Coherent structures in edge dynamics: how the tail wags the dog

Mingyun Cao, P.H. Diamond

Department of Physics, University of California, San Diego

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Contents

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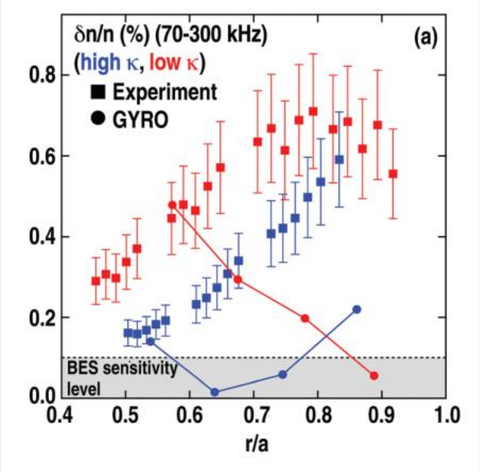
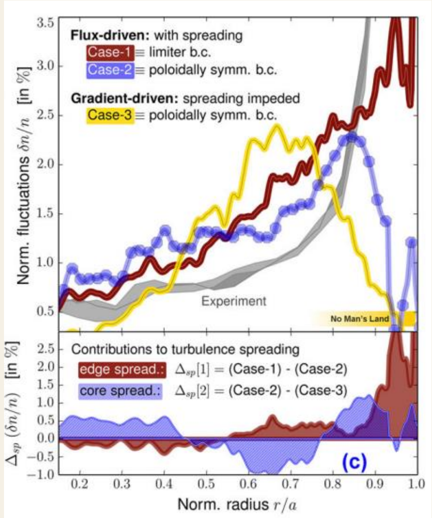
Motivation 1: shortfall problem

- The fluctuation level predicted by **local** gyrokinetic simulation is lower than experimental observation in edge-core coupling region (no man's land). (C. Holland, 2011)
- No shortfall when edge is destabilized & turbulence spreading is enabled. (G. Dif-Pradalier, 2022)





- The story of "the tail wagging the dog" has a long history:

"… And, finally, we have a very strong activity at the plasma edge. It controls the transition from one mode of confinement to another and its influence extends well into the bulk plasma… " —B.B. Kadomtsev, 1992

- But, no concrete physical picture of inward spreading of edge turbulence.

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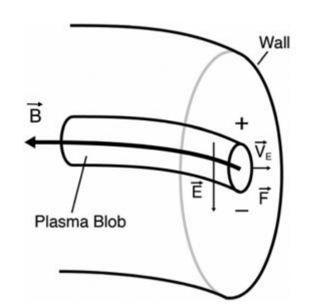
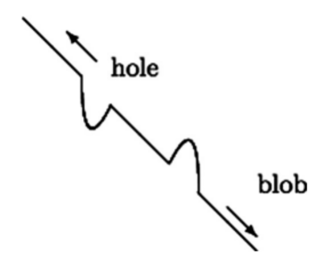
Motivation 1: shortfall problem

- Propagating pulses (avalanche) is a specific form of turbulence spreading.
- Example in MFE: the cross-field transport of blobs in the scrape-off layer.
  - Theory of blob motion is well-established [1] ⇒ a physical picture of outward spreading.
- In avalanche theory:
  - Particle conservation[2] ⇒ when there is a blob, there must be a hole.
  - Joint reflection symmetry[3] ⇒ hole moves inward while blob moves outward.
- There are millions of papers on blobs, but very little attention on holes.
  - Bursts of heat carried by blobs can destroy the plasma facing components.
  - You cannot stick the probes too deep into the main plasma.
- It doesn't mean holes are less important.
  - Is inward turbulence spreading mediated by holes?





1. D.A. D'Ippolito et al., 2011.
2. J.R. Myra et al., 2018.
3. P.H. Diamond, T.S. Hahm, 1995.

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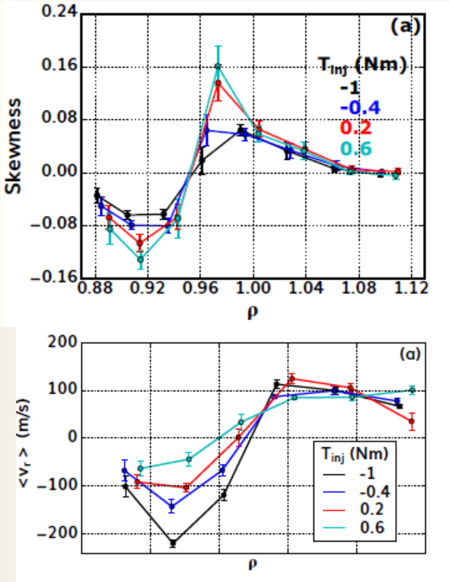
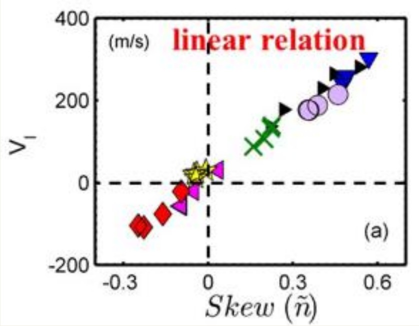
Motivation 2: recent experiments

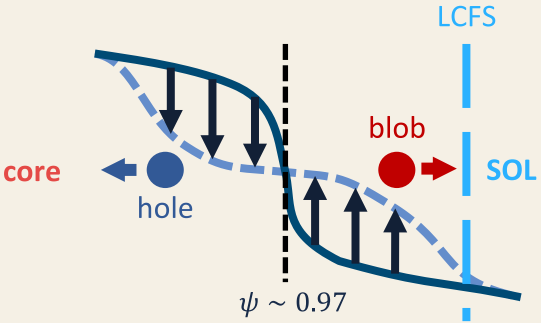
- Now the situation has been changed.
  - The development of beam emission spectroscopy (BES) provides a channel for us to study density holes.
  - Probes are also applicable on devices operating in lower temperature.



Filipp et al. on DIII-D (BES)[1]



Ting et al. on J-TEXT (probe)[2]



Blobs move outward, cross LCFS
holes move inward, stay in the main plasma

Lessons learned:

1. Turbulence spreading in edge plasma is non-diffusive. (See Ting's topical plenary talk in the CD-8 session)
2. Blobs and holes are created **in pairs** from edge gradient relaxation events (GREs) **close to LCFS**.

1. F. Khabanov et al., 2024, NF.
2. T. Long et al., 2024, NF.

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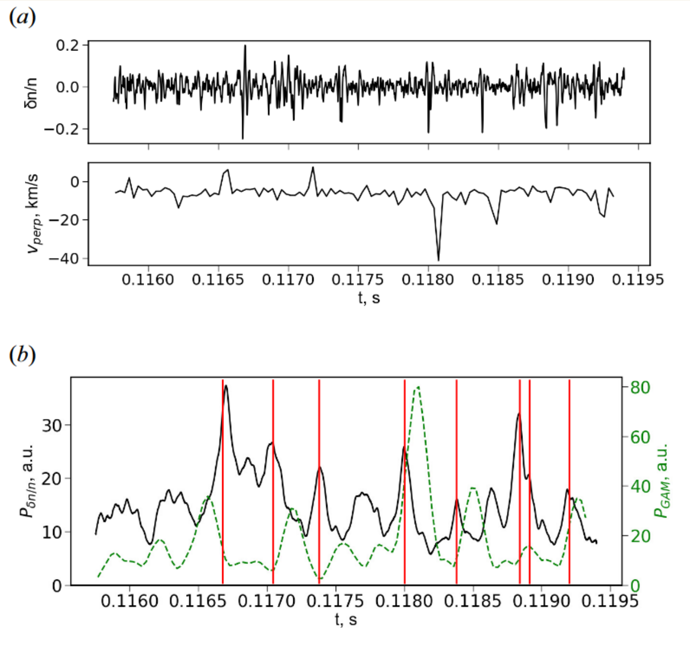
Motivation 2: recent experiments

- More evidence on how an inward moving hole affects boundary dynamics



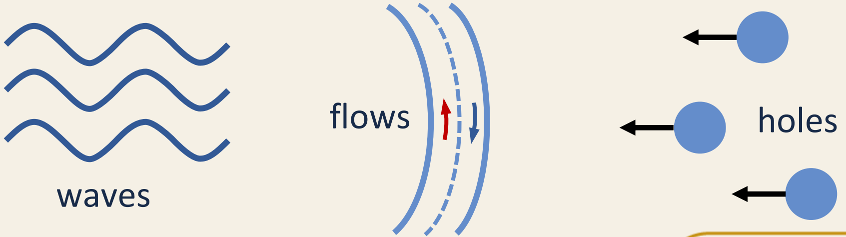
Alsu et al. on MAST (BES)[1]

- Recall Alsu's talk: bursts of zonal flow power follow bursts of density fluctuation power due to the propagation of density holes.

  ⇒ coherent structures (holes) can drive zonal flow.

- Message: inward moving density holes are important components of edge turbulence.

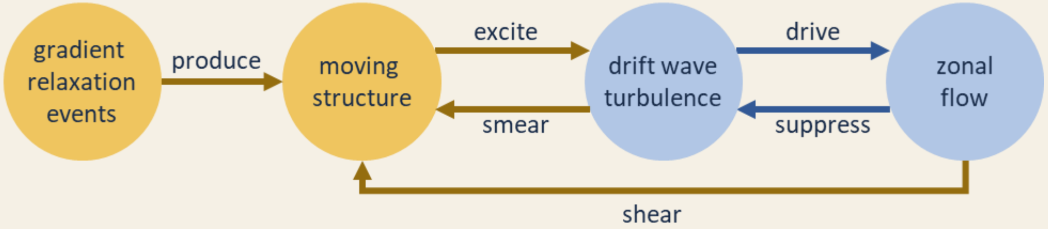
  ⇒ **Need a model to probe into the other half of the story, i.e., figuring out the role hole plays in edge dynamics.**



1. A. Sladkomedova et al., 2024, JPP.

Motivation & Preview

- Questions we aim to address:
  1. **mechanism and shearing rate of hole-driven flow?**
  2. **back-reaction of (ambient) turbulence and flow on the hole?**
  3. **contribution of the hole to the turbulence level in no man's land?**



- Takeaways:
  - **A moving hole can excite drift wave turbulence, and hence drive zonal flow.**
  - **The shearing rate of the flow driven by the hole $\gtrsim$ the ambient shearing rate.**
  - **The (ambient) turbulence and shear flow can smear the hole, and thus constrain the hole lifetime.**
  - **The inward turbulence intensity flux induced by holes and the width of no man's land are estimated.**

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Model Development

- Three incentives for the model

Association

- A moving charge can emit electromagnetic wave → a moving hole can be treated as a macro particle emitting waves.

Observation

- Radial propagation speed of holes is comparable to the diamagnetic drift velocity in the strong spreading scenario.[1]

Demand

- Connect to previous theoretical models of coherent structure, e.g., two-field model for structure convection.[2]

- Inference: a hole moving in a background plasma can excite drift waves ⇒ start from Hasegawa-Wakatani model.

1. T. Long et al., 2024, NF.
2. O.E. Garcia et al., 2005, PoP.

Model Development

- Hasegawa-Wakatani model (with curvature drive):

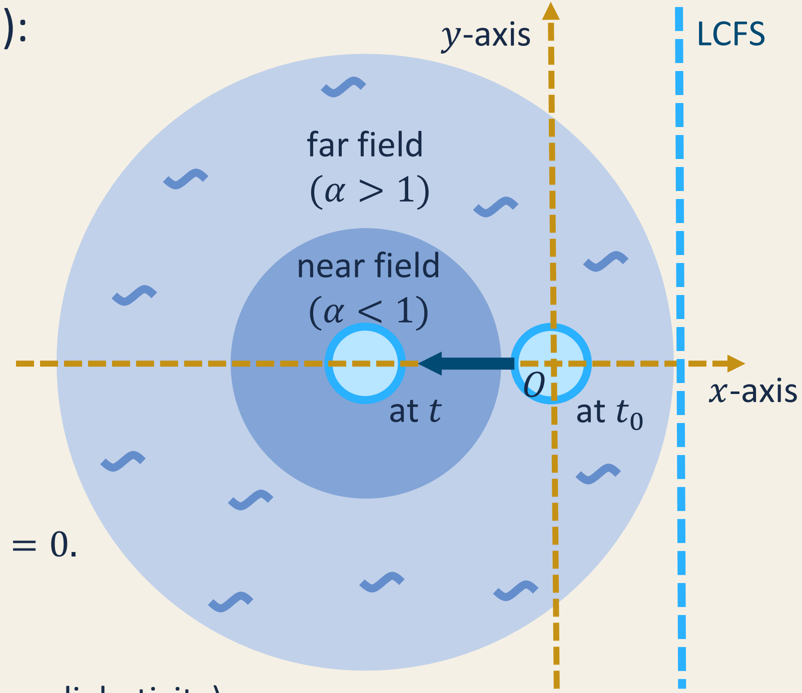
$$\frac{d}{dt}\nabla_\perp^2\varphi + \frac{2\rho_s}{R_c}\frac{1}{n_0}\frac{\partial n}{\partial y} = D_\parallel \nabla_\parallel^2 \left(\frac{n}{n_0} - \varphi\right)$$

$$\frac{1}{n_0}\frac{dn}{dt} = D_\parallel \nabla_\parallel^2 \left(\frac{n}{n_0} - \varphi\right)$$

- Divide the whole space into two parts:
  - Near field regime: close to the structure, $\alpha < 1$
    ($\alpha > 1 \rightarrow$ no density mixing $\rightarrow$ no structure formation)

    $\Rightarrow$ Two-field model[1]: $\frac{d}{dt}\nabla_\perp^2\varphi + \frac{2\rho_s}{R_c}\frac{1}{n_0}\frac{\partial n}{\partial y} = 0, \qquad \frac{1}{n_0}\frac{dn}{dt} = 0.$
  - Far field regime: far away from the structure, $\alpha > 1$

    $\Rightarrow$ Hasegawa-Mima equation: $\frac{d}{dt}\nabla_\perp^2\varphi - \frac{1}{n_0}\frac{dn}{dt} = 0$ (α: adiabaticity)



1. O.E. Garcia et al., 2005, PoP.

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Model Development

- Target: the turbulence excited by a moving hole $\Rightarrow$ focus on the far field regime ($\alpha > 1$)
- Coherent structure (hole) enters the model via profile modulation ($n = n_0 + n_h + \tilde{n}$):

$$\frac{d}{dt}(\nabla_\perp^2 \varphi - \varphi) - v_* \frac{\partial \varphi}{\partial y} = \boxed{\frac{1}{n_0}\frac{dn_h}{dt}} \rightarrow \text{source} \qquad n_h = 2\pi n_0 h \Delta x \Delta y \delta(x + u_x t)\delta(y - u_y t)H(t)H(\tau_h - t)$$

h: magnitude; Δx, Δy: spatial extent;
u_x, u_y: convection speed; τ_h: lifetime

- The workflow of the calculation procedure:

Get the Green's func of the linearized H-M eqn and then solve φ of the far field → Calculate the Reynolds stress, the shearing rate of the structure-driven flow, the turbulence intensity flux to NML → Compare the shearing rate of the structure-driven flow to that of the ambient flow

- The linearization of the H-M eqn (i.e., bare propagator) is strictly valid in the $Ku < 1$ regime. For $Ku \gtrsim 1$, recall the problem of wave propagating in a random medium $\Rightarrow$ renormalized propagator.

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Results: three limiting cases

- Two challenges:
  - The Green's function is complicated:

$$G = -\int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} \exp\left(s\tau + \frac{v_* \chi}{2s}\right) \frac{1}{2\pi s} \mathrm{K}_0\left[\left(1 + \left(\frac{v_*}{2s}\right)^2\right)^{1/2} \rho\right].$$

  - Holes move in both poloidal and radial directions.
- Solution: consider three limiting cases:

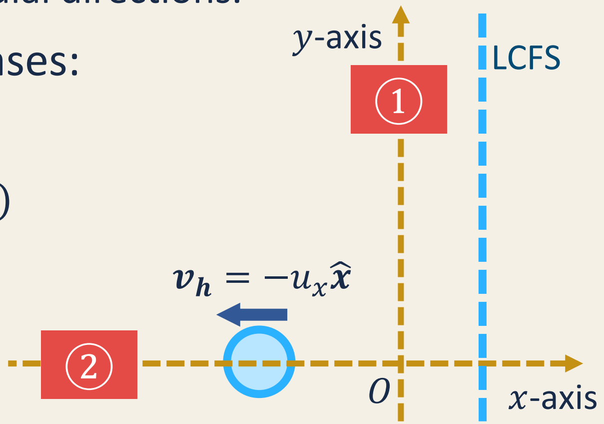
a) Radially moving hole $(u_y = 0)$:

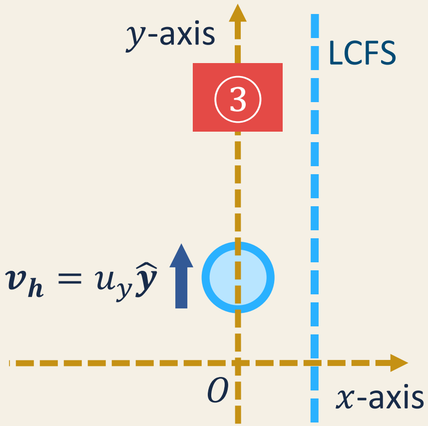
1) away from the x-axis $(|y| \gg |x|)$

2) near x-axis $(|x| \gg |y|)$

b) Poloidally moving hole $(u_x = 0)$:

3) near y-axis $(|y| \gg |x|)$





Results: solutions

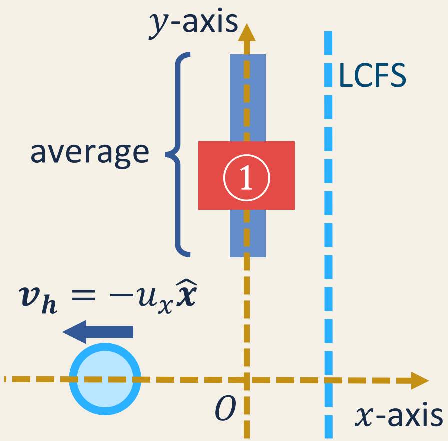
- In the limit of $\tau \to \infty$, the asymptotic form of the Green's function is

$$G \approx -\frac{1}{2\pi}\frac{1}{\sqrt{v_*\rho\tau}}\cos\left[\sqrt{2v_*(\rho-\chi)\tau}\right].$$

$$\tau = t - t'$$
$$\chi = y - y'$$
$$\rho = |\boldsymbol{r} - \boldsymbol{r}'|$$

  - For causality: the influence of the hole should be confined to $\rho \lesssim v_*\tau$.
- Case 1: radially moving hole, away from x-axis
  - Spatial-temporal ordering:

$$x' \sim d_{pe}(= u_x\tau_h) \sim \Delta x \sim \Delta y \sim x \ll y,$$
$$1/\omega_{ci} \ll 1/\omega_* \ll t' \sim \tau_h \ll t.$$



  - Electrostatic potential φ:

$$\varphi = \frac{-2h\Delta x\Delta y}{v_*u_x\tau_h t}\sin\left[\left(\frac{v_*t}{y}\right)^{\frac{1}{2}}\frac{d_{pe}}{2}\right]\cos\left[\left(\frac{v_*t}{y}\right)^{\frac{1}{2}}\left(x+\frac{d_{pe}}{2}\right)\right]$$

  - $\tilde{\boldsymbol{v}} = -\nabla\varphi \times \hat{\boldsymbol{z}} \Rightarrow \omega_s^h = -\int \langle \tilde{v}_x\tilde{v}_y\rangle'' dt$ $\quad$ $\langle\ \rangle$: local poloidal average

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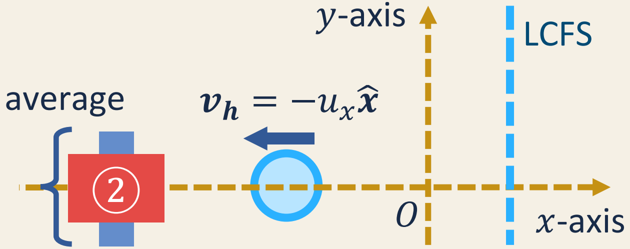
Results: solutions, cont'd

- Case 2: radially moving hole, near x-axis
  - Spatial temporal ordering:

$$y', y \to 0 \ll x' \sim d_{pe} \sim \Delta x \sim \Delta y \ll x$$
$$1/\omega_* \ll t' \sim \tau_h \ll t.$$

  - Electrostatic potential φ:

$$\varphi \approx \frac{\sqrt{2}h\Delta x\Delta y}{v_* u_x \tau_h t} 2\cos\left([-2v_* t(x+y)]^{1/2} + \left[\frac{-v_* t}{2(x+y)}\right]^{1/2} \frac{u_x \tau_h}{2} \right) \sin\left(\left[\frac{-v_* t}{2(x+y)}\right]^{1/2} \frac{u_x \tau_h}{2} \right)$$

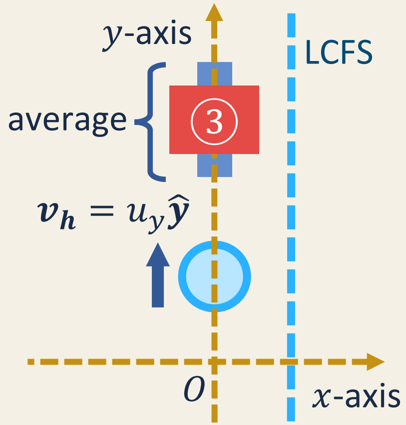


- Case 3: poloidally moving hole, near y-axis
  - spatial-temporal ordering:

$$\frac{v_* t}{1+k^2} \gg y \gg \left| \left(u_y - \frac{v_*}{1+k^2}\right) \tau_h \right|, \left| ky - \frac{k v_* t}{1+k^2} \right| \gg k^2 x^2, 1/\omega_* \ll t' \sim \tau_h \ll t$$

  - Electrostatic potential φ

$$\varphi \approx -\frac{\pi h \Delta x \Delta y}{2k_0 u_y \tau_h} \mathrm{J}_0 \left\{ \left[\left(k_0 y - \frac{k_0 v_* t}{1+k_0^2} \right)^2 + k_0^2 x^2 \right]^{1/2} \right\}$$



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Results: shearing rate of hole-driven flow

- Summary of the spatial-temporal orderings and shearing rates of the flow in these three cases :

Case	Spatial-temporal ordering	ω_s^h/ω_s^a	If $v_F^a \sim v_*$, $\Delta_F^a \sim 10\rho_s$
$\boldsymbol{v_h} = -u_x\hat{\boldsymbol{x}}$ away from x-axis	$x' \sim d_{pe}(= u_x\tau_h) \sim \Delta x \sim \Delta y \sim x \ll y$ $1/\omega_* \ll t' \sim \tau_h \ll t$	$\frac{\omega_s^h}{\omega_s^a} \sim \left(\frac{h\Delta x\Delta y}{v_* u_x \tau_h a}\right)^2 \frac{\Delta_F^a}{v_F^a/v_*}$	$\frac{\omega_s^h}{\omega_s^a} \sim 10h^2$
$\boldsymbol{v_h} = -u_x\hat{\boldsymbol{x}}$ near x-axis	$y', y \to 0 \ll x' \sim d_{pe} \sim \Delta x \sim \Delta y \ll x$ $1/\omega_* \ll t' \sim \tau_h \ll t$	$\frac{\omega_s^h}{\omega_s^a} \sim \left(\frac{h\Delta x\Delta y}{v_* u_x \tau_h}\right)^2 \frac{2\ln(a/v_*)\,\Delta_F^a}{x^3 v_F^a/v_*}$	$\frac{\omega_s^h}{\omega_s^a} \sim (10h)^2 \quad \left(\frac{x}{\rho_s} \sim 10^2\right)$
$\boldsymbol{v_h} = u_y\hat{\boldsymbol{y}}$ near y-axis	$\frac{v_* t}{1+k^2} \gg y \gg \left\lvert\left(u_y - \frac{v_*}{1+k^2}\right)\tau_h\right\rvert$ $\left\lvert ky - \frac{k v_* t}{1+k^2}\right\rvert \gg k^2x^2,\ 1/\omega_* \ll t' \sim \tau_h \ll t$	$\frac{\omega_s^h}{\omega_s^a} \sim \frac{\pi(1+k_0^2)}{4k_0}\left(\frac{h\Delta x\Delta y}{v_* u_y \tau_h}\right)^2 \frac{x}{a^3}\frac{\Delta_F^a}{v_F^a/v_*}$	$\frac{\omega_s^h}{\omega_s^a} \sim h^2 \quad \left(\frac{x}{\rho_s} \sim 10, k_0 = 1\right)$

- As $h = n_h/n_0 \in (0.1,1)$, in all cases, ω_s^h is comparable with ω_s^a (or even larger).

x', y', t': integration coordinates; x, y, t: far-field coordinates; v_F^a: ambient flow velocity; Δ_F^a: ambient flow width
a: minor radius; ω_s^h: shearing rate of the structure-driven flow; ω_s^a: shearing rate of the ambient flow
In dimensionless form: $v_*/c_s \sim u_x/c_s \sim u_y/c_s \sim 10^{-2}$, $a/\rho_s \sim 10^3$, $\omega_{ci}\tau_h \sim 10^3$;

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Results: estimate of hole lifetime

- What are the effects of turbulence and flow on the structure?

  $\Rightarrow$ Turbulence & flow can smear the structure, thus constraining the structure lifetime.

- Consider a diffusion model:

$$\partial_t n_h = D\nabla_\perp^2 n_h$$

- A practical definition of the lifetime of a coherent structure:

  $\Rightarrow$ When h decays by half, structure is vanished $\Rightarrow \tau_h = 2\Delta x^2/D$.

- According to Kubo formalism, when there is no shear (purely diffusive regime):

$$D = \int_0^\infty \langle v(0)v(t)\rangle dt \approx \sum_k \frac{|v_k|^2}{Dk_\perp^2} \Rightarrow D \sim \tilde{v} l_{mix}$$

  Between Bohm and gyro-Bohm, $l_{mix} \sim L_n \rho_*^\delta \quad \delta \in (0,1) \Rightarrow D \sim D_B \rho_*^\delta$.

  when there is shear and $\omega_s > Dk_\perp^2$ (shearing dominant regime):

$$D \sim \tilde{v}^{3/2}\left(k_y \omega_s\right)^{-1/2} = \frac{(c_s \rho_s)^{3/2}\, \rho_*^{\delta/2}}{(\omega_s L_n^2)^{1/2}}$$

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Results: estimate of hole lifetime

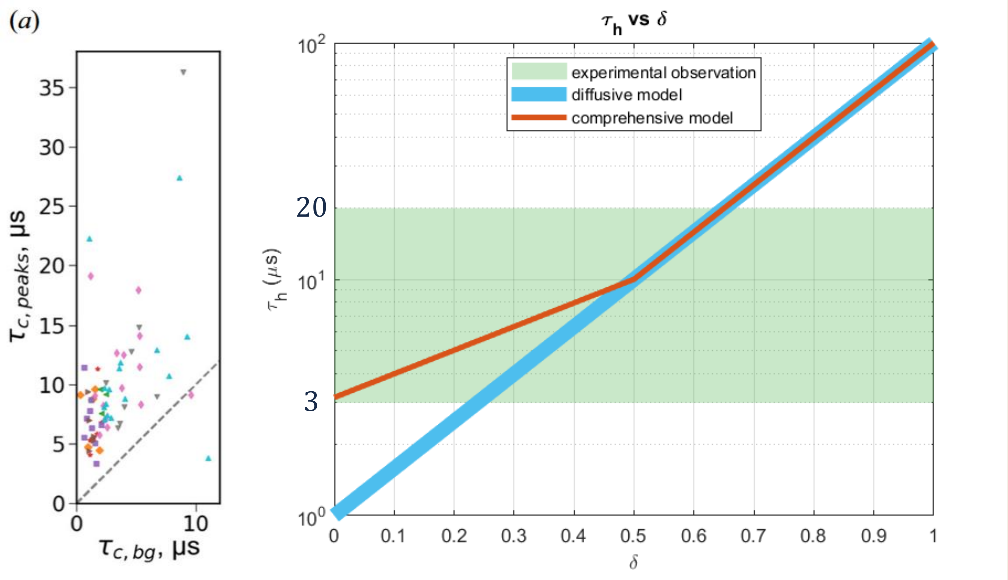
- If $\rho_* = \rho_s/L_n \sim .01,\ \omega_s^a/\omega_* \sim \rho_*^{1/2}$, $\omega_*/\omega_{ci} \sim \rho_*$, then based on $\omega_s/Dk_\perp^2$, D in two different regimes are:
  - Purely diffusive regime

    $(\omega_s^a < Dk_\perp^2$ or $\frac{1}{2} < \delta < 1)$

    $D/D_B = \rho_*^\delta, \qquad \tau_h \propto \rho_*^{-\delta}.$
  - Shearing dominant regime

    $(\omega_s^a > Dk_\perp^2$ or $0 < \delta < \frac{1}{2})$

    $D/D_B \sim \rho_*^{(1+2\delta)/4}, \qquad \tau_h \propto \rho_*^{-(1+2\delta)/4}.$
- Our estimate: $\tau_h \sim 3 - 100\ \mu s$.
- Experiment[1]: $\tau_h \sim 3 - 20\ \mu s$.



- This simple estimate brackets the experiment observation of the hole lifetime (correlation time) reasonably well.

1. A. Sladkomedova et al., 2024, JPP.

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Results: hole-induced turbulence production

- Each hole provides a turbulence intensity bursts $\Delta I \Rightarrow$ the turbulence intensity flux induced by the holes:

$$\Gamma = \sum_{i,j} u_x \Delta I \exp\left[-\frac{(y-il)^2}{2\Delta y^2}\right] \exp\left[-\frac{(t-j\tau_w)^2}{2\tau_h^2}\right] \sim \sum_{i,j} u_x \Delta I 2\pi \Delta y \tau_h \delta(y-il)\delta(t-j\tau_w).$$

$$\langle\bar{\Gamma}\rangle\Big|_{\rho_1} \approx 2\pi \left(\frac{h\Delta x \Delta y}{u_x \tau_h}\right)^2 \frac{1}{v_* \tau_h^2} \frac{N\Delta y}{L_y} \frac{\tau_h}{\tau_w}. \quad \text{(poloidal and time average)}$$

- The budget equation for turbulence intensity:

$$\frac{\partial}{\partial t}\langle \tilde{v}^2 \rangle = -\frac{\partial}{\partial x}\langle \bar{\Gamma} \rangle + \kappa \langle \tilde{v}\tilde{n} \rangle \quad (\kappa\text{: curvature}) \Rightarrow$$

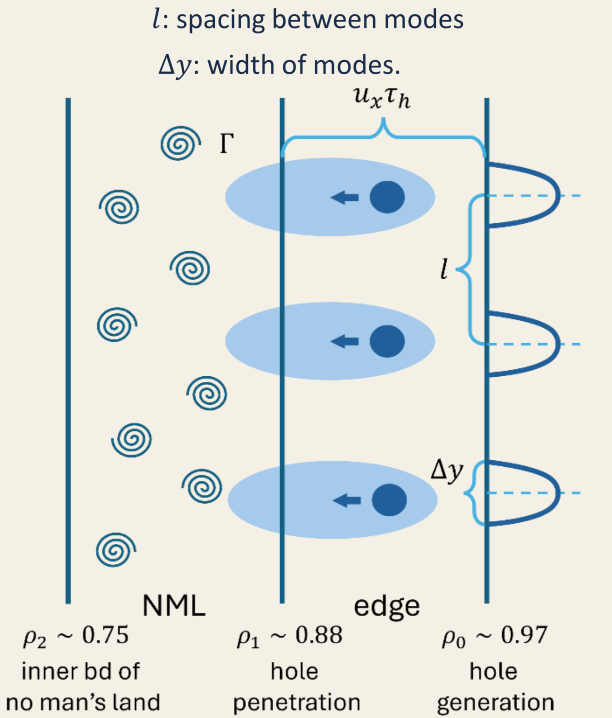
- The ratio of the turbulence intensity flux induced by holes to the total local production in no man's land is

$$R_a = \frac{\langle\bar{\Gamma}\rangle|_{\rho_1}}{\int_{\rho_2}^{\rho_1} \kappa \langle \tilde{v}\tilde{n} \rangle dr} \approx \frac{\langle\bar{\Gamma}\rangle|_{\rho_1}}{\kappa \langle \tilde{v}\tilde{n} \rangle w_{NML}}$$

- In no man's land, $Ra \sim 1 \Rightarrow$ defines the width of the no man's land:

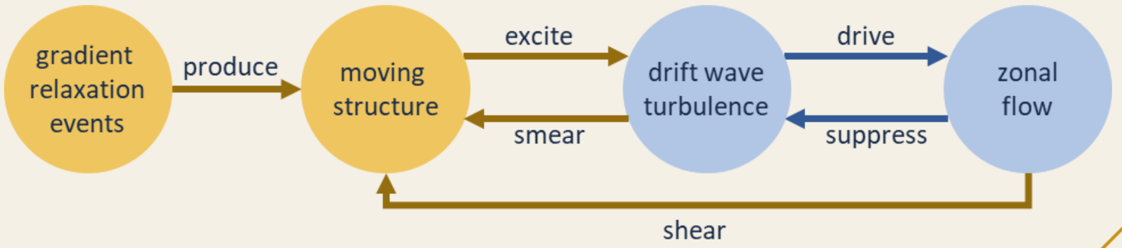
$$w_{NML} \sim \frac{2\pi}{\kappa \langle \tilde{v}\tilde{n} \rangle} \left(\frac{h\Delta x \Delta y}{u_x \tau_h}\right)^2 \frac{1}{v_* \tau_h^2} \frac{N\Delta y}{L_y} \frac{\tau_h}{\tau_w}$$

- For $\Delta x \sim \Delta y \sim 10$, $u_x \sim v_* \sim 10^{-2}$, $\tau_h \sim 10^3$, $l \sim 10^3$, $\tilde{v} \sim \tilde{n} \sim 10^{-2}$, $\kappa/2\pi \sim 10^{-4}$, $h \sim .1 \rightarrow$ **$w_{NML} \sim 10^2 \rho_s$**, kind of reasonable.



Conclusion

- Hole and blob are generated in pairs from GRE close to LCFS.
- A realistic physical picture of how the tail (edge) wags the dog (core): turbulence could be excited by the inward moving holes and spread to the no man's land.
- More specifically, we obtain:
  - An estimate of the turbulence intensity flux induced by holes and the spatial extent of no man's land. This may account for the turbulence level in the no man's land and thus address the "shortfall" problem.
  - The shearing rate of the zonal flow driven by a moving hole. ω_s^h is comparable with or even larger than ω_s^a.
  - An estimate of the lifetime based on a simple diffusion model. This estimate can fit the experimental observation quite well.



Future

- We suggest several possible directions for future research:
  - **For theories:**
    1. the net effect of holes on edge transport? Need an estimate of the magnitude of the turbulence excited by holes.
    2. constrain the upper limit of the hole lifetime further. Holes lose energy as emitting waves $\rightarrow$ decay faster than the diffusion model predicted. Need an estimate of the reaction force exerted on the hole by the turbulence field.
    3. or, consider the self-scattering of the hole? Currently we only consider the ambient turbulence and flow. Can refer to the equation for the 1D phase space coherent structure.
  - **For experiments:** observe the correlation between the frequency of GREs (the birth rate of the coherent structures) and the turbulence level in no man's land.
  - **For simulations:** include GREs into edge turbulence simulations, as inward moving hole energizing the edge. The profile-driven simulations miss this point.

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Thank you!