The role of coherent structures in the dynamics of edge turbulence

Mingyun Cao, P.H. Diamond

Department of Physics, University of California, San Diego

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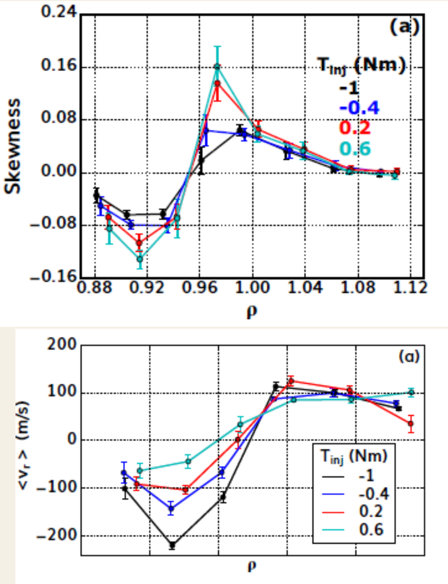
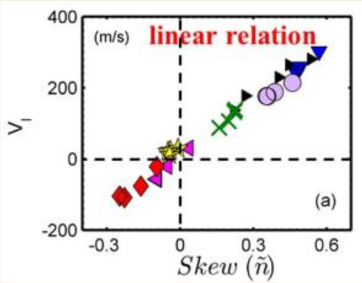
Contents

Motivation

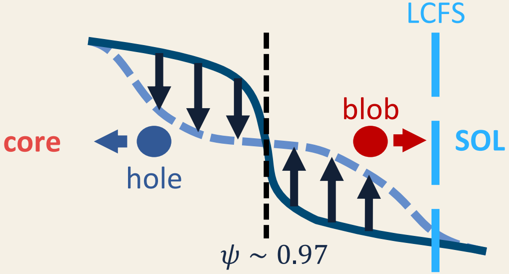
- Most studies of coherent structures focused on blobs (technical difficulties in hole diagnostics).
- From avalanche theory: when there is a blob, there must be a hole ⇒ half the story is missed.



Filipp et al. on DIII-D (BES)[1]



Ting et al. on J-TEXT (probe)[2]



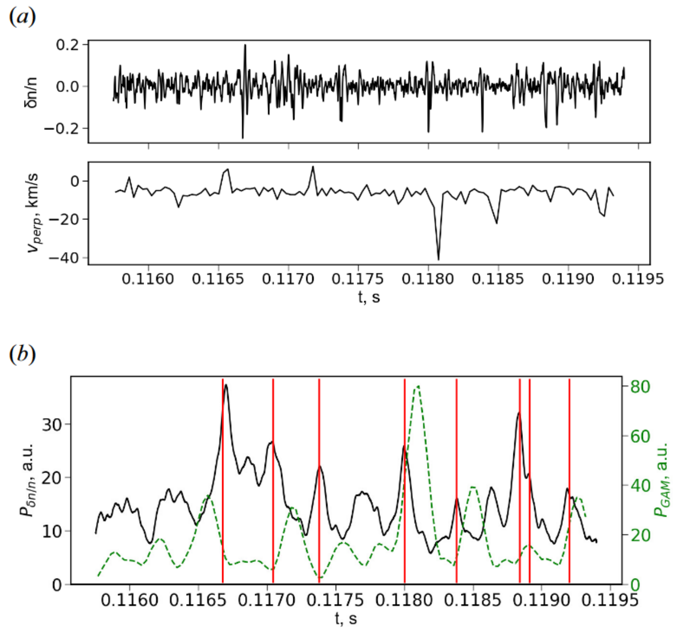
Blobs move outward, cross LCFS
holes move inward, stay in the main plasma

- Lessons learned from BES: blobs and holes are created **in pairs** from edge gradient relaxation events (GREs) **close to LCFS**.
- Hole moves inward and energizes the edge plasma ⇒ the idea of “edge influencing core” can be traced to Kadomtsev in 1992.[3]

1. F. Khabanov et al., 2024, NF.
2. T. Long et al., 2024, NF.
3. B.B. Kadomtsev, 1992, PPCF.

Motivation

- Holes move inward and thus affect the edge dynamics. But in which way?



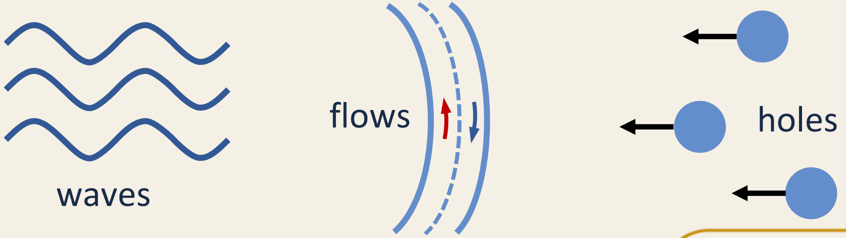
Alsu et al. on MAST (BES)[1]

- Bursts of zonal flow power follow bursts of density fluctuation power due to the propagation of density holes.

  ⇒ coherent structures (holes) can drive zonal flow.

- Message: inward moving density holes are important components of edge turbulence.

  ⇒ **Need a model to probe into the other half of the story, i.e., figuring out the role hole plays.**



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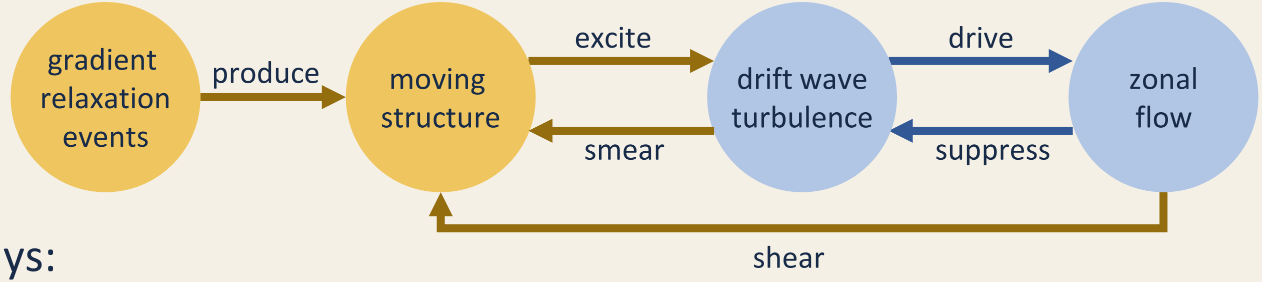
1. A. Sladkomedova et al., 2024, JPP.

Motivation & Preview

- Questions we aim to address:
  1. mechanism and shearing rate of structure-driven flow?
  2. how (ambient) turbulence and flow affect the structure?
  3. effect of coherent structures on the edge transport?



- Takeaways:
  - A moving coherent structure (hole) can excite drift wave turbulence, and hence drive zonal flow.
  - The shearing rate of the flow driven by the structure is comparable with that of the ambient shear.
  - The ambient turbulence and shear flow can smear the coherent structures, and thus constrain the structure lifetime.

Model Development

- Three incentives for the model

- A moving charge can emit electromagnetic wave → a moving hole can be treated as a macro particle emitting waves.

Association

- Radial propagation speed of holes is comparable to the diamagnetic drift velocity in the strong spreading scenario.[1]

Observation

- Connect to previous theoretical models of coherent structure, e.g., two-field model for structure convection.[2]

Demand

- Inference: a hole moving in a background plasma can excite drift waves ⇒ start from Hasegawa-Wakatani model.

1. T. Long et al., 2024, NF.
2. O.E. Garcia et al., 2005, PoP.

Model Development

- Hasegawa-Wakatani model (with curvature drive):

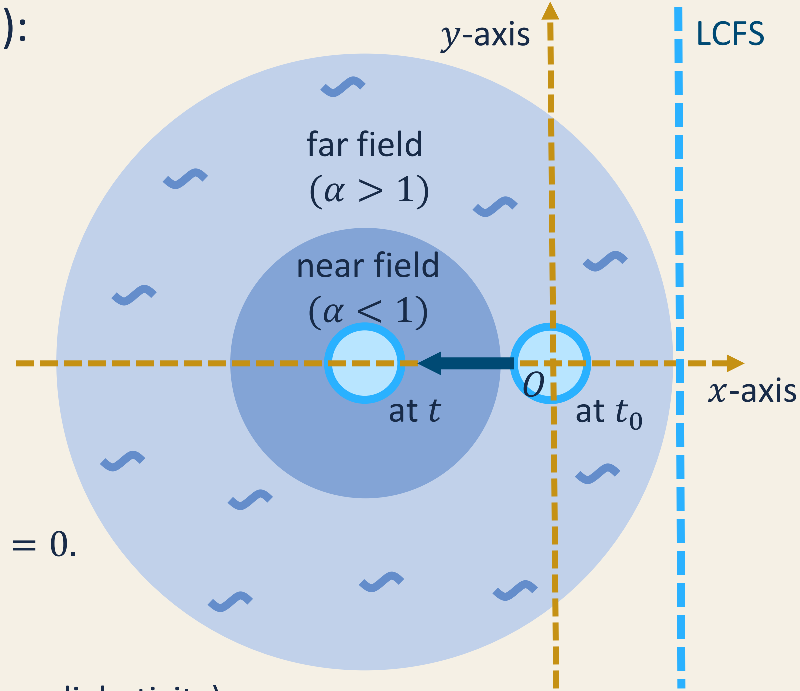
$$\frac{d}{dt}\nabla_\perp^2\varphi + \frac{2\rho_s}{R_c}\frac{1}{n_0}\frac{\partial n}{\partial y} = D_\parallel\nabla_\parallel^2\left(\frac{n}{n_0} - \varphi\right)$$

$$\frac{1}{n_0}\frac{dn}{dt} = D_\parallel\nabla_\parallel^2\left(\frac{n}{n_0} - \varphi\right)$$

- Divide the whole space into two parts:
  - Near field regime: close to the structure, $\alpha < 1$
    ($\alpha > 1$ → no density mixing → no structure formation)

    ⇒ Two-field model[1]: $\frac{d}{dt}\nabla_\perp^2\varphi + \frac{2\rho_s}{R_c}\frac{1}{n_0}\frac{\partial n}{\partial y} = 0, \qquad \frac{1}{n_0}\frac{dn}{dt} = 0.$
  - Far field regime: far away from the structure, $\alpha > 1$

    ⇒ Hasegawa-Mima equation: $\frac{d}{dt}\nabla_\perp^2\varphi - \frac{1}{n_0}\frac{dn}{dt} = 0$ (α: adiabaticity)



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1. O.E. Garcia et al., 2005, PoP.

Model Development

- Focus on the far field regime ($\alpha > 1$)
- Coherent structure (hole) enters the model via profile modulation ($n = n_0 + n_h + \tilde{n}$):

$$\frac{d}{dt}\left(\nabla_\perp^2\varphi - \varphi\right) - v_*\frac{\partial\varphi}{\partial y} = \boxed{\frac{1}{n_0}\frac{dn_h}{dt}} \longrightarrow \text{source} \qquad n_h = 2\pi n_0 h \Delta x \Delta y \delta(x + u_x t)\delta\big(y - u_y t\big)H(t)H(\tau_h - t)$$

h: magnitude; Δx, Δy: spatial extent;
u_x, u_y: convection speed; τ_h: lifetime

- The workflow of the calculation procedure:

Get the Green's func of the linearized H-M eqn and then solve φ of the far field

Calculate the Reynolds stress and then the shearing rate of the structure-driven flow

Compare the shearing rate of the structure-driven flow to that of the ambient flow

- The linearization of the H-M eqn (i.e., bare propagator) is strictly valid in the $Ku < 1$ regime. For $Ku \gtrsim 1$, recall the problem of wave propagating in a random medium $\Rightarrow$ renormalized propagator.

Results

- Two challenges:
  - The Green's function is complicated:

$$G = -\int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} \exp\left(s\tau + \frac{v_*\chi}{2s}\right) \frac{1}{2\pi s} \mathrm{K}_0\left[\left(1 + \left(\frac{v_*}{2s}\right)^2\right)^{1/2} \rho\right].$$

  - In reality, holes move in both poloidal and radial directions.
- Solution: consider three limiting cases:

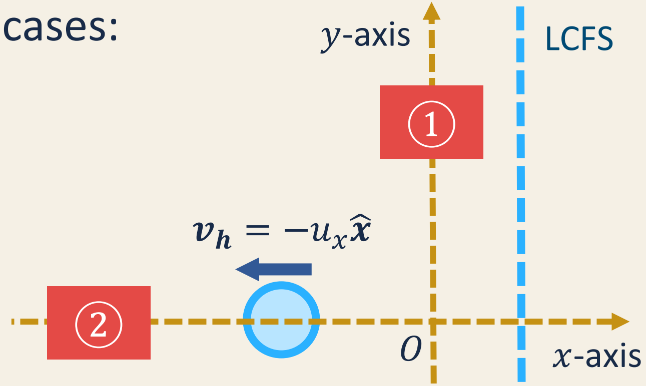
  a) Radially moving hole $(u_y = 0)$:

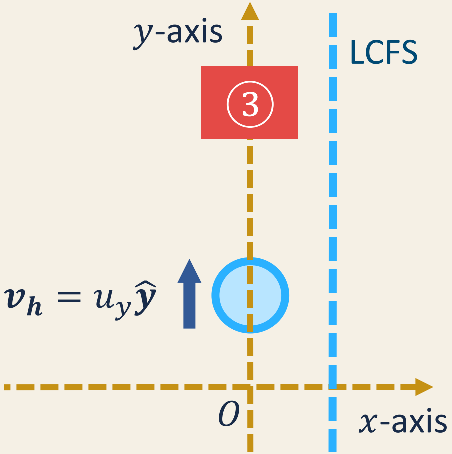
    1) away from the x-axis ($|y| \gg |x|$)

    2) near x-axis ($|x| \gg |y|$)

  b) Poloidally moving hole $(u_x = 0)$:

    3) near y-axis ($|y| \gg |x|$)





- Poloidally moving structure⇒indication of structure-wave resonance.

Results

- Spatial-temporal orderings and shearing rates of the flow in these three cases are:

Case	Spatial-temporal ordering	ω_s^h/ω_s^a	If $v_F^a \sim v_*$, $\Delta_F^a \sim 10\rho_s$
$\boldsymbol{v_h} = -u_x\hat{\boldsymbol{x}}$ away from x-axis	$x' \sim d_{pe}(= u_x\tau_h) \sim \Delta x \sim \Delta y \sim x \ll y$ $1/\omega_* \ll t' \sim \tau_h \ll t$	$\frac{\omega_s^h}{\omega_s^a} \sim \left(\frac{h\Delta x\Delta y}{v_* u_x \tau_h a}\right)^2 \frac{\Delta_F^a}{v_F^a/v_*}$	$\frac{\omega_s^h}{\omega_s^a} \sim 10h^2$
$\boldsymbol{v_h} = -u_x\hat{\boldsymbol{x}}$ near x-axis	$y', y \to 0 \ll x' \sim d_{pe} \sim \Delta x \sim \Delta y \ll x$ $1/\omega_* \ll t' \sim \tau_h \ll t$	$\frac{\omega_s^h}{\omega_s^a} \sim \left(\frac{h\Delta x\Delta y}{v_* u_x \tau_h}\right)^2 \frac{2\ln(a/v_*)\,\Delta_F^a}{x^3 v_F^a/v_*}$	$\frac{\omega_s^h}{\omega_s^a} \sim (10h)^2 \ \left(\frac{x}{\rho_s} \sim 10^2\right)$
$\boldsymbol{v_h} = u_y\hat{\boldsymbol{y}}$ near y-axis	$\frac{v_* t}{1+k^2} \gg y \gg \left\lvert\left(u_y - \frac{v_*}{1+k^2}\right)\tau_h\right\rvert$ $\left\lvert ky - \frac{k v_* t}{1+k^2}\right\rvert \gg k^2x^2,\ 1/\omega_* \ll t' \sim \tau_h \ll t$	$\frac{\omega_s^h}{\omega_s^a} \sim \frac{\pi(1+k_0^2)}{4k_0}\left(\frac{h\Delta x\Delta y}{v_* u_y \tau_h}\right)^2 \frac{x}{a^3}\frac{\Delta_F^a}{v_F^a/v_*}$	$\frac{\omega_s^h}{\omega_s^a} \sim h^2 \ \left(\frac{x}{\rho_s} \sim 10, k_0 = 1\right)$

- As $h = n_h/n_0 \in (0.1,1)$, in all cases, ω_s^h is comparable with ω_s^a (or even larger).

x', y', t': integration coordinates; x, y, t: far-field coordinates; v_F^a: ambient flow velocity; Δ_F^a: ambient flow width
a: minor radius; ω_s^h: shearing rate of the structure-driven flow; ω_s^a: shearing rate of the ambient flow
In dimensionless form: $v_*/c_s \sim u_x/c_s \sim u_y/c_s \sim 10^{-2}$, $a/\rho_s \sim 10^3$, $\omega_{ci}\tau_h \sim 10^3$;

Results

- What are the effects of turbulence and flow on the structure?

  ⇒Turbulence & flow can smear the structure, thus constraining the structure lifetime.

- Consider a diffusion model:

  $$\partial_t n_h = D\nabla_\perp^2 n_h$$

  When h decays by half, structure is vanished ⇒

  $$\tau_h = 2\Delta x^2/D$$

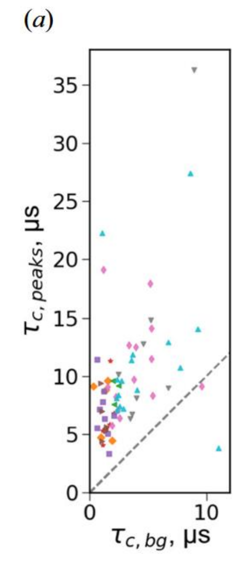
  If $\rho_* = \rho_s/L_n \sim .01,\ \omega_s^a/\omega_* \sim \rho_*^{1/2},\ \omega_*/\omega_{ci} \sim \rho_*,$ $l_{mix} = L_n\rho_*^{\delta}$, then

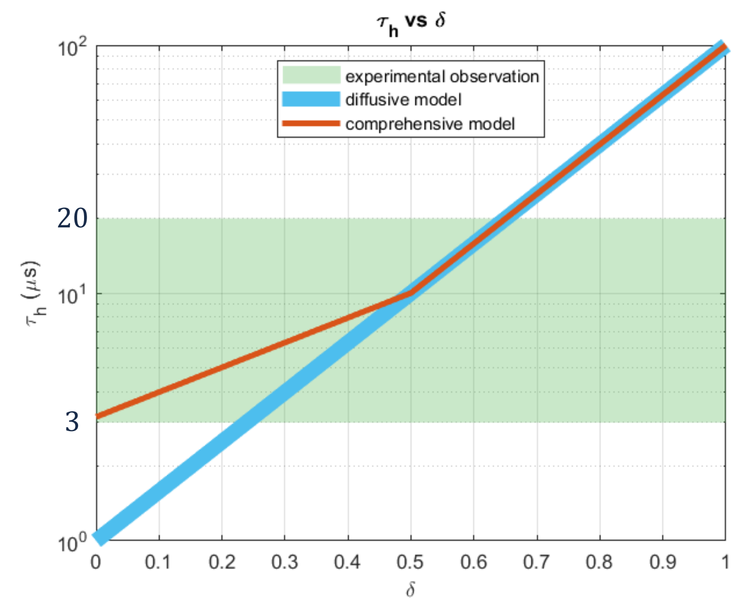
- Purely diffusive regime ($\omega_s^a < Dk_\perp^2$ or $\frac{1}{2} < \delta < 1$)

  $$D/D_B = \rho_*^{\delta}, \qquad \tau_h \propto \rho_*^{-\delta}.$$

- Shearing dominant regime ($\omega_s^a > Dk_\perp^2$ or $0 < \delta < \frac{1}{2}$)

  $$D/D_B \sim \rho_*^{(1+2\delta)/4}, \qquad \tau_h \propto \rho_*^{-(1+2\delta)/4}.$$





- This simple estimate brackets the experiment observation[1] of the structure lifetime (correlation time) reasonably well.

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1. A. Sladkomedova et al., 2024, JPP.

Conclusion & Future

- Hole and blob are generated in pairs from GRE close to LCFS. Ways in which holes affect the edge dynamics are:
  - The inward moving hole can excite drift wave turbulence and thus drive zonal flow.
    - This may account for the "anomalous" turbulence level in no man's land, and hence address the "shortfall problem".
    - The zonal flow driven by coherent structures may modify the edge shear.
  - Ambient turbulence and zonal flow can smear the coherent structure and thus constrain its lifetime.
    - The estimate of the lifetime based on a simple diffusion model can fit the experimental observation quite well.
- We suggest several possible directions for future research:
  - **For theories:**
    1. the net effect of holes on edge transport? Need an estimate of the magnitude of the turbulence excited by holes.
    2. an estimate of the contribution of holes to the turbulence level in no man's land. Comparison with local production?
    3. constrain the upper limit of the hole lifetime further. Holes lose energy as emitting waves → decay faster than the diffusion model predicted. The reaction force exerted on the hole by the turbulence field?
  - **For experiments:** observe the correlation between the frequency of GREs and the turbulence level in no man's land.
  - **For simulations:** include GREs into edge turbulence simulations, as inward moving hole energizing the edge.

Thank you!