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Instability and Turbulent Relaxation in a Stochastic Magnetic Field[1]

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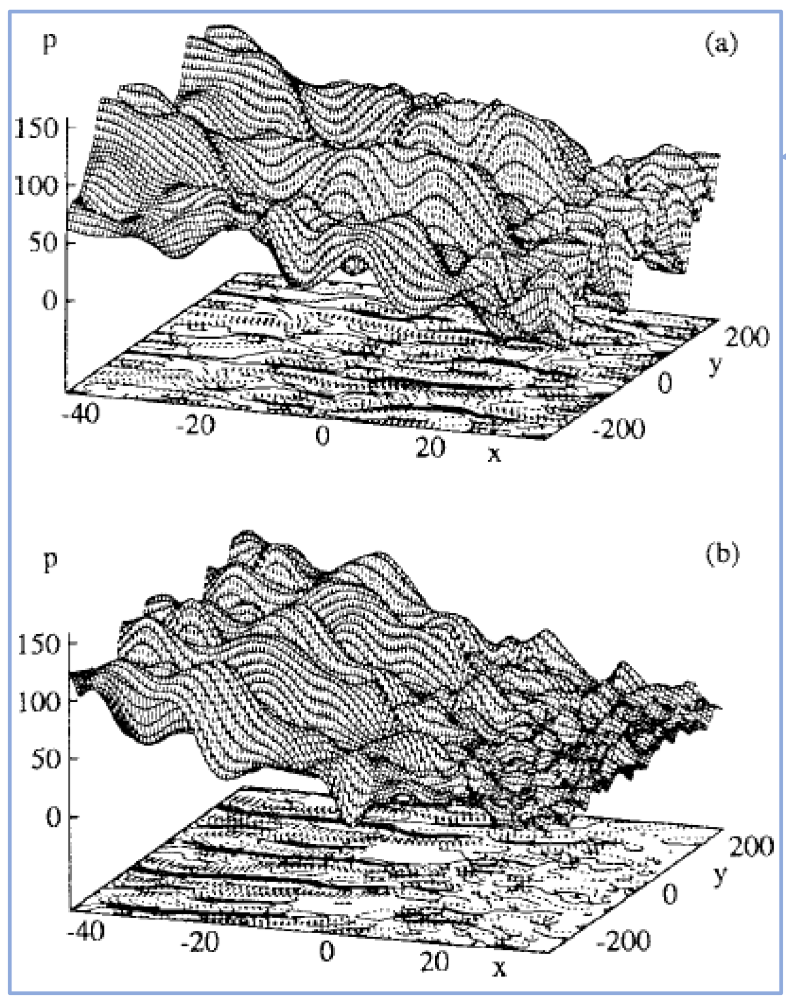
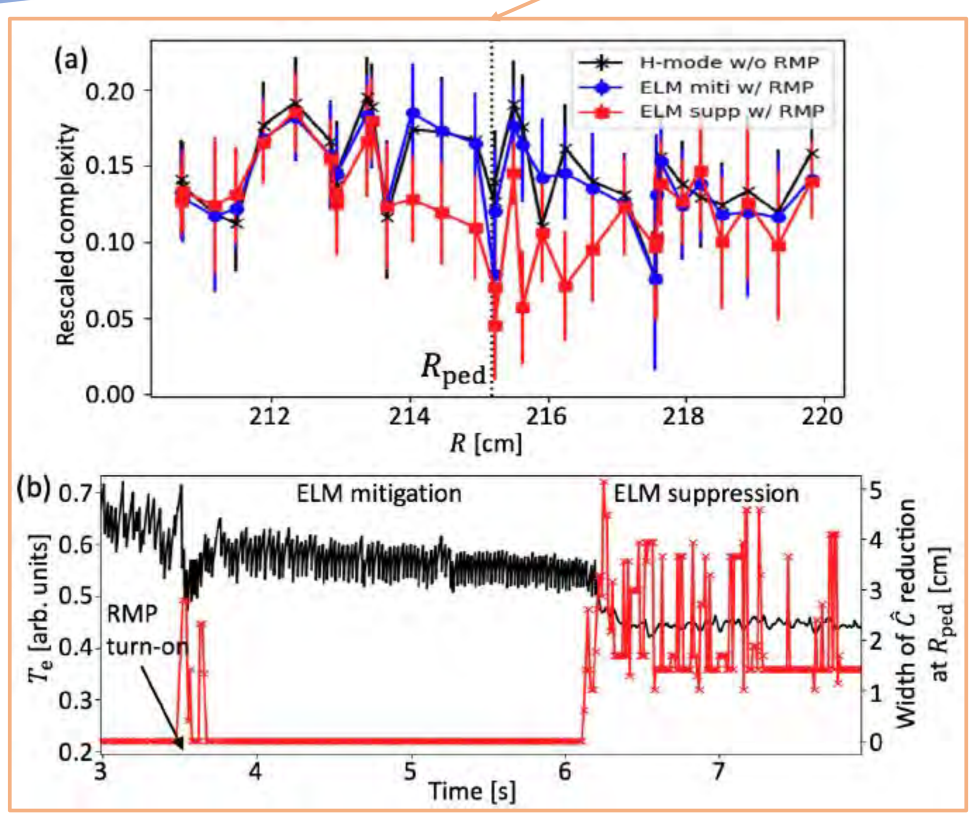
Motivation

Trend: Syntheses of **good confinement** & **optimal power handing**
Key question: Instability and turbulence in a stochastic magnetic field

Hints from simulations & experiments





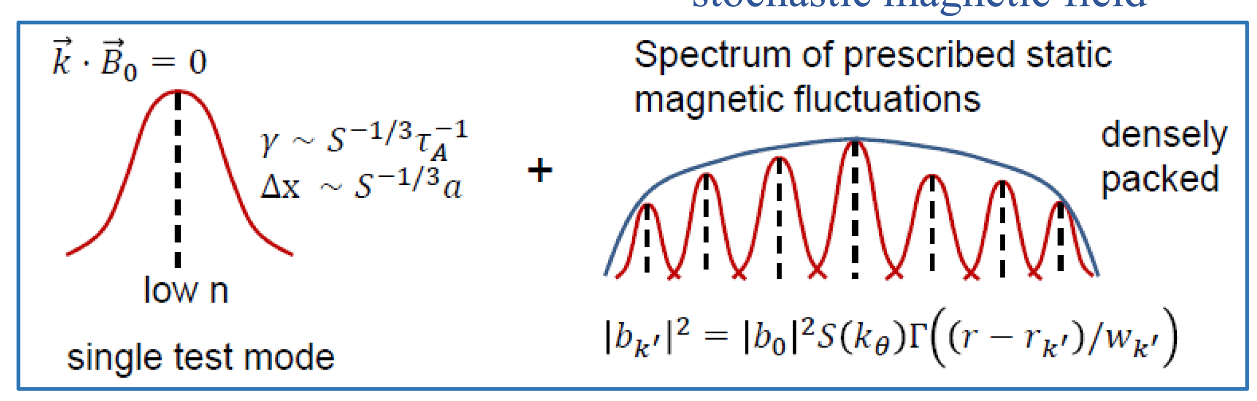
Previous simulations of resistive ballooning modes in a stochastic magnetic field by Beyer et al.[2] shows the emergence of small-scale structures and increased spatial roughness of the turbulence field.

Recent experimental results reported by Minjun Choi et al.[3] indicate that the effect of stochasticity on pedestal turbulence is to reduce its Jensen-Shannon complexity and predictability—i.e., the distribution of turbulence becomes more random compared with the natural ELM free case.

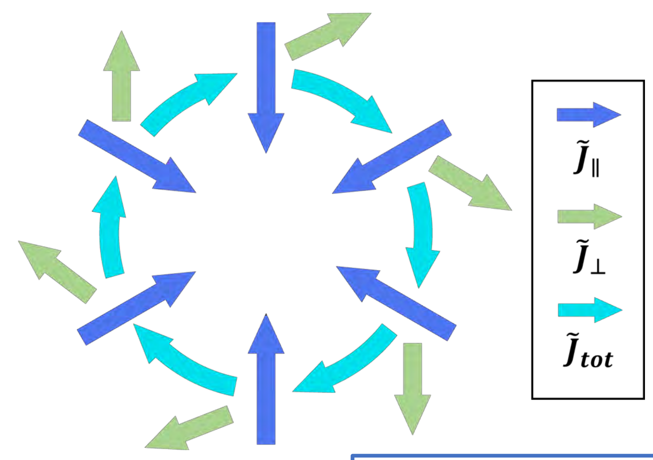
Novelty

Our model is supposed to
- maintain $\nabla \cdot \boldsymbol{J} = 0$ at all orders
- connect micro and macro scales
- be tractable → **resistive interchange**

The story is: dynamics of a low-$\boldsymbol{k}$ resistive interchange mode and turbulent relaxation in a high-$\boldsymbol{k}$ ambient and static background stochastic magnetic field



However, things are not so simple…
Stochastic magnetic field induces a current density fluctuation along the perturbed field line $\tilde{\boldsymbol{J}}_\parallel$, which is not divergence-free. So to keep $\nabla \cdot \tilde{\boldsymbol{J}} = 0$, a potential fluctuation $\tilde{\varphi}$ must be driven to produce a $\tilde{\boldsymbol{J}}_\perp$. This idea is similar to that in Kadomtsev and Pogutse's paper[4].

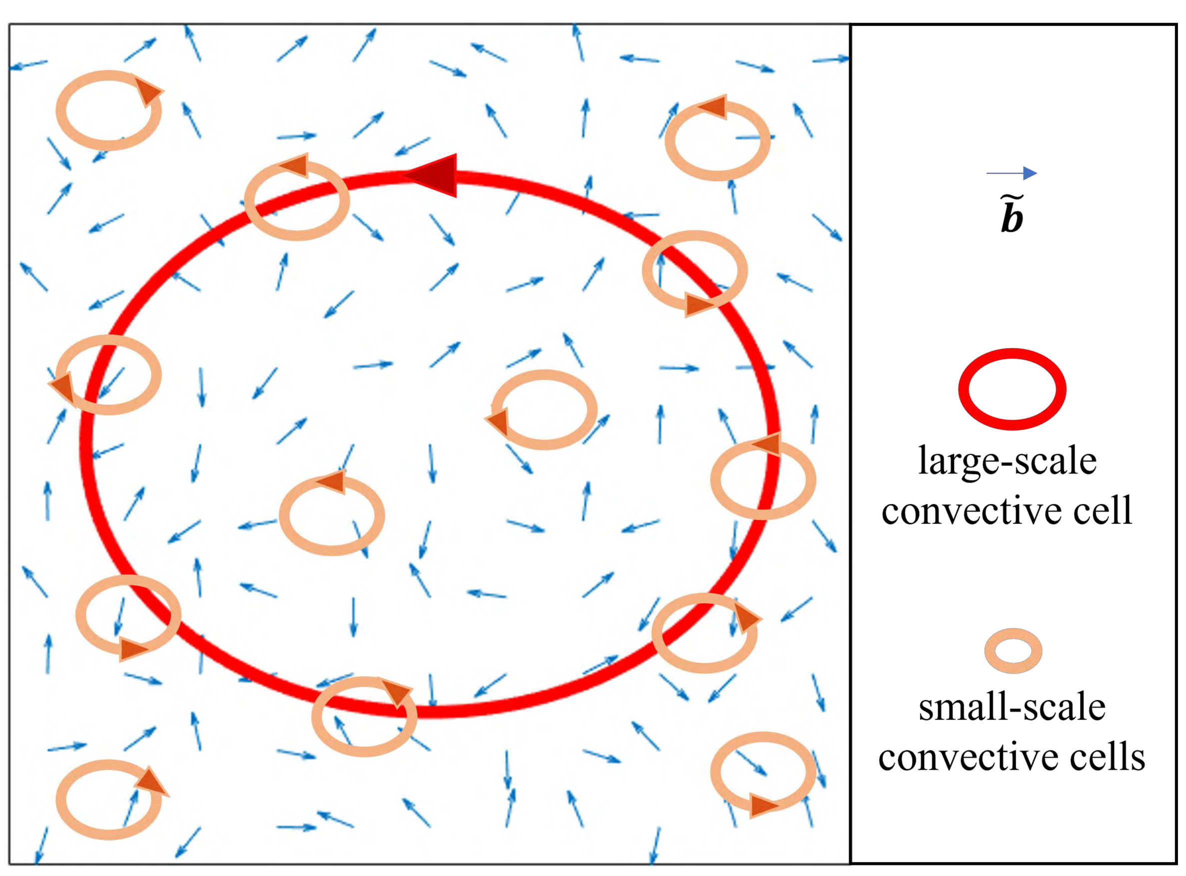


Analogy	C&D	K&P
Goal	$\gamma_{\boldsymbol{k}}^{(1)}$	$\langle q_r \rangle_{NL}$
Base state	$\bar{\varphi}$	$\bar{T}$
Stochastic Quantity	$\tilde{\boldsymbol{b}}$	$\tilde{\boldsymbol{b}}$
Constraint	$\nabla \cdot \boldsymbol{J} = 0$	$\nabla \cdot \boldsymbol{q} = 0$
Resulting fluctuations	$\boxed{\tilde{\varphi}}$	$\tilde{T}$

Intrinsic Multi-Scale Microturbulence

Model

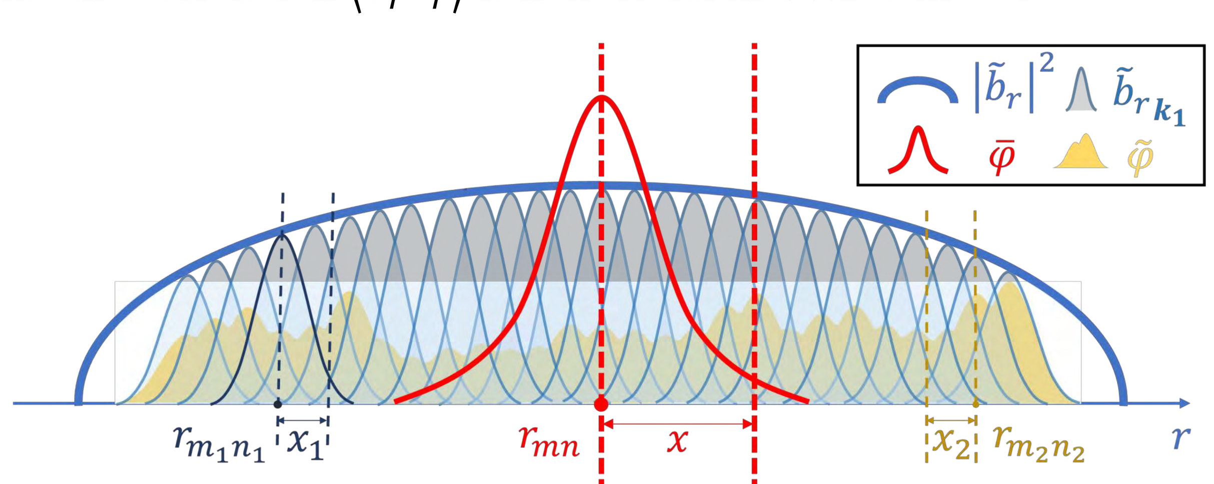
The actual system owns an intrinsically multi-scale nature and contains three players: a large-scale single cell (large red ellipse), a prescribed background stochastic magnetic field (small blue arrows), and small-scale convective cells (small orange ellipsis), i.e., *the intrinsic multi-scale microturbulence*.



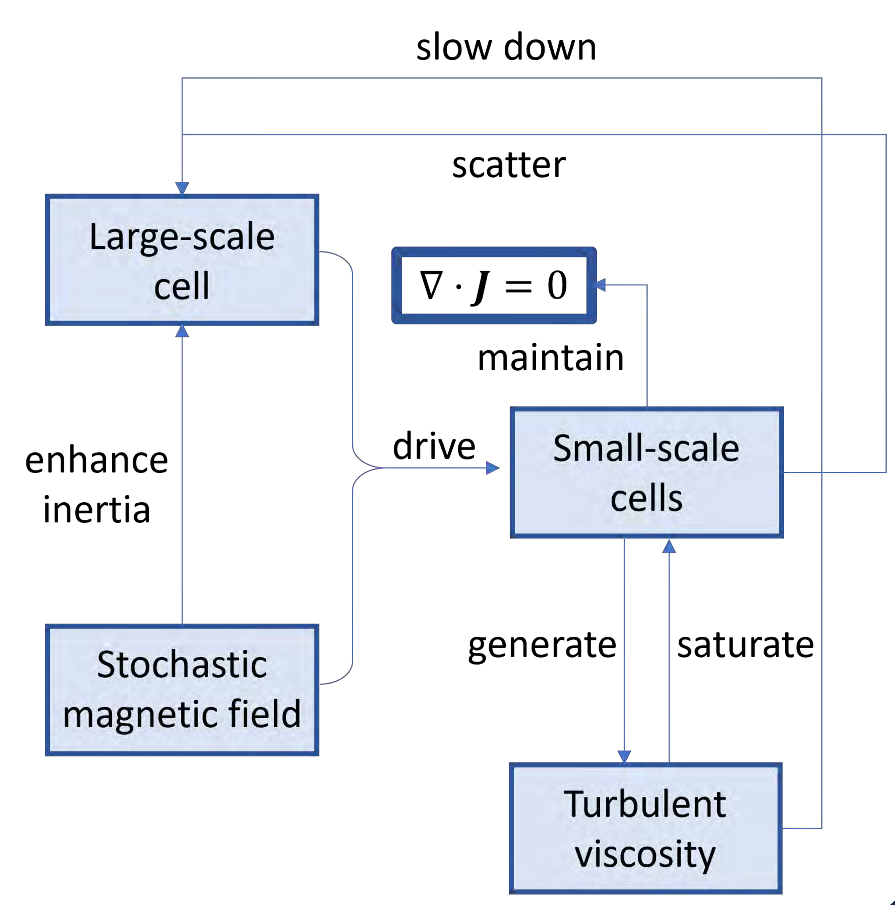
Small-scale cells are adiabatically modulated by the beat of large-scale cell and stochastic magnetic field, i.e.

$$\frac{\partial \tilde{\varphi}}{\partial t} + \lambda \tilde{\varphi} = \widehat{D}[\tilde{b}_r \bar{\varphi}] \quad (1)$$

where λ is the effective friction and $\widehat{D}$ denotes the drive of $\tilde{b}_r \bar{\varphi}$ beat. Since equation (1) is similar in form to Langevin equation, it implies a fluctuation-dissipation balance and shows the dual identities of $\tilde{\boldsymbol{b}}$: on the one hand, it serves as an external noise to excite $\tilde{\varphi}$; on the other hand, microturbulence also generates a turbulent background and the resultant turbulent viscosity damps these small-scale cells (that's where λ comes from). And in equation (1), $\tilde{\varphi}$ and $\tilde{b}_r$ are tightly related so it is no surprise to find that the correlation $\langle \tilde{b}_r \tilde{v}_r \rangle$ is non-trivial in C&D's model.



Equation (1) also indicates that macro and micro scales are connected. Therefore, the system incorporates multi-scale feedback loops, which couple the dynamics of the large-scale envelope and small-scale cells, as shown below:



Conclusions

Broadly applicable findings:
1. Maintaining $\nabla \cdot \boldsymbol{J} = 0$ at all orders reveals that microturbulence is driven at small scales by the beat of small-scale magnetic perturbations $\tilde{\boldsymbol{b}}$ and large-scale electrostatic potential $\bar{\varphi}$.
2. The microturbulence in turn modifies the large-scale mode via an effective flow viscosity and thermal diffusivity, as well as electrostatic scattering. Thus, dynamics has a disparate scale interaction.
3. The stochastic magnetic field produces a magnetic braking effect, which exerts a drag on large-scale vorticity. This effect is similar in structure to the nonlinear $\boldsymbol{J} \times \boldsymbol{B}$ force identified by Rutherford[5], but in our case, it is produced by magnetic perturbations.
4. A non-trivial correlation between the electrostatic turbulence and the ambient stochastic field—i.e., $\langle \tilde{b}_r \tilde{v}_r \rangle \neq 0$ is shown. Thus, the velocity fluctuations 'lock on' to the ambient static magnetic perturbations. This will necessarily affect the statistics of the turbulence.

Detailed calculations:
1. The net effect of stochastic magnetic fields is to reduce resistive interchange growth—i.e., a trend towards stabilization. The increment is calculated.
2. Turbulent viscosity and turbulent thermal diffusivity driven by the microturbulence are calculated.
3. The width of magnetic islands when magnetic braking effect becomes significant is calculated, which differs from Rutherford's result by a factor of $k_\theta^2/k_{2\theta}^2$.
4. The correlation $\langle \tilde{b}_r \tilde{v}_r \rangle$ is calculated explicitly.

Future

- Consider the couplings between large-scale modes, as opposed to a single mode case.
- Extend this analysis to a kinetic description of microinstabilities.
- Look at effects of stochastic magnetic field $\tilde{\boldsymbol{b}}$ on twisted slicing modes, i.e., include toroidicity (**ongoing work**).

References

1. Cao, M. and Diamond, P.H., 2022. Instability and turbulent relaxation in a stochastic magnetic field, *Plasma Physics and Controlled Fusion*.
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4. Kadomtsev, B.B. and Pogutse, O.P., 1979. Electron heat conductivity of the plasma across a 'braided' magnetic field. Plasma
5. Rutherford, P.H., 1973. Nonlinear growth of the tearing mode.