The ubiquitous zonal flows
(and corrugations)

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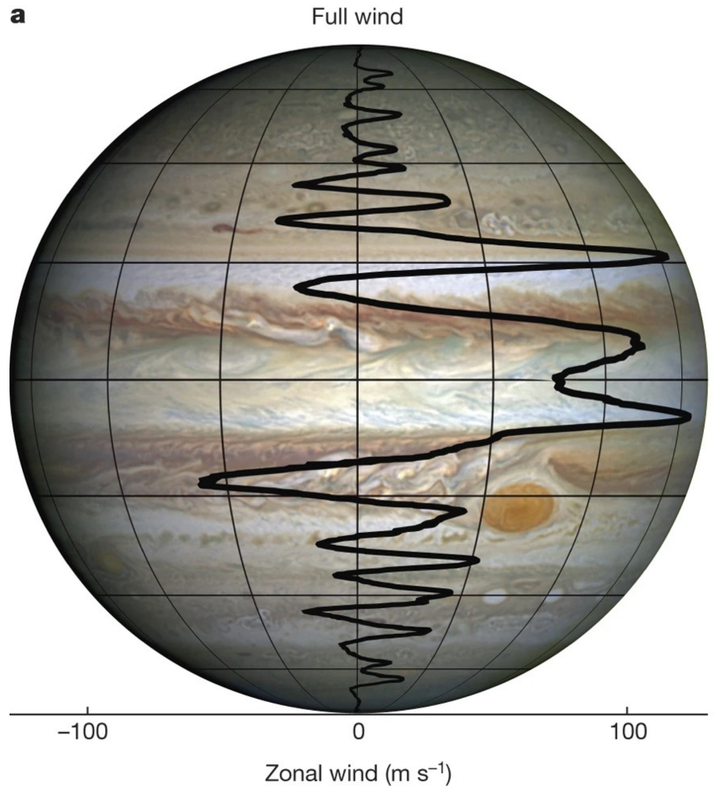
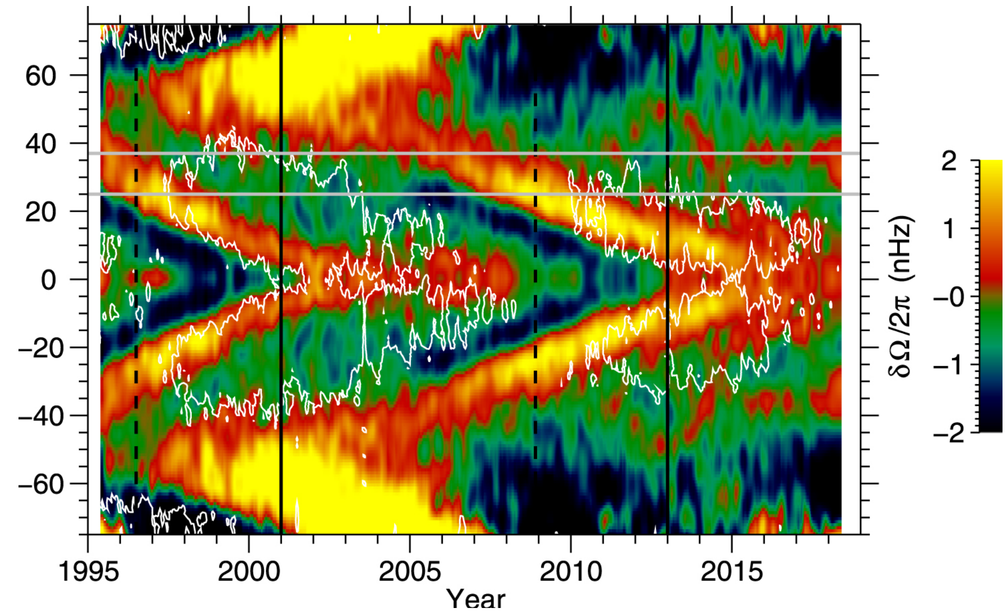
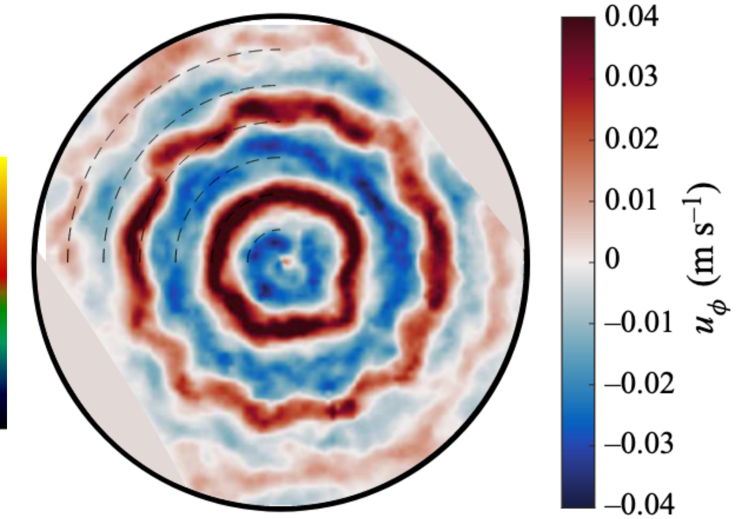
Zonal flows are ubiquitous



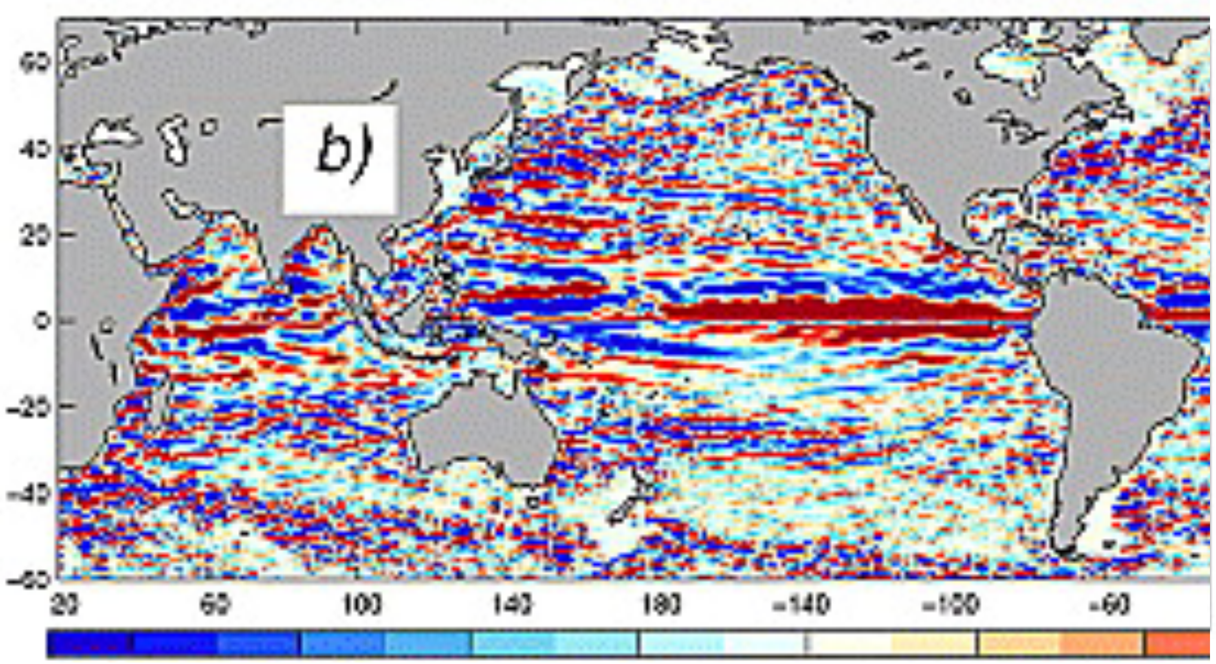
ZFs in Jupiter



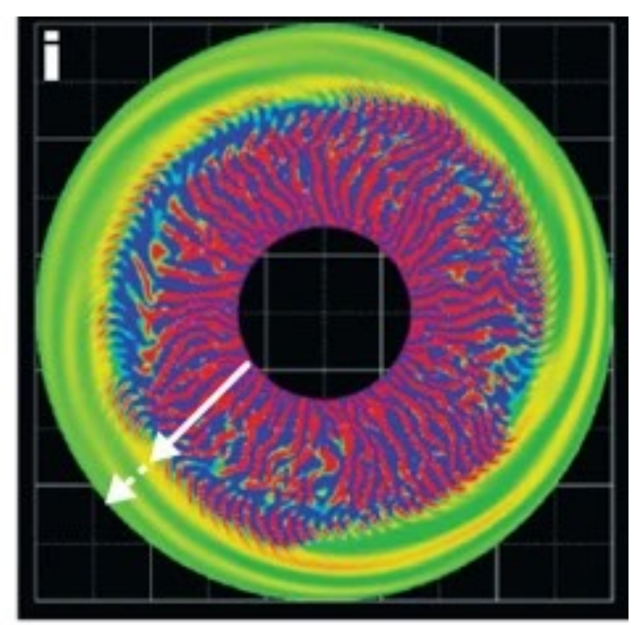
Migrating ZFs in convection zone of Sun



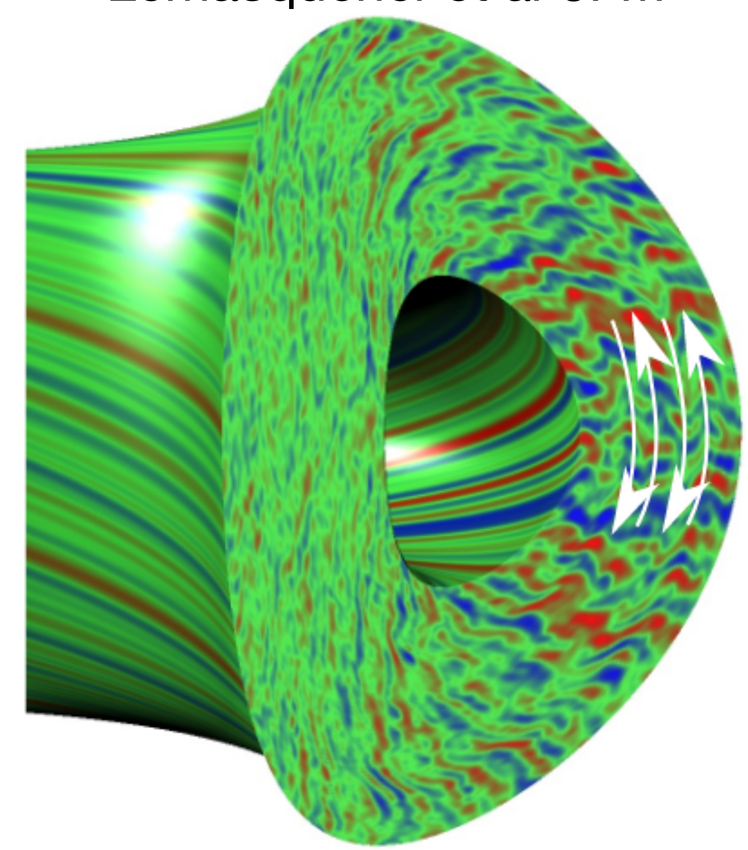
ZFs in rotating liquid column experiment Lemasquerier *et al* JFM



ZFs in ocean Maximenko *et al* GRL 2005



ZFs in liquid interior of Earth outer core T Miyagoshi *et al* Nature 2010



ZFs in tokamak

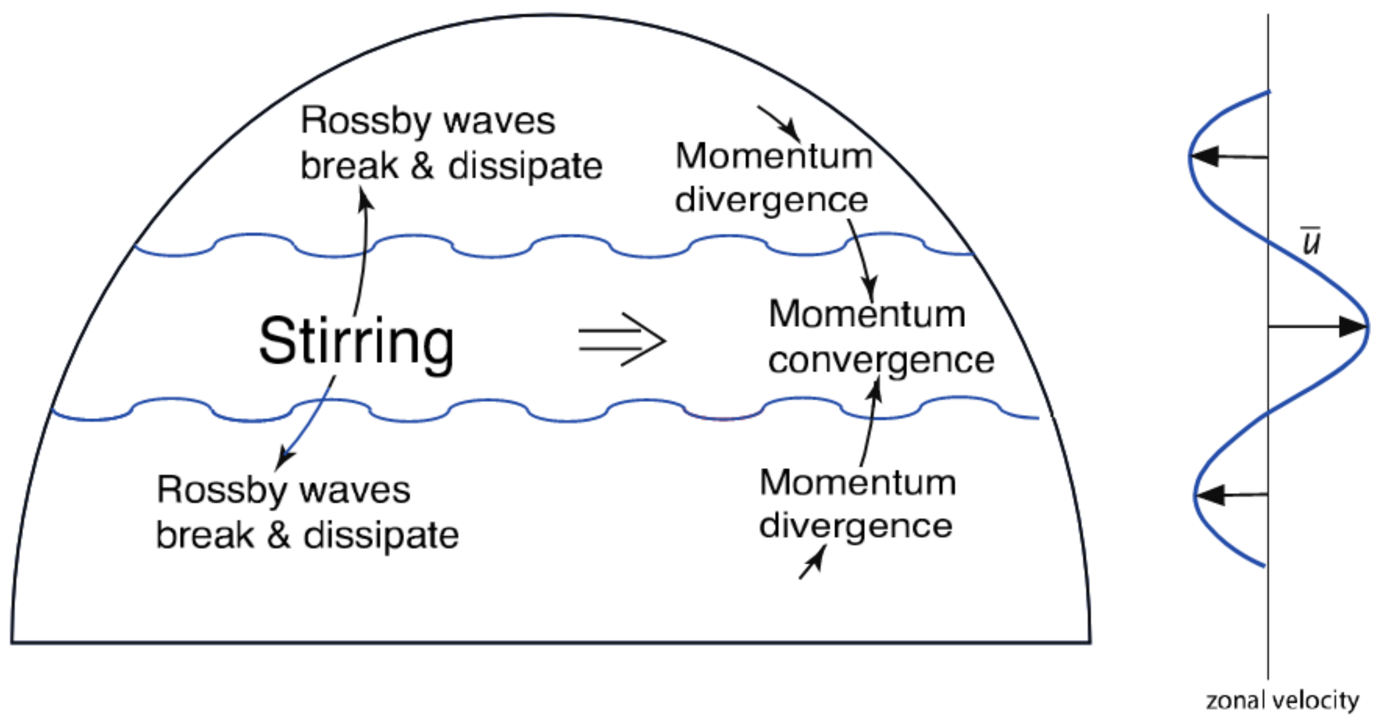
Outline

- Why zonal flows are ubiquitous?
- What determines the ZF pattern: Momentum theorem
- Shearing effects
  - Turbulence decorrelation, Induced diffusion , Modulational instability
- Unifying zonal flows and corrugations
  - Noise + modulations
  - Zonal cross correlation- staircase
- Feedback loops (noise effects)
- Conclusions

Why zonal flows are ubiquitous?

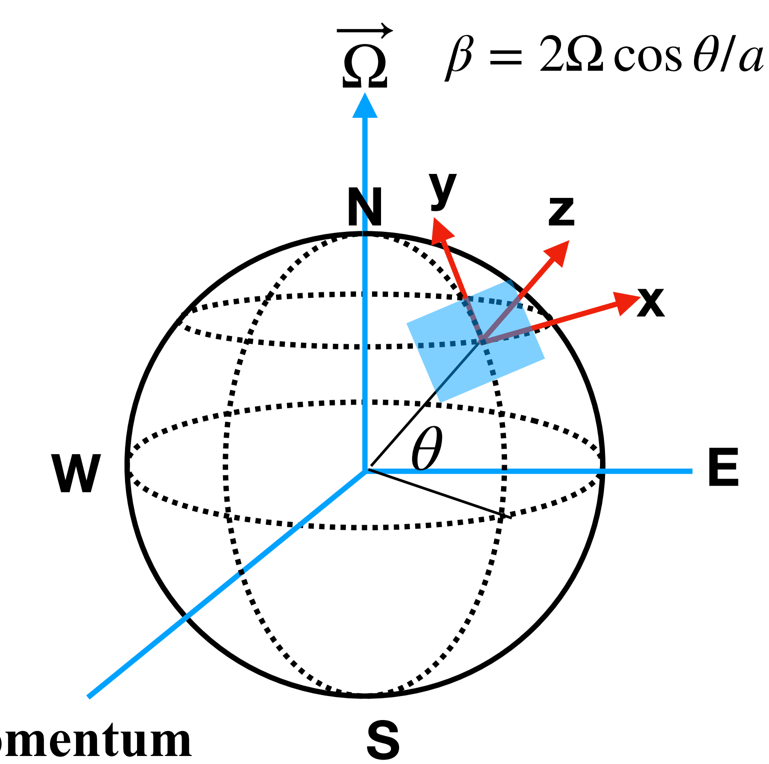
GFD Perspective

- Mid latitude zonal circulation [G K Vallis Book]



- Stirring in mid-latitudes (by baroclinic eddies) generates Rossby waves that propagate away.

- Momentum converges in the region of stirring, producing eastward flow there and weaker westward flow on its flanks.



Rossby waves on beta-plane

- Frequency $\omega = -\dfrac{\beta k_x}{k_x^2 + k_y^2}$, Group velocity $v_{gy,k} = \dfrac{2\beta k_x k_y}{\left(k_x^2 + k_y^2\right)^2}$
- Momentum flux: $\left\langle v_y v_x \right\rangle = -\sum_{\vec{k}} \frac{1}{2} k_x k_y \left|\psi\right|^2 = -\sum_{\vec{k}} v_{gy,k} \mathscr{P}_k$

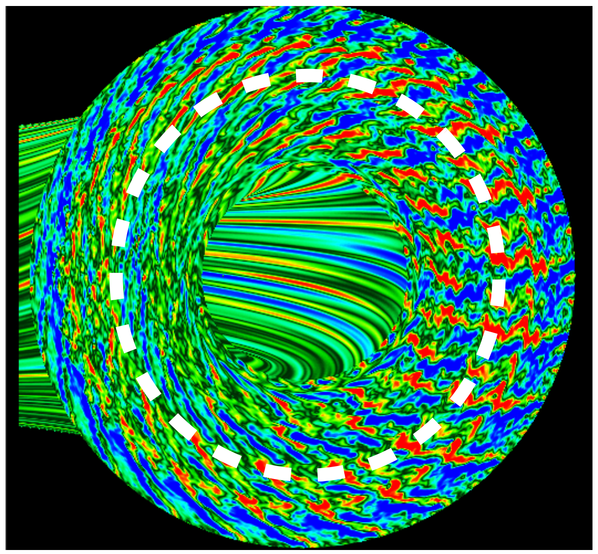
$\boxed{\text{•Outgoing waves} \Longrightarrow \text{Incoming momentum flux}}$

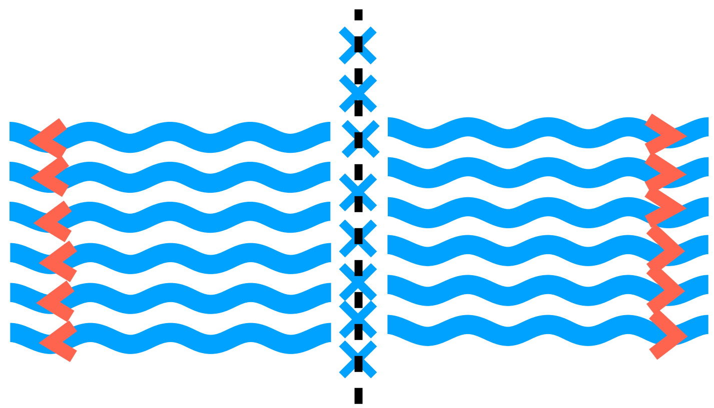
Pseudomomentum

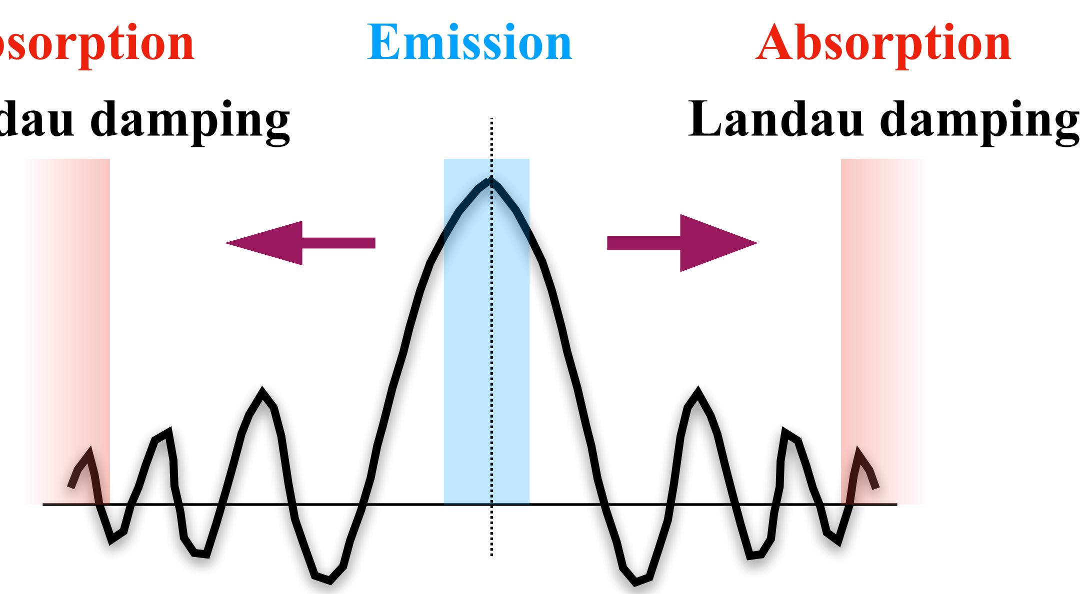
Why zonal flows are ubiquitous?

MFE perspective

- Drift wave instability is localized at mode rational surfaces $\vec{k} \cdot \vec{B} = 0$.





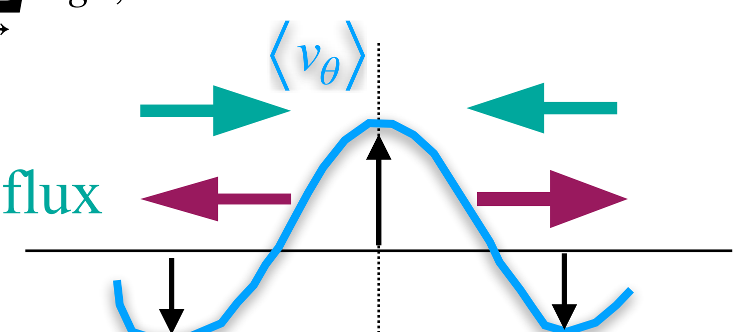


- DW frequency $\omega = \dfrac{v_{\star e} k_y}{1 + k_\perp^2 \rho_s^2}$, $\Longrightarrow$ $v_{gr} = -\dfrac{2\rho_s^2 v_{\star e} k_x k_y}{(1 + k_\perp^2 \rho_s^2)^2}$, $v_{\star e} < 0 \Longrightarrow v_{gr} > 0$ for $k_x k_y > 0 \rightarrow$ outgoing wave energy

- Momentum flux: $\langle v_{E,r} v_{E,\theta} \rangle = -\dfrac{c}{B^2} \sum_{\vec{k}} k_x k_y \left| \phi_k \right|^2 = -\sum_{\vec{k}} v_{gx,k} \mathscr{P}_k < 0$

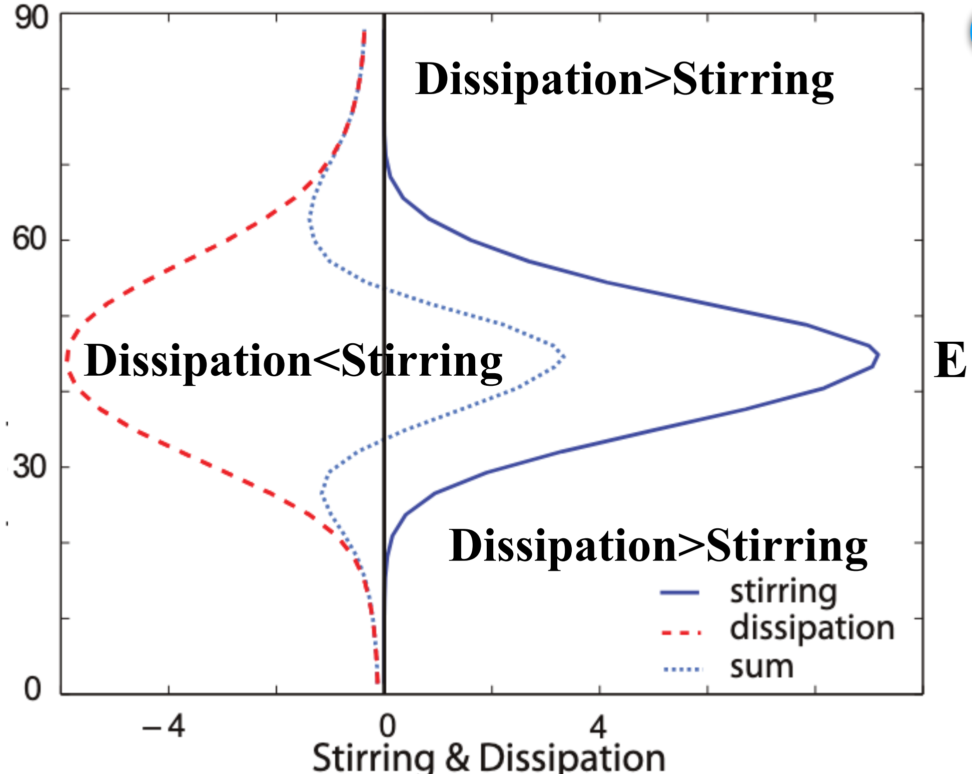
- Outgoing wave energy flux $\rightarrow$ incoming wave momentum flux

- Zonal Flow spins up at the DW excitation region $\vec{k} \cdot \vec{B} = 0$



What determines the zonal flow pattern?

GFD Perspective



- Meridionally confined stirring.
- dissipation broader than that of forcing.
- stirring + dissipation → zonal wind (east-ward) in the region of the stirring.

Zonal flow scale ~ wave- turbulence cross over scale ~ Rhines scale

$$\vec{v}\cdot\vec{\nabla}\zeta : \beta v_y \rightarrow l_R = \left(\frac{U}{\beta}\right)^{1/2}$$

1.Vorticity: $\partial_t\zeta + \vec{v}\cdot\vec{\nabla}\zeta + \beta v_y = F_\zeta - D_\zeta$

2.Momentum theorem: $\partial_t\left(\mathscr{P} + \langle v_x\rangle\right) = \gamma^{-1}\left[\left\langle\tilde{\zeta}\tilde{F}_\zeta\right\rangle - \left\langle\tilde{\zeta}\tilde{D}_\zeta\right\rangle - \partial_y\left\langle\tilde{v}_y\tilde{\zeta}^2\right\rangle\right] - r\langle v_x\rangle$

Drive Dissipation PE flux/ Turbulence spreading Drag

Pseudomomentum

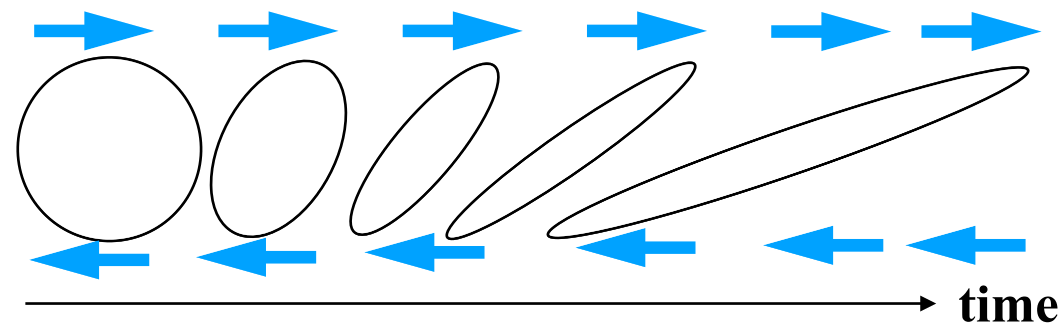
$$\mathscr{P} = \left\langle\tilde{\zeta}^2\right\rangle/2\gamma$$

$$\gamma = \beta - \partial_{yy}\langle v_x\rangle$$

- **Non-acceleration theorem**: Absent driving (flux) and local PE decrement →can not accelerate/maintain ZF with stationary fluctuations!

Shearing effects

- Coherent shearing [Taylor, Dupree, BDT 90]
  - Radial scattering + mean shear V_E' → hybrid decorrelation $k_r D_\perp^2 \rightarrow (k_\theta^2 V_E'^2 D_\perp/3)^{1/3} = 1/\tau_c$



- Wave kinetics:
  - $$\frac{\partial k_r}{\partial t} = -\frac{\partial\left(\omega + k_\theta V_E\right)}{\partial r}$$
  - Mean shearing: $k_r = k_{r0} - k_\theta V_E' \tau_c$
  - Random shearing:

➡ induced diffusion $\left\langle \delta k_r^2 \right\rangle = D_{kk}\tau$, $D_{kk} = \sum_q k_\theta^2 \left| \tilde{V}_{E,q}' \right|^2 \tau_{k,q}$

N_k: Wave action density

- Mean field WKE $$\frac{\partial \langle N_k \rangle}{\partial t} = \frac{\partial}{\partial k_x}\left[D_{kk} \frac{\partial \langle N_k \rangle}{\partial k_x} \right] + \langle \gamma_k N_k \rangle - \langle C(N_k) \rangle$$

Shearing effects

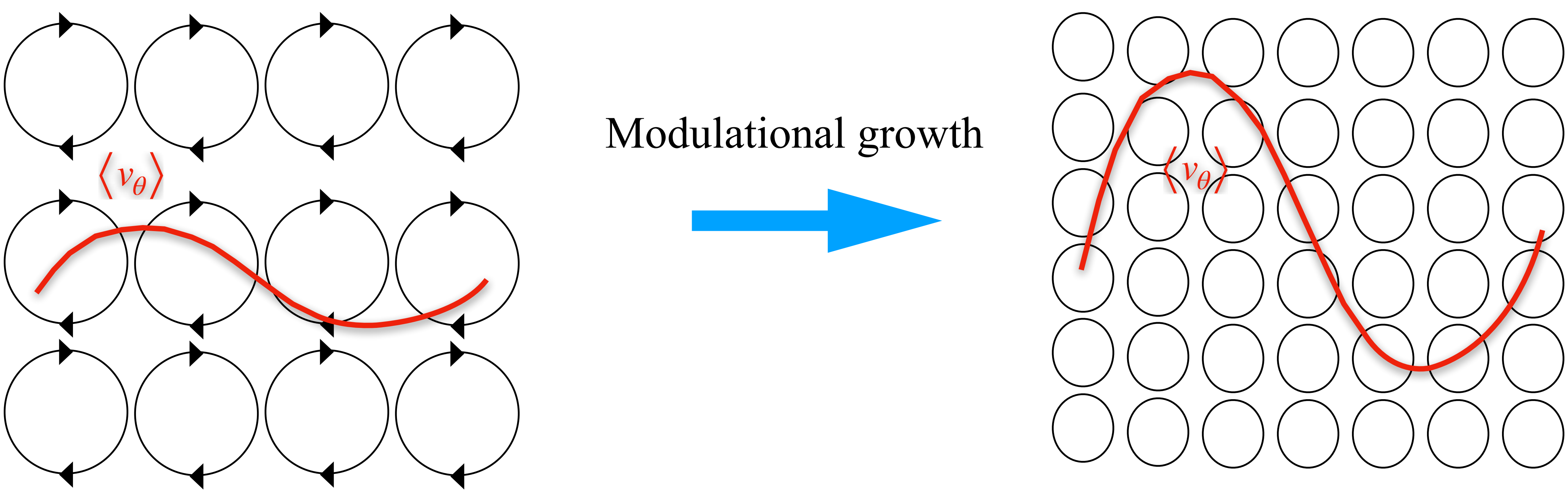
- Zonal flow shearing damps wave energy $\mathscr{E} = \int d\vec{k}\,\omega \langle N \rangle : \frac{\partial}{\partial t}\mathscr{E} = -\int d\vec{k}\, v_{gr,k} D_{kk} \frac{\partial}{\partial k_r} \langle N \rangle$

- Reduction of turbulence energy appears as growth of zonal flow

$$\frac{\partial}{\partial t}\delta\left\langle v_\theta \right\rangle + \frac{\partial}{\partial r}\delta\left\langle v_r v_\theta \right\rangle = -\nu_c \delta\left\langle v_\theta \right\rangle$$

$$\delta\left\langle v_r v_\theta \right\rangle = \int d\vec{k}\, k_r k_\theta \delta \left|\phi_k\right|^2 = \int d\vec{k}\, k_r k_\theta C \delta N_{k,q} = \boxed{\int d\vec{k}\, k_y^2 C_k \mathscr{R}^{(r)}_{k,q} k_x \frac{\partial \left\langle N_k \right\rangle}{\partial k_x}} \frac{\partial \delta \left\langle v_\theta \right\rangle}{\partial r}$$

  - Modulational growth due to -ve turbulent viscosity for $k_x \frac{\partial \left\langle N_k \right\rangle}{\partial k_x} < 0$



Noise meets modulation

- Almost all theoretical models of zonal flow generation divide cleanly into:

  1. Zonal flow dielectric or screening response, with occasional mention of polarization beat noise. [RH 1998, HR, 1999].

     $$\frac{\partial}{\partial t}\left\langle\left|\phi_q\right|^2\right\rangle = \frac{2\tau_c \boxed{\left\langle\left|S_q\right|^2\right\rangle}}{\chi(q)}$$

     S_q = **Polarization flux**

     $$\chi = \chi_{cl} + \chi_{neo} = \left\{1 + \frac{q^2}{\epsilon^2}\right\} k_r^2 \rho_i^2 \quad ;$$

     -overlooks modulational mechanism - "negative viscosity"

  2. Modulational stability calculations ignore noise emission.

- Iteraction? → Coupled spectral evolution - DIA closure

  -What of profile corrugations ?

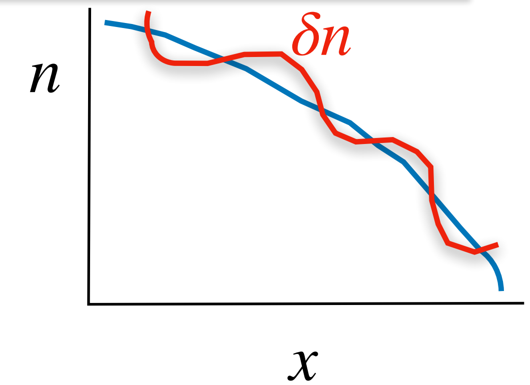
  -What of zonal shears and corrugations alignment - staircases?

  -Noise effect on feedback processes ?

  -Macroscopics - L→H transition?

Profile corrugations meet zonal flows

- Wave kinetics:



  - $\partial_t k_r = -\partial_r \left(\omega + k_\theta V_E\right); \quad \omega = \omega(n', T', \cdots)$

  - Freq. modulation due to profile corrugations $\boxed{\delta\omega = \dfrac{\partial\omega}{\partial n'}\delta n'' + \dfrac{\partial\omega}{\partial T'}\delta T'' + \cdots}$

  - Random shearing: $\rightarrow$ induced diffusion $\left\langle \delta k_r^2 \right\rangle = D_{kk}\tau$,

$$D_{kk} = \sum_q k_\theta^2 \left| \tilde{V}'_{E,q} + \boxed{\frac{\partial\omega}{\partial n'}\delta n'' + \frac{\partial\omega}{\partial T'}\delta T'' + \cdots} \right|^2 \tau_{k,q}$$

➡Induced diffusion depends on the relative alignment of zonal shear and profile corrugations!

➡Interferes fluctuation energy and zonal flow saturation !

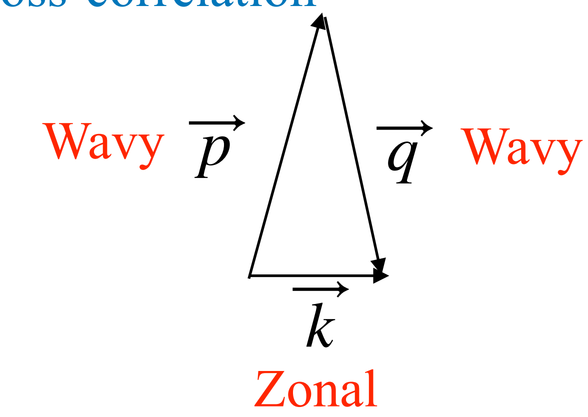
➡… unified theory of zonal flow and corrugations needed! Use Hasegawa- Wakatani model at the simplest level of description.

Spectral evolution of zonal intensity

For the zonal mode $k_y = k_\parallel = 0$ and $k_x \neq 0$

Zonal density - potential cross-correlation

$$\left(\frac{\partial}{\partial t} + 2\mu k_x^2\right)\left\langle \left|\phi_k\right|^2 \right\rangle + 2\eta_{1k}^{zonal}\left\langle \left|\phi_k\right|^2 \right\rangle + \Re\left[2\eta_{2k}^{zonal}\boxed{\left\langle n_k \phi_k^\star \right\rangle}\right] = F_{\phi k}^{zonal}$$



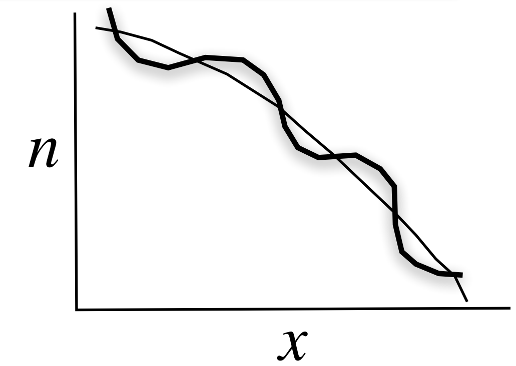
[Singh, Diamond PPCF 2021]

- NL damping rate: $\eta_{1k}^{zonal} \propto k_x^2$ and becomes -ve for $\dfrac{\partial I_q}{\partial q_x} < 0 \rightarrow$ transfer to large scales by negative turbulent viscosity!
  - Zonal growth is maximum when the adiabaticity parameter $\alpha_q \rightarrow \infty$ $\Longrightarrow$Non-adiabatic fluctuations inhibit transfer to large scales !
- Cross transfer rate: $\eta_{2k}^{zonal,(r)} > 0$ ALWAYS for $\dfrac{\partial I_q}{\partial q_x} < 0 \Longrightarrow$ Forward transfer when $\Re\left\langle n_k \phi_k^\star \right\rangle < 0$, backward transfer when $\Re\left\langle n_k \phi_k^\star \right\rangle > 0$
- Noise: always +ve and of envelope scale! $F_{\phi k}^{zonal} = 4\sum_q \Pi_q^2 \Theta_{k,-q,q}^{(r)}$; Reynolds stress $\Pi_q = q_y q_x I_q$. Noise/Modulation = $q_x^2 I_q / k_x^2 I_k$ = Turbulent KE/ Zonal KE.

Spectral evolution of density corrugations

[Singh, Diamond PPCF 2021]

$$\left(\frac{\partial}{\partial t} + 2D_n k^2\right)\left\langle \left|n_k\right|^2 \right\rangle + 2\zeta_{1k}\left\langle \left|n_k\right|^2 \right\rangle + \Re\left[2\zeta_{2k}\left\langle n_k^\star \phi_k \right\rangle\right] = F_{nk}$$



Corrugations damping rate, +ve $\sim 1/\alpha_q^2$

Cross transfer rate, +ve $\sim 1/\alpha_q^2$

Advection noise, +ve $\sim 1/\alpha_q^2$

- Density cascade forward in k_x !
- Corrugations become weaker as the response become more adiabatic.
- Corrugation is determined by noise vs diffusion balance.
- Important for the nonlinear dynamics underlying staircases. Forward cascade in k_x-space is supporting the idea of (inhomogeneous) mixing in real space.

Zonal cross-correlation(ZCC)

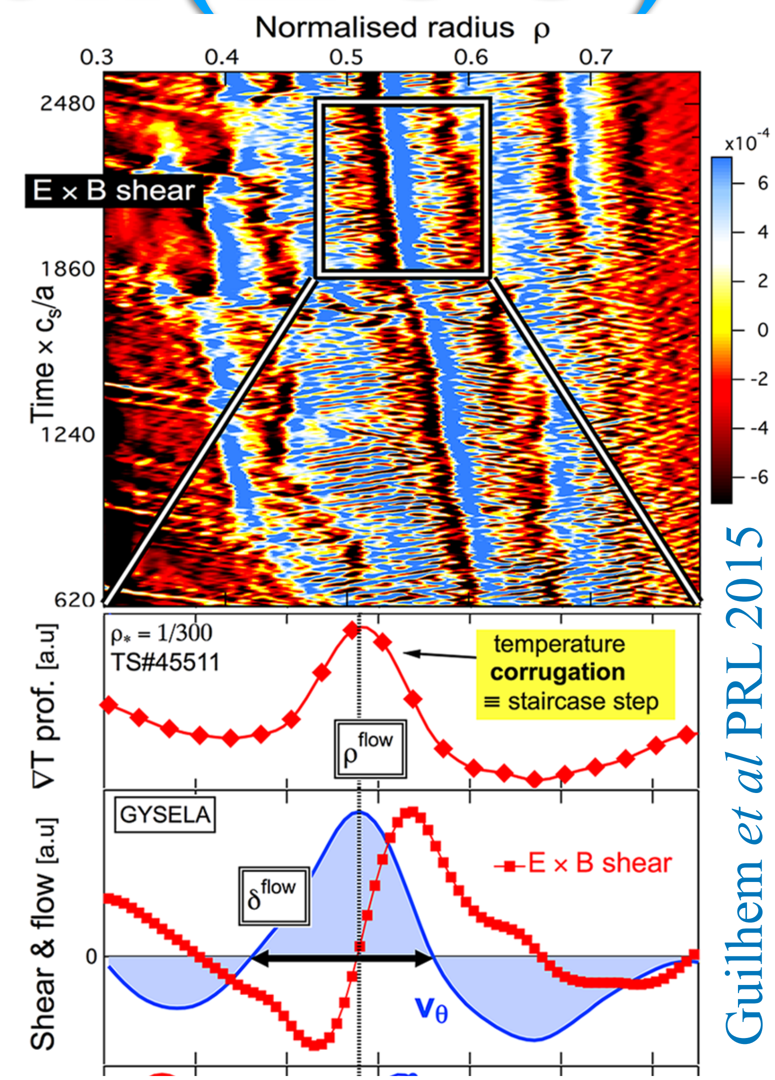
[Singh, Diamond PPCF 2021]

- Significant for layering or staircase structure - ZF and ∇T are aligned in staircase!

- When do zonal density and zonal potential align?

$$\Re\left\langle n_k \phi_k^\star \right\rangle = \frac{2\eta_{2k}^{(r)} \left\langle \left| n_k \right|^2 \right\rangle + 2\zeta_{2k}^{(r)} \left\langle \left| \phi_k \right|^2 \right\rangle}{-\left(\mu + D_n\right) k_x^2 - 2\xi_{1k}^{(r)}}; \quad \xi_{1k}^{(r)} = \eta_{1k} + \zeta_{1k}$$

- Zonal density and potential are correlated (anti-correlated) when the modulational growth of zonal flow more (less) than modulational damping of corrugations.



Guilhem *et al* PRL 2015

- Imposing physically bounded solution for $\Im\left\langle n_k \phi_k^\star \right\rangle = 0$ fixes the sign of $\left(\mu + D_n\right) k_x^2 + 2\xi_{1k}^{(r)} > 0$. Thus $\Re\left\langle n_k \phi_k^\star \right\rangle < 0$.

- Hence ZCCs in real space are: $\left\langle \overline{n}\,\overline{\phi} \right\rangle < 0$, $\left\langle \overline{n}\, \nabla_x^2 \overline{\phi} \right\rangle > 0$, $\left\langle \nabla_x \overline{n}\, \nabla_x^2 \overline{\phi} \right\rangle = 0$

- $\left\langle \nabla_x \overline{n}\, \nabla_x^3 \overline{\phi} \right\rangle > 0$: ***zonal density jumps are co-located with the zonal vorticity jumps.***

- $\left\langle -\nabla_x \overline{n}\, \nabla_x \overline{\phi} \right\rangle > 0$: ***density gradient peaks are co-located with the zonal flow peaks.***

Summary of zonal flow and corrugations interaction

Process	Impact	Key physics
(a) Zonal flow - Vorticity equation - Polarization charge flux		
Polarization noise	Seeds zonal flow	Polarization flux correlation, +ve definite
Zonal flow response (comparable to noise)	Drives zonal shear using DW energy	Non-local inverse transfer in k_x , -ve viscosity
Zonal shear straining of small scale	Regulates waves via straining	Stochastic refraction straining waves, induced diffusion to high k_x
(b) Density corrugations - Density equation - Particle flux		
Density advection beat noise	Seeds density corrugation	Advection beats due to non-adiabatic electrons.
Density corrugations response	Damps and regulates density corrugations	Non-local forward transfer in k_x +ve diffusivity, turbulent mixing weak for $\alpha >> 1$
Zonal shear straining of small scale	Regulates waves via straining	Stochastic refraction straining waves, induced diffusion to high k_x
(c) Zonal cross-correlation - Vorticity and density transport processes		
ZCC response	Sets corrugation - shear layer correlation; staircase states	Growth of zonal intensity must exceed the modulational damping rate of corrugation

Feedback loop with nonlinear zonal noise

[Singh, Diamond PPCF 2021]

Feedback + Noise → revisit predator - prey

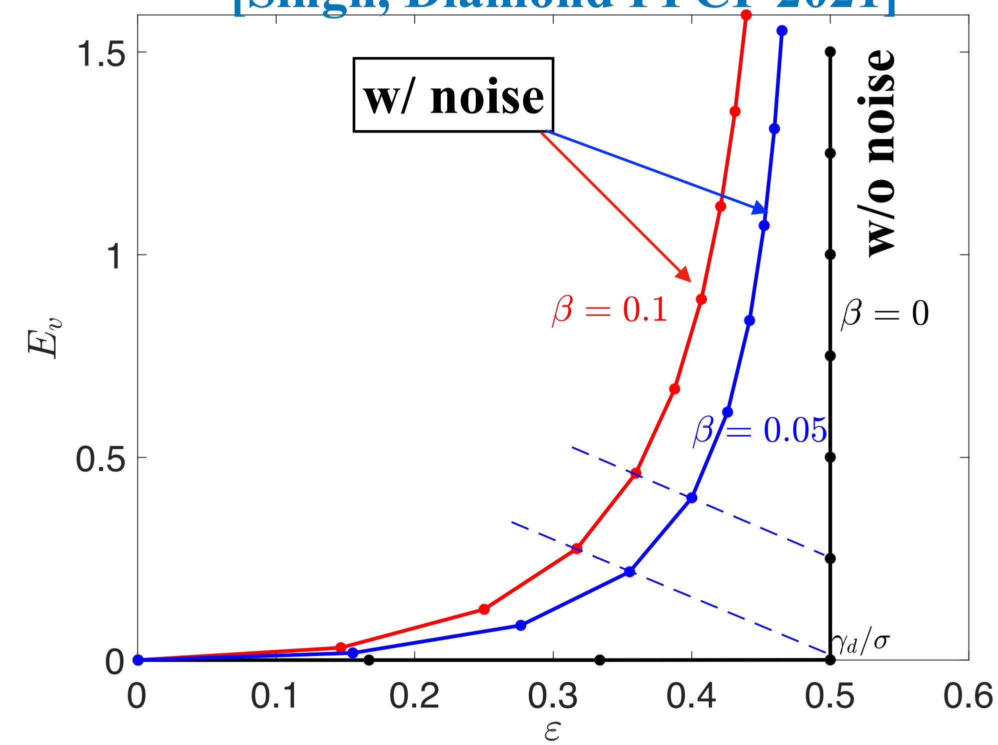
Induced diffusion

Turbulence energy $\mathcal{E}$: $$\frac{\partial \mathcal{E}}{\partial t} = \gamma\mathcal{E} - \sigma E_v \mathcal{E} - \eta \mathcal{E}^2$$
(Prey)

Nonlinear damping

Zonal flow energy E_v: $$\frac{\partial E_v}{\partial t} = \sigma \mathcal{E} E_v - \gamma_d E_v + \beta \mathcal{E}^2$$
(Predator)

Modulational growth



<u>Without noise:</u>

- Threshold in growth rate $\gamma > \eta\gamma_d/\sigma$ for appearance of stable zonal flows.
- Turbulence energy increases as γ/η below the threshold, until it locks at γ_d/σ, at the threshold.
- Beyond the threshold, turbulence energy remains locked at $\frac{\gamma_d}{\sigma}$ while the zonal flow energy continues to grow as $\sigma^{-1}\eta\left(\gamma/\eta - \gamma_d/\sigma\right)$.

<u>With noise:</u>

- Both zonal and turbulence <u>co-exist at any growth rate</u> - No threshold in growth rate for zonal flow excitation.
- Turbulence energy never hits the old modulational instability threshold, absent noise!
- Turbulence energy ↓ and zonal flow energy ↑:- Noise feeds energy into zonal flow!

L - H transition

How does zonal noise affect the dynamics of L-H transition ?

KD03 + Noise

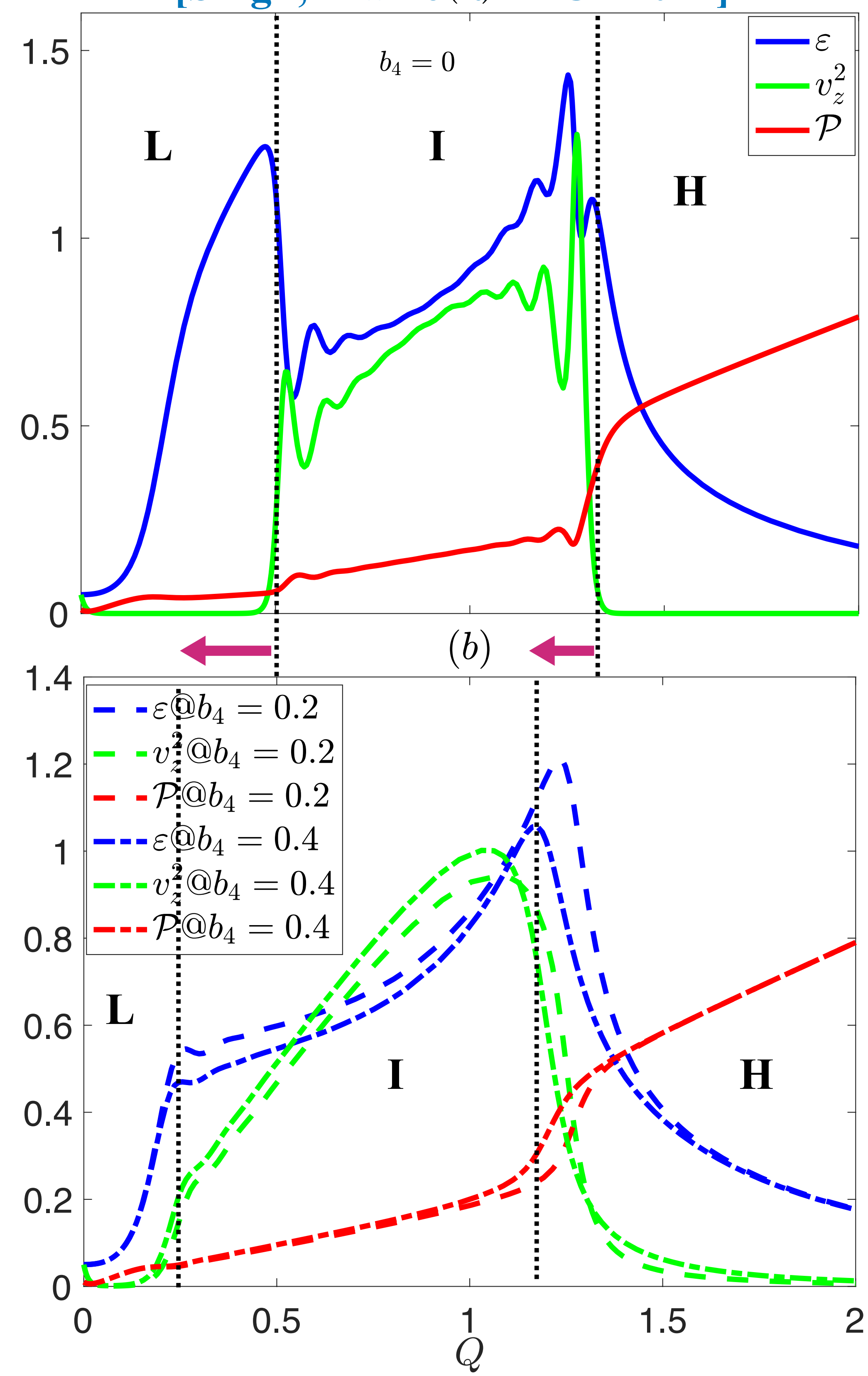
Without noise:

- Critical power, set by modulational instability threshold.
- Zonal flows exist only within the I-phase.

With Noise: No ZF threshold in Q!

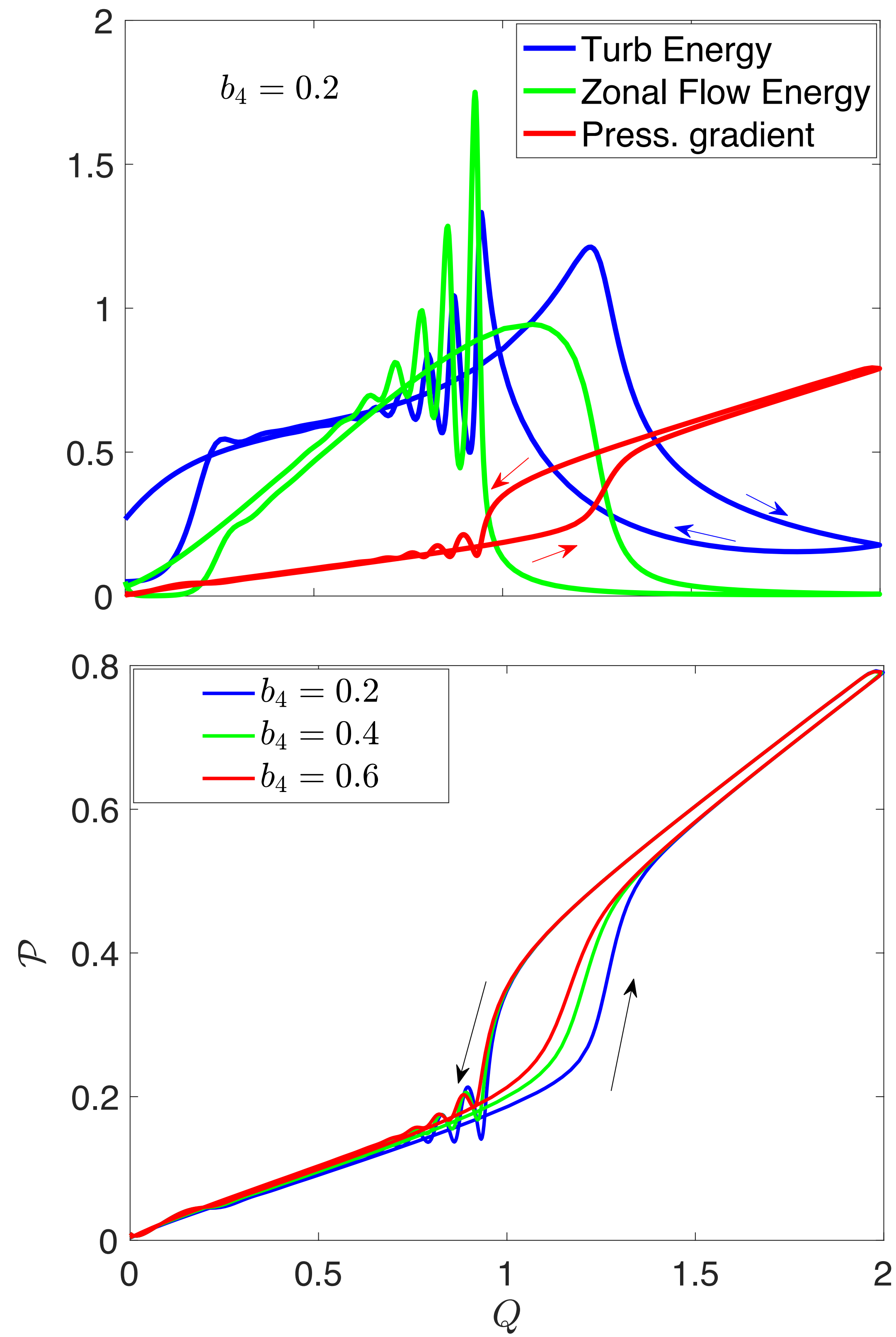
- Turbulence level ↓, no overshoot, zonal flow ↑.
- I-Phase oscillations ↓.
- H-mode power threshold ↓. Noise couples more fluctuation energy to zonal flows.

[Singh, Diamond PPCF 2021]



Noise effect on L-H-L hysteresis

- A cyclic power ramp exhibits hysteresis.
- The I-phase in the back transition is more oscillatory than that in the forward transition.
- Hysteresis with noise is robust w.r.t the variations in the initial conditions.
- Threshold power for both forward and backward transition decreases with noise such that the area enclosed the hysteresis curve decreases with noise.



Conclusions I

- Unified theory of zonal modes (flows and corrugations) encompassing both noise and modulations.

  - Vorticity flux corr.→ ZF noise. density flux corr.→ corrugation noise.

  - Bi-directional transfer: KE energy to large scales, internal energy to small scales. Turbulent viscosity -ve but turbulent diffusivity +ve.

  - The effective zonal viscosity goes negative only for an energy spectrum which decays sufficiently rapidly in k_r i.e., $\partial E/\partial k_r < 0$ and $\left|\partial E/\partial k_r\right| < \left|\partial E/\partial k_r\right|_{crit}$.

  - Zonal cross-correlation $\Re\left\langle n_k \phi_k^\star \right\rangle < 0 \rightarrow \left\langle \nabla \bar{n} \nabla^3 \overline{\phi} \right\rangle > 0$ i.e., zonal density gradient jumps are colocated with zonal vorticity jumps.

Conclusions II

Implications:

- Polarization beat noise and modulational effects are comparable intrinsically (both driven by Reynolds stress!). The synergy of the two mechanisms is stronger than either alone.
  - Expands the range of zonal flow activity relative to that predicted by modulational instability calculations.
  - Increases branching ratio of zonal flow energy to turbulence energy.
- Interaction of zonal noise and modulation: → significant effect on feedback processes !
  - L-H transition: Noise eliminates the threshold for zonal flow excitation, and so expands the predicted range of the intermediate phase, drastically reduces the turbulence overshoot.
  - The energy transfer to zonal flow is accelerated which lowers the threshold for L-H transition.

Future directions

- Understanding interaction of corrugations with avalanches:
  - Corrugations in state of high *ZCC* sustained as localized transport barriers, staircases etc. localized by accompanying shear flow?
  - Corrugations in state of low *ZCC* likely to overturn, and drive avalanches, as in running sandpile?
  - Relevant for TEM turbulence. Does the density gradient state consist of standing corrugations , running avalanches or mixtures thereof ?
- Effect of noise on staircase? (have been considered only in context of Mean Field theory).
- Relation between *ZCC* and the staircase structure: Does the physics of *ZCC* set the relative positions of corrugations and shear layer? Is there a single *ZCC* for staircase state ? Or a band ?
- GFD: Are zonal wind and temperature profile correlated? ZCC and weather pattern?