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UC San Diego

Theory of Pedestal Microturbulence with RMP-induced Stochasticity [1]

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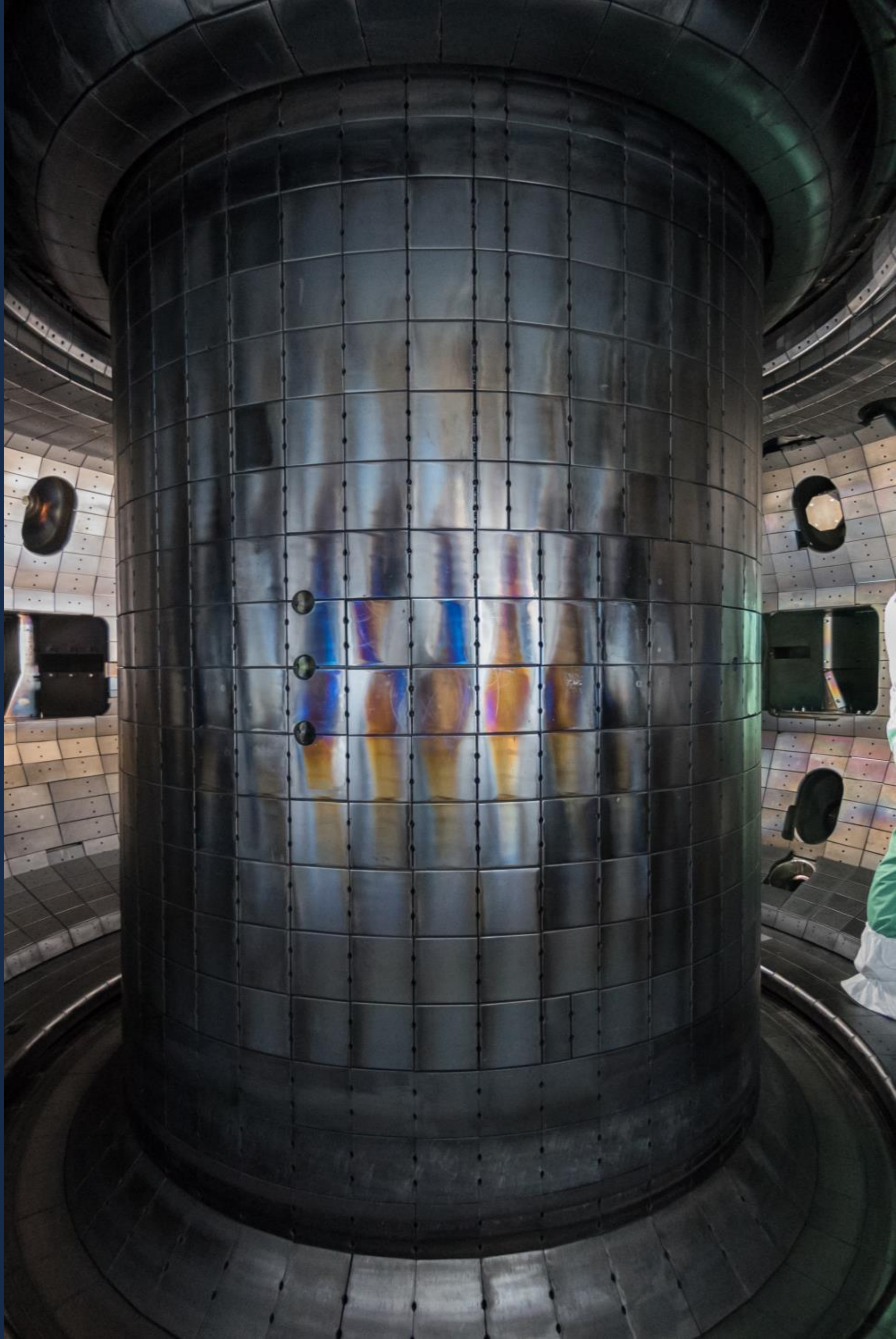
AAPPS-DPP 2022

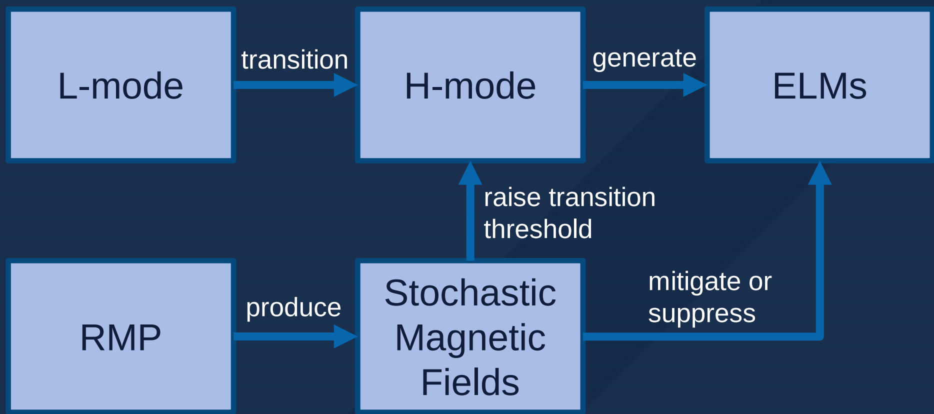
1. Cao, Mingyun, and Patrick H. Diamond. *Plasma Physics and Controlled Fusion* 64, no. 3 (2022): 035016.

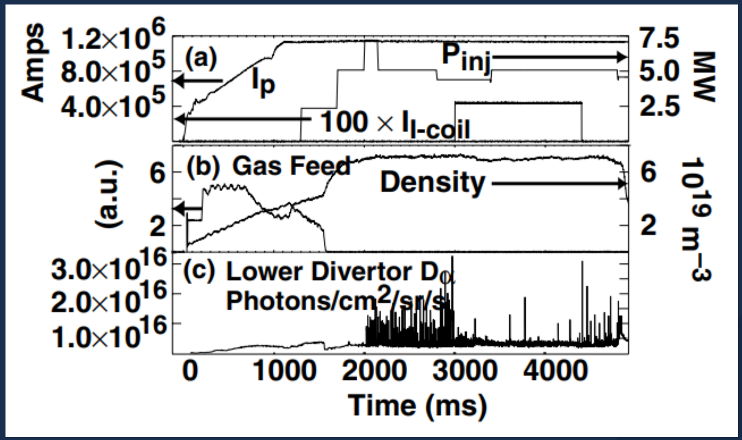
OUTLINE

- Motivation: the Application of RMP
- Observations : Questions arising from Sims & Expts
- Model Development: A Multi-scale Feedback Loop
- Results: Theoretical Predictions & Answers to Questions
- Conclusion: What We Have Learned & What We Will Do

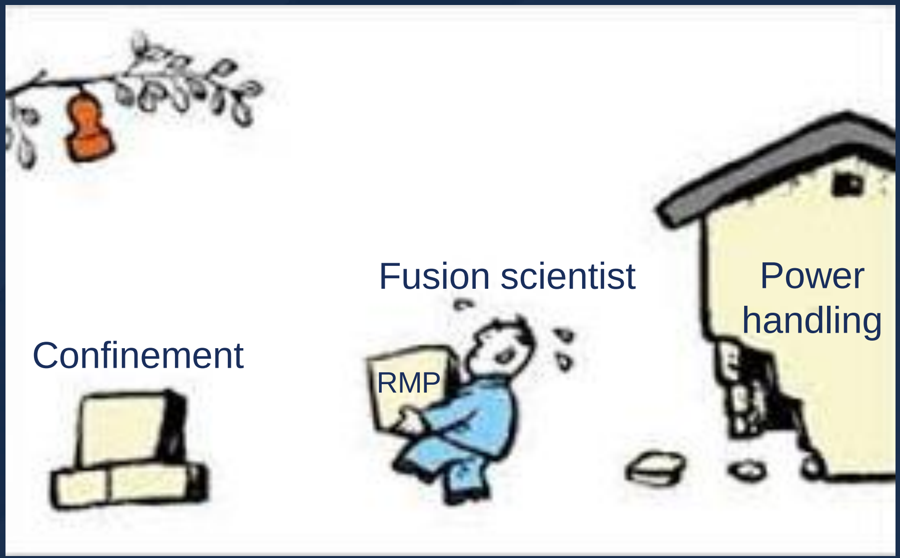


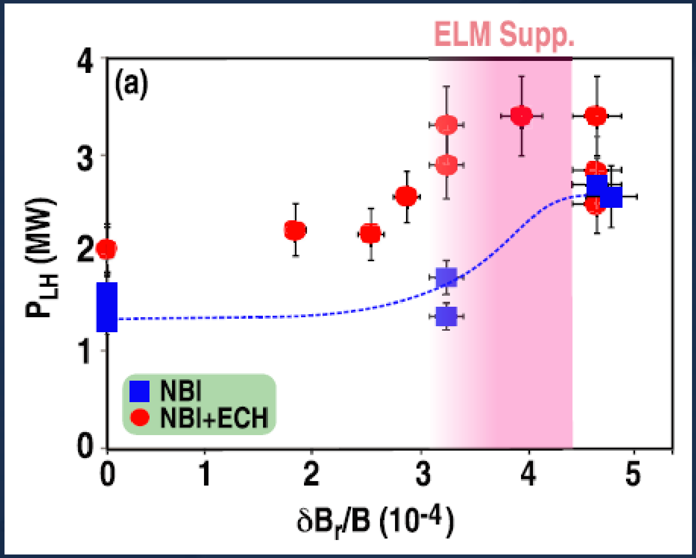
Motivation: the Application of RMP





Suppression of ELM by using RMP [1]





RMP raises the L-H transition power threshold [2]

1. Evans, T. E., et al. *Journal of nuclear materials* 337 (2005): 691-696.
2. Schmitz, L., et al. *Nuclear Fusion* 59, no. 12 (2019): 126010.

Motivation: the Application of RMP

- **A new trend:** A trade-off between good confinement and good power handling.

Stochastic Magnetic Fields $\xrightarrow{\text{modify}}$ Plasma Instabilities $\xrightarrow{\text{drive}}$ Plasma Turbulence

- **A basic question:** How does a stochastic magnetic field modify the instability process?

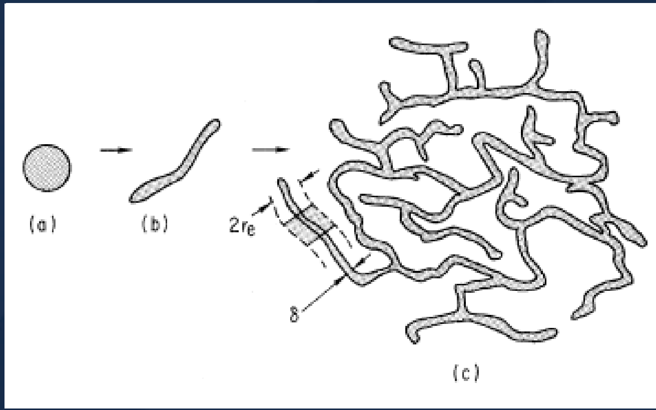
- **Origin:** Interest in stochastic field transport in the late 1970's [1,2]

- **Early research:** Tearing mode in braided magnetic field [3]

- **Point:** Effect of stochastic magnetic field enters as anomalous dissipation by hyper-resistivity μ. Ohm's law of resistive MHD is revised as

$$E_{\|} = \eta J_{\|} - \mu \nabla_{\perp}^{2} J_{\|}$$

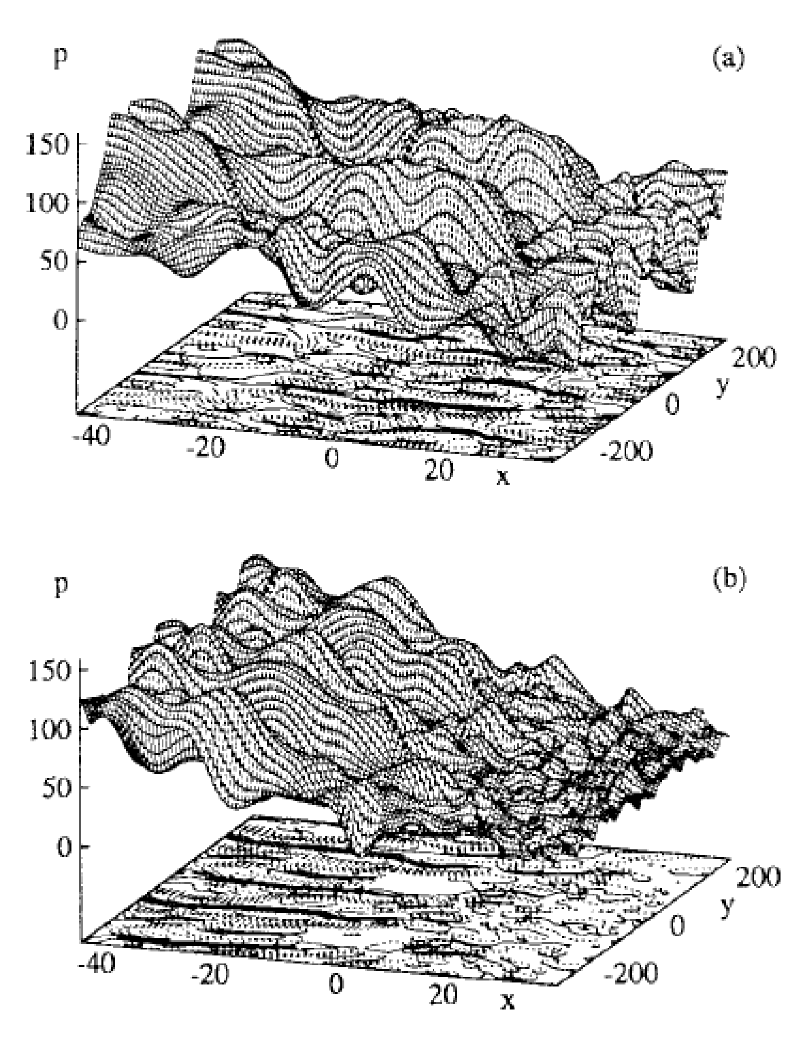
- **Unsolved questions:**
  1. Quasi-neutrality is not maintained at all orders
  2. Lack of, or too simple, micro-macro feedback



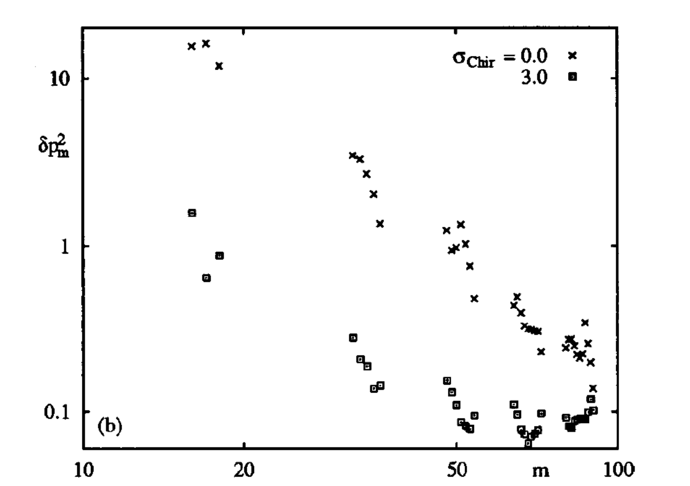
Rechester and Rosenbluth's test particle model [1]

1. A.B. Rechester, and M.N. Rosenbluth, 1977
2. B.B. Kadomtsev, O.P. Pogutse, 1979.
3. P.K. Kaw, E.J. Valeo, and P.H. Rutherford, 1979.

Questions Arising from Simulations



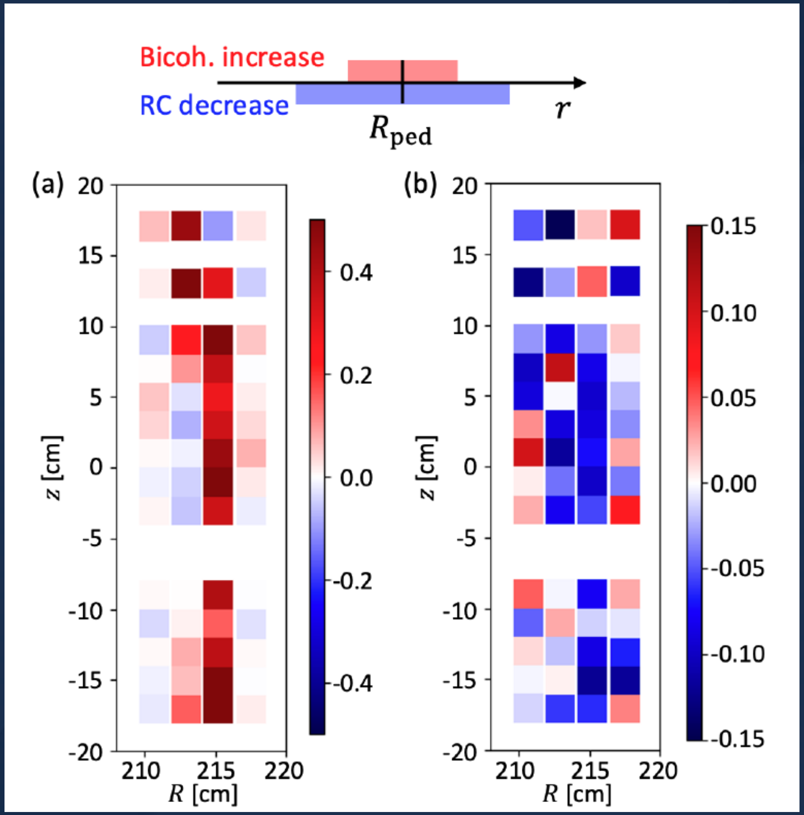
- Simulations of resistive ballooning modes in a stochastic magnetic field. [1]
- Increased small-scale structures and spatial roughness of the pressure fluctuation profile in stochastic region.
- Stronger suppression of Large-scale fluctuations than small-scale fluctuations.

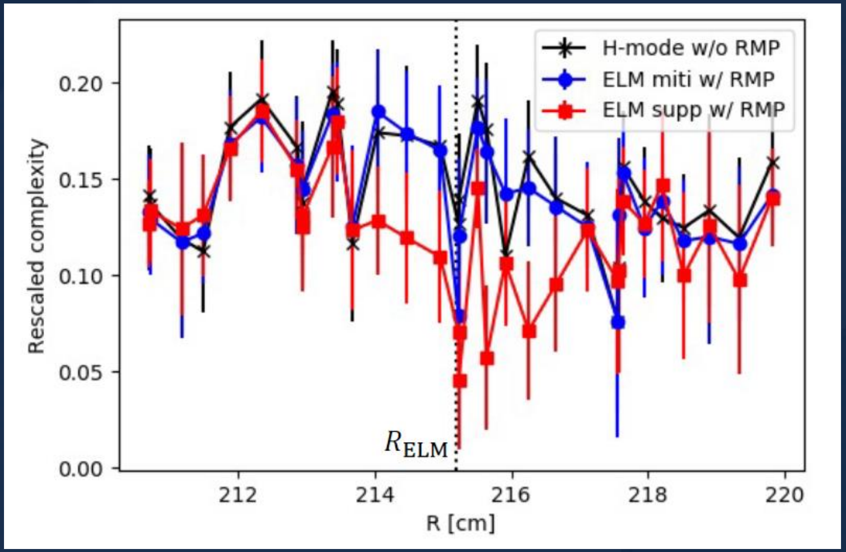


Why?

1. Beyer, P., Xavier Garbet, and Philippe Ghendrih. *Physics of Plasmas* 5, no. 12 (1998): 4271-4279.

Questions Arising from Experiments





Rescaled complexity $C_{JS} \in [-1,1]$ tells the statistics and predictability of a turbulence. E.g., for white noise, $C_{JS} = 0$, for logistic map, $C_{JS} = 1$.

- Experimental study on the fluctuations with the stochastic magnetic field. [1]
- An increase in the bicoherence of the temperature fluctuation → increased nonlinear coupling.
- A reduction in the Jensen-Shannon complexity → turbulence distribution becomes more random.

Why?

1. Choi, Minjun J., et al. *arXiv preprint arXiv:2102.10733* (2021).

Possible Answer: A Microturbulence

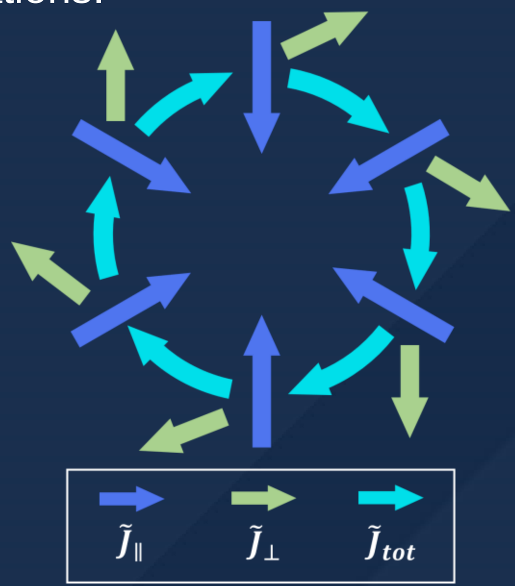
- **Constraint:** Quasi-neutrality (**$\nabla \cdot \boldsymbol{J} = 0$**) at all scales!
- **Effect:** Introduction of $\tilde{\boldsymbol{b}}$ leads to parallel current density fluctuations.

$$J_\parallel = -\frac{\mathbf{1}}{\eta_\parallel}\nabla_\parallel^{(0)}\boxed{\bar{\varphi}}\,\boldsymbol{b}_\mathbf{0}$$

electrostatic potential

$$J_\parallel = -\frac{1}{\eta_\parallel}\left[\nabla_\parallel^{(0)} + \tilde{\boldsymbol{b}}\cdot\nabla_\perp\right]\bar{\varphi}(\boldsymbol{b}_\mathbf{0} + \tilde{\boldsymbol{b}})$$

$$\widetilde{\nabla_\parallel J_\parallel} = -\frac{1}{\eta_\parallel}\left\{\nabla_\parallel^{(0)}\left[(\tilde{\boldsymbol{b}}\cdot\nabla_\perp)\bar{\varphi}\right] + \left(\tilde{\boldsymbol{b}}\cdot\nabla_\perp\right)\nabla_\parallel^{(0)}\bar{\varphi}\right\} \neq 0$$



- **Insights from the classic:** Kadomtsev and Pogutse'78 [1]:

Analogy	K&P	C&D
Goal	$\langle q_r \rangle_{NL}$	$\gamma_{\boldsymbol{k}}^{(1)}$
Base State	$\bar{T}$	$\bar{\varphi}$
Stochastic quantity	$\tilde{\boldsymbol{b}}$	$\tilde{\boldsymbol{b}}$
Constraint	$\nabla \cdot \boldsymbol{q} = 0$	$\nabla \cdot \boldsymbol{J} = 0$
Resulting Fluctuations	$\tilde{T}$	$\boxed{\tilde{\varphi}}$

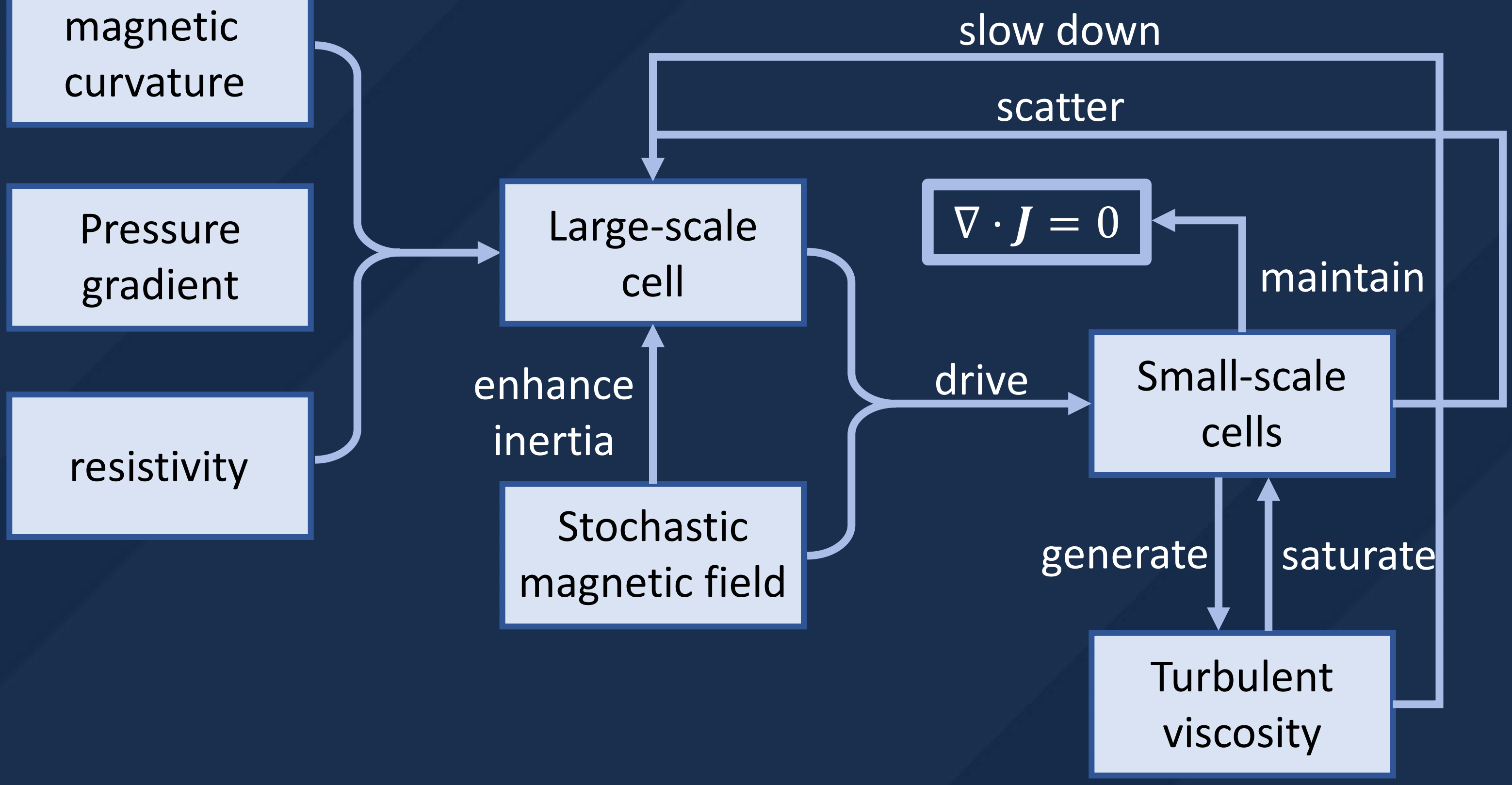
A current density fluctuation $\tilde{\boldsymbol{J}}_\perp$ must be driven to balance $\tilde{\boldsymbol{J}}_\parallel$, so that the total current density fluctuation $\tilde{\boldsymbol{J}}_{tot}$ is divergence free.

Intrinsic Multi-Scale Microturbulence → increased small-scale structure & nonlinear coupling?

1. B. B. Kadomtsev, and O. P. Pogutse, 1979.

Model Development

A Multi-scale Feedback Loop



Model Development

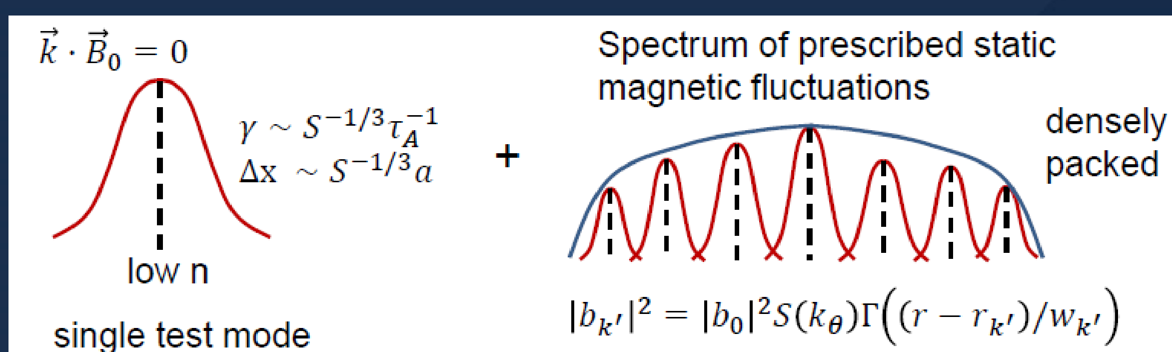
- Our model is supposed to
  - ➢ maintain $\nabla \cdot J = 0$ at all scales
  - ➢ be tractable ⟶ resistive interchange mode
  - ➢ connect micro and macro scales
  - ➢ be generic
- **Formulation:**

$$\begin{cases} -(\rho_0/B_0^2)\partial_t \nabla_\perp^2 \bar{\varphi} - (\kappa/B_0)\partial_y \bar{p}_1 + \boldsymbol{b_0} \cdot \nabla J_\parallel = 0 \longrightarrow \boxed{\nabla \cdot \boldsymbol{J} = 0} \\ \bar{E}_\parallel = \eta_\parallel \bar{J}_\parallel = -\boldsymbol{b_0} \cdot \nabla \bar{\varphi} \\ \partial_t \bar{p}_1 - (\nabla \bar{\varphi} \times \hat{\boldsymbol{z}})/B_0 \cdot \nabla p_0 = 0 \end{cases}$$

- **Externally-prescribed (static) magnetic perturbations:**

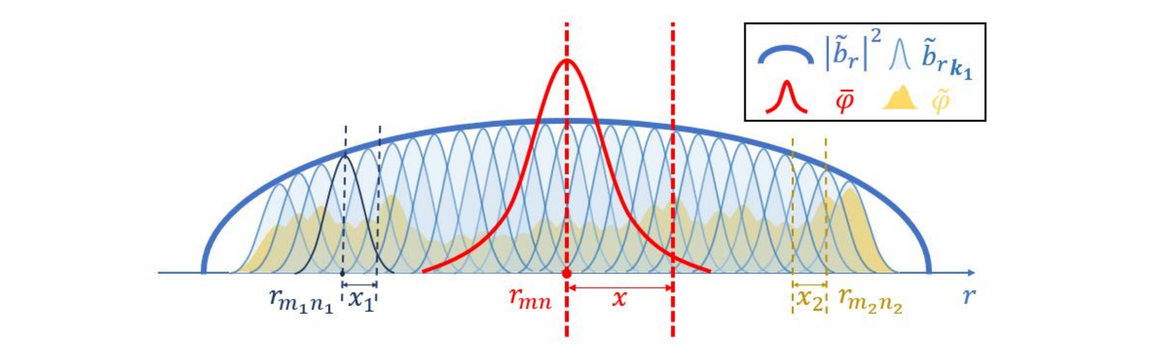
$$\tilde{\boldsymbol{b}} = \tilde{\boldsymbol{B}}_\perp / B_0 = \sum_{m_1 n_1} \tilde{\boldsymbol{b}}_{\boldsymbol{k_1}}(x') e^{i(m_1\theta - n_1\phi)} . (x' = r - r_{m_1 n_1})$$

- **Parallel gradient operator:** $\nabla_\parallel = \tilde{\boldsymbol{b}}_{\boldsymbol{0}} \cdot \nabla \longrightarrow \nabla_\parallel = \nabla_\parallel^{(0)} + \tilde{\boldsymbol{b}}_\perp \cdot \nabla_\perp$.



Model Development

- **New character:** Microturbulence



- **Modified model:**

$$\left(\frac{\partial}{\partial t}+\tilde{\boldsymbol{v}}\cdot\nabla\right)\nabla_\perp^2(\bar{\varphi}+\tilde{\varphi})=\frac{\eta S}{\tau_A}\nabla_\parallel J_\parallel-\frac{\kappa B_0}{\rho_0}\frac{\partial(\bar{p}_1+\tilde{p}_1)}{\partial y},$$

$$\left(\frac{\partial}{\partial t}+\tilde{\boldsymbol{v}}\cdot\nabla\right)(\bar{p}_1+\tilde{p}_1)-\frac{\nabla(\bar{\varphi}+\tilde{\varphi})\times\hat{\boldsymbol{z}}}{B_0}\cdot\nabla p_0=0,$$

$$\eta J_\parallel=-\nabla_\parallel(\bar{\varphi}+\tilde{\varphi}).$$

- **Observation:** A multi-scale problem.
- **Technique:** Method of averaging

$$\bar{A}=\langle A\rangle=\langle\bar{A}+\tilde{A}\rangle=\left(\frac{1}{2\pi}\right)^2\iint d\theta d\phi e^{-i(m\theta-n\phi)}A.$$

Model Development

- Separation of different scales:

$$① \left[\frac{\partial}{\partial t} + \widetilde{\boldsymbol{v}}\cdot\nabla\right]\nabla_\perp^2\bar{\varphi} = -\frac{S}{\tau_A}\Big[\nabla_\parallel^{(0)^2}\bar{\varphi} + \underbrace{\left(\nabla_\perp\cdot\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}\rangle\right)\cdot\nabla_\perp\bar{\varphi}}_{(a)} + \underbrace{\left\langle\nabla_\parallel^{(0)}\tilde{b}\cdot\nabla_\perp\tilde{\varphi}\right\rangle}_{(b)} + \underbrace{\left\langle(\tilde{b}\cdot\nabla_\perp)\nabla_{\parallel'}^{(0)}\tilde{\varphi}\right\rangle}_{(c)}\Big] - \frac{gB_0}{\rho_0}\frac{\partial\bar{p}_1}{\partial y},$$

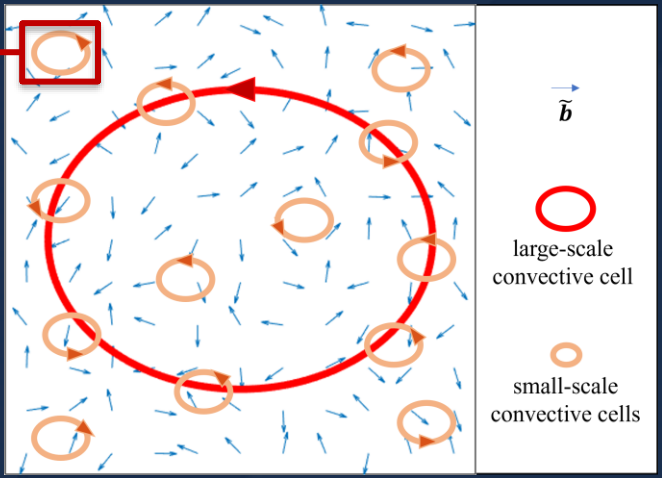
$$② \left[\frac{\partial}{\partial t} + \widetilde{\boldsymbol{v}}\cdot\nabla\right]\nabla_\perp^2\tilde{\varphi} = -\frac{S}{\tau_A}\Big[\nabla_\parallel^{(0)^2}\tilde{\varphi} + \underbrace{(\tilde{b}\cdot\nabla_\perp)\nabla_\parallel^{(0)}\bar{\varphi}}_{(\alpha)} + \underbrace{\nabla_\parallel^{(0)}(\tilde{b}\cdot\nabla_\perp)\bar{\varphi}}_{(\beta)}\Big] - \frac{gB_0}{\rho_0}\frac{\partial\tilde{p}_1}{\partial y}, \longrightarrow \text{Relate } \tilde{\varphi} \text{ to } \tilde{\boldsymbol{b}}$$

$$③ \left[\frac{\partial}{\partial t} + \widetilde{\boldsymbol{v}}\cdot\nabla\right]\bar{p}_1 - \frac{\nabla\bar{\varphi}\times\hat{\mathbf{z}}}{B_0}\cdot\nabla p_0 = 0,$$

$$④ \left[\frac{\partial}{\partial t} + \widetilde{\boldsymbol{v}}\cdot\nabla\right]\tilde{p}_1 - \frac{\nabla\tilde{\varphi}\times\mathbf{z}}{B_0}\cdot\nabla p_0 = 0,$$

replaced by $\boxed{-\nu_T\nabla_\perp^2 \text{ or } \chi_T\nabla_\perp^2}$ ⟶ **Very Important!**

leads to



- Some assumptions/observations:

1. *$\bar{\varphi}$: low $\boldsymbol{k}$, slow interchange approximation*
2. *$\tilde{\varphi}$: high $\boldsymbol{k}_2$, fast interchange approximation*
3. *The beat of $\widetilde{\mathbf{b}}$ and $\bar{\varphi}$ drives $\tilde{\varphi}$*
4. *$\tilde{\varphi}$ reacts on the evolution of $\bar{\varphi}$*

} A feedback loop

$$\boldsymbol{B_0} = B_\phi\widehat{\boldsymbol{\phi}} + B_\theta(r)\widehat{\boldsymbol{\theta}}, \qquad \tau_A = a(4\pi\rho_0)^{1/2}/B_0, \qquad S = \tau_R/\tau_A, \qquad \tau_R = 4\pi a^2/\eta$$
$$L_p = |(1/p_0)(dp_0/dr)|^{-1}, \qquad L_s = s/Rq, \qquad s = |(r/q)(dq/dr)|, \qquad k_\theta = m/r_{mn}$$

Results: $\tilde{v}_r$ phase *`locks on'* to $\tilde{b}_r$

$$\left[\frac{\partial}{\partial t} - \boxed{\nu_T} \nabla_\perp^2\right] \underbrace{\nabla_\perp^2 \tilde{\varphi} + \frac{S}{\tau_A} \nabla_\parallel^{(0)^2} \tilde{\varphi} + \frac{g B_0}{\rho_0} \frac{\partial \tilde{p}_1}{\partial y}}_{\text{fast interchange}} = \underbrace{[\underbrace{(\tilde{b} \cdot \nabla_\perp) \nabla_\parallel^{(0)} \bar{\varphi}}_{(\alpha)} + \underbrace{\nabla_\parallel^{(0)} (\tilde{b} \cdot \nabla_\perp) \bar{\varphi}}_{(\beta)}]}_{\text{slow interchange}}$$

$\boxed{\tilde{\boldsymbol{b}} \text{ is static!}}$

$\boldsymbol{\nu_T}$ **is required to saturate the growth of $\tilde{\varphi}$ on a short time scale**

- **Results:** The linear response of $\tilde{\varphi}$ to $\tilde{b}_r$ in the limit of $\gamma_{\boldsymbol{k}} \ll \nu_T k_{2\theta}^2$ (on the macro time scale)

$$\tilde{\varphi}_{k_2} \approx -i \frac{k_\theta}{L_S} \frac{S}{\tau_A} \sum_l \frac{\psi_{k_2}^l (x_2)}{\Lambda_{k_2}^l - \Lambda_{k_2}} \bar{\varphi}_k (x = 0) \int \psi_{k_2}^l \tilde{b}_{r(k_2 - k)} dx_2' ,$$

where $\psi_{\boldsymbol{k_2}}^l$ is the eigen solution, $\Lambda_{\boldsymbol{k_2}} = 1/\chi_T \tau_p \tau_\kappa - \nu_T k_{2\theta}^4$, $\Lambda_{\boldsymbol{k_2}}^l = \sqrt{8 \nu_T S k_{2\theta}^2 / \tau_A L_s^2} (l + 1/2)$.

- **Implication:** A non-trivial correlation

$$\langle \tilde{b}_r \tilde{v}_r \rangle = \pi^{\frac{1}{2}} \frac{\tilde{k}_\theta R r_{mn}}{L_S^3 B_0} \frac{S}{\tau_A} \bar{\varphi}_k (0) \times \int dk_{2\theta} |k_{2\theta}| k_{2\theta} \frac{c^2 Z^2 (k_\theta - k_{2\theta}) w_{k_2} o_{k_2}^2}{\Lambda_{k_2}^0 - \Lambda_{k_2}},$$

- **Reminder:** The statistics of the turbulence is affected by $\tilde{\boldsymbol{b}}$ (Minjun Choi).

Results: the Slow Down of the Large-scale Cell

- Main equation

$$-\frac{S}{\tau_{\mathrm{A}}}\frac{k_\theta^2}{L_{\mathrm{s}}^2}\frac{d^2}{dk_x^2}\hat{\bar{\varphi}}_k(k_x)+\gamma_k k_x^2\hat{\bar{\varphi}}_k(k_x)-\frac{\kappa p_0}{L_{\mathrm{p}}\rho_0}\frac{k_\theta^2}{\gamma_k}\hat{\bar{\varphi}}_k(k_x)=\hat{H}_0\hat{\bar{\varphi}}_{\boldsymbol{k}}(k_x)$$

turbulent viscosity damping

electrostatic scattering

$$\left.\begin{aligned}&=\boxed{-\nu_T k_x^4\hat{\bar{\varphi}}_k(k_x)-\frac{\kappa p_0\chi_T k_\theta^2}{\rho_0 L_{\mathrm{p}}\gamma_k^2}k_x^2\hat{\bar{\varphi}}_k(k_x)}-\frac{S}{\tau_{\mathrm{A}}}\left|\tilde{b}_r\right|^2 k_x^2\hat{\bar{\varphi}}_k(k_x)\\&\boxed{-\left(\frac{S}{\tau_{\mathrm{A}}}\right)^2\frac{Rk_\theta^2}{L_{\mathrm{s}}^3}\bar{\varphi}_k(0)\left[i\sqrt{2\pi}\delta^{(1)}(k_x)+r_{mn}\sqrt{2\pi}\delta(k_x)\right]I}\end{aligned}\right\}=\hat{H}_1\hat{\bar{\varphi}}_{\boldsymbol{k}}(k_x).$$

- The first-order growth rate correction:

$$\gamma_k^{(1)}=\frac{\int_{-\infty}^{\infty}\hat{\bar{\varphi}}_k^{(0)}\hat{H}_1\hat{\bar{\varphi}}_k^{(0)}dk_x}{\int_{-\infty}^{\infty}\hat{\bar{\varphi}}_k^{(0)}\left[\partial_{\gamma_k^{(0)}}\hat{H}_0\right]\hat{\bar{\varphi}}_k^{(0)}dk_x}=-\frac{5}{6}\underbrace{\hat{\nu}_T}_{\boldsymbol{TBD}}\left(\frac{\tau_{\mathrm{p}}\tau_\kappa}{\tau_{\mathrm{A}}^2}\right)^{\frac{1}{3}}S^{\frac{2}{3}}\tilde{k}_\theta^{\frac{2}{3}}-\frac{1}{3}\frac{S}{\tau_{\mathrm{A}}}\left|\tilde{b}_r\right|^2-\frac{2\sqrt{2}}{3}\frac{\hat{I}S^{\frac{4}{3}}\tilde{k}_\theta^{\frac{4}{3}}}{\left(\tau_{\mathrm{p}}\tau_\kappa\tau_{\mathrm{A}}^4\right)^{\frac{1}{3}}}<0,$$

where $\hat{\nu}_T=\nu_T/L_s^2,\ \hat{I}=IRr_{mn}/L_s^3, \tilde{k}_\theta=k_\theta L_s$.

Physics?

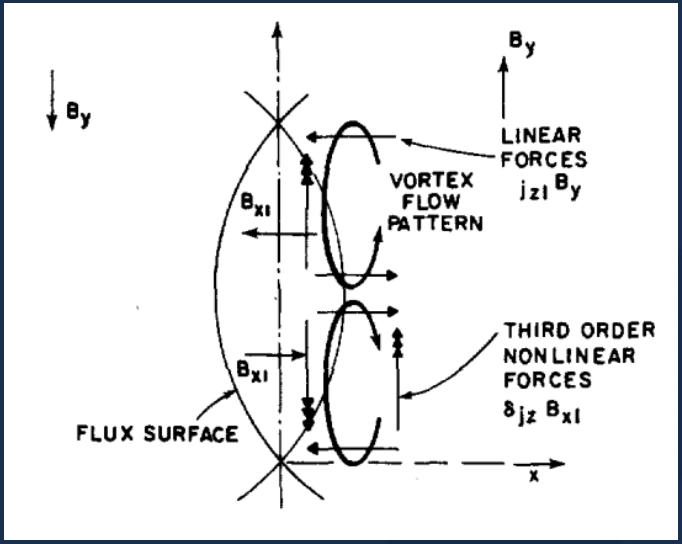
- **Reminder:** Stronger suppression of large-scale fluctuations. (P. Beyer)

Results: Magnetic Braking Effect

Focusing on the third term, main equation becomes

$$-\frac{S}{\tau_{\mathrm{A}}}\frac{k_\theta^2}{L_{\mathrm{s}}^2}\frac{d^2}{dk_x^2}\bar{\varphi}_{\boldsymbol{k}}+\left[\frac{S}{\tau_{\mathrm{A}}}\left|\tilde{b}_r\right|^2+\boxed{\gamma_{\boldsymbol{k}}}\right]k_x^2\bar{\varphi}_{\boldsymbol{k}}-\frac{\kappa p_0}{L_{\mathrm{p}}\rho_0}\frac{k_\theta^2}{\gamma_{\boldsymbol{k}}}\bar{\varphi}_{\boldsymbol{k}}=0.$$

Inertia term $\rho\partial_t\nabla_\perp^2\bar{\varphi}$



$o_{\boldsymbol{k}_2}\sim\Delta x$

- **Effect:** Enhances the plasma inertia and opposes the growth of $\bar{\varphi}$ ⟶ ***Magnetic braking effect***.
- **Results:**
  1. Corrected growth rate of the ground state

$$\gamma_{\boldsymbol{k}}^{(1)}=\frac{\tilde{k}_\theta}{S|\tilde{b}_r|}\frac{\tau_A}{\tau_p\tau_\kappa}\propto\frac{1}{|\tilde{b}_r|}.$$

  2. Balancing the third term with the linear term, the critical width of magnetic islands

$$o_{\boldsymbol{k}_2}\sim\left[\boxed{\frac{k_\theta^2}{k_{2\theta}^2}}(\Delta x)^4\right]^{\frac{1}{4}}$$

k_θ: large-scale cell
$k_{2\theta}$: small-scale cells

character of multi-scale system, different with Rutherford's result $o_{\boldsymbol{k}_2}\sim\Delta x$ [1]

1. Rutherford, Paul Harding. *The Physics of Fluids* 16, no. 11 (1973): 1903-1908.

Results: Turbulent Viscosity

- **The last piece:** The calculation of the turbulent viscosity ν_T
- **Strategy:** Nonlinear closure theory

$$\nu_T = \sum_{\boldsymbol{k_2}} \left|\tilde{v}_{\boldsymbol{k_2}}\right|^{\boldsymbol{2}} \tau_{\boldsymbol{k_2}} \,.$$

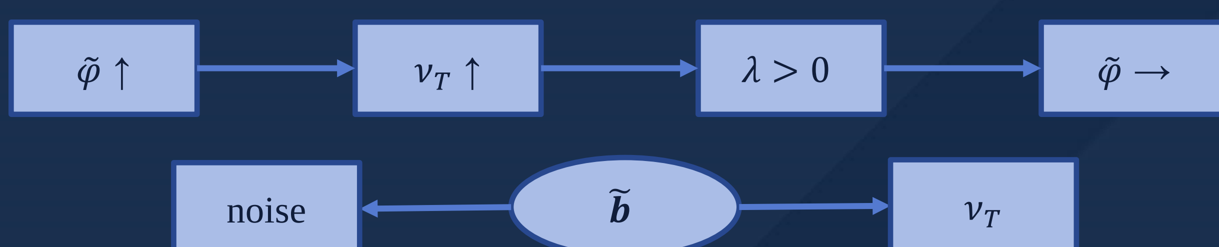
- **Result:** In the limit of $\nu k_{2\theta}^2 - \left(1/\tau_p \tau_\kappa\right)^{1/2} \gg 0$, the scaling of the turbulent viscosity is

$$\nu_T = \left[\pi^{\frac{1}{2}} \frac{R r_{mn}}{B_0^2} \frac{\tilde{k}_\theta^2}{L_\mathrm{s}^5} \left(\frac{S}{\tau_\mathrm{A}}\right)^2 \bar{\varphi}_{\boldsymbol{k}}^2(0) \int dk_{2\theta} \frac{c^2 Z^2 w_{\boldsymbol{k_2}} o_{\boldsymbol{k_2}}^2}{|k_{2\theta}|^5 \gamma_{\boldsymbol{k_2}}^{(0)}}\right]^{\frac{1}{3}} .$$

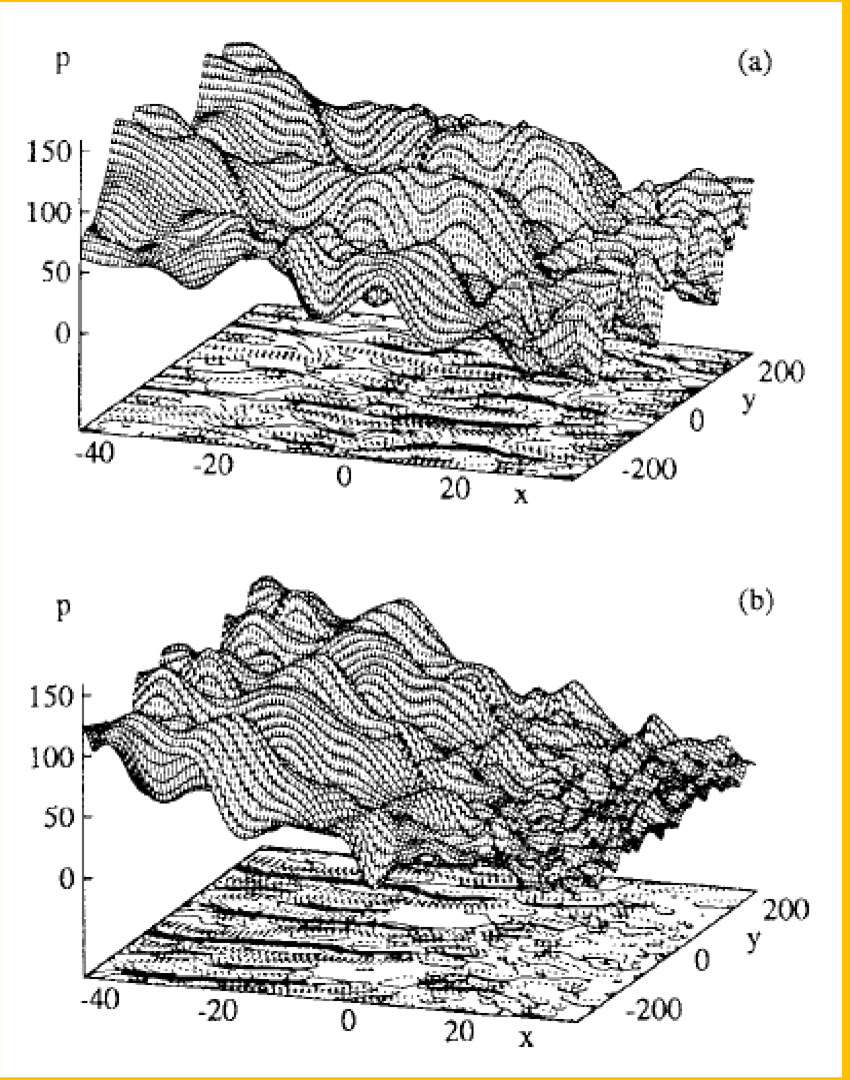
- **Analysis:** Equation (2) can be simplified to

$$\frac{\partial \tilde{\varphi}}{\partial t} + \lambda\, \tilde{\varphi} = \underbrace{\hat{D}\left[\tilde{b}_r \bar{\varphi}\right]}_{drive} . \longrightarrow \text{Langevin equation!}$$

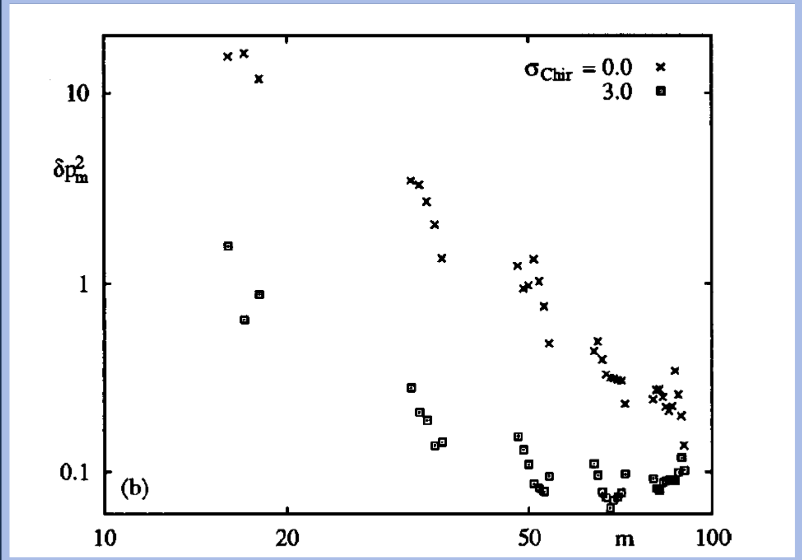
$$\nu_T k_{2\theta}^2 - \left(1/\tau_p \tau_\kappa\right)^{1/2} > 0$$



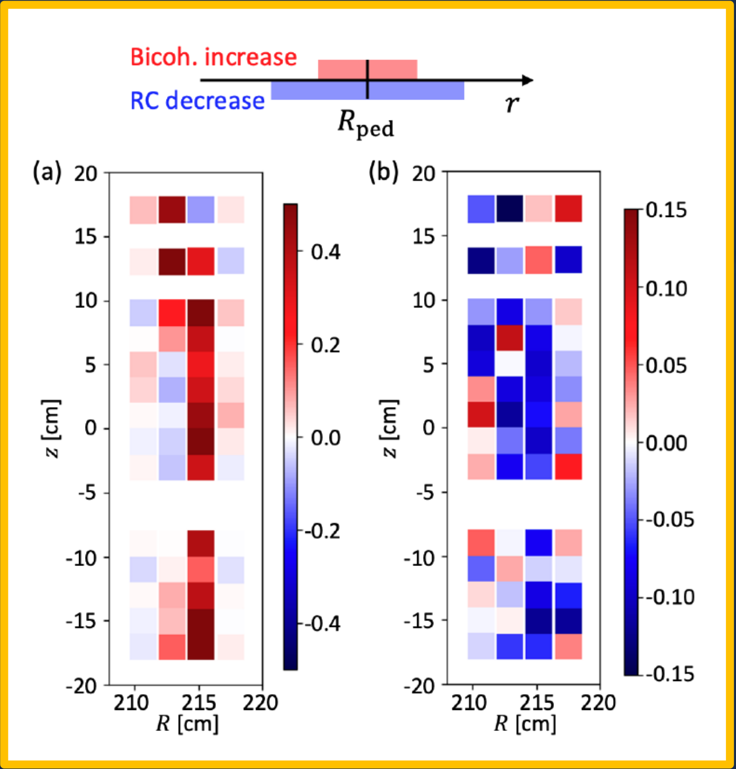
Answers to Questions from Simulations



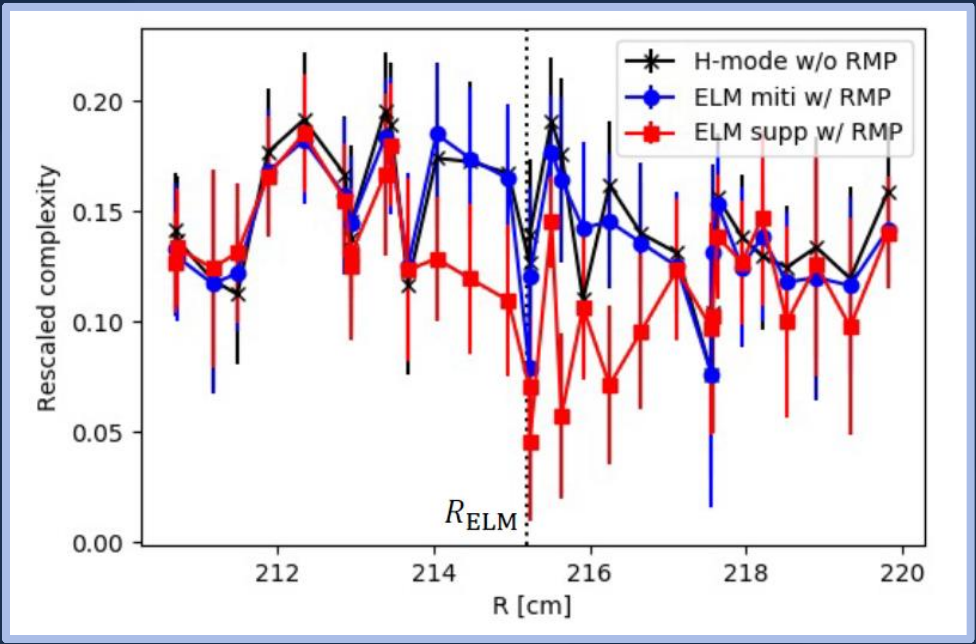
- Appearance of small-scale structures and increased spatial roughness with stochastic magnetic field → the generation of the small-scale convective cells;
- Stronger suppression of large-scale fluctuations in the stochastic region → large-scale cell tends to be stabilized by the stochastic magnetic field.



Answers to Questions from Experiments



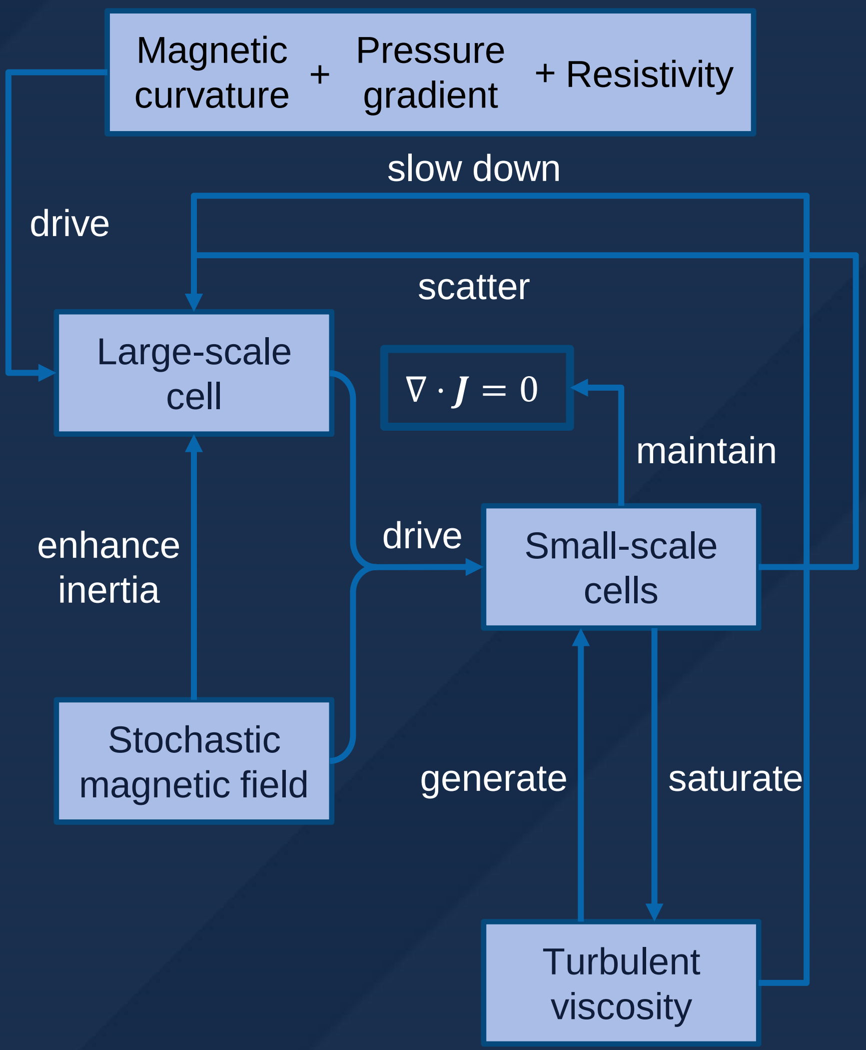
- Increased bicoherence in the pedestal turbulence → small-scale convective cells potentially open the possibility of increased nonlinear transfer, by increasing the number of triad interactions.



- Reduced C_{JS} of the temperature fluctuation in the ELM suppression phase with RMP → *electrostatic turbulence phase 'locks on to the stochastic magnetic field'*. In other words, turbulence becomes more `random' because its statistics is now correlated with the stochastic magnetic field.

Conclusion: What We Have Learned

- $\nabla \cdot \boldsymbol{J} = 0$ is maintained at all scales, which reveals that electrostatic convective cells must be driven by $\tilde{\boldsymbol{b}}\bar{\varphi}$ beat.
- Large scale and small scale are connected. As small-scale convective cells are modulated by large-scale mode, large-scale mode is modified by small-scale cells through turbulent viscosity ν_T and electrostatic scattering.
- Stochastic magnetic field produces a magnetic braking effect, which enhances the plasma inertia and exerts a drag on large-scale mode. This is similar in structure to Rutherford's nonlinear $\boldsymbol{J} \times \boldsymbol{B}$ forces[1], but in our case, it's produced by stochastic magnetic perturbations.
- We get a non-trivial $\langle \tilde{b}_r \tilde{v}_r \rangle$. The velocity fluctuations $\tilde{\boldsymbol{v}}$ 'lock on' to the magnetic perturbations $\tilde{\boldsymbol{b}}$.



Conclusion: What We Have Learned

- Correlation $\langle \tilde{b}_r \tilde{v}_r \rangle$ is calculated explicitly:

$$\langle \tilde{b}_r \tilde{v}_r \rangle = \pi^{\frac{1}{2}} \frac{\tilde{k}_\theta R r_{mn}}{L_S^3 B_0} \frac{S}{\tau_A} \bar{\varphi}_k(0) \times \int dk_{2\theta} |k_{2\theta}| k_{2\theta} \frac{c^2 Z^2 (k_\theta - k_{2\theta}) w_{k_2} o_{k_2}^2}{\Lambda_{k_2}^0 - \Lambda_{k_2}}$$

- The increment in the growth rate of the large-scale mode is calculated:

$$\gamma_{\boldsymbol{k}}^{(1)} = -\frac{5}{6} \hat{\nu} \left(\frac{\tau_p \tau_\kappa}{\tau_A^2} \right)^{1/3} S^{2/3} \tilde{k}_\theta^{2/3} - \frac{1}{3} \frac{S}{\tau_A} \left| \tilde{b}_r \right|^2 - \frac{2\sqrt{2}}{3} \frac{\hat{I} S^{4/3} \tilde{k}_\theta^{4/3}}{\left(\tau_p \tau_\kappa \tau_A^4 \right)^{1/3}}.$$

As $\gamma_{\boldsymbol{k}}^{(1)}$ is negative definite, the net effect of $\tilde{\boldsymbol{b}}$ is to reduce resistive interchange growth.

- The criterion when magnetic braking effect becomes significant is given. When the width of magnetic islands satisfies

$$o_{\boldsymbol{k_2}} \sim \left[\frac{k_\theta^2}{k_{2\theta}^2} (\Delta x)^4 \right]^{1/4}.$$

Unlike Rutherford's result, here we have an extra factor $(k_\theta / k_{2\theta})^2$, which reflects the multi-scale nature of this problem.

- The scaling of the turbulent viscosity (or turbulent thermal diffusivity) is calculated:

$$\nu = \left[\pi^{\frac{1}{2}} \frac{R r_{mn}}{B_0^2} \frac{\tilde{k}_\theta^2}{L_s^5} \left(\frac{S}{\tau_A} \right)^2 \bar{\varphi}_{\boldsymbol{k}}^2(0) \int dk_{2\theta} \frac{c^2 Z^2 w_{\boldsymbol{k_2}} o_{\boldsymbol{k_2}}^2}{|k_{2\theta}|^5 \gamma_{\boldsymbol{k_2}}^{(0)}} \right]^{1/3}$$

Conclusion: What We Will Do

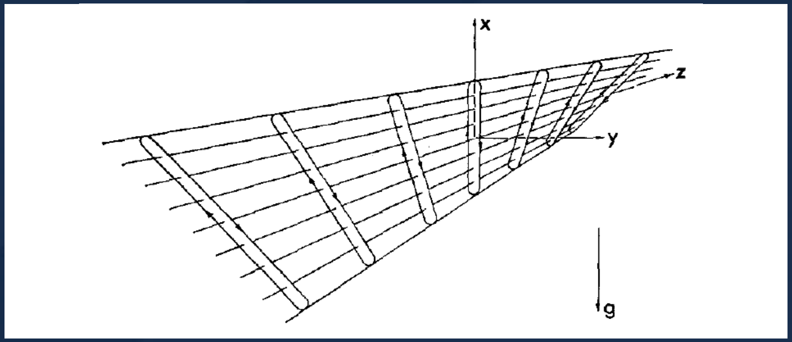
- Toroidicity effect

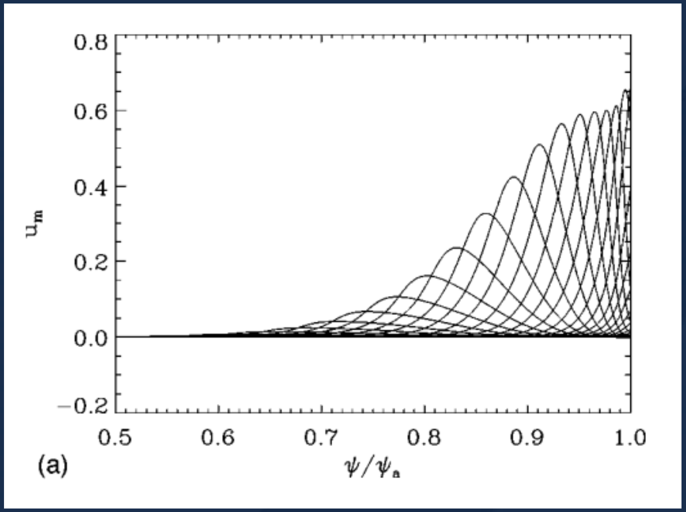
  In tokamak, the poloidal symmetry is broken by *toroidicity effect*. This fact introduces *poloidal coupling* of a series of harmonics, which results in ballooning mode.

  Both twisted slicing mode and ballooning mode can be considered as wave packets [1,2]. So twisting slicing mode is a particularly clear realization of ballooning.

  Idea: studying resistive ballooning mode in a stochastic magnetic field.

  Tool: ballooning mode representation.





- Zonal flow

  Zonal flow plays a crucial role in L-H transition. The observed enhancement of the transition power threshold implies that zonal flow screening or collapse could be a possible scenario. Therefore, it is essential to couple zonal flow to current model.

1. Roberts, K. V., and J. B. Taylor. *The Physics of Fluids* 8, no. 2 (1965): 315-322.
2. Wilson, H. R., P. B. Snyder, et al. *Physics of Plasmas* 9, no. 4 (2002): 1277-1286.

Thank you