and turb.
  - Key issues (L→H transition with RMP, island, stellarator, etc)
- Mean field model $\langle E_r \rangle$—follow radial force balance
  - Key fundamentals:

    — *phases*
    — *instability in stochastic field*
  - Turbulent transport

    — *Particle transport*
    — *Momentum transport (poloidal and toroidal)*
    — *Ion heat transport*
- Applications
  - L-H transition with $\tilde{b}^2$, 0D at present
- Implications and future work

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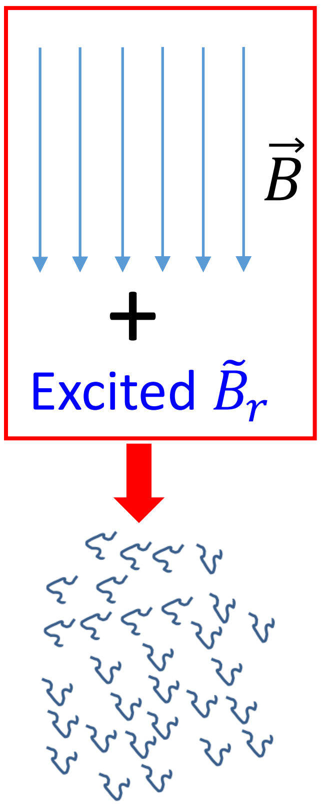
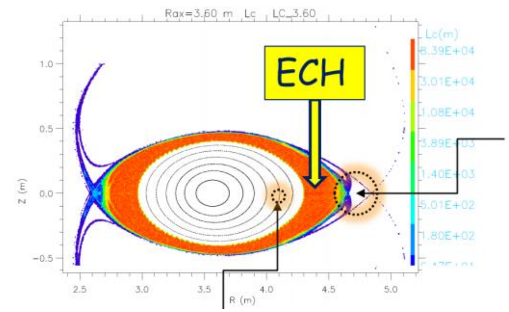
Stochastic magnetic field

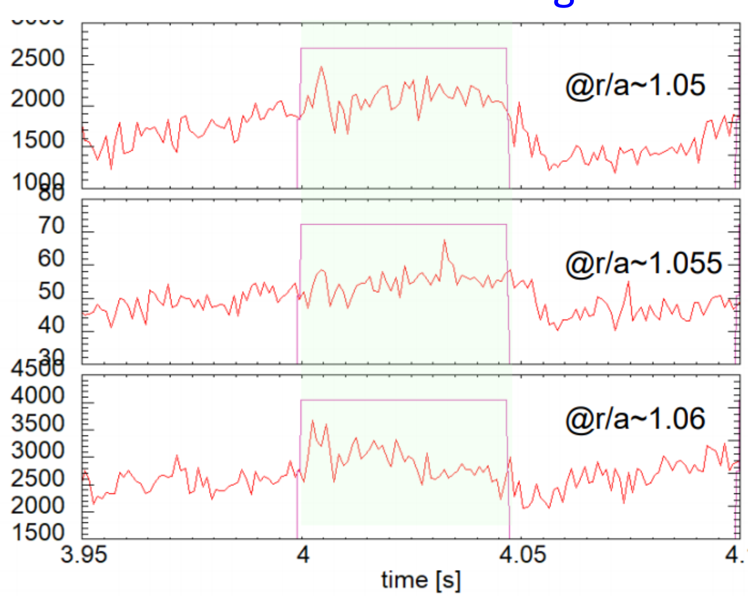
- Stochastic field : chaos of magnetic field lines (RMP, island, stellarator,……)

✓ **Interaction and co-existence** of stochastic magnetic field and turbulence



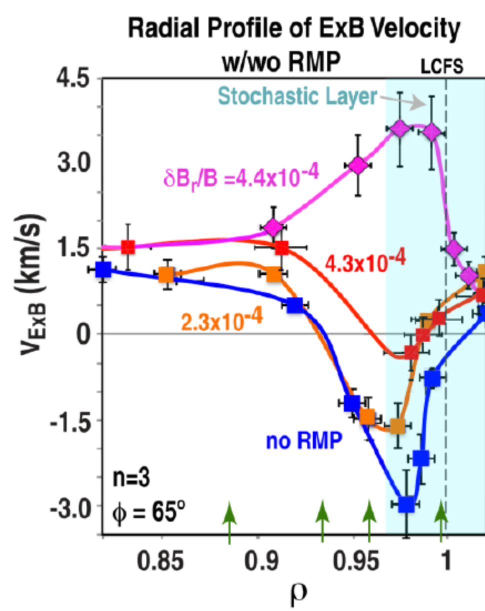


Turbulence in stochastic region



T. Tokuzawa, APTWG 2021

L→H occurs in stochastic layer



L. Schmitz et al, NF 2019

Motivation

- Stochastic field is important for boundary control in fusion device

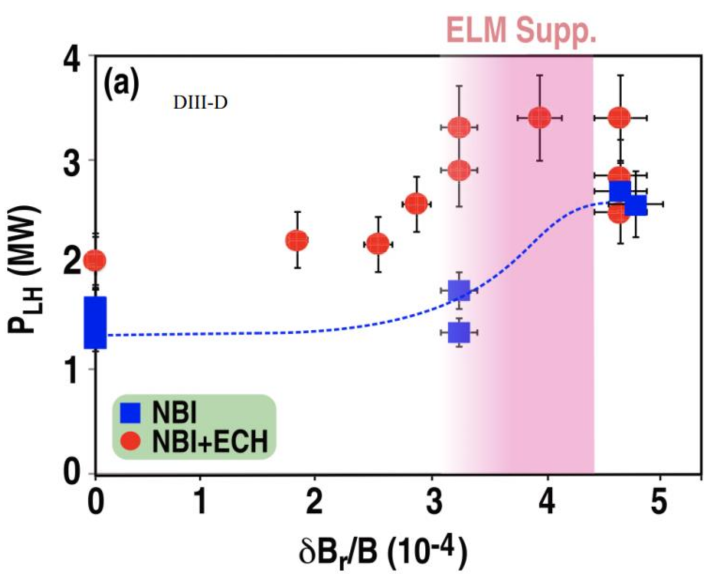
Shear Flow → Edge-Localized Mode (ELM) → ELM Burst

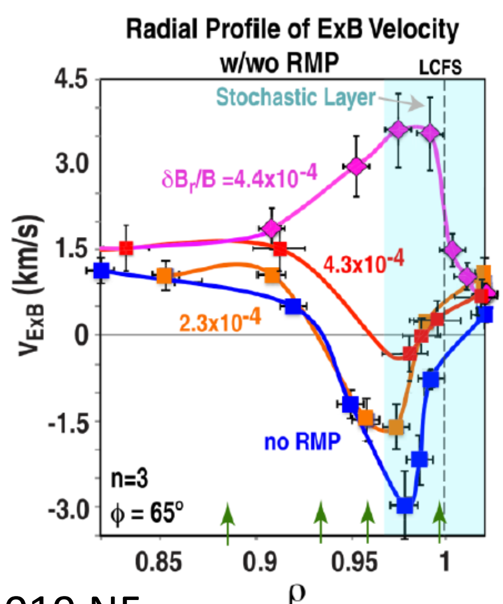
Boundary Control: Resonant Magnetic Perturbation (RMP)

Suppress (by inducing magnetic perturbation)

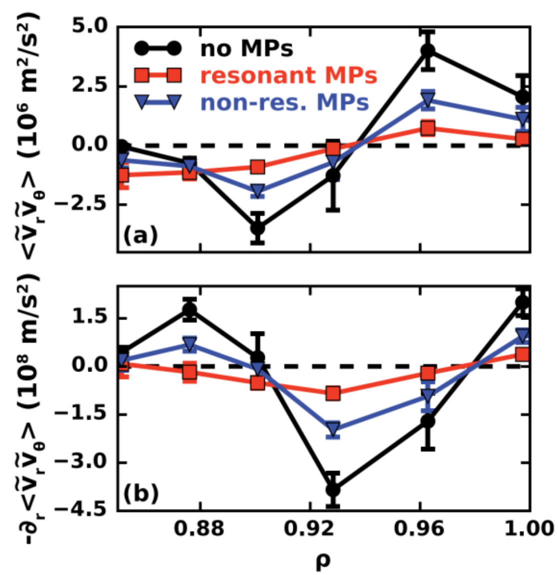
Stochasticity: Overlapping of Magnetic islands.

Trade off: RMPs controls gradients and mitigates ELM, but raise the **power threshold**.





Schmitz 2019 NF



Kriete 2020 PoP

Motivation

Need theory for turbulent transport in stochastic B field:

(current focus: L→H transition)

Key physics (All interconnected) :

- Direct effect of stochastic field on turbulence[1] → generation of micro convective cell

  [1] M. Y. Cao, P9, this meeting

- Dephasing effect[2]→ quenches poloidal Reynolds stress and generation of ZF

  [2] C. C. Chen, O 3.2, this meeting

- Particle transport[3] → density pump-out
- Momentum transport (V_θ and V_ϕ)[3]→ intrinsic torque at edge
- Heat transport[3] → L-H power threshold

  [3] P.H. Diamond, O3.1, this meeting

Note: Previous focused on electron heat transport (Manz 2020), we focus on flow, particle and ion heat transport.

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Our goal – understand effects of stochastic field on $\langle E_r \rangle$ and $\langle v_E' \rangle$

- More specific: how stochastic B-field affects $\langle v_E' \rangle$?

$$\underbrace{E_r}_{\langle v_E' \rangle} = \underbrace{\frac{1}{enB}\frac{\partial}{\partial r}P_{\rm i}}_{\text{Heat, particles}} + \underbrace{v_\phi B_\theta - v_\theta B_\phi}_{\perp,\, \parallel \text{ flows} \rightarrow \text{momentum}}.$$

✓ Study turbulence, particle, momentum and heat transport to ascertain change of $\langle E_r \rangle$ due to stochastic B field.

✓ Goal is towards $\langle \boldsymbol{J_r} \rangle$ $\boxed{\leftrightarrow}$ $\langle \boldsymbol{E_r} \rangle$ relation— effective "Ohm's law"

- Stochastic B-field, externally excited but self-consistent within plasma (Ampere's law), enters $\langle \boldsymbol{J_r} \rangle$
- Take turbulence as electrostatic in L-mode (simple first step).

Ambipolarity breaking $\Rightarrow \langle J_r \rangle$

- Ambipolarity breaking due to stochastic field $\Rightarrow \langle J_r \rangle$

$$\langle J_r \rangle = \left\langle \vec{J}_{\parallel} \cdot \vec{e}_r \right\rangle = \frac{\langle \tilde{J}_{\parallel} \tilde{B}_r \rangle}{B} \qquad \langle J_{\parallel} \rangle = \langle J_{\parallel,e} \rangle + \langle J_{\parallel,i} \rangle$$

- From Ampere law: $\tilde{J}_{\parallel} = -\frac{c}{4\pi} \nabla^2 \tilde{A}_{\parallel}$

(Self- consistency)

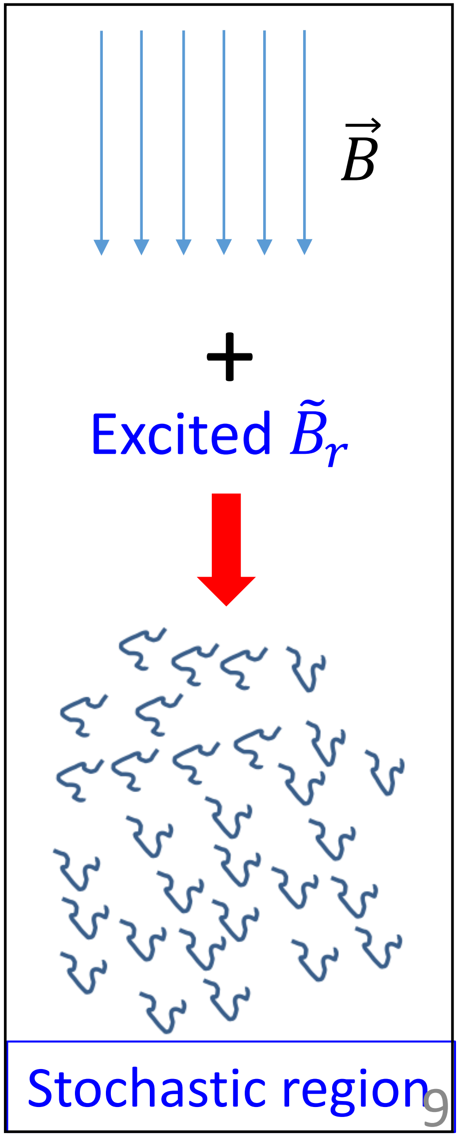
Stochastic field produces currents in plasmas

$$\langle J_r \rangle = \frac{\langle \tilde{J}_{\parallel} \tilde{B}_r \rangle}{B} = -\frac{c}{4\pi B} \left\langle \frac{\partial}{\partial y} \tilde{A}_{\parallel} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \tilde{A}_{\parallel} \right\rangle$$

$$= -\frac{c}{4\pi B} \frac{\partial}{\partial x} \left\langle \left(\frac{\partial}{\partial x} \tilde{A}_{\parallel} \right) \left(\frac{\partial}{\partial y} \tilde{A}_{\parallel} \right) \right\rangle = \frac{c}{4\pi B} \frac{\partial}{\partial x} \langle \tilde{B}_x \tilde{B}_y \rangle$$

$$= \frac{cB}{4\pi} \frac{\partial}{\partial x} \langle \tilde{b}_x \tilde{b}_y \rangle \Rightarrow \frac{cB_0}{4\pi} \frac{\partial}{\partial r} \boxed{\langle \tilde{b}_r \tilde{b}_\theta \rangle}$$ Maxwell stress

Note: Stochasticity excited externally (RMP) but Ampere's law must be satisfied in plasma. $\langle J_r \rangle$ tracks momentum, not heat transport. Phases?



Phase 1: Phase in Maxwell stress

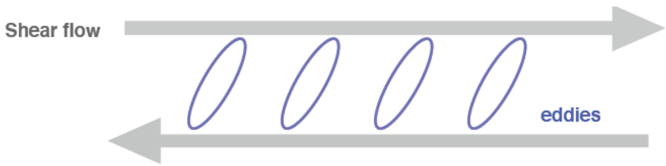
$$\frac{\partial A}{\partial t} + V \cdot \nabla A = \mu \boldsymbol{J}$$

$$\frac{\partial A}{\partial t} + V_E' \frac{\partial A}{\partial y} + \tilde{V} \cdot \nabla A = \mu \boldsymbol{J}$$

Shear flow   Fluctuation scattering

$\tilde{A}_k$ tilted by developing $\boldsymbol{E} \times \boldsymbol{B}$ flow, scattered by fluctuation.

Maxwell stress: $\langle \tilde{b}_r \tilde{b}_\theta \rangle = \frac{1}{B^2} \sum_k |\tilde{A}_k|^2 \langle k_r k_\theta \rangle$ phase set by $\langle k_r k_\theta \rangle$



$$\frac{\partial \boldsymbol{k_r}}{\partial t} = -\frac{\partial (\boldsymbol{k_\theta V_E})}{\partial r} \rightarrow \boldsymbol{k_r} = \boldsymbol{k_r^0} - \boldsymbol{k_\theta V_E' \tau_c}$$

$$\langle \tilde{b}_r \tilde{b}_\theta \rangle = -\frac{1}{B^2} \sum_k |\tilde{A}_k|^2 \langle k_\theta^2 V_E' \tau_c \rangle = \frac{1}{B^2} \sum_k |\tilde{B}_k|^2 \langle V_E' \tau_c \rangle$$

$\boldsymbol{E} \times \boldsymbol{B}$ shear aligns phases, regardless of mechanisms $\quad \tau_c = \left(\frac{k_\theta^2 V_E'^2 D_T}{3}\right)^{-1/3}$

Note: τ_c is coherence time (in shear field) of magnetic perturbation.

Tilting will tend to align turbulent RS and stochastic Maxwell stress.

Phase 2: Dephasing of Reynold stress

Without $\widetilde{b}_r$

Snell's law:

$$\frac{d}{dt}k_x = -\frac{\partial(\omega_0 + u_y k_y)}{\partial x} = -k_y\frac{\partial u_y}{\partial x}$$

Gives an non-zero $\langle k_x k_y \rangle$

$\rightarrow \langle \widetilde{u}_x \widetilde{u}_y \rangle \propto \langle k_x k_y \rangle$

shear flow

Self-feedback loop:

The $E \times B$ shear generates the $\langle k_x k_y \rangle$ correlation and hence support the non-zero Reynolds stress.

$$\langle \widetilde{u}_x \widetilde{u}_y \rangle \simeq \sum_k \frac{|\widetilde{\phi}_k|^2}{B_0^2}(k_y^2 \frac{\partial u_y}{\partial x}\tau_c)$$

The Reynold stress modifies the shear via momentum transport.

With $\widetilde{b}_r$

Snell's law:

$$\frac{d}{dt}k_x = -\frac{\partial \omega_k}{\partial x}$$

$$= -k_y\frac{\partial u_y}{\partial x} - \frac{1}{2}\frac{v_A^2 k_\perp^2}{\omega_D}\frac{\partial b^2}{\partial x}$$

Self-feedback loop is broken by b^2:

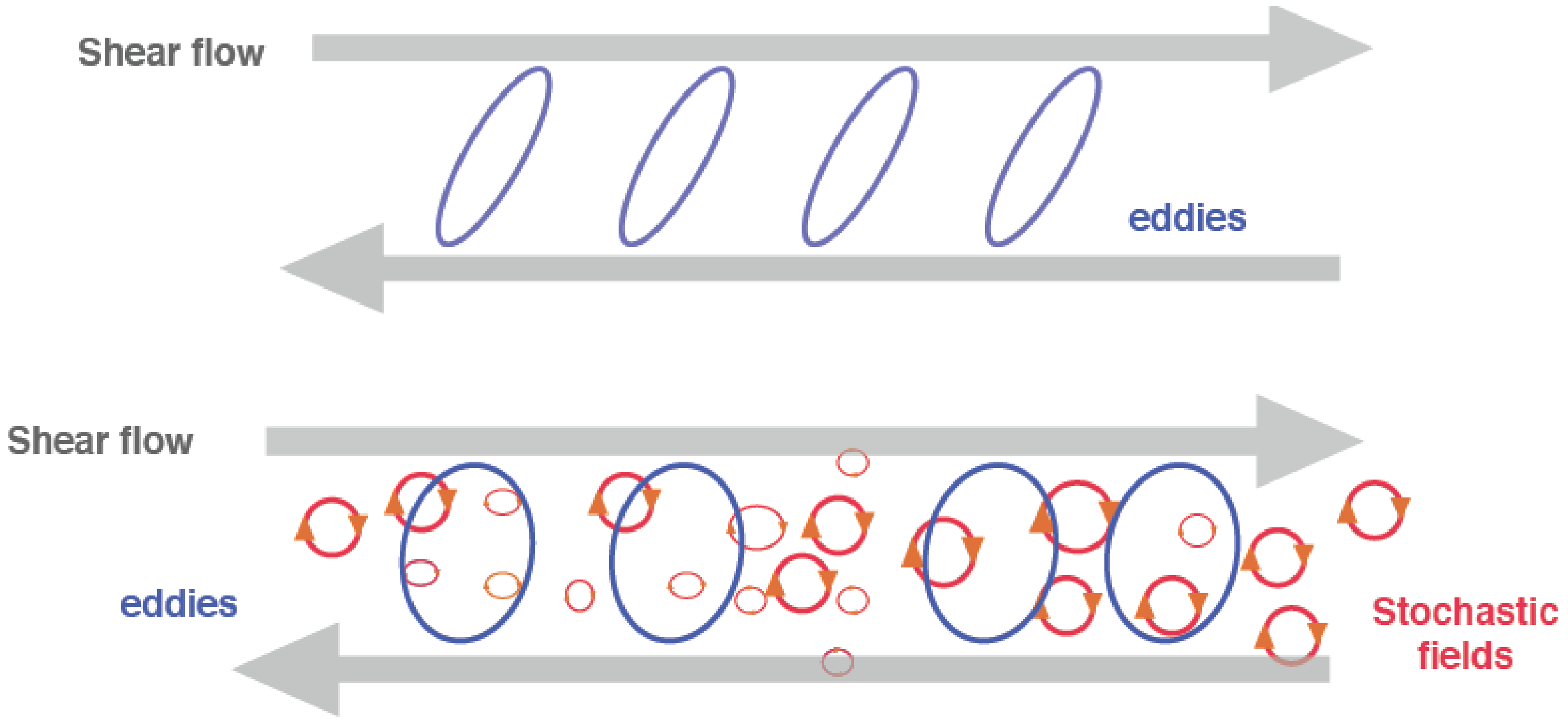
$$\langle \widetilde{u}_x \widetilde{u}_y \rangle \simeq \sum_k \frac{|\widetilde{\phi}_k|^2}{B_0^2}(k_y^2 \frac{\partial u_y}{\partial x}\tau_c + \frac{1}{2}k_y\frac{v_A^2 k_\perp^2}{\omega_D}\frac{\partial b^2}{\partial x}\tau_c)$$

Due to the Ensemble average eigen-frequency shift

Stochastic dephasing

More details: C.C. Chen, O 3.2, this meeting

Phase 2: Dephasing of Reynold stress



Stochastic fields interfere with shear-tilting feedback loop.

Through effect on phase correlation:

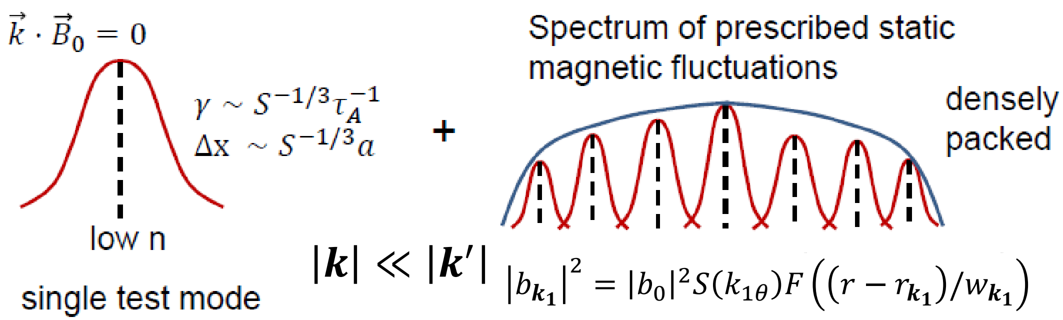
– $\langle V_E \rangle'$ tends to align phases.

– $\langle \tilde{b}^2 \rangle$ tends to break the alignment.

Instability affected by stochasticity— phase I

Common theme: A Simple Model $\nabla \cdot \boldsymbol{J} = 0$ (Kadomtsev and Pogutse '78)

A low-k single test mode + a high-k stochastic magnetic field background



If only $\tilde{\boldsymbol{b}}$ and $\bar{\varphi}$, $\nabla \cdot \boldsymbol{J} = 0$

is not guaranteed!

What is missing?

$$\frac{\partial}{\partial t}\nabla_\perp^2 \bar{\varphi} = -\frac{S}{\tau_A}\left(\nabla_\parallel^{(0)} + \tilde{\boldsymbol{b}} \cdot \nabla_\perp\right)^2 \bar{\varphi} - \frac{g B_0}{\rho_0}\frac{\partial \bar{p}}{\partial y}$$

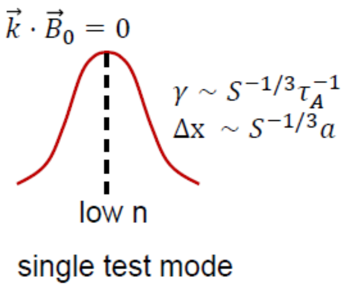
Analogy	Kadomtsev and Pogutse '78	This model
Base state	$\langle T(r) \rangle$	$\bar{\varphi}_{\boldsymbol{k}}$
External fluctuation	$\tilde{\boldsymbol{b}}$	$\tilde{\boldsymbol{b}}$
Constraint	$\nabla \cdot \boldsymbol{q} = 0$	$\nabla \cdot \boldsymbol{J} = 0$
Resulting fluctuation	$\tilde{T}$	$\tilde{\varphi}$

electrostatic potential fluctuation induced by $\tilde{\boldsymbol{b}}_{\boldsymbol{r}}$

See more details: M. Y. Cao, P9, this meeting

Instability affected by stochasticity — phase II

The actual model must be: See more details: M. Y. Cao, P9, this meeting



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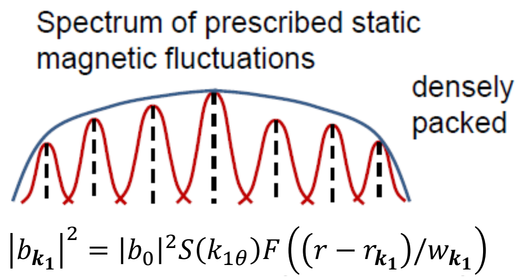
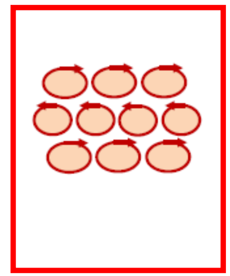
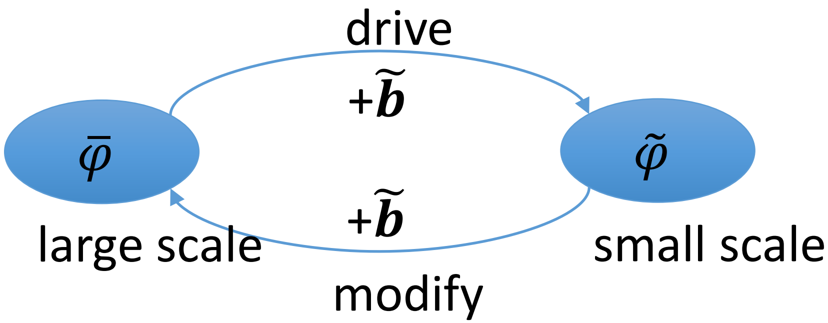
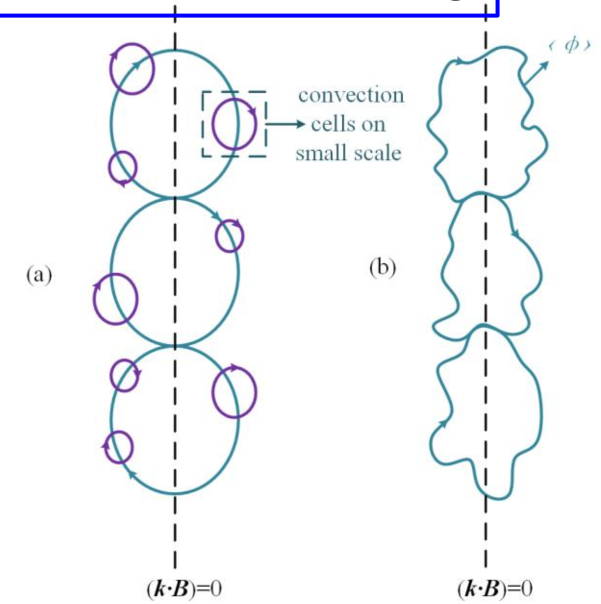


densely packed +



Small scale convective cells





Long wavelength cell in presence of short wavelength cells and $\tilde{\boldsymbol{b}}$.

Further, these small-scale convective cells drive a turbulent viscosity ν and a turbulent diffusivity χ.

- Interation develops $\langle \tilde{b}_r \tilde{\phi} \rangle \neq 0 \rightarrow$ small electrostatic fluctuations “lock on” to $\tilde{b}$.
- Intrinsically a multi-scale problem: $\bar{\varphi}$; $\tilde{\varphi}$ and $\tilde{\boldsymbol{b}}$

Are micro-cells the agent of RMP induced density “pump-out”?

Particle 1:Stochasticity contribution to particle flux

- For electron density : $\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\Gamma_\mathrm{e}) = S_p$

with $\Gamma_e = \boxed{-(D_{neo} + D_T)\frac{\partial n}{\partial r}} + \boxed{\Gamma_{e,\,stoch}}$ ←

- $D_{neo} = (m_e/m_i)^{1/2}\chi_{i,neo}$
- $D_T \sim bD_{GB}$ with b<1

$$S_p = \Gamma_a \frac{a - r + d_a}{L_{dep}^2}\exp\left(-\frac{(a + d_a - r)^2}{2L_{dep}^2}\right)$$

- The stochastic field can induce particle flux ($n_e = n_i$):

$$\Gamma_{e,\,stoch} = \frac{c}{4\pi eB}\langle \tilde{b}_r \nabla_\perp^2 \tilde{A}_\parallel \rangle + n\langle \tilde{V}_{\parallel,i}\tilde{b}_r\rangle$$

$\langle \tilde{b}_r \tilde{J}_\parallel \rangle$ $\langle \tilde{b}_r \tilde{J}_{\parallel,i} \rangle$

with

$$\frac{c}{4\pi eB}\langle \tilde{b}_r \nabla_\perp^2 \tilde{A}_\parallel \rangle = -\frac{cB}{4\pi e}\frac{\partial}{\partial r}\langle \tilde{b}_r \tilde{b}_\theta \rangle$$

✓ $\langle \tilde{b}_r \tilde{b}_\theta \rangle$ phasing via V_E' tilt.

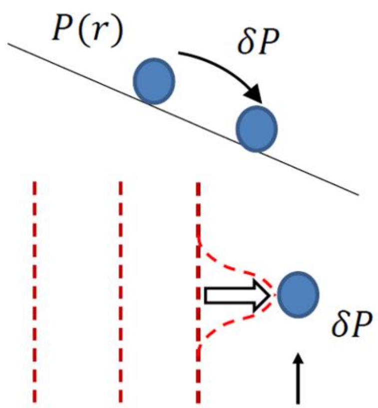
$n\langle \tilde{V}_{\parallel,i}\tilde{b}_r\rangle$: parallel ion flow along tilted field lines

- RMP induced density "pump-out"?
- Kinetic stress $\langle \tilde{b}_r \delta P \rangle$

Particle 2: Hybrid diffusivity

- **Heuristics**

P.H. Diamond, O 3.1, this meeting



- How is pressure balanced along field line?

  i) Build parallel pressure gradient

  <u>or</u>

  ii) Drive parallel flow, damped by turbulent mixing/viscosity

- <u>Structure</u> of $\left\{ \begin{matrix} \text{correlator} \\ \text{fluxes} \end{matrix} \right\}$ change !

$$\langle \tilde{b}_r \delta P \rangle \approx -D_{st}\, \partial \langle V_\parallel \rangle / \partial r$$

$$\boxed{\langle \tilde{b}_r \delta V_\parallel \rangle} \approx -D_{st} \partial \langle P \rangle / \partial r$$

Flux-gradient relation is changes by $\tilde{b}$

- Stochastic viscosity/diffusivity is <u>hybrid</u>

$$\boxed{D_{ST} = \sum_k c_s^2 \left| b_{r,k} \right|^2 / \, k_\perp^2 D_T}$$

Magnetic scattering,
with τ_{ck} set
by electrostatics

Flow 1: Stochastic B-field affects $\langle V_\theta \rangle$

- Poloidal momentum balance

Turbulence Reynold stress

Maxwell stress of stochastic field perturbation

$B_\phi \langle J_r \rangle$

$$\frac{\partial \langle V_\theta \rangle}{\partial t} = -\mu(\langle V_\theta \rangle - V_{\theta,neo}) - \frac{\partial}{\partial r}\left(\langle \tilde{V}_\theta \tilde{V}_r \rangle - \frac{1}{4\pi\rho} \langle \tilde{B}_r \tilde{B}_\theta \rangle \right)$$

- For SS: $$\langle V_\theta \rangle = V_{\theta,neo} - \frac{1}{\mu}\frac{\partial}{\partial r}\left(\langle \tilde{V}_\theta \tilde{V}_r \rangle - \frac{1}{4\pi\rho} \langle \tilde{B}_r \tilde{B}_\theta \rangle \right)$$
$$= V_{\theta,neo} - \frac{1}{\mu}\frac{\partial}{\partial r}\left(\frac{1}{B^2}\tau_c V_E' \frac{I}{1+\alpha V_E'^2} - \frac{B^2}{4\pi\rho}\tau_c' V_E' |\tilde{b}_r|^2\right)$$

with $\mu = \mu_{00}\left(1 + \frac{\nu_{CX}}{\nu_{ii}}\right)\nu_{ii} q^2 R^2$ $\quad V_{\theta,neo} \approx -1.17\frac{\partial T_i}{\partial r}$ $\quad \tau_c' = \tau_c$

- V_E' phasing via tilt tends to align turbulence and stochastic B-field, which counteracts the spin-up of $\langle V_\theta \rangle$.
- $\frac{\partial}{\partial r}|\tilde{b}_r|^2$, i.e., profile of stochastic enters → introduce stochastic layer width as novel scale

Flow 2: Stochastic B-field affects $\langle V_\phi \rangle$

- For V_ϕ: $\dfrac{\partial \langle V_\phi \rangle}{\partial t} + \nabla \cdot \langle \tilde{V}_r \tilde{V}_\phi \rangle = \dfrac{1}{\rho c} \langle J_r \rangle B_\theta + S_M$

$$\langle \tilde{V}_r \tilde{V}_\phi \rangle = -\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle, \quad \chi_\phi = \chi_T = \frac{\rho_s^2 c_s}{L_T}, \quad S_M = S_a \exp\left(-\frac{r^2}{2L_{M,dep}^2}\right)$$

Only consider diffusive term.

$$\Rightarrow \frac{\partial \langle V_\phi \rangle}{\partial t} = \frac{\partial}{\partial r}\left(\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle\right) + \frac{1}{4\pi\rho} \frac{B_\theta}{B} \frac{\partial}{\partial r} \langle \tilde{B}_r \tilde{B}_\theta \rangle + S_M$$

- For SS: $\dfrac{\partial}{\partial r}\left(\chi_\phi \dfrac{\partial}{\partial r} \langle V_\phi \rangle\right) = -\dfrac{V_{Ti}^2}{\beta} \dfrac{B_\theta}{B} \dfrac{\partial}{\partial r} \langle \tilde{b}_r \tilde{b}_\theta \rangle - S_M$

Stochasticity affects edge toroidal velocity, shear

$$\boxed{\frac{\partial}{\partial r} \langle V_\phi \rangle |_{r_{sep}} = -\frac{1}{\chi_\phi} \int_0^{r_{sep}} S_M dr - \frac{V_{Ti}^2}{\beta \chi_\phi} \frac{B_\theta}{B} \langle \tilde{b}_r \tilde{b}_\theta \rangle |_{r_{sep}}}$$

$B_\theta \langle J_r \rangle$

Integrated external torque

with $\langle \tilde{b}_r \tilde{b}_\theta \rangle = V_E' \tau_c' |\tilde{b}_r|^2$

✓ Force through radial current across separatrix.

Note: $\langle V_\phi \rangle'$ proportional to $|\tilde{b}_r|^2 / \chi_\phi$. Quenched $\chi_\phi \rightarrow$ stronger $\langle V_\phi \rangle'$ effects!

Flow 3: Intrinsic rotation and kinetic stress

From parallel acceleration:

$$\frac{\partial}{\partial t}u_z + (\mathbf{u}\cdot\nabla)u_z = -\frac{1}{\rho}\frac{\partial}{\partial z}p$$

\+

Stochastic Fields Effect

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z}^{(0)} + \underline{b}\cdot\underline{\nabla}_{\perp}$$

$$\rightarrow \frac{\partial}{\partial t}\langle u_z\rangle + \frac{\partial}{\partial x}\langle \tilde{u}_x\tilde{u}_z\rangle = -\frac{1}{\rho}\frac{\partial}{\partial x}\langle b\tilde{p}\rangle$$

Toroidal Reynolds Stress

Kinetic Stress

$$\langle \tilde{u}_x\tilde{u}_z\rangle = -\nu_{turb}\frac{\partial}{\partial x}\langle u_z\rangle + F_{z,res}\frac{\partial}{\partial x}\langle p\rangle$$

Turbulent viscosity

Toroidal Residual Stress

Influence intrinsic rotation

$$\nu_{turb} = \sum_{k\omega} |\tilde{u}_{x,k\omega}|^2 \frac{2C_s b^2 l_{ac} k^2}{\omega_{sh}^2 + (2C_s b^2 l_{ac} k^2)^2}$$

- The sound speed is the relevant speed (acoustic dynamics). Stochastic fields effect is weaker ($C_s D_M < v_A D_M$).
- $F_{z,res} \sim \sum_{k\omega} \frac{-k_z}{\omega_{sh}\rho}\nu_{turb,k\omega}$.

$F_{z,res}$ Requires symmetry breaking $\langle k_z k_y\rangle \neq 0$

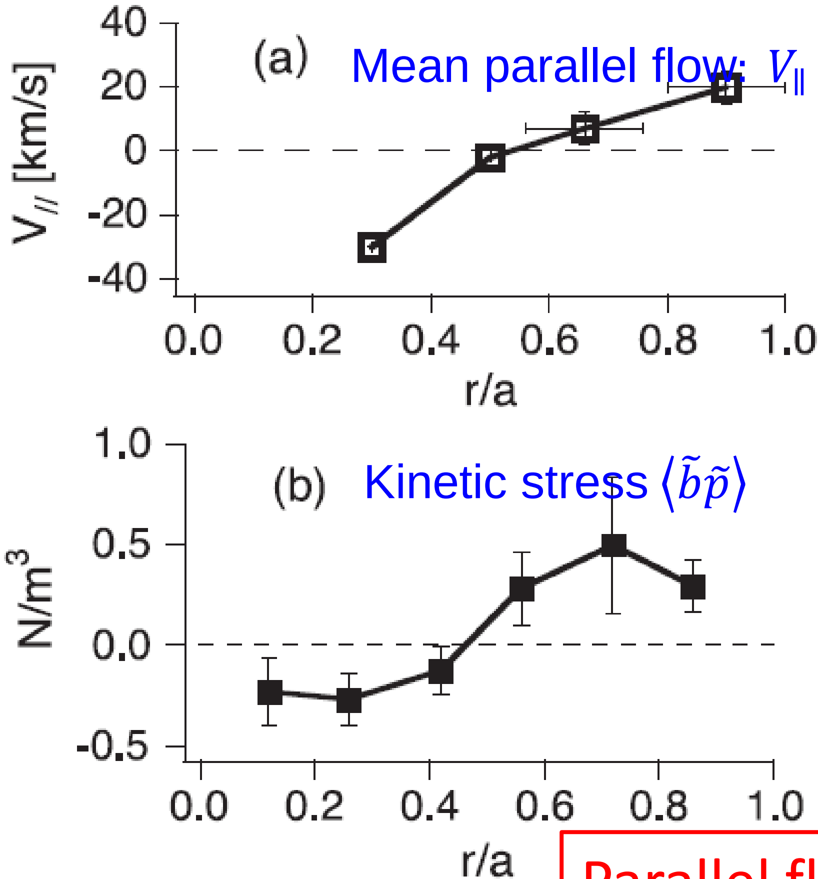
(Chen et al.,PoP **28**, 042301 (2021))

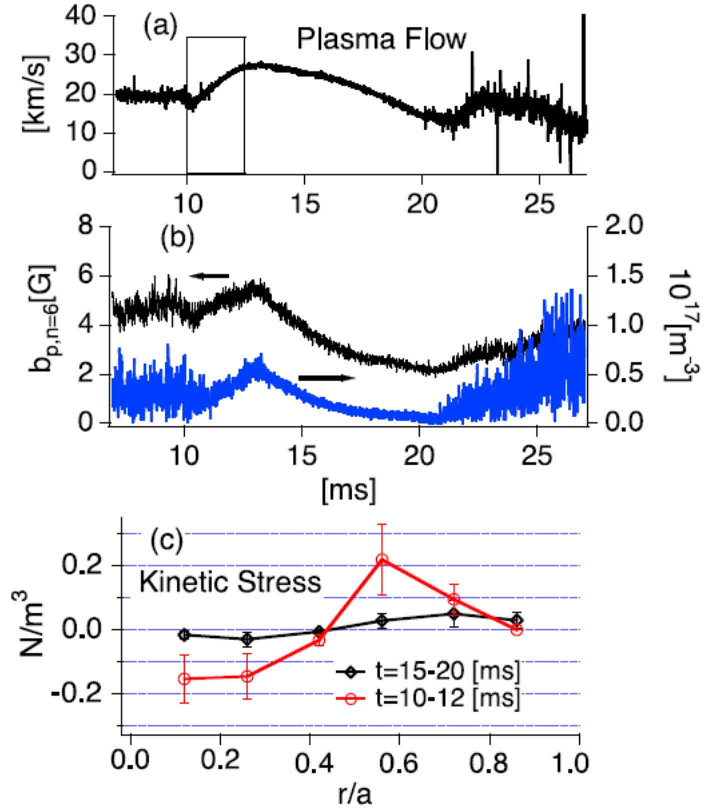
- Kinetic stress is stochastic field-induced viscous stress → significant drag on rotation.
- Stochastic field reduces the toroidal Reynold stress and the effect is modest.

Flow 4: Intrinsic rotation and kinetic stress

$$\partial_t \langle V_\parallel \rangle + \partial_r \langle \tilde{V}_r \tilde{V}_\parallel \rangle = -\frac{c_s^2}{\rho}\, \partial_x \langle b_r P \rangle$$

Kinetic stress $\langle \tilde{b}\tilde{p} \rangle$
→ affects momentum balance and intrinsic rotation





Parallel flow driven by kinetic stress, balanced by stochastic magnetic field induced diffusion

W. X. Ding, 2013 PRL, MST

Ion heat flux with stochastic field

- Heat flux induced by stochastic field :

$$Q_i = -\left(\chi_{i,neo} + \chi_{i,T}\right)\nabla T_i + Q_{i,stoch}$$

- $\chi_{i,neo} = \varepsilon^{-3/2} q^2 \rho_s^2 \nu_{ii}$
- $\nu_{ii} = \dfrac{n_0 Z^4 e^4 \ln \Lambda}{\sqrt{3}6\pi\varepsilon_0^2 m_i^{1/2} T_{i0}^{3/2}}$
- $\chi_{i,T} = \left(\dfrac{C_s^2 \tau_c}{1+\alpha V_E'^2}\right) * I$

$\sim \chi_{GB} * I$

- The stochastic field affects ion heat flux

$$Q_{i,stoch} = \int V_\parallel \langle \tilde{B}_r \delta f \rangle \left(V_\parallel^2 + V_\perp^2\right) = -\frac{\partial \langle T_i \rangle}{\partial r} \sqrt{\chi_{\parallel,i} \chi_{\perp,i}} \langle \tilde{b}_r^2 \rangle l_{ac} k_\perp^{RMS}$$

$$\propto -v_{th,i} D_{M,\,eff} \frac{\partial \langle T_i \rangle}{\partial r}$$

✓ Important as threshold power is directly related to heat flux.

Direct effect on ion heat flux is finite but not so large ($\langle \tilde{b}_r^2 \rangle l_{ac} k_\perp \ll 1$).

Towards an expression for $\langle V_E\rangle'$

- From Ohm's law, $\langle E_r\rangle$ and $\langle J_r\rangle$ are related :

$$\boxed{E_r = \frac{1}{enB}\frac{\partial}{\partial r}P_{\mathrm{i}} + v_\phi B_\theta - v_\theta B_\phi.}$$

$$\langle \tilde{J}_\parallel \tilde{B}_r\rangle \rightarrow \langle J_r\rangle \begin{cases} n_e \text{ via } \langle \tilde{b}_r J_\parallel\rangle \\ \langle V_\theta\rangle \text{ via } \langle J_r\rangle B_t \\ \langle V_\phi\rangle \text{ via } \langle J_r\rangle B_\theta \end{cases}$$

- Elements for $\boldsymbol{E}\times\boldsymbol{B}$ shear:

✓ $n\frac{\partial T_i}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rQ_i) = S_H$

Ion temperature

✓ $\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\underbrace{\Gamma_{\mathrm{e}}}_{\langle J_r\rangle}) = S_p$ Electron density

✓ $\langle V_\theta\rangle = V_{\theta,neo} + \frac{1}{\mu}\frac{\partial}{\partial r}\left(\frac{1}{B^2}\tau_c V_E' \frac{I}{1+\alpha V_E'^2} - \frac{B^2}{4\pi\rho}\tau_c' V_E' |\tilde{b}_r|^2\right)$ Poloidal flow

✓ $\frac{\partial}{\partial r}\langle V_\phi\rangle|_{r_{sep}} = -\frac{1}{\chi_\phi}\int_0^{r_{sep}} S_M dr - \frac{V_{Ti}^2}{\beta\chi_\phi}\frac{B_\theta}{B} V_E' \tau_c' |\tilde{b}_r|^2|_{r_{sep}}$ Toroidal flow

- $$\begin{aligned} \langle V_E\rangle' &= \frac{1}{eB}\frac{\partial}{\partial r}\langle \nabla P_i/n\rangle - \frac{\partial}{\partial r}\langle V_\theta\rangle + \frac{B_\theta}{B}\frac{\partial}{\partial r}\langle V_\phi\rangle \\ &= \frac{1}{eB}\frac{\partial}{\partial r}\langle \nabla P_i/n\rangle - \frac{\partial}{\partial r}\left[V_{\theta,neo} + \frac{1}{\mu}\frac{\partial}{\partial r}\left(\frac{1}{B^2}\tau_c V_E' \frac{I}{1+\alpha V_E'^2} - \frac{B^2}{4\pi\rho}\tau_c' V_E' |\tilde{b}_r|^2\right)\right] \\ &\quad + \frac{B_\theta}{B}\left[-\frac{1}{\chi_\phi}\int_0^{r_{sep}} S_M dr - \frac{V_{Ti}^2}{\beta\chi_\phi}\frac{B_\theta}{B} V_E' \tau_c' |\tilde{b}_r|^2|_{r_{sep}}\right] \end{aligned}$$

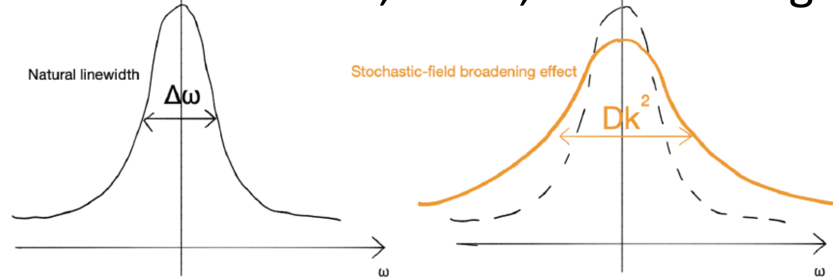
Outline

- Motivation and background
  - Why? → Interaction and co-existence of stochastic B field and turb.
  - Key issues (L→H transition with RMP, island, stellarator, etc)
- Mean field model $\langle E_r \rangle$—follow radial force balance
  - Key fundamentals:
    - — *phases*
    - — *instability in stochastic field*
  - Turbulent transport
    - — *Particle transport*
    - — *Momentum transport (poloidal and toroidal)*
    - — *Ion heat transport*
- Applications
  - L-H transition with $\tilde{b}^2$, 0D at present
- Implications and future work

Increment of LH threshold power

Concern: $\Delta\omega < Dk_\perp^2$, D=$v_A D_M$

More details: C.C. Chen, O 3.2, this meeting

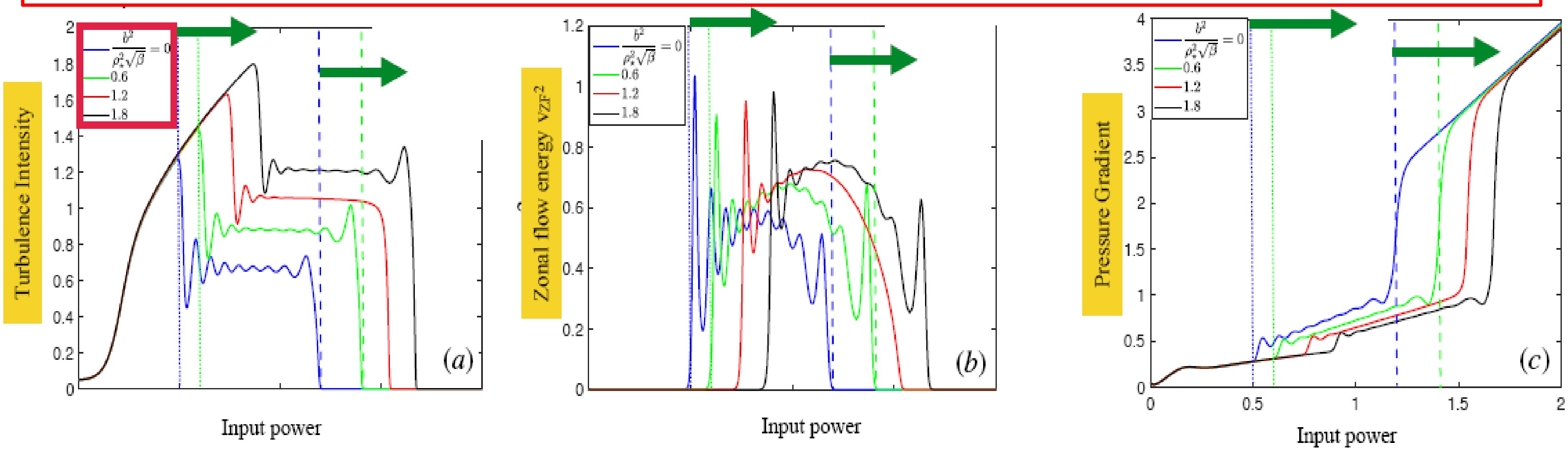


Turbulence decorrelation	Stochastic field induced scattering

$\Delta\omega \sim D_{ES} k_\perp^2$

Broadening parameter: $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} = 0.0, 0.2, 0.4, 0.6, 0.8...., 2.0$

Kim and Diamond model, 2003 PRL, predator: zonal flow, prey: turbulence



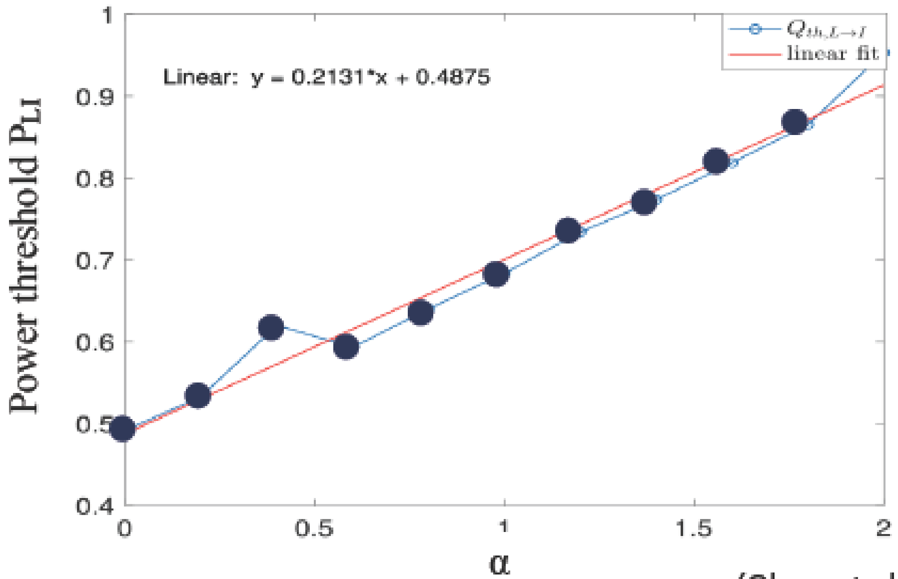
The threshold increase due to stochastic dephasing effect is seen in turbulence intensity, zonal flow, and pressure gradient.

Increment of LH threshold power

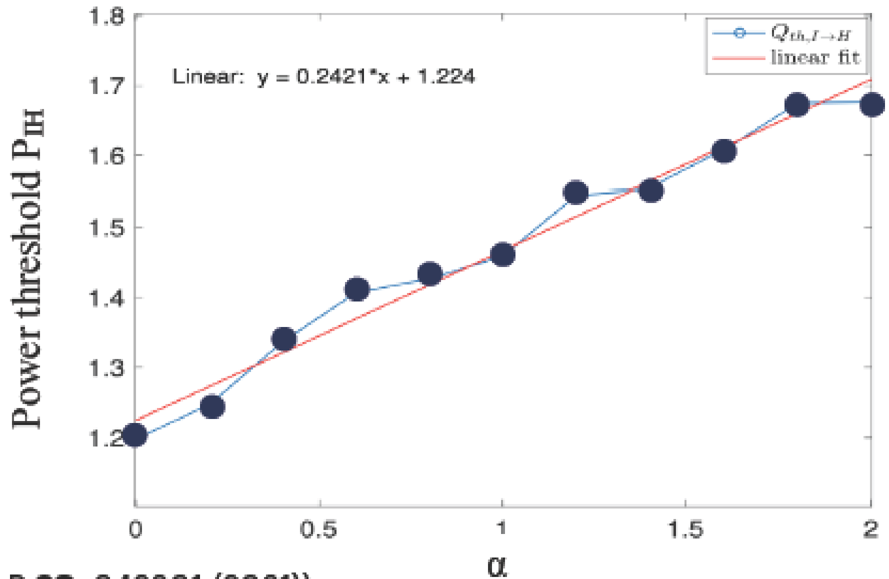
$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} = 0.0, 0.2, 0.4, 0.6, 0.8...., 2.0$$

More details: C.C. Chen, O 3.2, this meeting

P_{LI} v.s. α



P_{IH} v.s. α



(Chen et al.,PoP **28**, 042301 (2021))

- Testable prediction: The threshold power increase linearly with $\alpha \sim b^2/\rho_*^2$.
- Could compare directly with $\Delta\omega$ $(k_\perp^2 v_A D_M)$

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Results and implications

Topic	Goal	Key physical results	prediction
Reynold stress (C C Chen)	Flow shear evolution	Dephasing when $\Delta\omega \sim v_A D_M k_\perp^2$	Critical parameter $\alpha \sim b^2/\rho_*^2\sqrt{\beta}$
Parallel flow and ion heat transport (P.H. Diamond)	Calculate kinetic stress $\langle \tilde{b}_r \delta P \rangle$ in turbulence	Physical understanding of stochasticity-turbulence interaction	Hybrid stochastic field + turbulence viscosity
Instability evolution in stochastic field (M.Y. Cao)	Understand how prescribed $\tilde{b}$ affect instability evolution	Maintaining $\nabla \cdot J = 0$ forces generation of small scale cells by $\tilde{b}$	$\langle \tilde{b}_r \tilde{\phi} \rangle \neq 0$ →turbulence “lock on” to $\tilde{b}$.
Mean field theory for $\langle E_r \rangle$ (All)	Understand electric field shear evolution	Unified model including all transport channels	$\langle V_E \rangle'$ aligns stochastic field, particle flux due to $\tilde{b}$

Future work

- Towards to the 1D model to study the interplay in L-H transition.

- Related to experiments:

① Understand the relationship between the RMP effects on power threshold and micro-physics? [stress, fluctuations, transport...]

② How does the RMP change the evolution of the shear layer (Er well) ? How it builds up?

③ How the **cross-phase** of Reynolds stress change (evolution) vs RMP current?

④ How does the RMP change the **LCO**?

⑤ How **toroidal velocity** change at pedestal region?

- Related work in density limit, esp. effect of RMP on Er shear

Thanks for your attention!!!

Future work

Future work:

- Towards to the 1D model to study the interplay in L-H transition.
- Related to experiments: understand the relationship between the RMP effects on power threshold and micro-physics? RMP effects on evolution of the shear layer, LCO……
- Related work in density limit, esp. high density limit

Conclusion

Turbulence in stochastic field is important to many critical problems.

Particle flux

- ✓ Ambipolarity breaking $\Rightarrow \langle \tilde{b}_r \tilde{b}_\theta \rangle$, contribute to $\langle \boldsymbol{J_r} \rangle$, phase set by $\langle V_E \rangle'$
- ✓ Both amplitude and profile of $|b_r|^2$ matter.

Turbulence

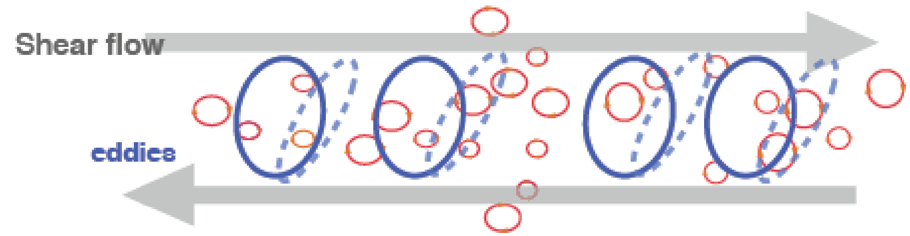
- ✓ To maintain $\nabla \cdot \boldsymbol{J} = 0$ at all scales for prescribed $\tilde{\boldsymbol{b}}$ and instability$\bar{\varphi}$, $\tilde{\varphi}$ (microscopic convective cells) generated by $\tilde{\boldsymbol{b}}$, yields a non-trivial $\langle \tilde{b} \tilde{\varphi} \rangle$, i.e., electrostatic turbulence 'locks on' to $\tilde{\boldsymbol{b}}$.

Momentum

- ✓ V_E' phasing $\Rightarrow$ stochastic $\langle \tilde{b}_r \tilde{b}_\theta \rangle$ opposes turbulence $\langle \tilde{V}_r \tilde{V}_\theta \rangle$,
  phase linked, counteracting V_θ
- ✓ Kinetic stress (residual stress) drive parallel flow (intrinsic rotation), which is balanced by turbulent χ_ϕ
- ✓ Toroidal flow effects significant for low external torque.

Conclusion and future work

- **Two effects opposes flow:**
  —Dephasing effect caused by stochastic fields reduces poloidal Reynolds stress (for $\Delta\omega < Dk_{\perp}^{2}$, D=$v_A D_M$)
  — **introduce maxwell stress** $\langle \tilde{b}_r \tilde{b}_\theta \rangle$



- P_{LH} increased due to the Reynold stress dephasing, in proportion to the broadening parameter.

Future work:

- ☐ Towards to the 1D model to study the whole effect on L-H transition.
- ☐ Related to experiments: understand the relationship between the RMP effects on power threshold and micro-physics? RMP effects on evolution of the shear layer, LCO……

A limiting case (further reduced)

- Consider simple flow shear + fluctuations → predator-prey type model

  ✓ Flow: $\frac{\partial V}{\partial t} = -\mu V - \frac{\partial}{\partial r}\left[\langle V_\theta \rangle' \tau_c c_s^2 \left(\varepsilon_F - \frac{|\tilde{b}_r|^2}{\beta}\right)\right]$

  Flow damping   Fluctuation intensity   Stochastic field, Maxwell stress

  ✓ Fluctuation energy

$$\frac{\partial \varepsilon_F}{\partial t} = \frac{\gamma \varepsilon_F}{1 + \alpha V_\theta'^2} - \sigma \varepsilon_F^2 \cong \gamma \varepsilon_F \left(1 - \alpha V_\theta'^2\right) - \sigma \varepsilon_F^2$$

- Follow P. H. Diamond (1994) →

  ✓ Slave fluctuation to flow shear

  ✓ 0D: identify edge length scale

A limiting case, cont'd

- Slave fluctuation to flow shear:

$$\frac{\partial \varepsilon_F}{\partial t} = -\mu\varepsilon_F + \frac{\varepsilon_F}{L^2}\left[\underbrace{\frac{\gamma/\sigma}{1+\alpha V_\theta'^2}}_{\text{Reynolds stress drive}} - \underbrace{\frac{|\tilde{b}_r|^2}{\beta}}_{\text{Maxwell stress}}\right]\tau_c c_s^2$$

- For steady state and fixed point, get simplified flow shear

$$V_\theta'^2 = \frac{1}{\alpha}\left[\frac{\frac{\tau_c c_s^2}{L^2}\gamma/\sigma}{\mu + \frac{|\tilde{b}_r|^2}{\beta L^2}\tau_c c_s^2} - 1\right]$$

✓ Growth rate $\gamma \sim \nabla P \Longrightarrow$ onset of flow shear

✓ Maxwell stress adds to damping

✓ Flow shear threshold is increased by stochastic field

✓ Layer width sets minimal L

✓ Likely a trigger of L→H transition at boundary of stochastic region?!