Momentum and Heat Transport in MHD Turbulence in Presence of Stochastic Magnetic Fields

Chang-Chun Chen
Defense

University of California San Diego, USA

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Outline

- Introduction:
  a. Importance of momentum Transport in quasi-2D astrophysical system and tokamak.
  b. Zonal flow formation/suppression, PV mixing.
  c. Comparison—β-plane and edge plasma physics.

- My research work/Results:
  a. **Stochastic-field induced** effect on momentum transport in the solar tachocline—β-plane MHD and the Rochester& Rosenbluth's proposal for describing stochastic field.
  (Chen et al. ApJ **892** (1), 2020) (Weak magnetization).
  b. Stochastic-field-induced effect on Poloidal momentum Transport at the edge of fusion devices (Strong magnetization)—Drift-wave turbulence (Chen et al. PoP **28**, 042301, 2021).
  c. Stochastic-field-induced effect on Parallel momentum and ion heat transport (Chen et al. PPCF **64**, 015006, 2022).

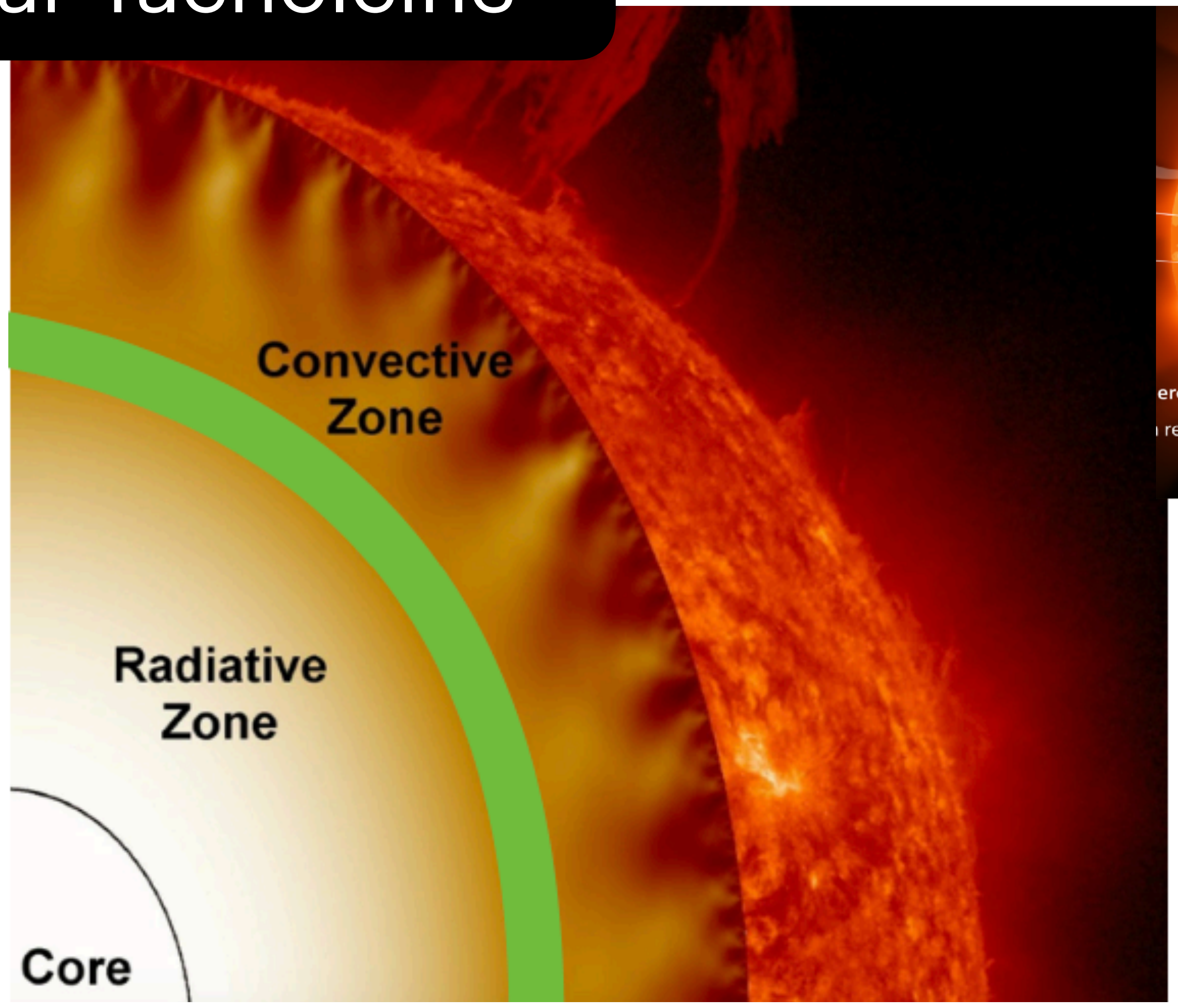
- Results:
  a. β-plane MHD: The momentum transport problem of the solar tachocline is due to the highly disordered magnetic field decorrelation effect.
  b. Stochastic fields can form a fractal, elastic network.
  c. The stochastic-field induced dephasing at the mean field intensity lower than that for the fully Alfvénization.

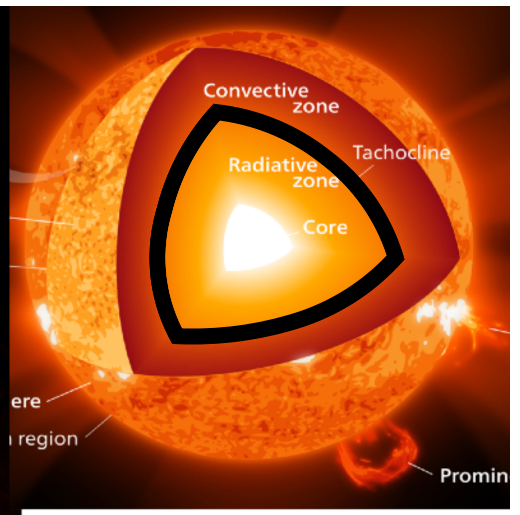
Outline

- Results:
  d. Poloidal momentum transport in stochastic B-field: Suppression of PV diffusivity and the shear-eddy tilting feedback loop.
  e. Power threshold increment for L-H transition. Intrinsic Rotation in presence of stochastic fields.
  f. Parallel and ion heat transport in stochastic B-field: In strong turbulence regime, the mean flow is driven by stochastic-turbulent scattering.
  g. We calculate the ***explicit form*** of the stochastic-field-induced transports which have different mechanisms in presence of strong/weak electrostatic turbulence.
- Future Work:
  a. Neutrals and Drift-Rossby-Alfvén turbulence:
     Ambipolar Diffusion
  b. Staircase and Mixing length in presence of stochastic fields

Momentum Transport in Astrophysical system and in tokamaks

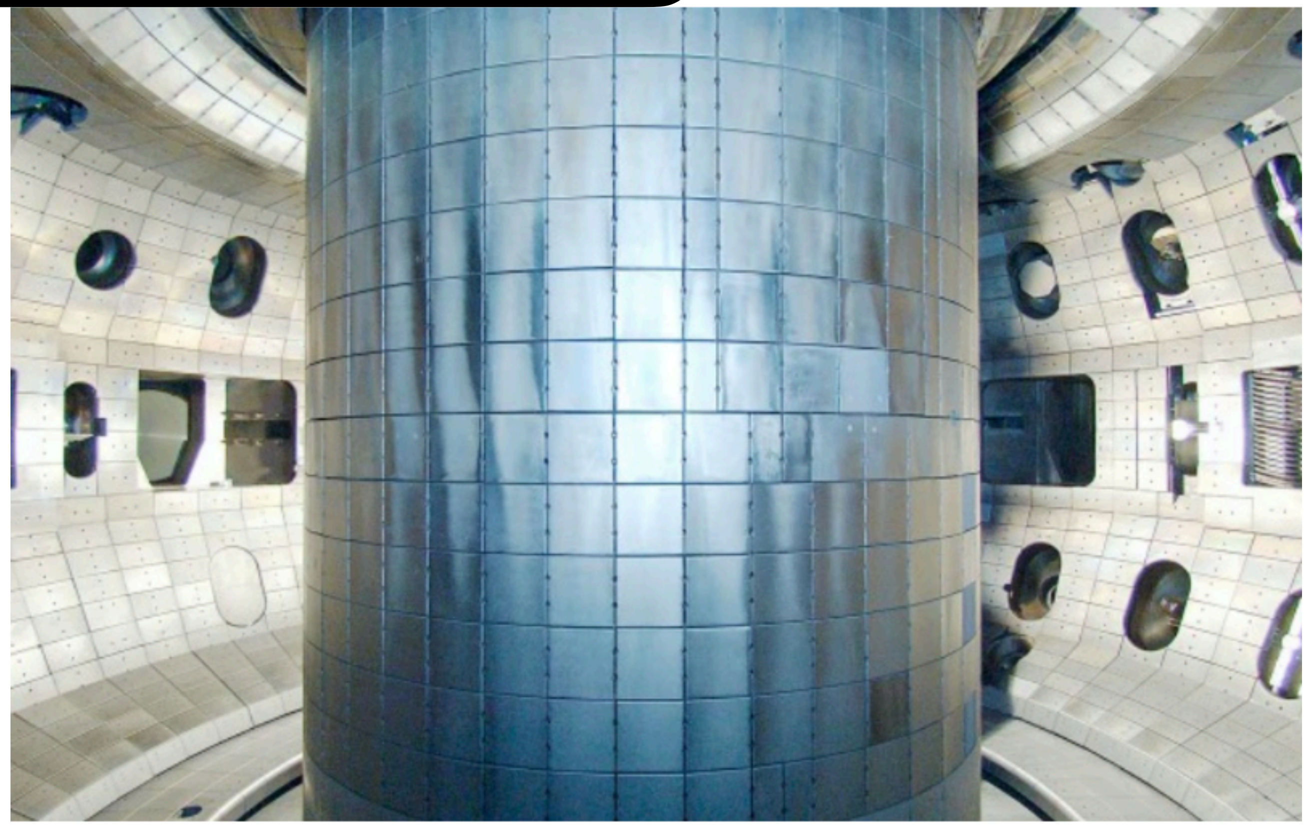
Solar Tacholcine





The solar tachocline inward spreding problem.

Tokamaks



Reynolds stress suppression during L-H transition in presence of RMPs.

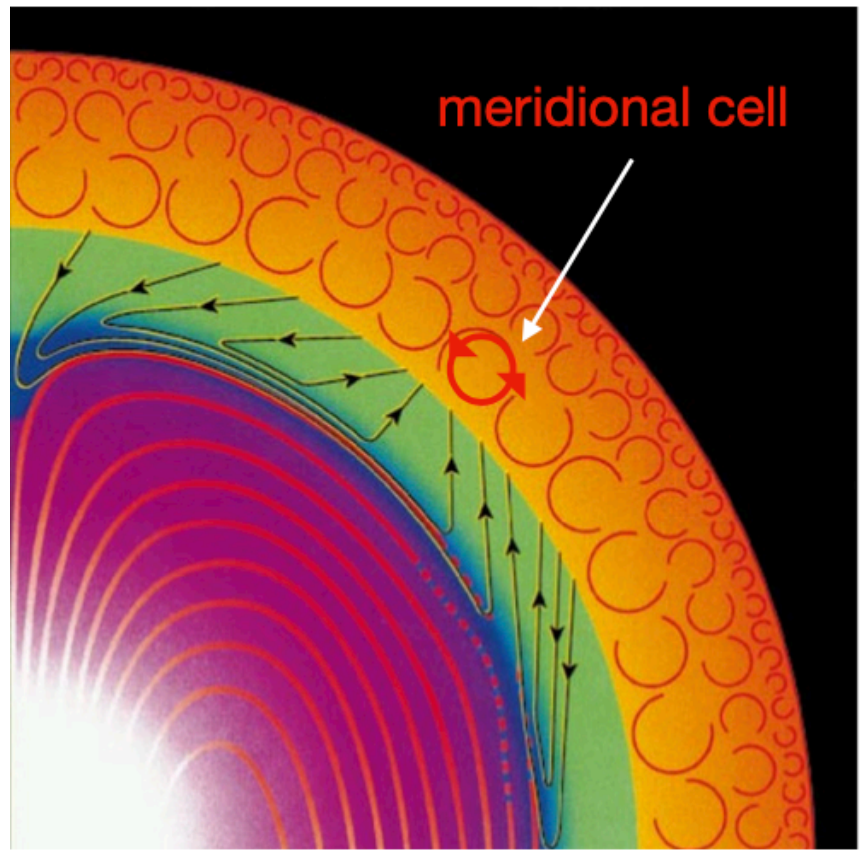
Jovian Atmosphere

Formation and evolution of the Rossby flow and the Great Red Spot.

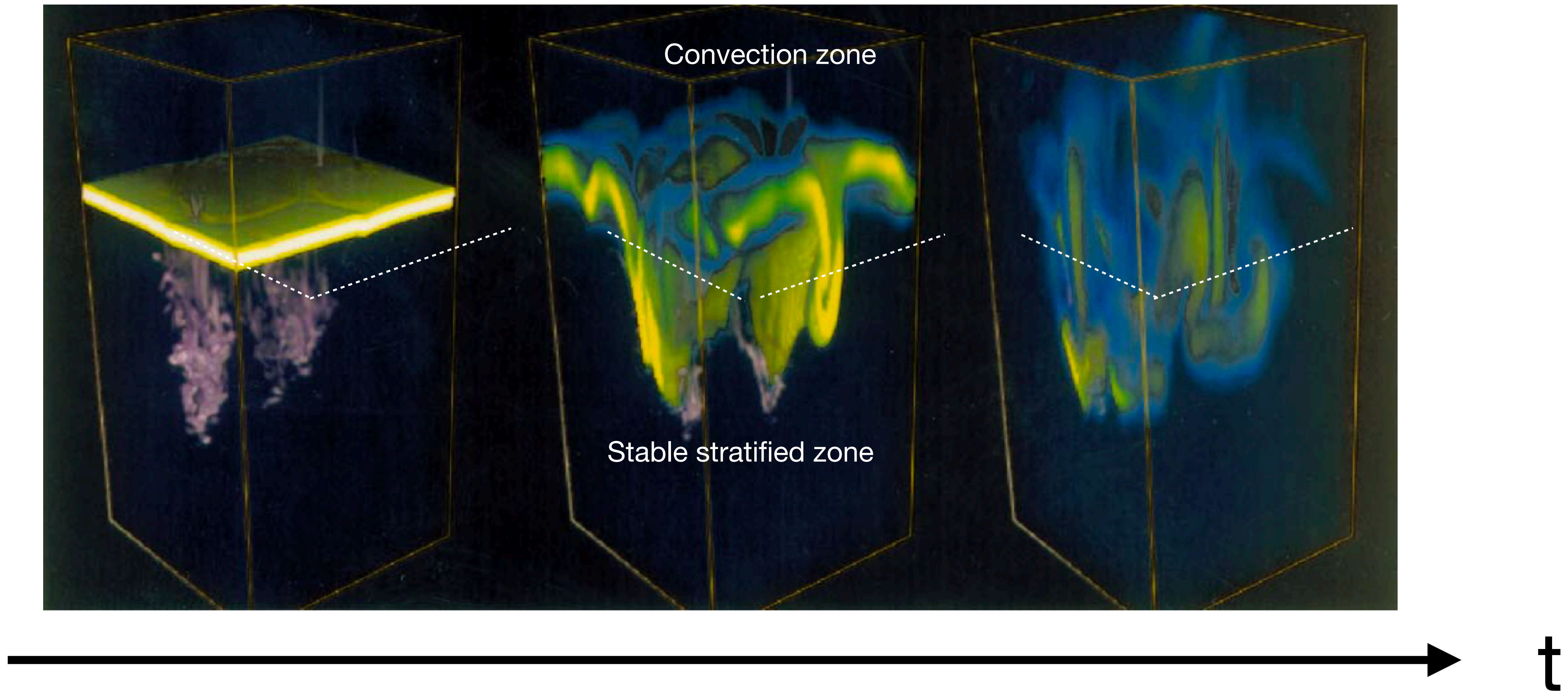
Momentum transport in presence of magnetic fields is a generic problem.

The Solar Tachocline and Disordered B Fields

- The solar tachocline (Weak mean magnetization):
  a. A layer between the convective and radiative zone.
  b. It is strongly stratified/Pancake-like structures.
  —>Incompressible rotating fluid in 2D layers: β-plane model
  c. Zonal Flow and Rossby Waves — as in the Jovian Atmosphere.
  d. Large magnetic perturbation $|\widetilde{B}| \gg B_0$.
  e. Meridional cells forms tachocline but will make it spread inward.

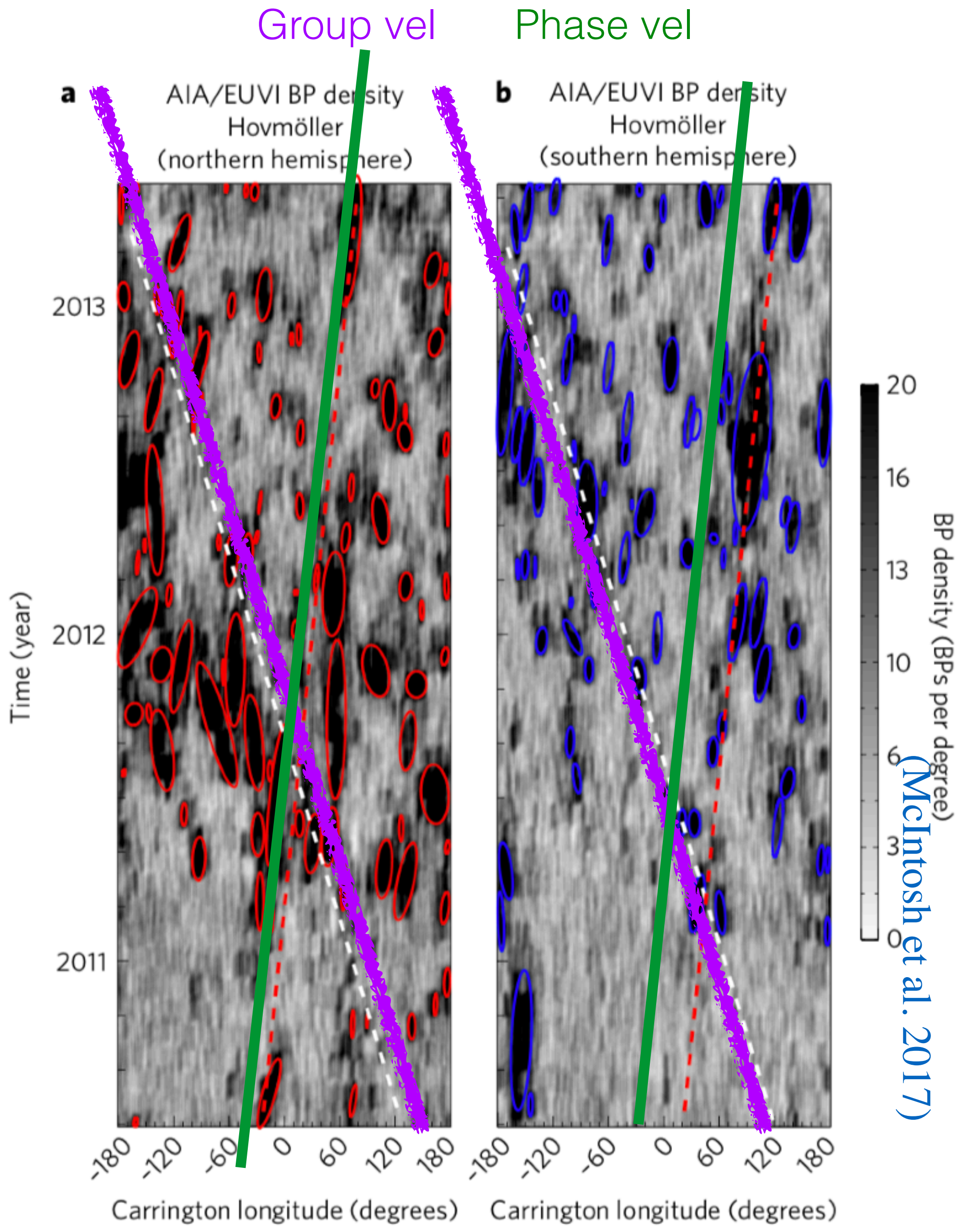


- Where does stochastic fields come from?



The stochastic magnetic field has been “pumped” from the convection zone into the stably stratified region.

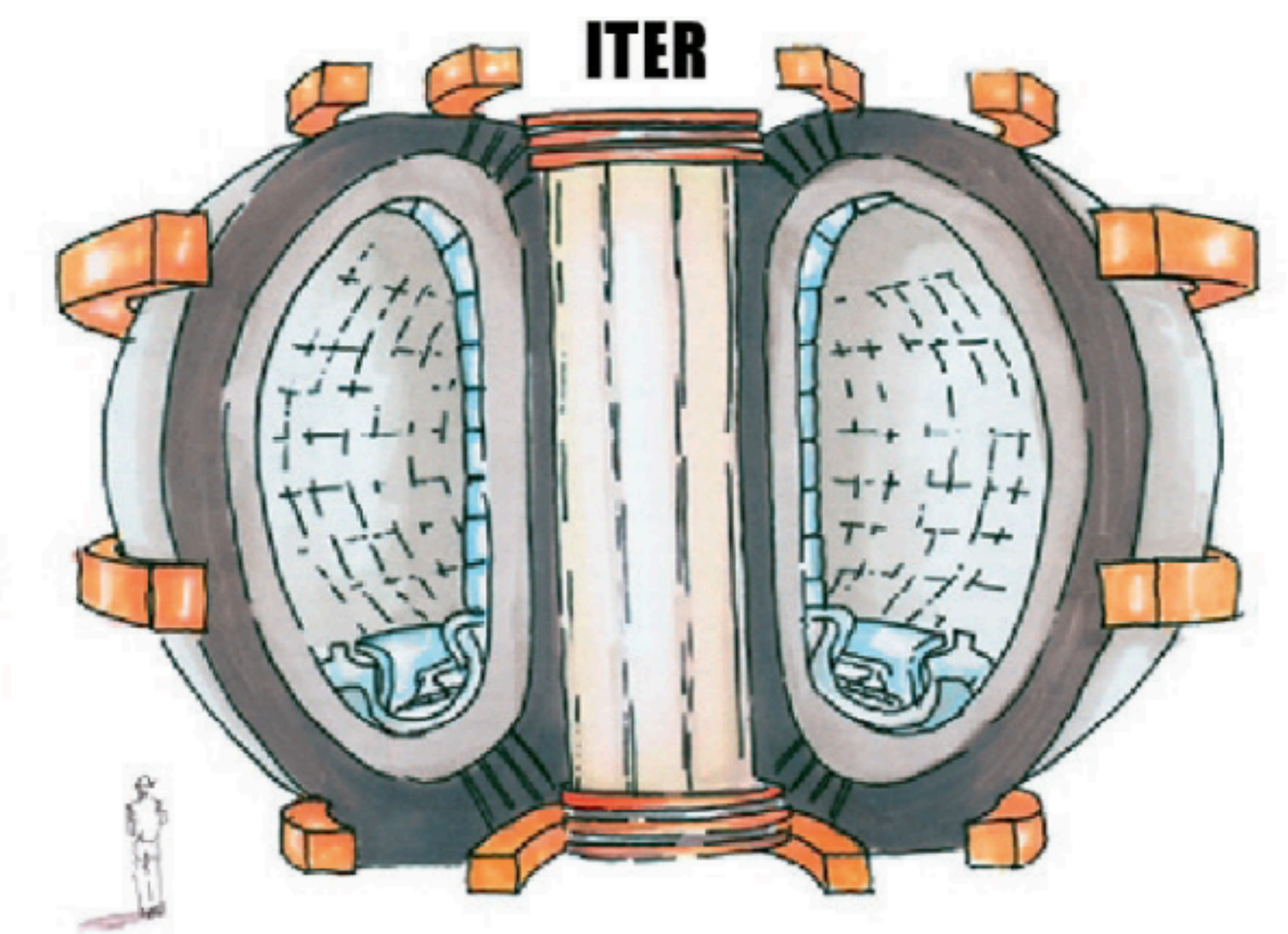
The drift-Rossby wave for the tachocline is Quasi-2D



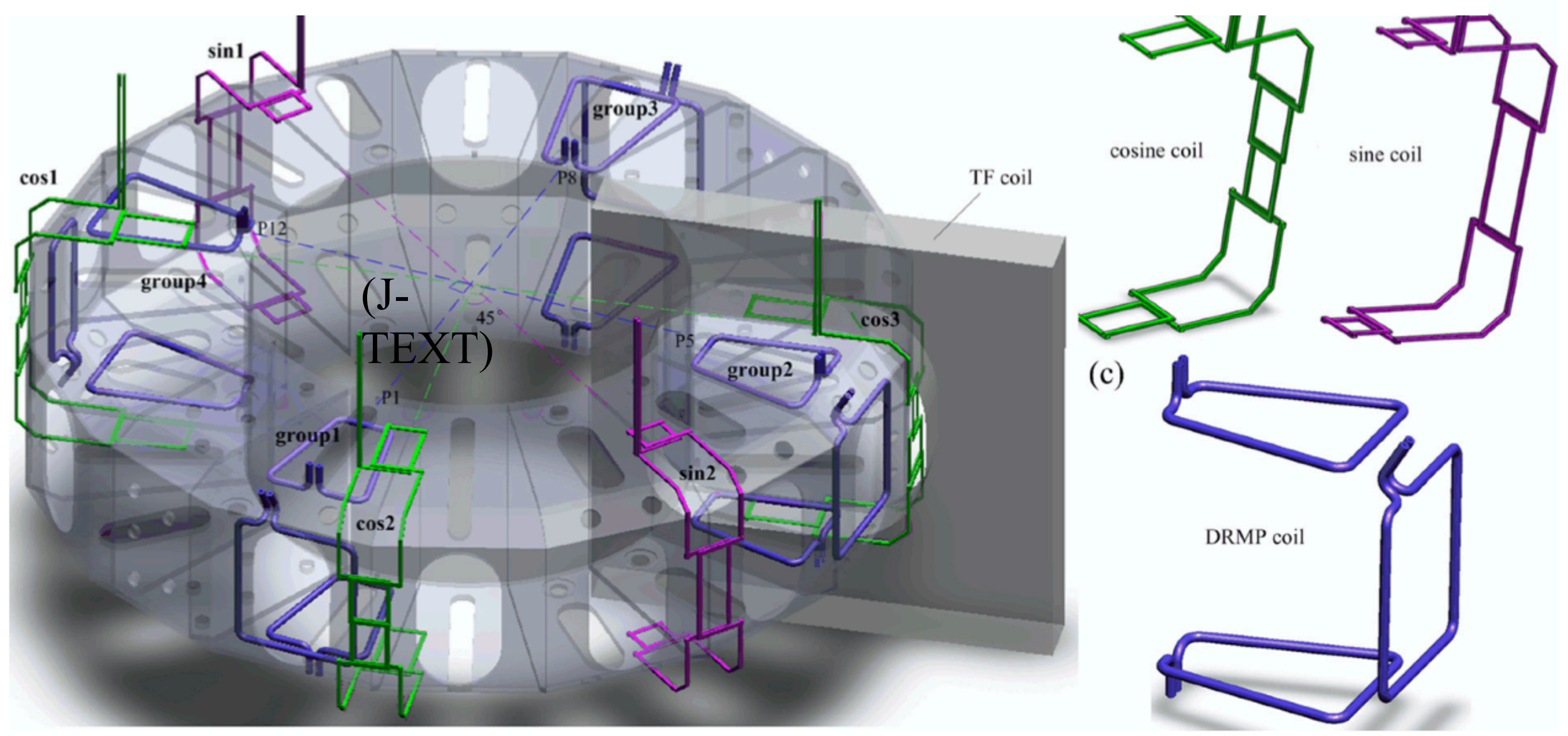
Fusion Devices and RMPs

- Where does **stochastic fields** come from? Resonant magnetic perturbation (RMP) is externally imposed, which raises L-H transition power threshold.

(Strong mean magnetization)



- ELMs are quasi-periodic relaxation events occurring at edge pedestal in H-mode plasma.
- ELMs can damage wall components of a fusion device.



3D geometry with $\underline{k} \cdot \underline{B}$ resonance leads to **quasi-2D** drift-Alfvén wave in presence of stochastic field.

PV mixing and Zonal Flow Formation

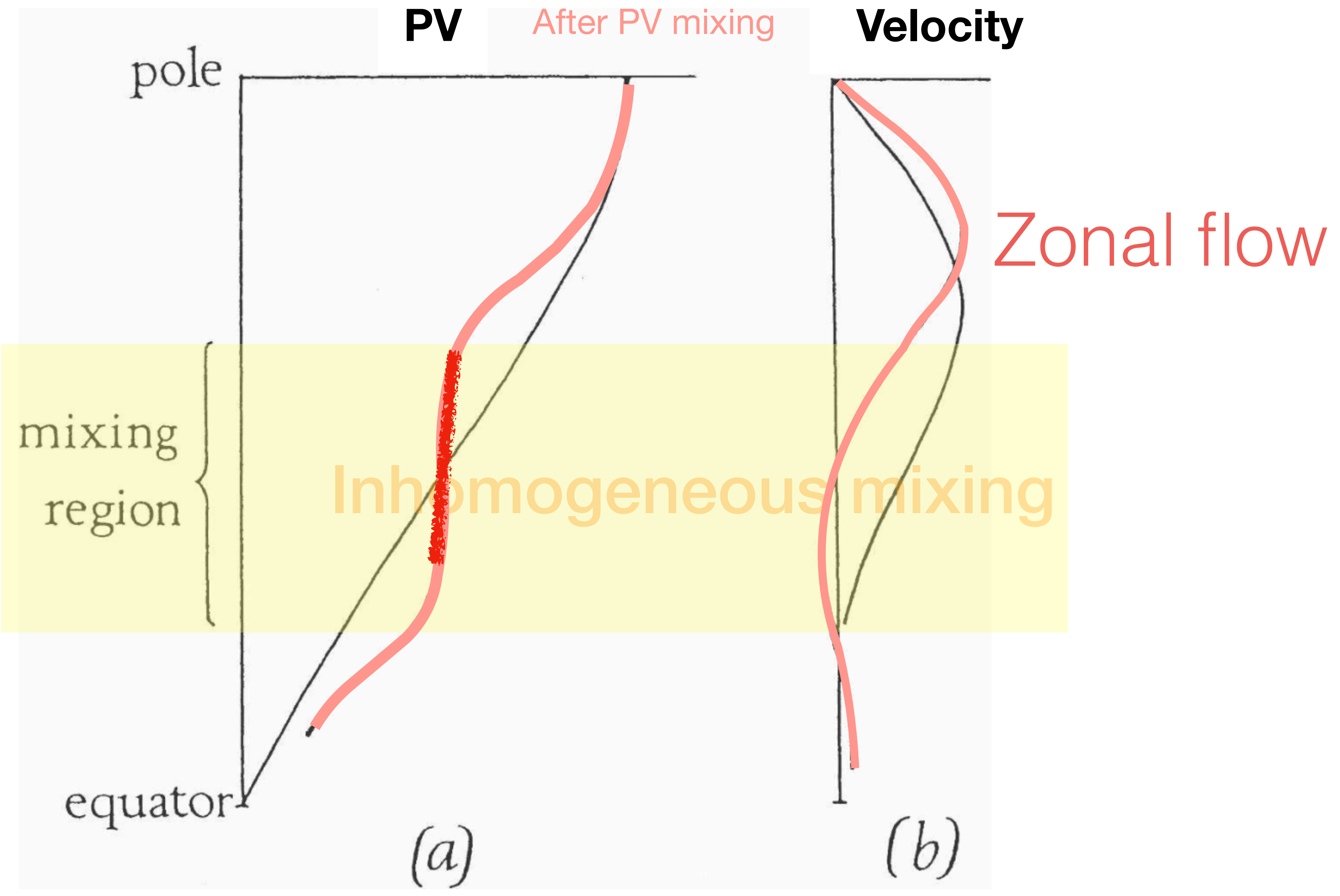
- What is Potential Vorticity (PV)?
  PV is a generalized vorticity.
  It is conserved along the fluid (conserved phase space density).

$PV \equiv \zeta \equiv \nabla \times \mathbf{v}$ **(pure 2D fluid)**

$PV \equiv \zeta + 2\Omega \sin \phi_0 + \beta y$ **(on the β-plane)**

$PV \equiv (1 - \rho_s^2 \nabla^2)\frac{|e|\phi}{T} + \frac{X}{L_n}$ **(Hasegawa-Mima eq. for tokamak)**

- Momentum transport and flow formation are determined by **inhomogeneous PV mixing**.



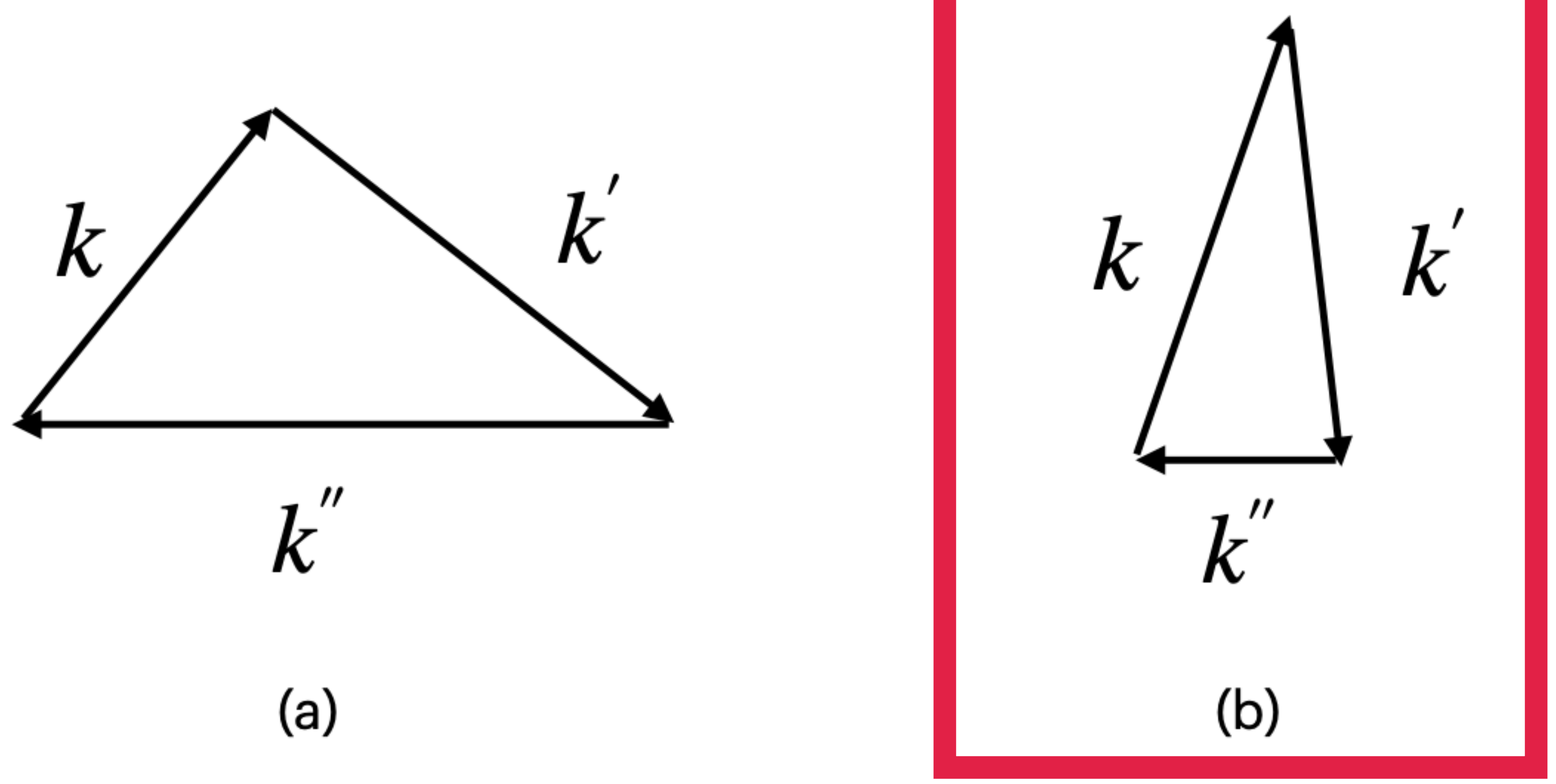
- How does zonal flow evolves?

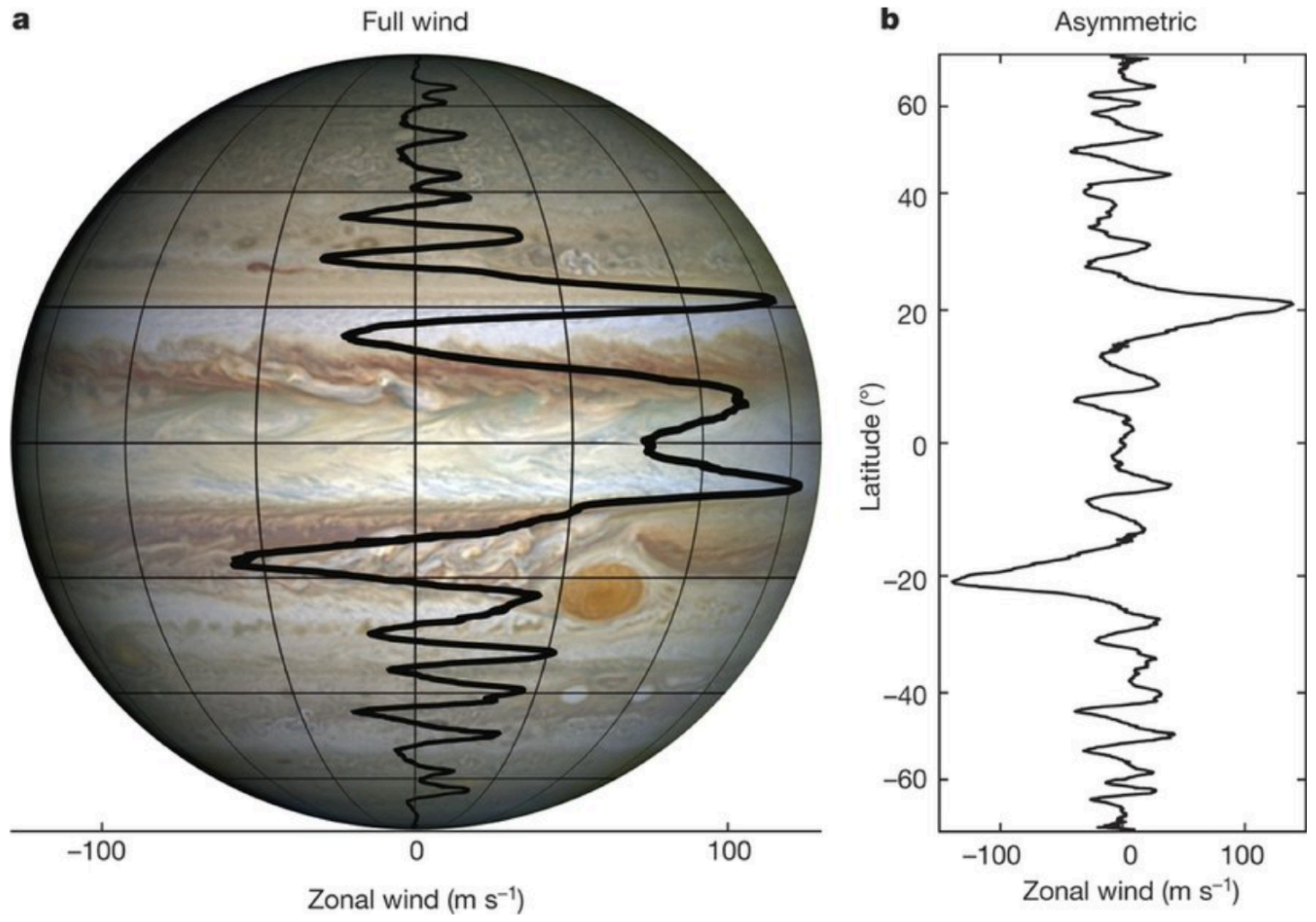
Taylor Identity: $\underbrace{\langle \tilde{u}_y \tilde{\zeta} \rangle}_{PV\ flux} = \underbrace{-\frac{\partial}{\partial y}\langle \tilde{u}_y \tilde{u}_x \rangle}_{Reynolds\ force}$

Evol. of zonal flow: $\frac{\partial}{\partial t}\langle u_x \rangle = \langle \tilde{u}_y \tilde{\zeta} \rangle = -\frac{\partial}{\partial y}\langle \tilde{u}_y \tilde{u}_x \rangle\,.$

PV mixing and Zonal Flow Formation

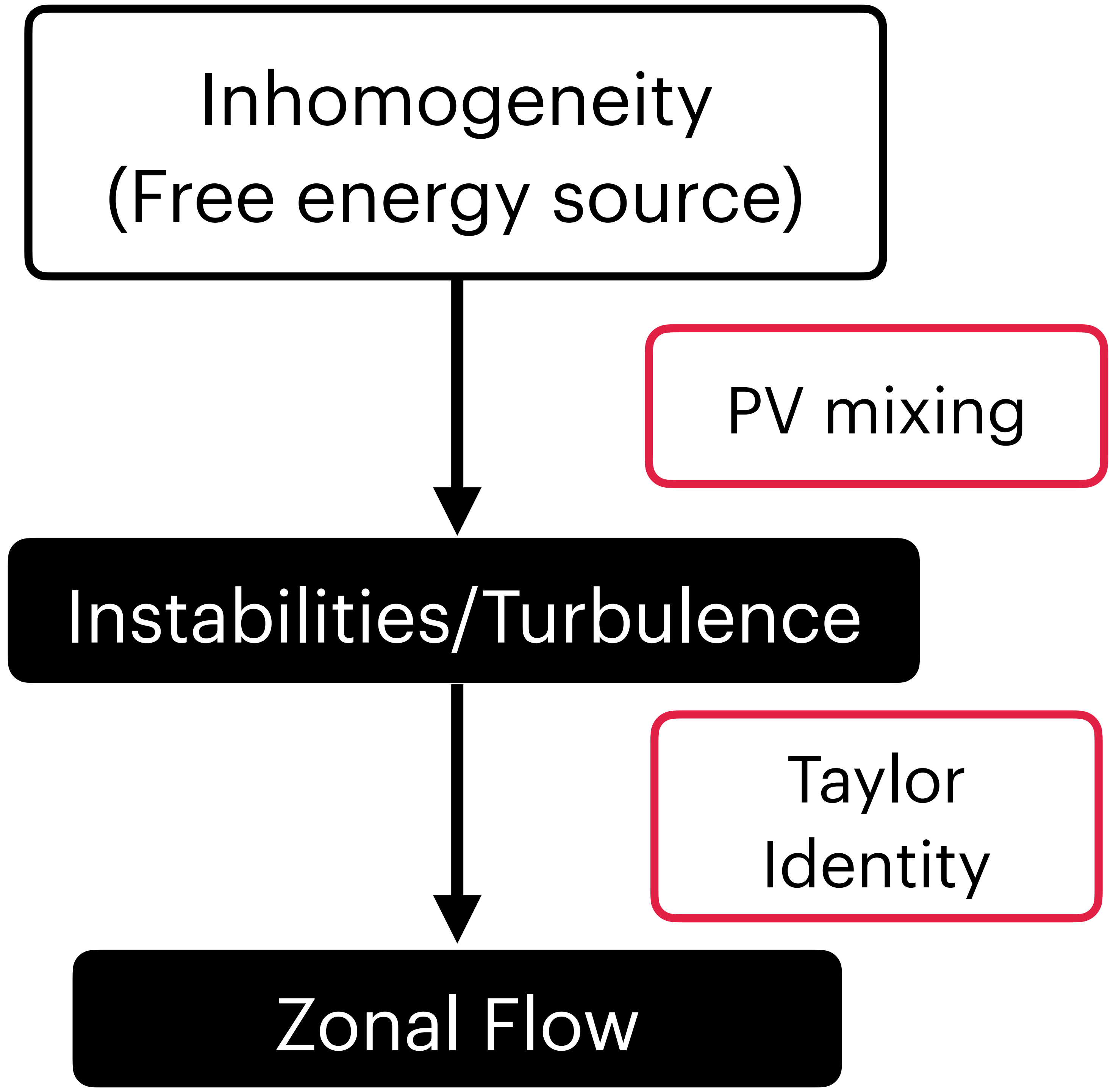
- Non-local wave-wave interaction:
  The resultant wave (zonal flow mode) has wavenumber much smaller than that of the other two waves.





Strongly nonlinear processes like PV mixing and wave breaking yield turbulent PV flux, and form a large-scale zonal flow.

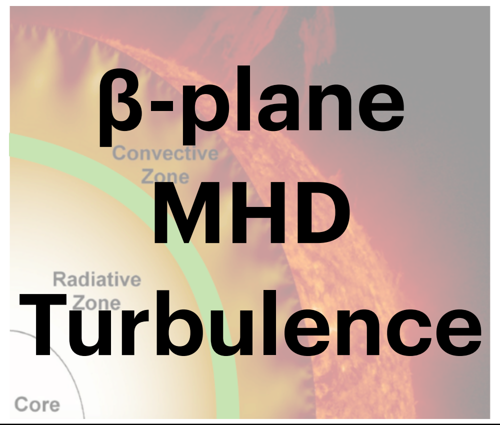
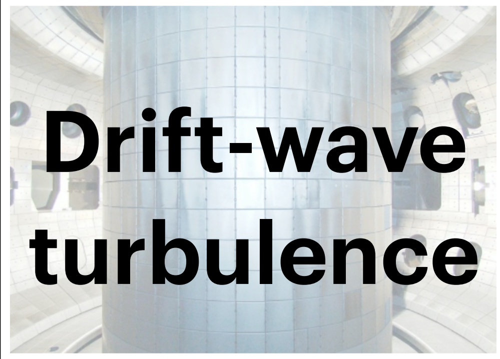
Turbulence and Zonal Flow

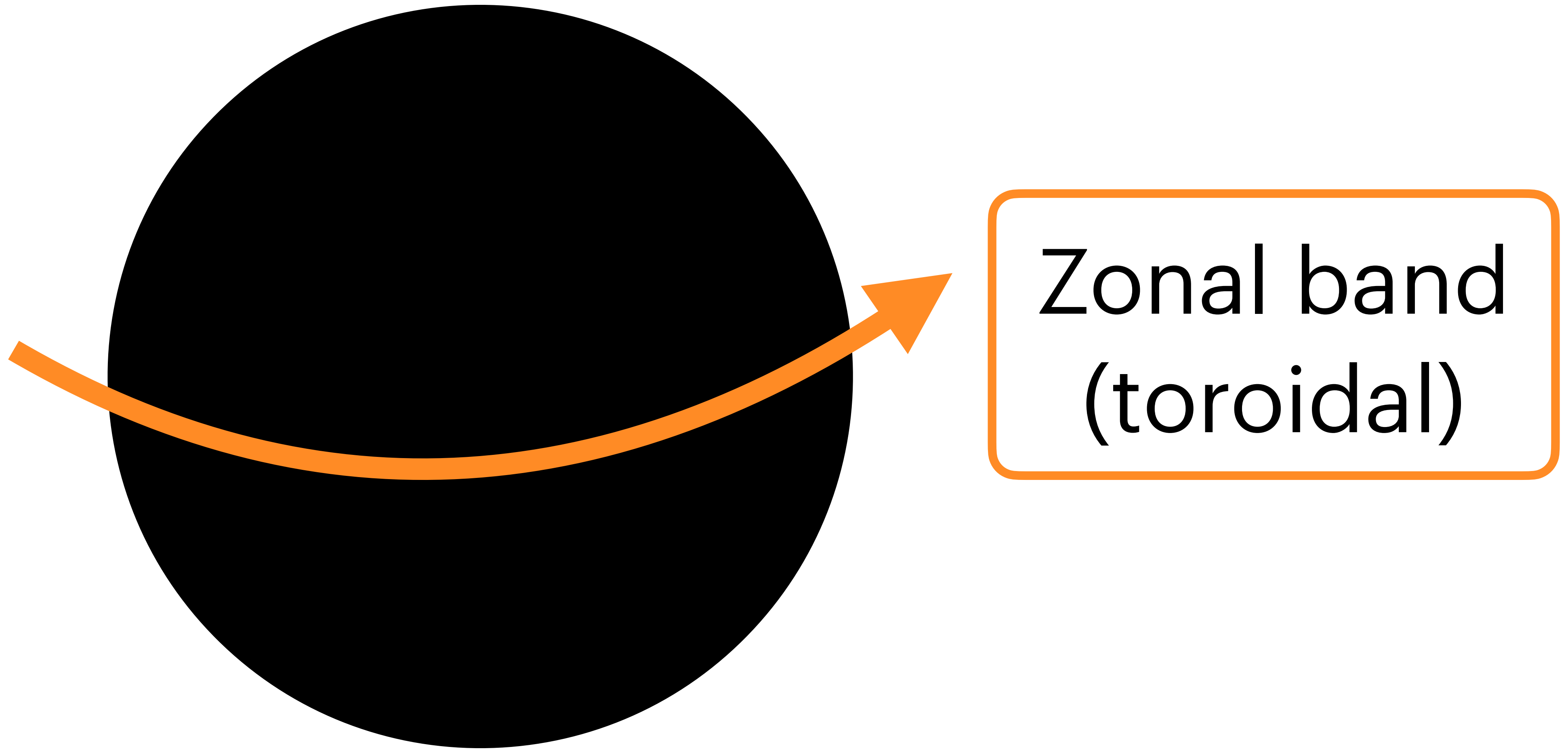


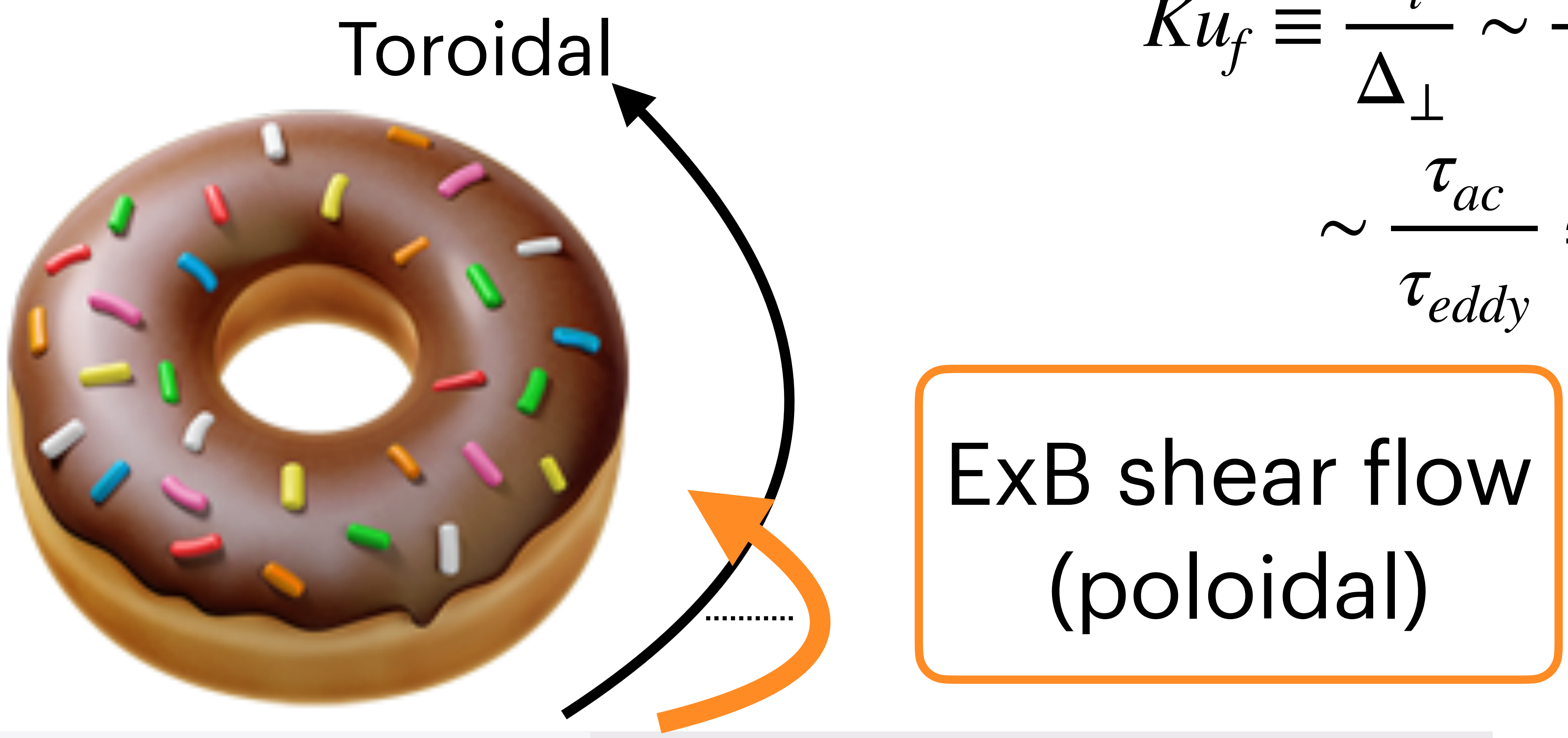
Predator-Prey Model

Turbulences is the prey, growing from the free energy source (i.e. the environment nutrients).
While the zonal flow (i.e. the predator) 'is fed' upon turbulence.

Physical System Summary

	Linear Wave	Conserved PV	inhomogeneity (free energy source)	Rossby # (Ro)	Reynolds # (Re)	Mag. Reynolds # (Rm)	Zonal Flow	Fluid Kubo # (Ku_{fluid})
 **β-plane MHD Turbulence**	Rossby-Alfvén waves	$PV = -\nabla^2\psi + \beta y$	β, N (Rossby Parameter and flow buoyancy)	0.1~1	$Re = \frac{\text{inertial}}{\text{viscous}} = 10^{16} - 10^{17}$	$Rm = \frac{\text{mag. advection}}{\text{mag. diffusion}} = 10^5 - 10^6$	Jets, zonal bands (toroidal)	$Ku_{fluid} \lesssim 1$
 **Drift-wave turbulence**	ExB Dirft waves	$PV = -\nabla^2\phi + n$	$\nabla n\ , \nabla T$ (particle and temp. gradient)	$Ro \ll 1$	$Re \simeq 10 - 100$ But the damping is the Landau damping.	$Rm = \frac{Lv_A}{\eta} = 10^5 - 10^7$	$E \times B$ shear flow (poloidal)	$Ku_{fluid} < 1$



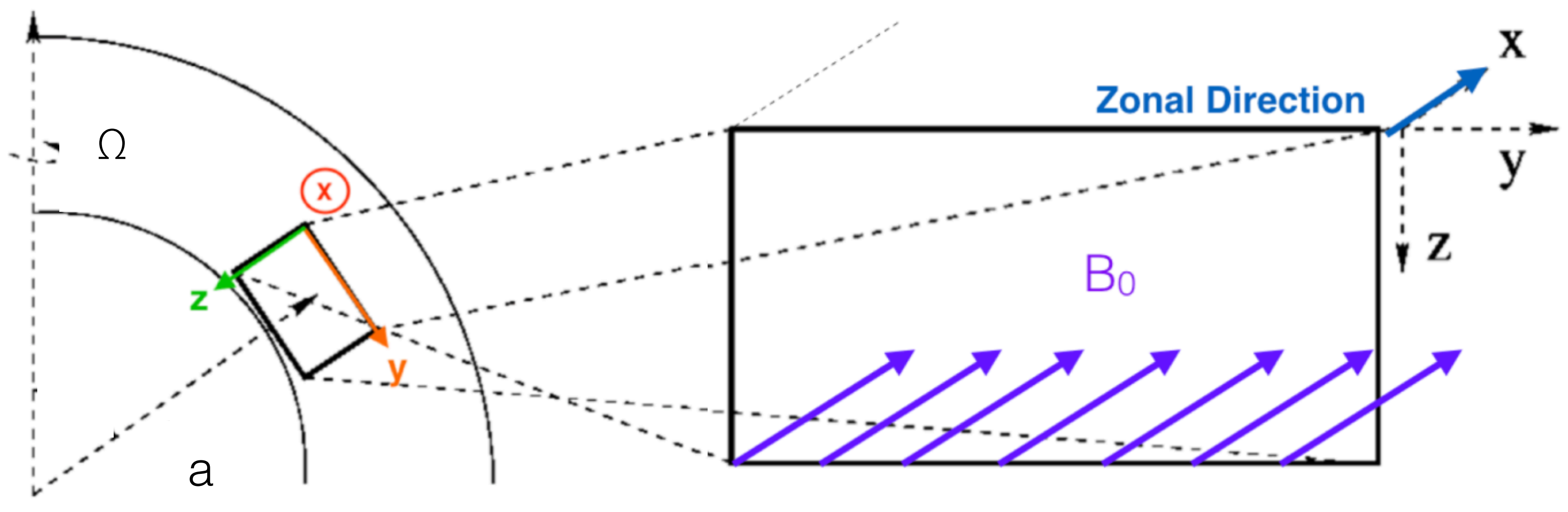


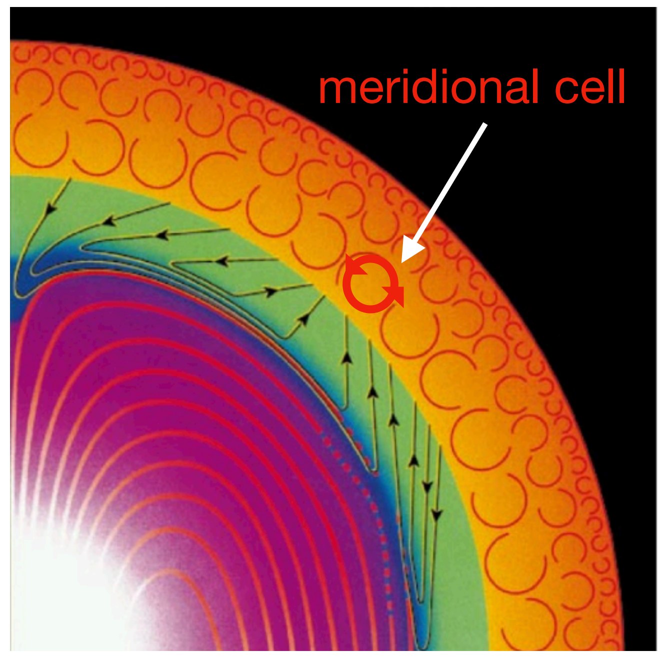
$$Ku_f \equiv \frac{\delta_l}{\Delta_\perp} \sim \frac{\tilde{v}\tau_{ac}}{\Delta_\perp} \sim \frac{\tau_{ac}}{\tau_{eddy}} \simeq 1,$$

Momentum transport in the solar tachocline—
β-plane MHD

My Research — The Solar Tachocline

- Setup:
  x—toroidal (zonal)
  y—poloidal (latitudinal)
  z—radial
  β—Rossby parameter ∝ rotation





- The existence of the tachocline is inferred from the helioseismology, but there is no direct observational evidence.

  a. Spiegel & Zahn (1992)— Spreading of the tachocline is opposed by turbulent viscous diffusion of momentum in latitude.
  b. Gough & McIntyre (1998)— Spreading of the tachocline is opposed by a hypothetical fossil field in the radiational zone.

These two models ignore the "likely" strong stochasticity of the tachocline magnetic field.

$$\beta = \frac{df}{dy}\Big|_{\phi_0} = 2\Omega cos(\phi_0)/a$$

Derivative of angular frequency f (Coriolis parameter)
latitude
rotation
radius

"At the heart of this argument is the role of the fast turbulent processes in redistributing angular momentum on a long timescale."

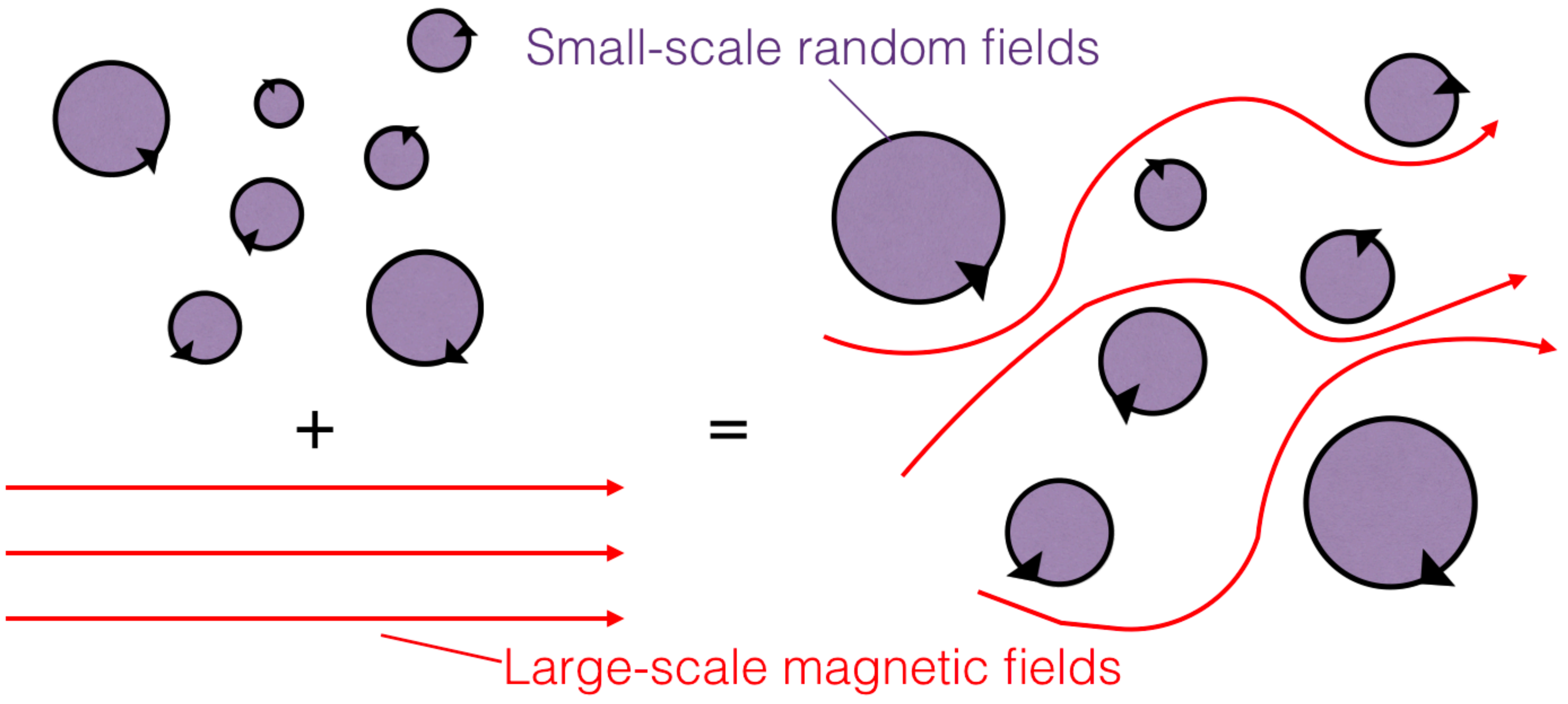
— (Tobias et al. 2007)

How we describe the stochastic magnetic fieldar Tachocline

- Fluid Kubo number: $$Ku_f \equiv \frac{\delta_l}{\Delta_\perp} \sim \frac{\tilde{v}\tau_{ac}}{\Delta_\perp} \sim \frac{\tau_{ac}}{\tau_{eddy}} \simeq 1,$$
  τ_{ac} ← Auto correlation time
  τ_{eddy} ← Eddy turnover time
- Magnetic Kubo number: $$Ku_{mag} \equiv \frac{\delta_l}{\Delta_{eddy}} = \frac{l_{ac}|\widetilde{\mathbf{B}}|}{\Delta_{eddy}B_0}$$ $$Ku = \begin{cases} < 1, & \textbf{Quasi-linear theory} \\ > 1, & \textbf{Quasi-linear theory fails} \end{cases}$$
- Zel'dovich, 1983 proposed a physical picture of the stochastic fields:

The large-scale magnetic field is distorted by the small-scale fields. The system thus is the **'soup' of cells** threaded by sinews of open field line.

A weak mean field— might lead to a large magnetic Kubo number, **since** $|\widetilde{B}^2|/B_0^2 \gg 1.$



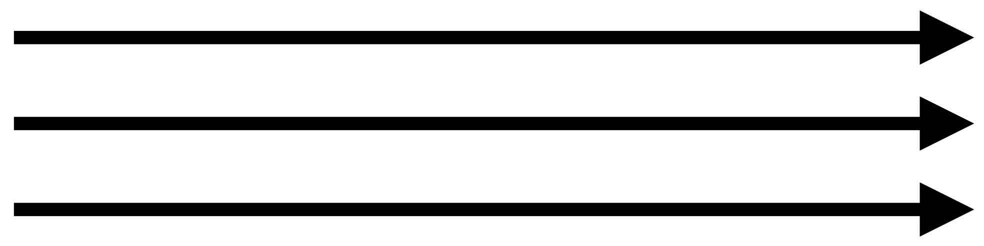
How we describe the stochastic magnetic field Tachocline

- The system is strongly nonlinear and simple quasi-linear method **fails**.

A "frontal assault" on calculating PV transport in an ensemble of tangled magnetic fields is a daunting task.

Goal: Find an analytical model beyond quasi-linear (QL) theory that can describe the stochastic-field-induced effect.

- How to describe the stochastic fields? Rechester & Rosenbluth (1978) suggested replacing the "full" problem with one where waves, instabilities, and transport are studied in the presence of an ensemble of prescribed, static, stochastic fields.



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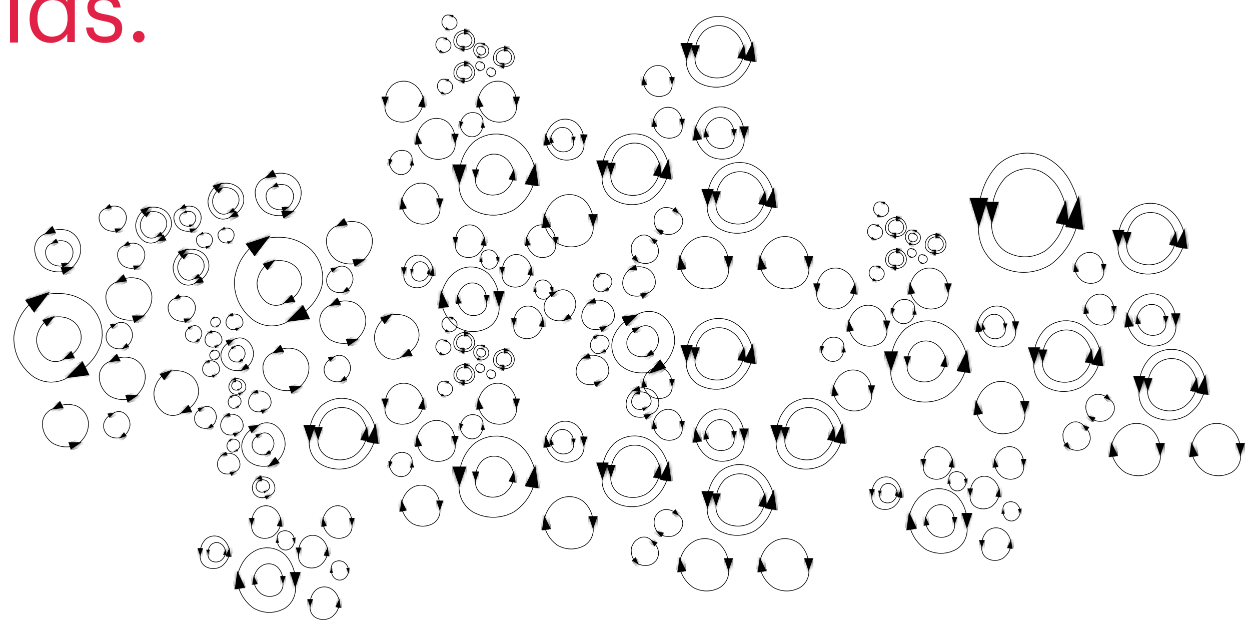


- Assumptions for stochastic field:
  1. Amplitudes of random fields distributed statistically.
  2. Auto-correlation length of fields is small so that ku_{mag} is small.

(delta correlation $l_{ac} \rightarrow 0$, such that $Ku_{mag} \equiv \frac{\tilde{u}\tau_{ac}}{\Delta_{eddy}} = \frac{l_{ac}|\tilde{\mathbf{B}}|}{\Delta_{\perp}B_0} < 1$, even $\tilde{B}/B_0 > 1$.)

→ Quasi-linear Theory becomes valid.

β–plane MHD Turbulence—Order of Scale

- Two-average method:

1.
$$\bar{F} = \int dR^2 \int dB_r \cdot P_{(B_{r,x},B_{r,y})} F$$

2.
$$\langle\ \rangle = \frac{1}{L}\int dx \frac{1}{T}\int dt$$

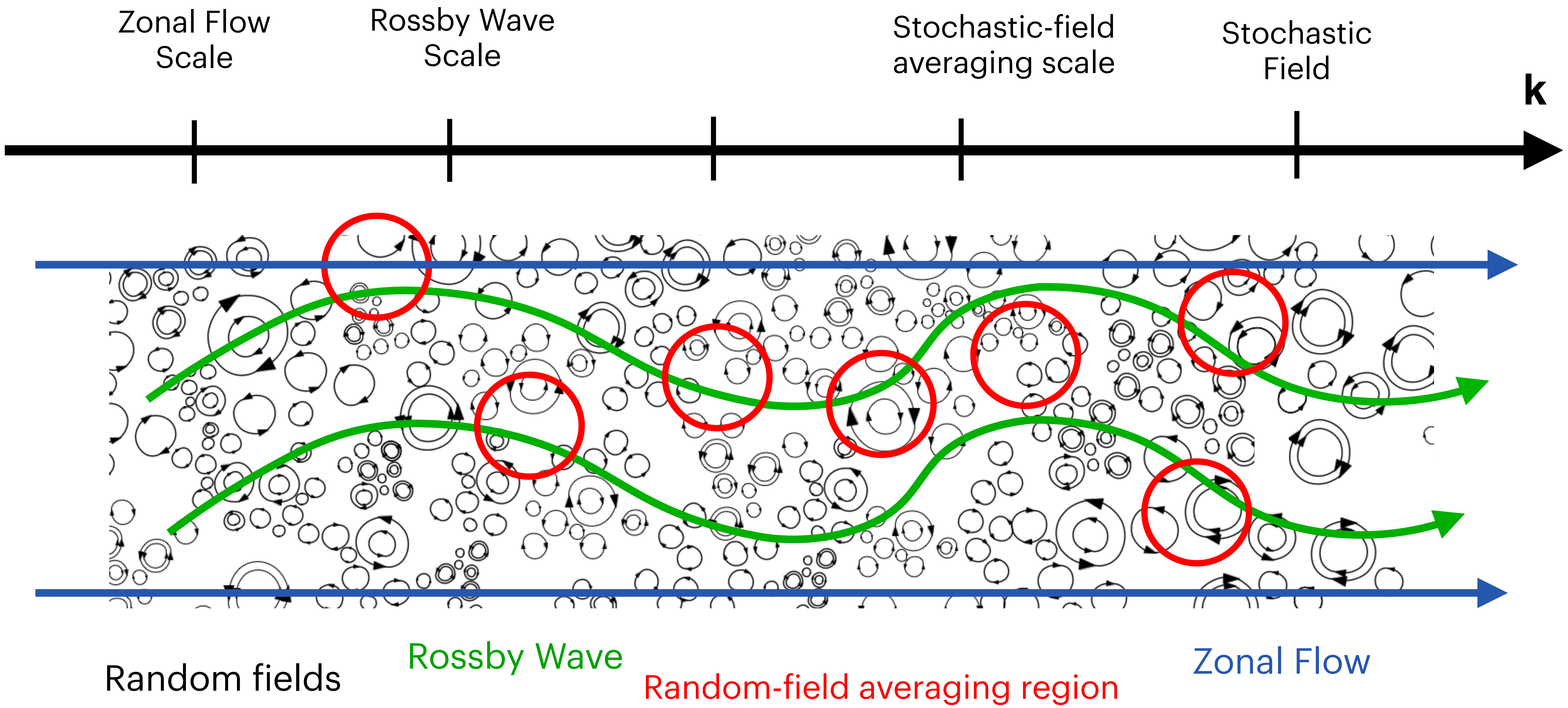
Ensemble average over the zonal scales

- Two main equation:

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_\perp \cdot \nabla_\perp\right)\zeta - \beta\frac{\partial \psi}{\partial x} = -\frac{(\mathbf{B}\cdot\nabla)(\nabla^2 A_z)}{\mu_0 \rho} + \nu(\nabla \times \nabla^2 \mathbf{u})$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_\perp \cdot \nabla_\perp\right)A = B_0\frac{\partial \psi}{\partial x} + \eta \nabla^2 A,$$

$l_{RM} = \sqrt{\frac{v_{x,A}}{\beta}}$ separates the Rossby and eddy regime



(Chen et al.,ApJ **892**, 24 (2020))

potential field	$\mathbf{A} = \mathbf{A}_l + \widetilde{\mathbf{A}} + \mathbf{A}_{st}$
magnetic f ield	$\mathbf{B} = \mathbf{B}_l + \widetilde{\mathbf{B}} + \mathbf{B}_{st}$
magnetic current	$\mathbf{J} = \mathbf{0} + \widetilde{\mathbf{J}} + \mathbf{J}_{st},$
stream function	$\psi = \langle\psi\rangle + \widetilde{\psi}$
flow velocity	$\mathbf{u} = \langle\mathbf{u}\rangle + \widetilde{\mathbf{u}}$
vorticity	$\zeta = \langle\zeta\rangle + \widetilde{\zeta}$

The Solar Tachocline Results

(Chen et al.,ApJ **892**, 24 (2020))

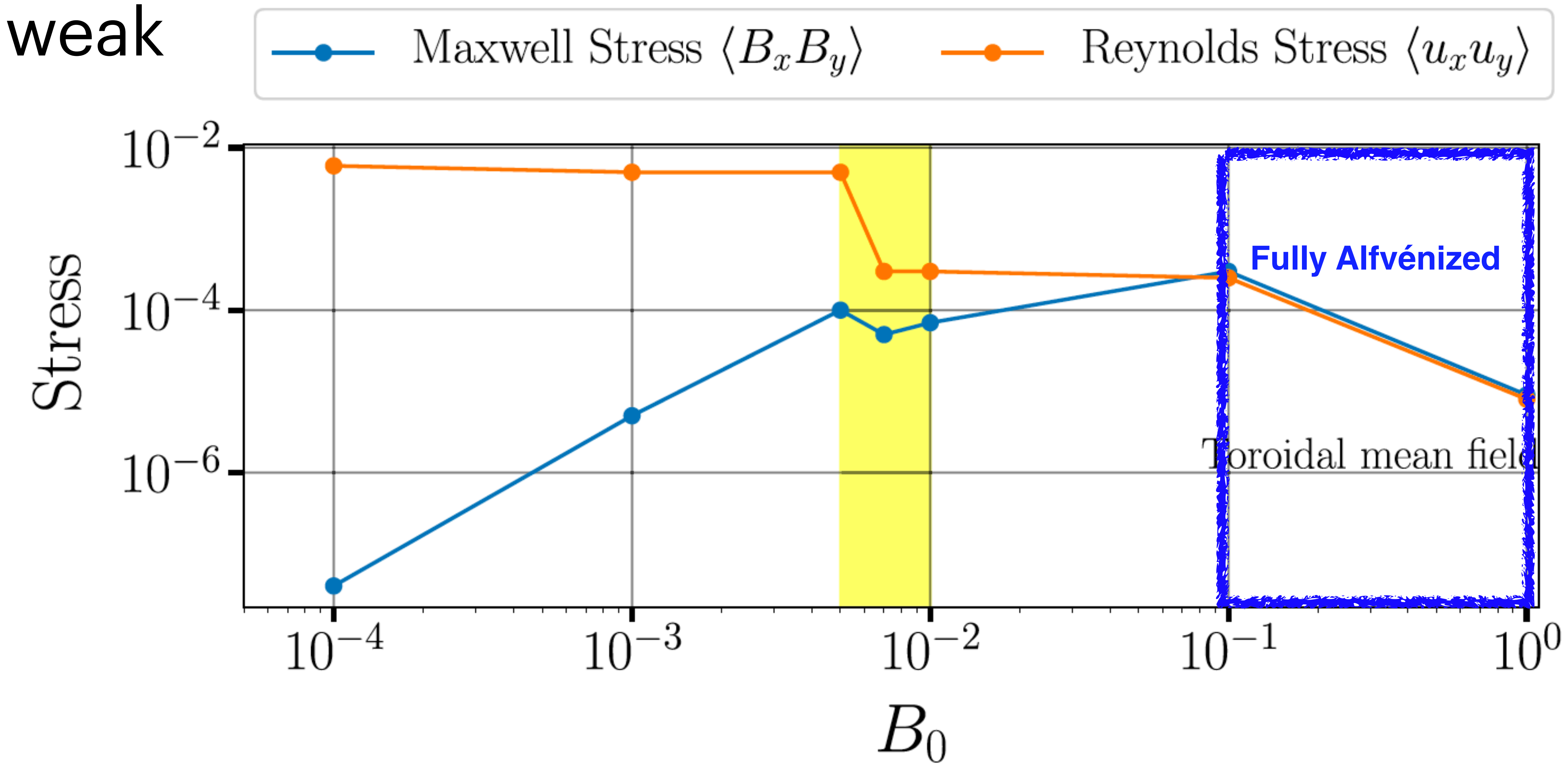
- Reynolds stress suppression when mean field is weak (before the system is fully Alfvénized).

$$\overline{\Gamma} = \langle \widetilde{u}_y \zeta \rangle = -\frac{\partial}{\partial y}\langle \widetilde{u}_y \widetilde{u}_x \rangle$$

PV flux

$$= -\sum_k |\widetilde{u}_{y,k}|^2 \frac{\nu k^2 + \frac{\overline{B^2_{st,y}} k_y^2}{\mu_0 \rho \eta k^2}}{\omega^2 + \left(\nu k^2 + \frac{\overline{B^2_{st,y}} k_y^2}{\mu_0 \rho \eta k^2}\right)^2}\left(\frac{\partial}{\partial y}\overline{\zeta} + \beta\right)$$

Reynolds stress dephasing ➔ PV flux suppression



The stochastic-field induced dephasing at the mean field intensity lower than that for the fully Alfvénization.

- Random magnetic fields have an effect on both the **PV flux** and the **magnetic drag: Two effects!**

$$\frac{\partial}{\partial t}\langle u_x \rangle = \langle \overline{\Gamma} \rangle - \frac{1}{\eta \mu_0 \rho}\langle \overline{B^2_{st,y}} \rangle \langle u_x \rangle + \nu \nabla^2 \langle u_x \rangle$$

PV flux   Magnetic drag

Stochastic fields dephase the Reynolds stress and hence suppress the zonal flow.

The Solar Tachocline Results

- Muti-scale dephasing:

$$\bar{\Gamma} = -\sum_k |\tilde{u}_{y,k}|^2 \frac{\nu k^2 + (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\eta k^2}{\omega^2 + \eta^2 k^4} + \frac{\overline{B_{st,y}^2} k^2}{\mu_0 \rho \eta k^2}}{\left(\omega - (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\omega}{\omega^2 + \eta^2 k^4}\right)^2 + \left(\nu k^2 + (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\eta k^2}{\omega^2 + \eta^2 k^4} + \frac{\overline{B_{st,y}^2} k^2}{\mu_0 \rho \eta k^2}\right)^2} \left(\frac{\partial}{\partial y}\zeta + \beta\right)$$

The large- and small-scale magnetic fields have a synergistic effect on the cross-phase in the Reynolds stress.

- Dispersion relation of the Rossby-Alfvén wave with stochastic fields:

(mean square) (Square mean)

$$\left(\omega - \omega_R + \frac{i\overline{B_{st,y}^2} k^2}{\mu_0 \rho \eta k^2} + i\nu k^2\right)\left(\omega + i\eta k^2\right) = \frac{B_{0,x}^2 k_x^2}{\mu_0 \rho}$$

$$\frac{\text{sping constant}}{\text{dissipation}} = \frac{\overline{B_{st}^2} k^2 / \mu_0 \rho}{\eta k^2}$$

(Chen et al.,ApJ **892**, 24 (2020))

Drag+dissipation effect: This implies that the tangled fields and fluids define a resisto-elastic medium.

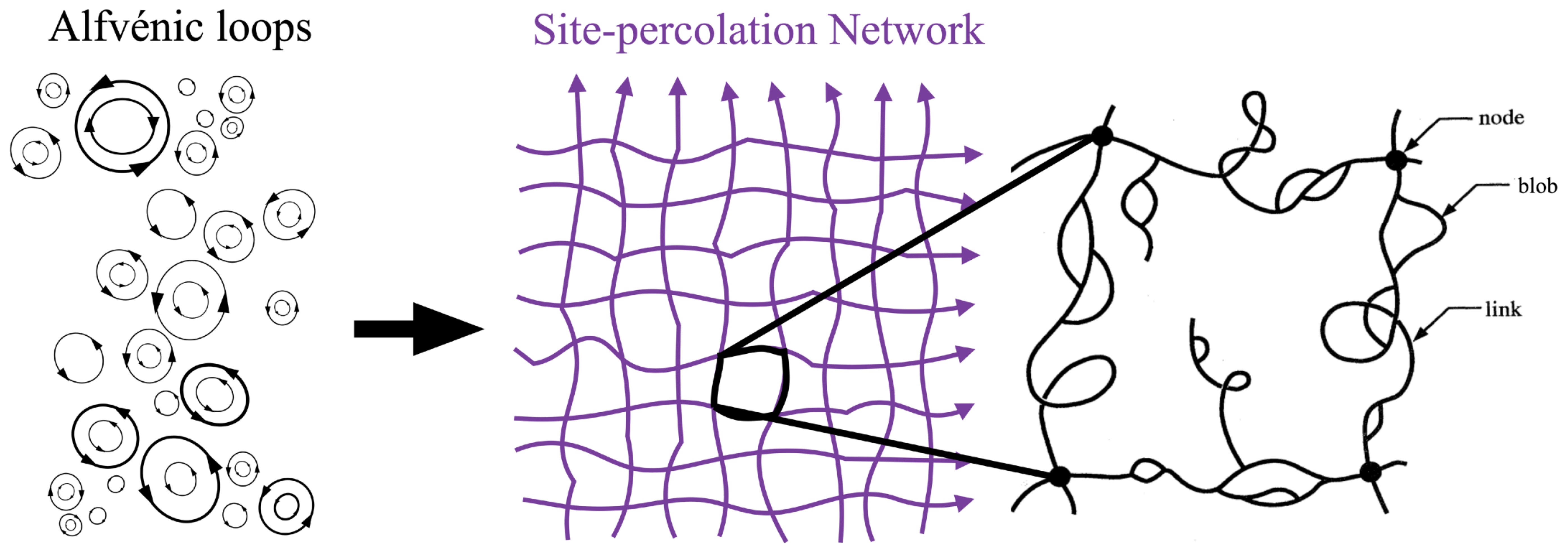
β-plane MHD Turbulence—Results

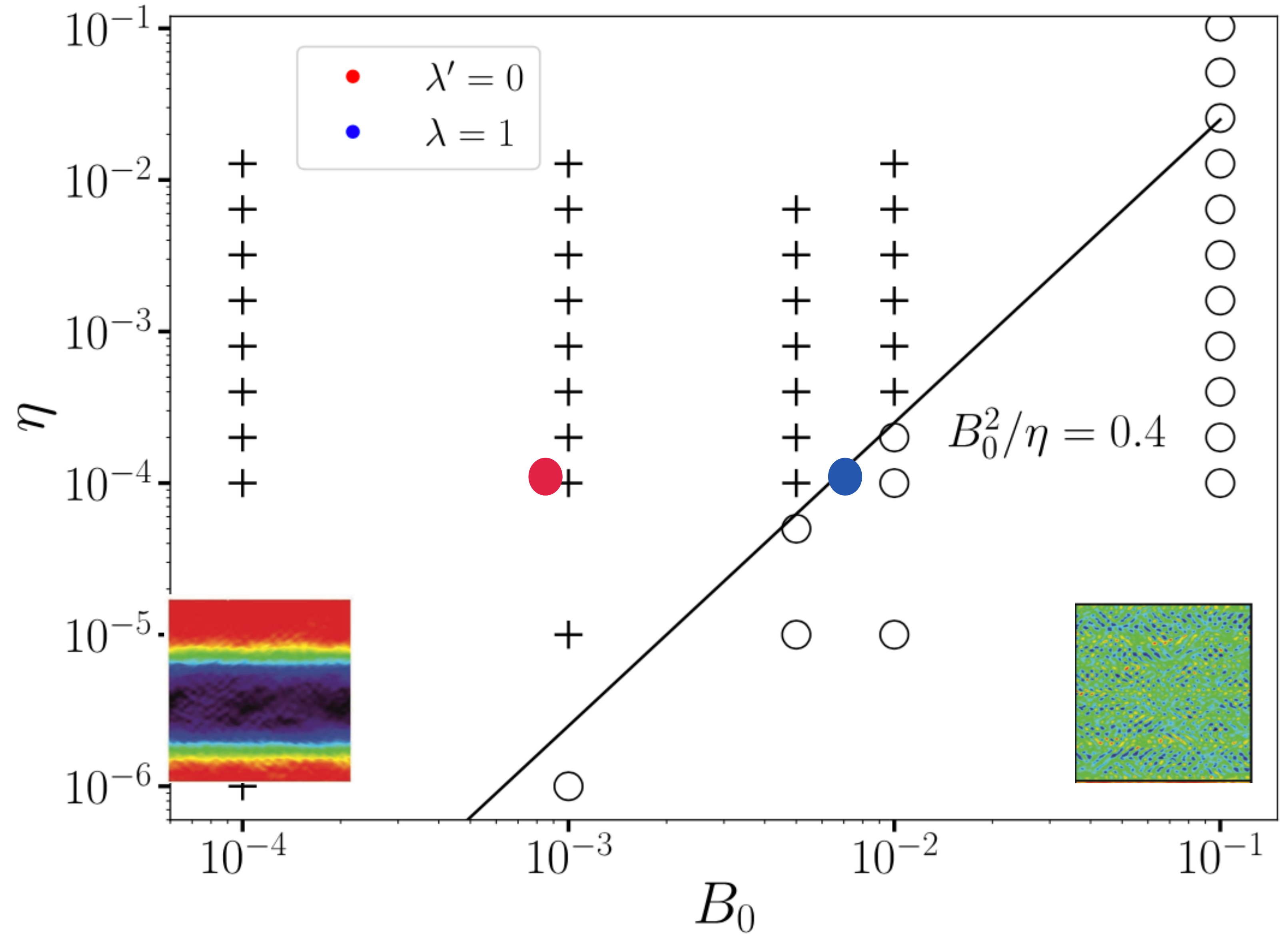
- Stochastic magnetic fields form a resisto-elastic medium:

Spring constant

$$\omega^2 + i(\alpha + \eta k^2)\omega - \left(\frac{\overline{B_{st,y}^2} k_y^2}{\mu_0 \rho} + \frac{B_0^2 k_x^2}{\mu_0 \rho}\right) = 0$$

Two contributions: $\overline{B_{st}^2} > B_0^2$





- We obtain the **Dimensionless parameters**
The transition parameter λ = 1, where the waves is critically damped and successfully predict the transition line.

$$\lambda = \left|\frac{\omega_{im}}{\omega_{re}}\right| \simeq \frac{\eta k^2 \omega_A^2}{\omega_R^3}, \text{ in the limit } \omega_R \gg \omega_A$$

(Chen et al.,ApJ **892**, 24 (2020))

β-plane MHD Turbulence—Conclusions

- Reynolds stress will undergo decoherence at levels of field intensities well below that of Alfvénization (where Maxwell stress balances the Reynolds stress).

$$\frac{\partial}{\partial t}\langle u_x\rangle = \langle\overline{\Gamma}\rangle - \frac{1}{\eta\mu_0\rho}\langle\overline{B^2_{st,y}}\rangle\langle u_x\rangle + \nu\nabla^2\langle u_x\rangle$$

PV flux   Magnetic drag

- Turbulent momentum transport in the tachocline is suppressed by the enhanced memory of **stochastically induced elasticity**.
  Both Spiegel & Zahn (1992) and Gough McIntyre (1998) models for the solar Tachocline are not correct. These two models both ignore strong stochastic fields of the tachocline.
  The truth here is 'neither pure nor simple' (apologies to Oscar Wilde).

Future Work

- This network can be fractal (multi-scale) and **intermittent**
  a. packing fractional factor: $\overline{B^2_{st}} \rightarrow p\overline{B^2_{st}}$)
  b. "fractons" (Alexander & Orbach 1982).

Momentum Transport at the edge of fusion devices—Drift-wave turbulence

Part I: Poloidal Reynolds Stress Dephasing

Why we study stochastic fields in fusion device?

Shear Flow

↓

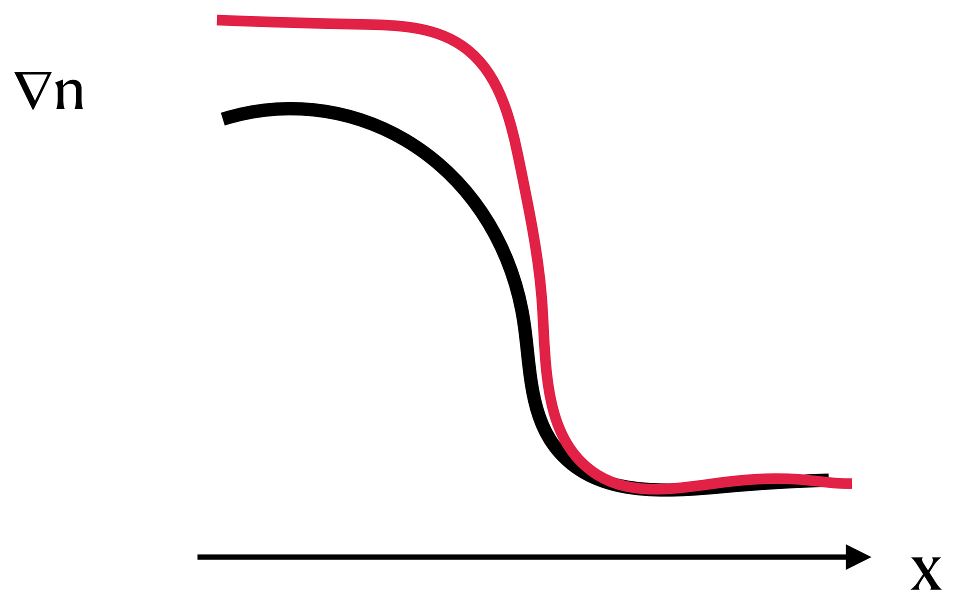
Quench Turbulence

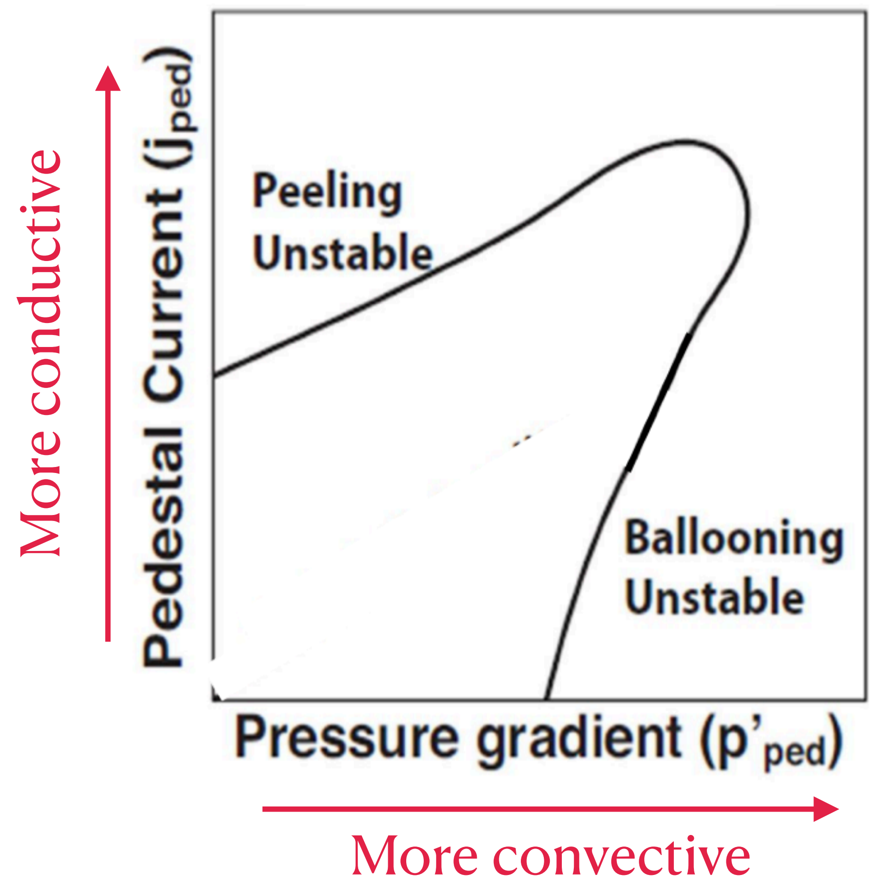
↓

Pedestal Formation

↓

Edge-Localized Mode (ELM)





Edge-Localized Mode (ELM)

↓

Edge gradient ↑

↓

Peeling-ballooning mode

↓

ELM Burst

Suppress (by inducing magnetic perturbation)

Boundary Control: Resonant Magnetic Perturbation (RMP)

- ELMs are quasi-periodic relaxation events occurring at edge pedestal in H-mode plasma.
- ELMs can damage wall components of a fusion device.

Stochastic field effect is important for boundary control

Shear Flow

Edge-Localized Mode (ELM)

ELM Burst

Boundary Control: Resonant Magnetic Perturbation (RMP)

Suppress (by inducing magnetic perturbation)

Stochasticity: Overlapping of Magnetic islands.

Trade off: RMPs controls gradients and mitigates ELM, but raise the **power threshold**.

Key Questions:

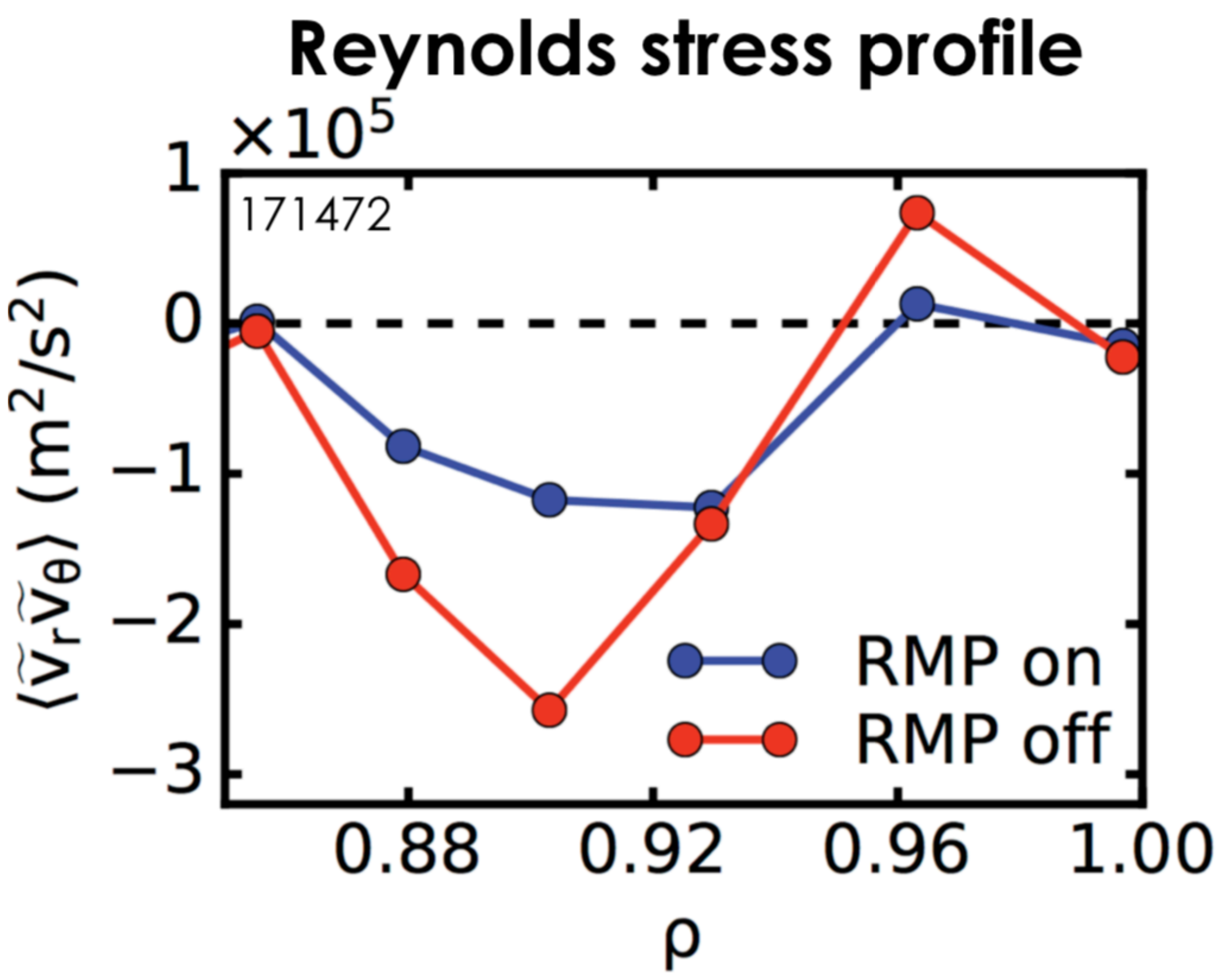
How RMPs influence the Reynolds stress and hence suppress the zonal flow?

How stochastic fields increase the power threshold of L-H transition?

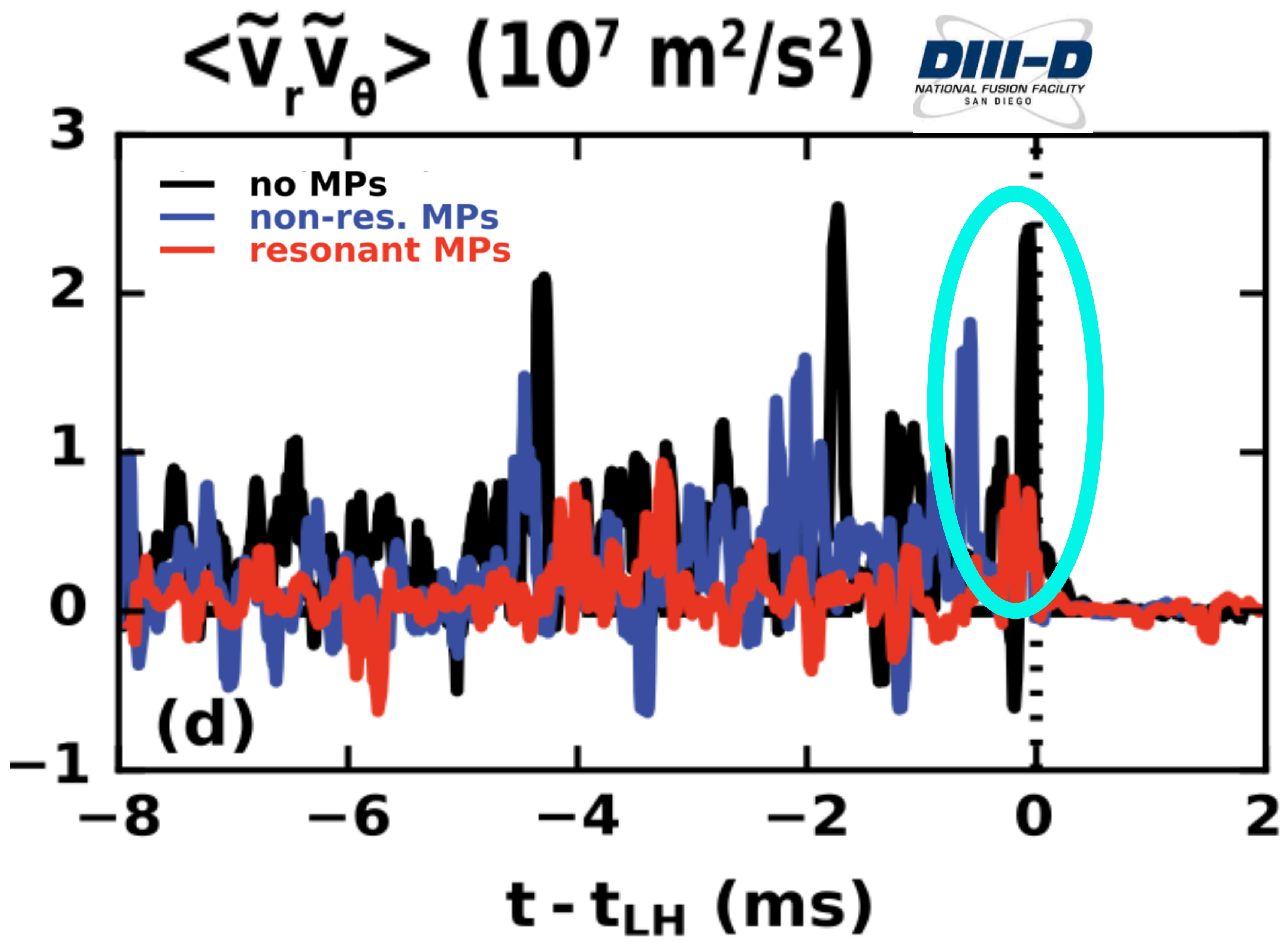
We examine the physics of stochastic fields interaction with zonal flow near the edge.

(Chen et al.,PoP **28**, 042301 (2021))

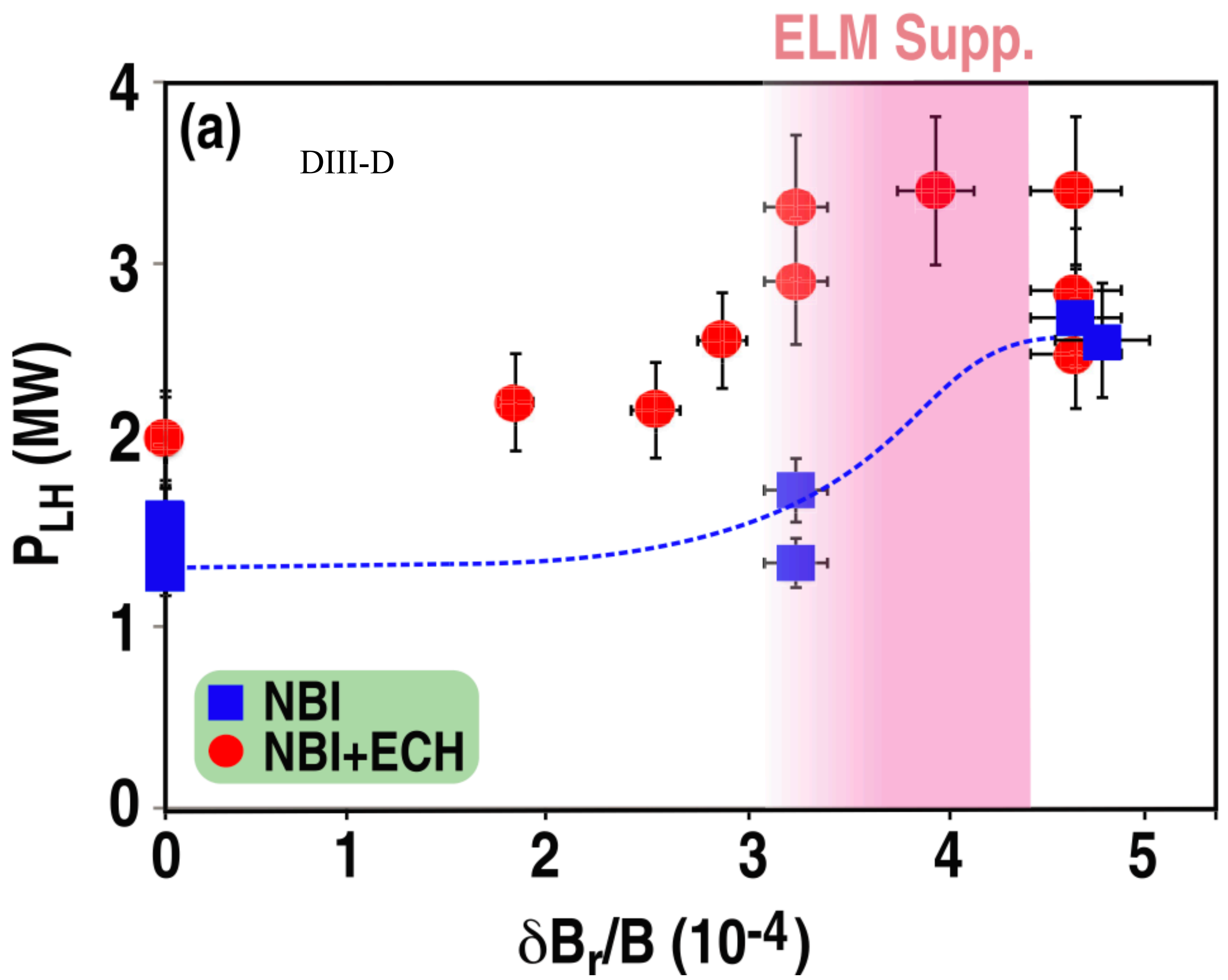
Experimental Results with RMP for L–H Transition — fluctuations



(D. Kriete et al, PoP **27** 062507 (2020))



(D. Kriete et al, PoP **27** 062507 (2020))



(L. Schmitz et al, NF **59** 126010 (2019))

DIII-D Experimental results: RMPs lower the Reynolds stress and increase the power threshold of L-H transition.

This section: Mean poloidal (zonal flow) in stochastic fields.

$$\langle E_r \rangle = \frac{\nabla \langle p_i \rangle}{ne} - \langle \mathbf{u} \rangle \times \langle \mathbf{B} \rangle$$

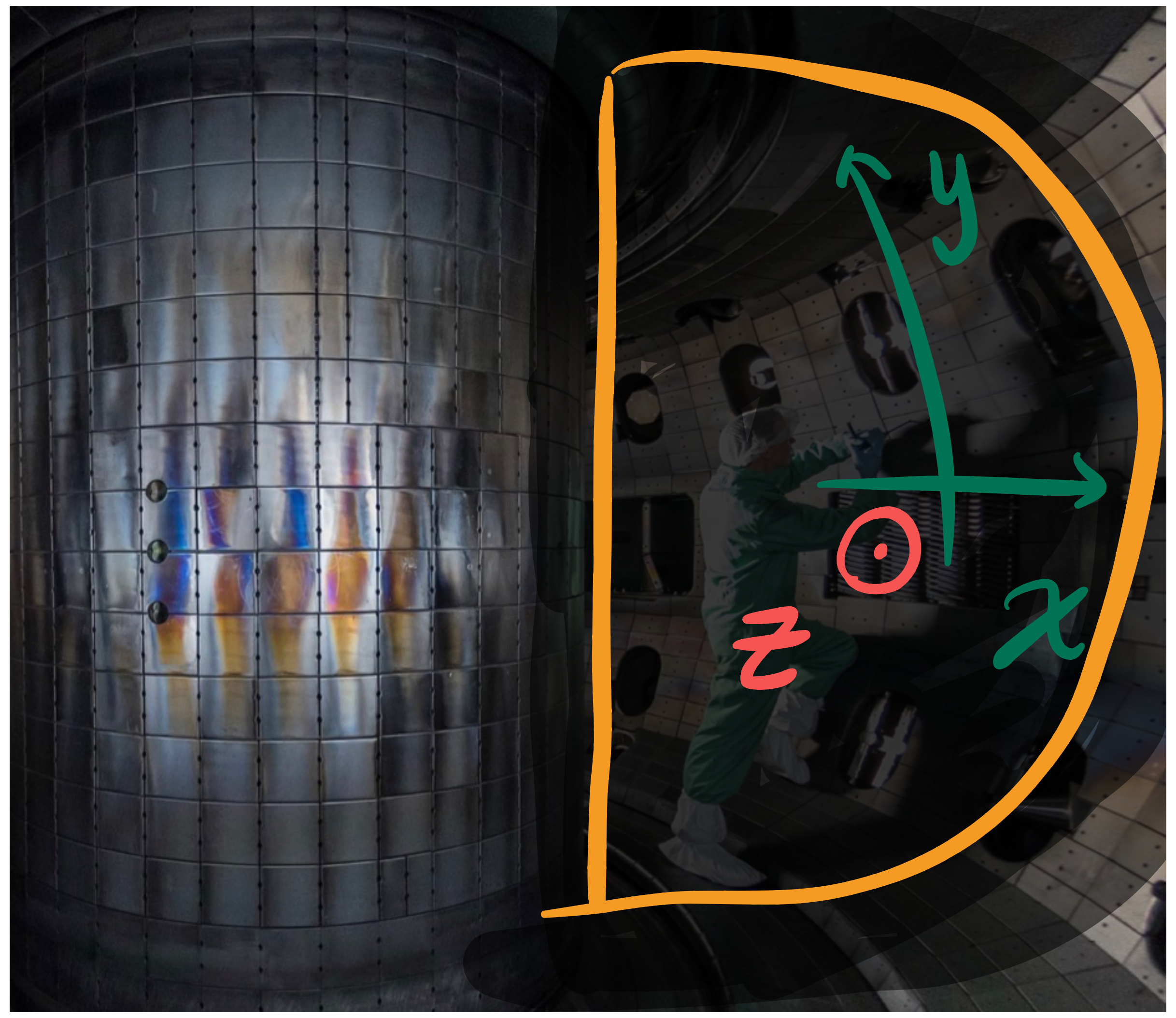
Next section: mean toroidal flow in stochastic fields.

Model

Small fluid and magnetic Kubo number.

1. Cartesian coordinate: strong mean field B_0 is in z direction (3D).
2. Rechester & Rosenbluth (1978): waves, instabilities, and transport are studied in the presence of **external excited, static, stochastic fields.**
3. $\underline{k} \cdot \underline{B} = 0$ (or $k_{\parallel} = 0$) **resonant at rational surface in third direction —** $\omega \to \omega \pm v_A k_z$, **and** Kubo number: $Ku_{mag} = \dfrac{l_{ac}|\widetilde{\mathbf{B}}|}{\Delta_{\perp} B_0} < 1$).
4. Four-field equations —
   (a) Potential vorticity equation—vorticity $-\nabla^2\psi \equiv \zeta$
   (b) Induction equation — $\mathbf{A}$, $\mathbf{J}$
   (c) Pressure equation — $\mathbf{p}$
   (d) Parallel flow equation — $\mathbf{u_z}$

Well beyond HM model



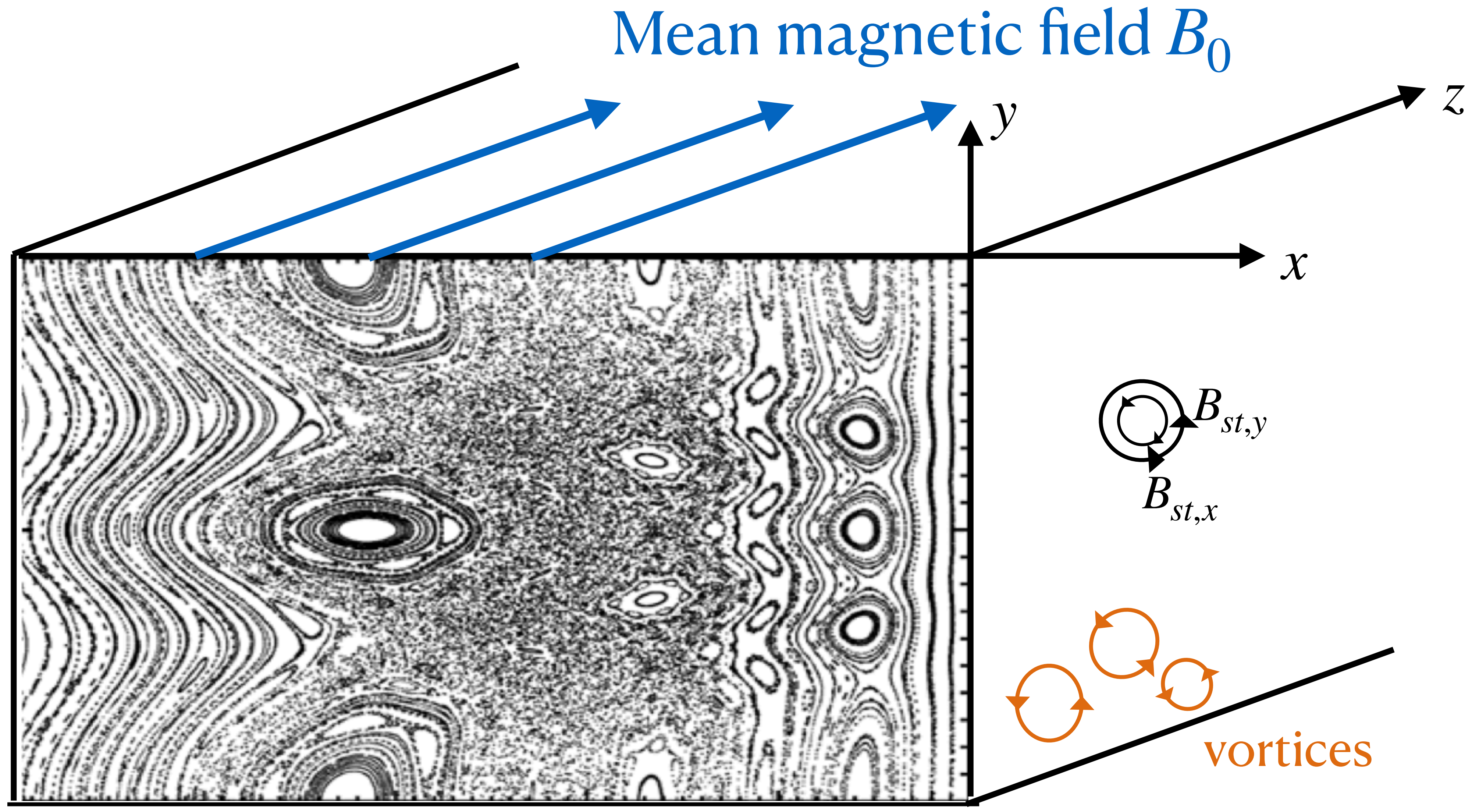
We use mean field approximation:

$\zeta = \langle \zeta \rangle + \tilde{\zeta},$ Perturbations produced by turbulences

$$where\ \langle\ \rangle = \frac{1}{L}\int dx \frac{1}{T}\int dt \qquad \langle \zeta \rangle = \frac{\partial v_{E\times B}}{\partial x} \quad (E \times B \text{ shear})$$

ensemble average over the zonal scales

We define rms of normalized stochastic field $b \equiv \sqrt{\overline{(B_{st}/B_0)^2}}$

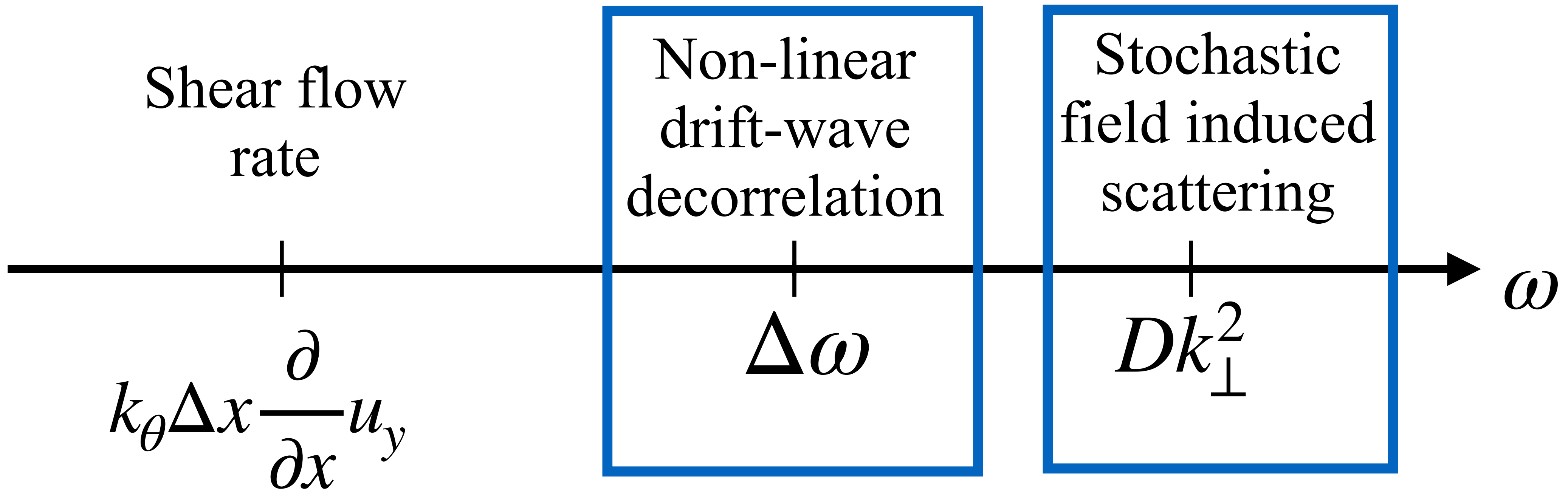


Magnetic islands overlapping forms stochastic

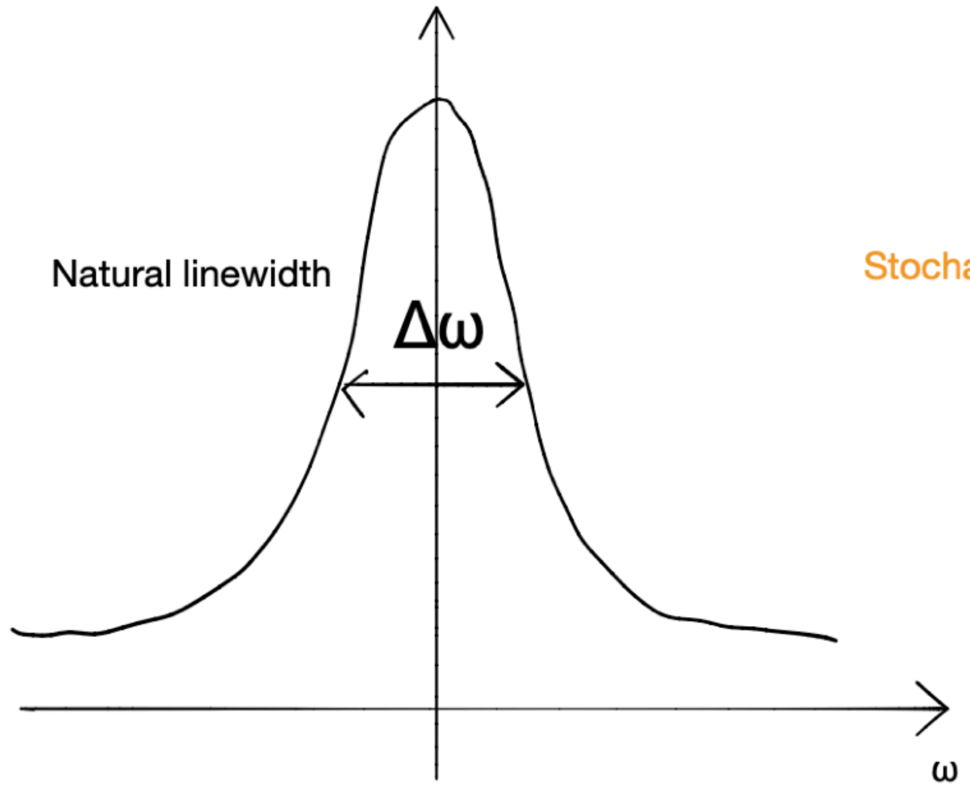
(Chen et al.,PoP **28**, 042301 (2021))

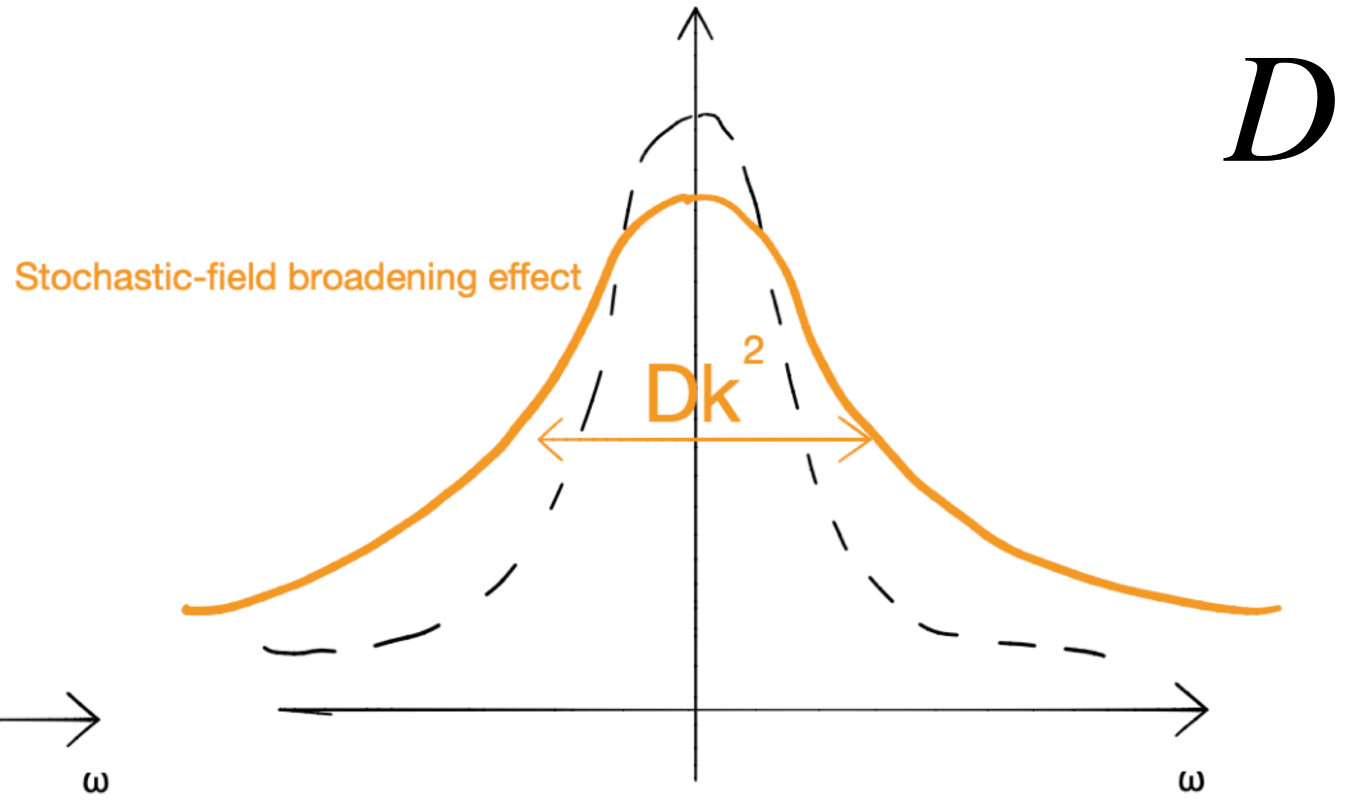
When does stochastic field effect becomes significant?

We consider timescales: (Chen et al.,PoP **28**, 042301 (2021))



Stochastic field decoherence beats the self-decoherence.





$$D \equiv v_A D_M = v_A \sum_k \pi \delta(k_z) b_k^2 \propto B_{st}^2$$

(Independent of B_0)

Magnetic diffusivity

Auto-correlation length l_{ac} ($\propto v_A$)

Perturbations propagate ultimately **in** $\perp$ (along stochastic fields)
$\rightarrow$ characteristic velocity (v_A) emerges from the calculation of $\underline{\nabla} \cdot \underline{J} = 0$

Dimensionless Parameters

How `stochastic' is magnetic field?

Alfvénic Dispersion		Stochastic broadening
$v_A/L_{\|\|}$	v.s	$Dk_\perp^2$

(excited by drift-Alfvénic coupling)

Ku_{mag} **(Magnetic Kubo number)**

$$\equiv \frac{\textbf{stochastic field scattering length}}{\textbf{perpendicular magnetic fluctuation size}} = \frac{l_{ac} b}{\Delta_{\textbf{eddy}}} \lesssim 1,$$

(for a b given)

Two dimensionless Parameters: $Dk_\perp^2 > \Delta\omega$

$$\begin{cases} l_{ac} \simeq Rq \\ \epsilon \equiv L_n/R \sim 10^{-2} \\ \beta \simeq 10^{-2} - 10^{-3} \\ \rho_* \equiv \frac{\rho_s}{L_n} \simeq 10^{-2} - 10^{-3} \end{cases}$$

1.

$$b^2 \equiv \left(\frac{\delta B_r}{B_0}\right)^2 > \sqrt{\beta}\rho_*^2 \frac{\epsilon}{q} \sim 10^{-8}$$

Criterion for stochastic fields effect important to L-H transition.

2.

Broadening parameter

$$\alpha \equiv \frac{b^2}{\rho_*^2 \sqrt{\beta}} \frac{q}{\epsilon} > 1$$

$\alpha = 1$:
stochastic broadening = natural linewidth

Decoherence of eddy tilting feedback

Snell's law:

$$\frac{d}{dt}k_x = -\frac{\partial(\omega_0 + u_y k_y)}{\partial x} = -k_y \frac{\partial u_y}{\partial x}$$

Gives an non-zero $\langle k_x k_y \rangle$

$\rightarrow \langle \tilde{u}_x \tilde{u}_y \rangle \propto \langle k_x k_y \rangle$

shear flow

Self-feedback loop:

The $E \times B$ shear generates the $\langle k_x k_y \rangle$ correlation and hence support the non-zero Reynolds stress.

$$\langle \tilde{u}_x \tilde{u}_y \rangle \simeq \sum_k \frac{|\tilde{\phi}_k|^2}{B_0^2}\left(k_y^2 \frac{\partial u_y}{\partial x} \tau_c\right)$$

The Reynold stress modifies the shear via momentum transport.

Shear flow reinforce the self-tilting.

Dispersion relation of drift-Alfvén coupling

$$\omega^2 - \omega_D \omega - k_\parallel^2 v_A^2 = 0$$

+

Stochastic Fields Effect

$$k_\parallel = k_\parallel^{(0)} + \underline{b}_\perp \cdot \underline{k}_\perp$$

$$\omega = \omega_D + \delta\omega$$

$$(\omega_D + \delta\omega)^2 - \omega_D(\omega_D + \delta\omega) - (k_\parallel + \underline{b} \cdot \underline{k}_\perp)^2 v_A^2 = 0$$

eigen-frequency shift

$$\delta\omega \simeq \frac{v_A^2}{\omega_D}\left(2k_\parallel \underline{b} \cdot \underline{k}_\perp + (\underline{b} \cdot \underline{k}_\perp)^2\right)$$

$$\omega_D \text{ (drift wave turbulence frequency)} \equiv \frac{k_y \rho_s C_s}{L_n}$$

Decoherence of eddy tilting feedback

Expectation frequency:

$$\left\langle \delta\omega \simeq \frac{v_A^2}{\omega_0}(2k_{\parallel}\underline{b}\cdot\underline{k}_{\perp} + (\underline{b}\cdot\underline{k}_{\perp})^2) \right\rangle$$

Ensemble average of eigen-frequency shift

$$\langle\delta\omega\rangle \simeq \frac{v_A^2}{\omega_0}\langle(\underline{b}\cdot\underline{k}_{\perp})^2\rangle = \frac{1}{2}\frac{v_A^2}{\omega_0}b^2k_{\perp}^2$$

$$\omega = \omega_D + \delta\omega$$

$$\langle\omega\rangle \simeq \omega_D + \frac{1}{2}\frac{v_A^2}{\omega_D}b^2k_{\perp}^2$$

Snell's law:

$$\frac{d}{dt}k_x = -\frac{\partial\omega_k}{\partial x}$$

$$= -k_y\frac{\partial u_y}{\partial x} - \frac{1}{2}\frac{v_A^2k_{\perp}^2}{\omega_D}\frac{\partial b^2}{\partial x}$$

Due to the Ensemble average eigen-frequency shift

Self-feedback loop is broken by b^2:

$$\langle\tilde{u}_x\tilde{u}_y\rangle \simeq \sum_k \frac{|\widetilde{\phi}_k|^2}{B_0^2}(k_y^2\frac{\partial u_y}{\partial x}\tau_c + \frac{1}{2}k_y\frac{v_A^2k_{\perp}^2}{\omega_D}\frac{\partial b^2}{\partial x}\tau_c)$$

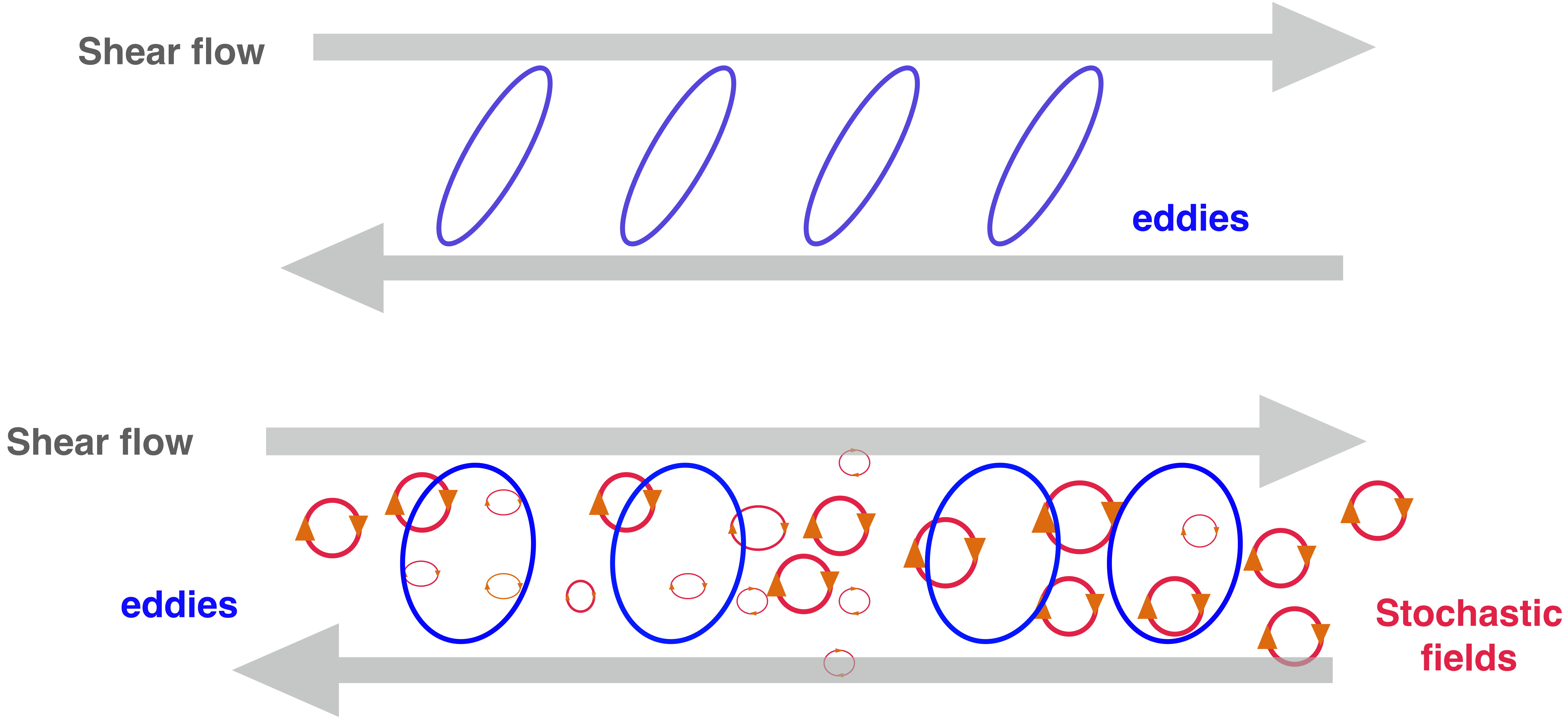
Stochastic dephasing

Stochastic fields (random ensemble of elastic loops) act as elastic loops and resist the tilting of eddies.

→change the cross-phase btw $\tilde{u}_x$ and $\tilde{u}_y$.

(Chen et al.,PoP **28**, 042301 (2021))

Decoherence of eddy tilting feedback



Stochastic fields interfere with shear-tilting feedback loop.

Results—Suppression of PV diffusivity

The ensemble average Reynolds force $\frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle$:

$$\text{PV flux} = \langle \tilde{u}_x \tilde{\zeta} \rangle = \frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle = -D_{PV}\frac{\partial}{\partial x}\langle \zeta \rangle + F_{res}\kappa \frac{\partial}{\partial x}\langle p \rangle$$

PV diffusivity — Residual Stress — Curvature

Suppressed by stochastic fields

Taylor Identity: $\langle \tilde{u}_x \tilde{\zeta} \rangle = \frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle$

$\langle \zeta \rangle = \frac{\partial v_{E\times B}}{\partial x}$ $(E \times B \text{ shear})$

$$D_{PV} = \sum_{k\omega} |\tilde{u}_{x,k\omega}|^2 \frac{v_A b^2 l_{ac} k^2}{\bar{\omega}^2 + \left(v_A b^2 l_{ac} k^2\right)^2}$$

PV transport will be suppressed by stochastic fields via decoherence.

$$F_{res} \simeq \sum_{k\omega} \frac{-2k_y}{\bar{\omega}\rho} D_{PV,k\omega}$$

$$\bar{\omega} \equiv \omega - \langle u_y \rangle k_y$$

$$\text{Zonal flow acceleration} = \frac{\partial}{\partial t}\langle u_y \rangle = D_{PV}\frac{\partial}{\partial x}\langle \zeta \rangle - F_{res}\kappa \frac{\partial}{\partial x}\langle p \rangle$$

Zonal flow acceleration is slowed down by the stochastic field.

This stochastic dephasing effect is insensitive to turbulent modes, e.g. ITG, TEM,...etc.

Macroscopic Impact

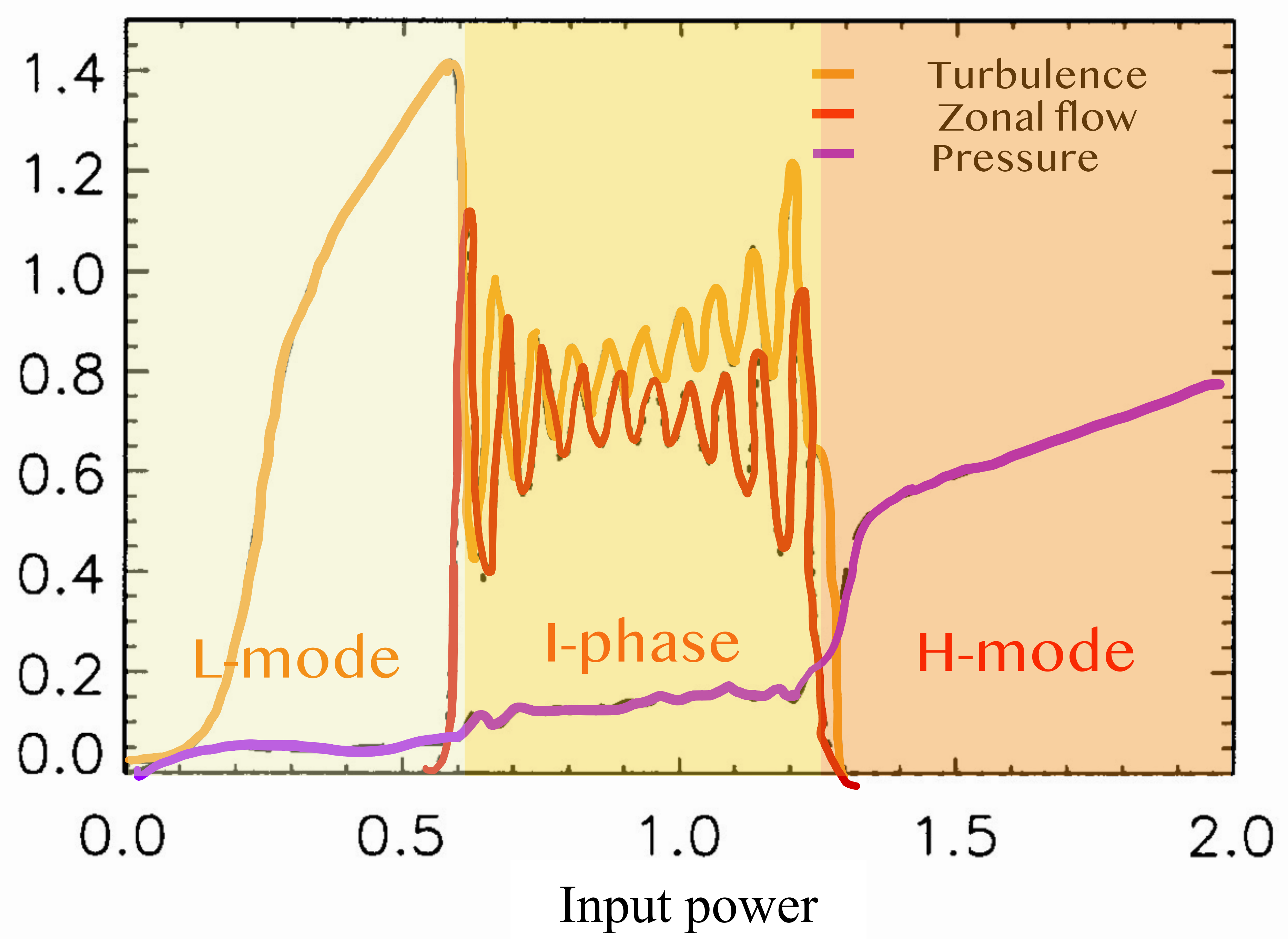
Results — Increment of P_{LH}

Stochastic field stress dephasing effect requires: $\Delta\omega \leq k_\perp^2 D$ (where $D = D_M v_A$).

This gives **Broadening parameter** (α): $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} > 1$

α quantifies the strength of stochastic dephasing.

$$\begin{cases} l_{ac} \simeq Rq \\ \epsilon \equiv L_n/R \sim 10^{-2} \\ \beta \equiv \frac{P_{thermal}}{P_{mag}} \simeq 10^{-2\sim-3} \\ \rho_* \equiv \frac{\text{gyro-radius}}{\text{density scale length}} \\ \equiv \frac{\rho_s}{L_n} \simeq 10^{-2\sim-3} \\ q(\text{stafety factor}) \equiv \frac{rB_t}{RB_p} \end{cases}$$



Kim-Diamond Model

(Kim & Diamond, PoP **10**, 1698 (2003))

This reduce model for the L-H transition is useful for testing trends in power threshold increment induced by stochastic fields.

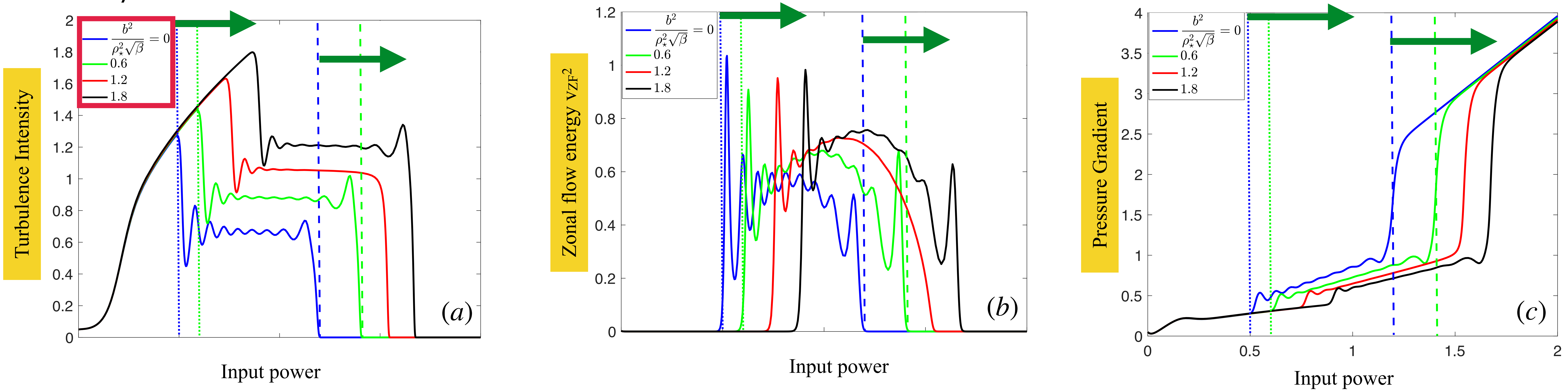
Predator: zonal flow
prey: turbulence

We expect stochastic fields to raise L-H transition thresholds.

Results — Increment of P_{LH}

$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} = 0.0, 0.2, 0.4, 0.6, 0.8...., 2.0$$

$\alpha \neq 0$

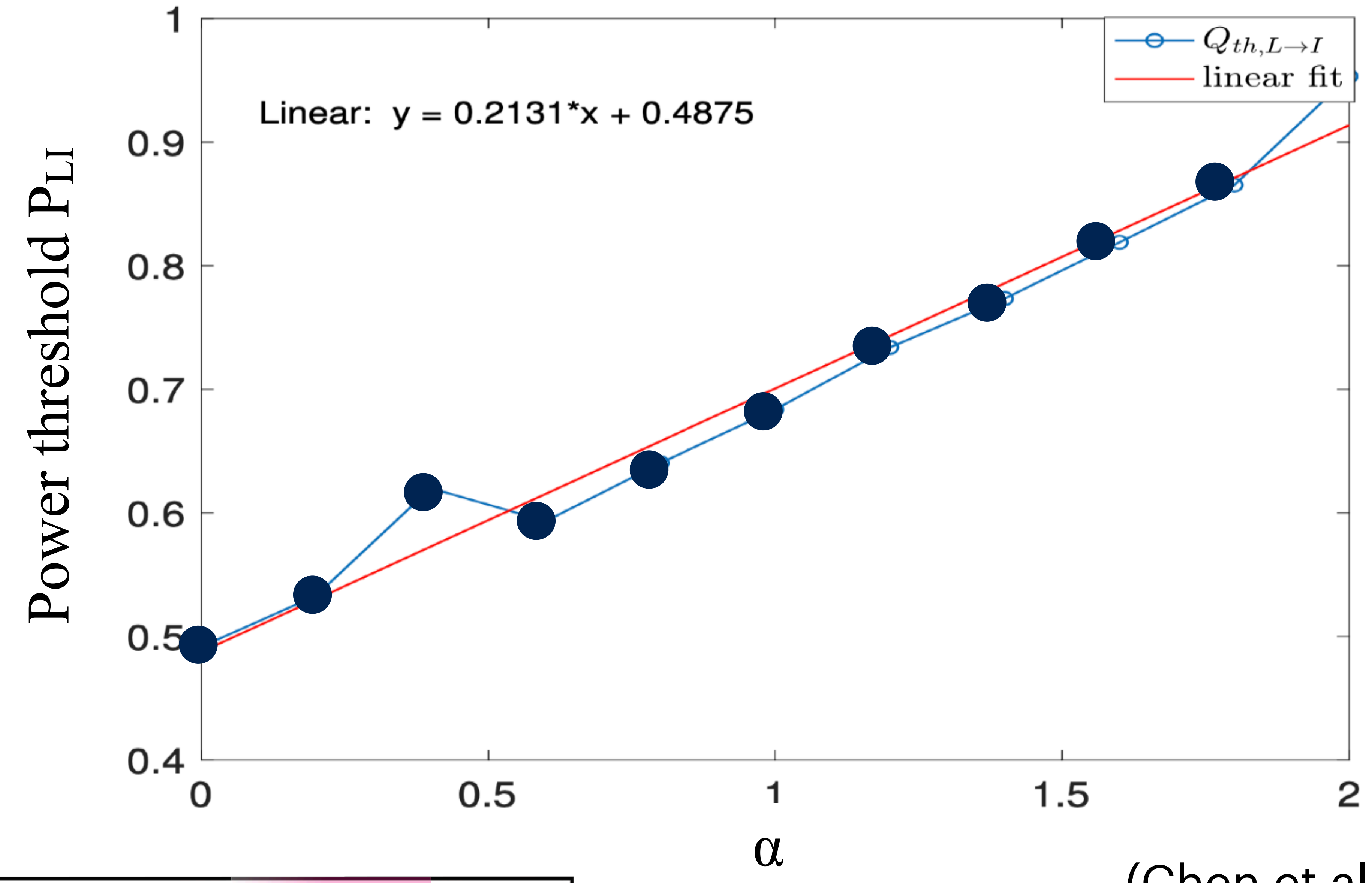


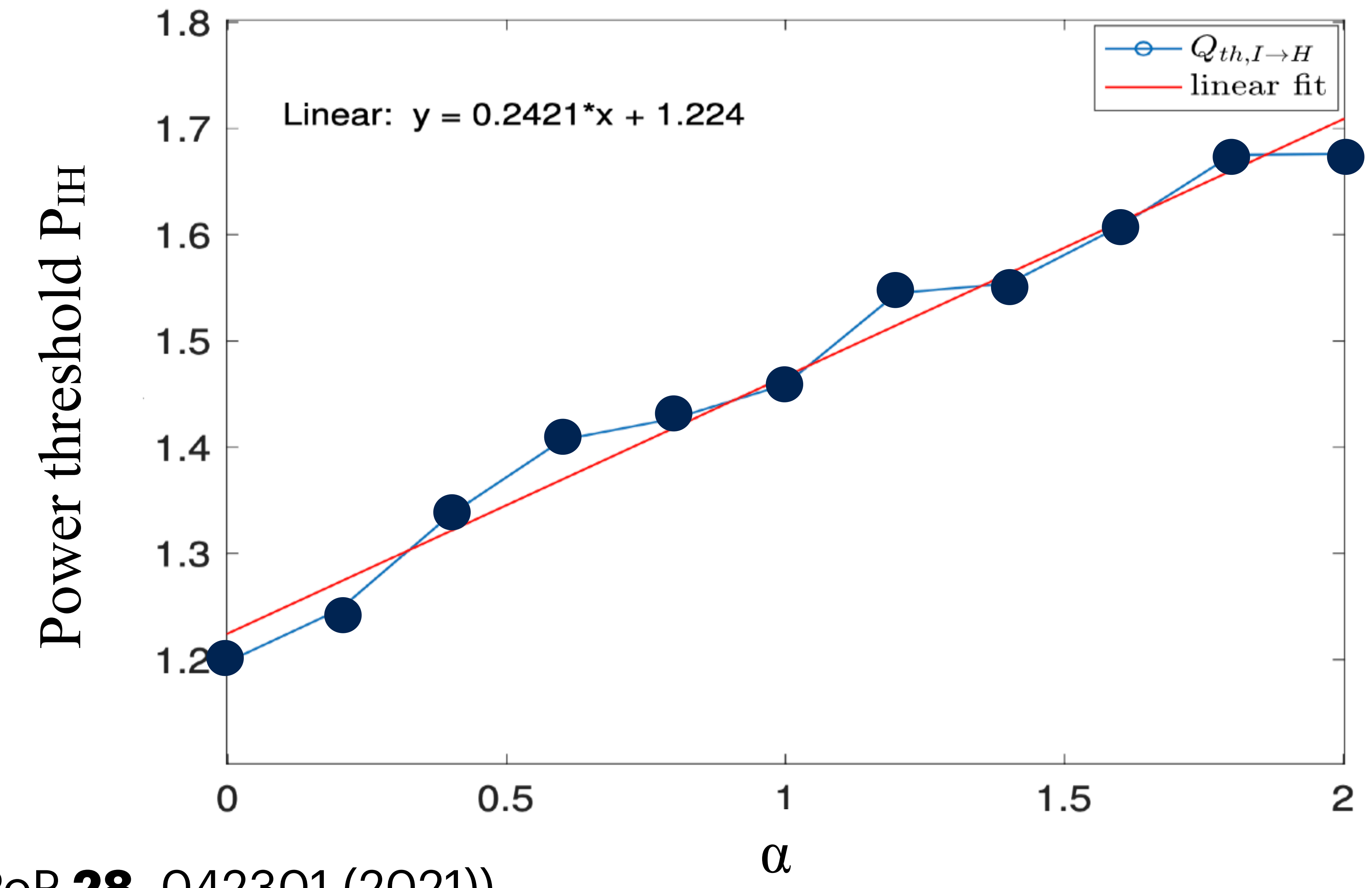
The threshold increase due to stochastic dephasing effect is seen in turbulence intensity, zonal flow, and pressure gradient.

(Chen et al.,PoP **28**, 042301 (2021))

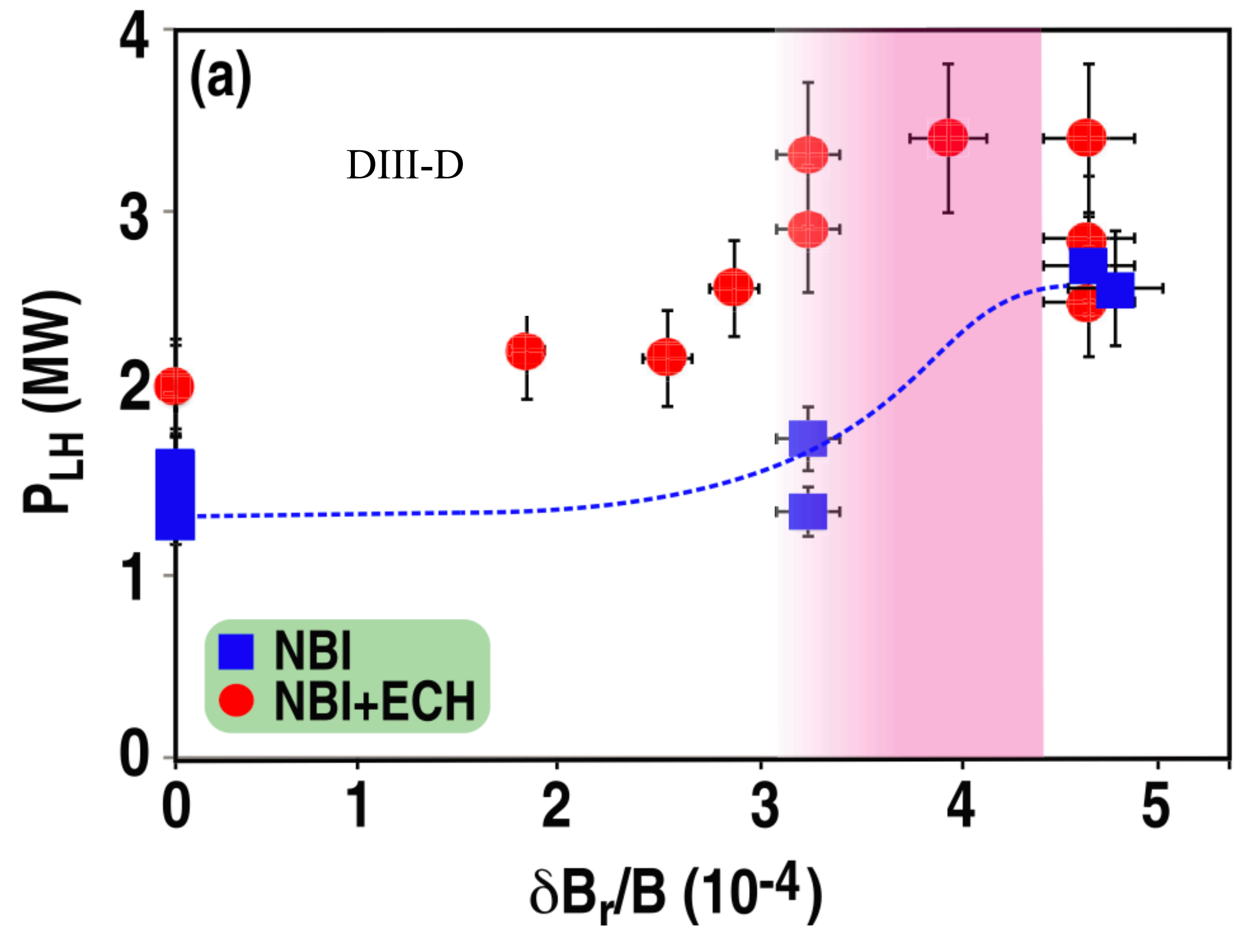
Results — Increment of P_{LH}

Stochastic field stress dephasing effect requires: $\Delta\omega \leq k_\perp^2 D$





(Chen et al.,PoP **28**, 042301 (2021))



(L. Schmitz et al, NF **59** 126010 (2019))

Broadening parameter

$$\alpha \equiv \frac{b^2}{\sqrt{\beta\rho_*^2}}\frac{q}{\epsilon}$$

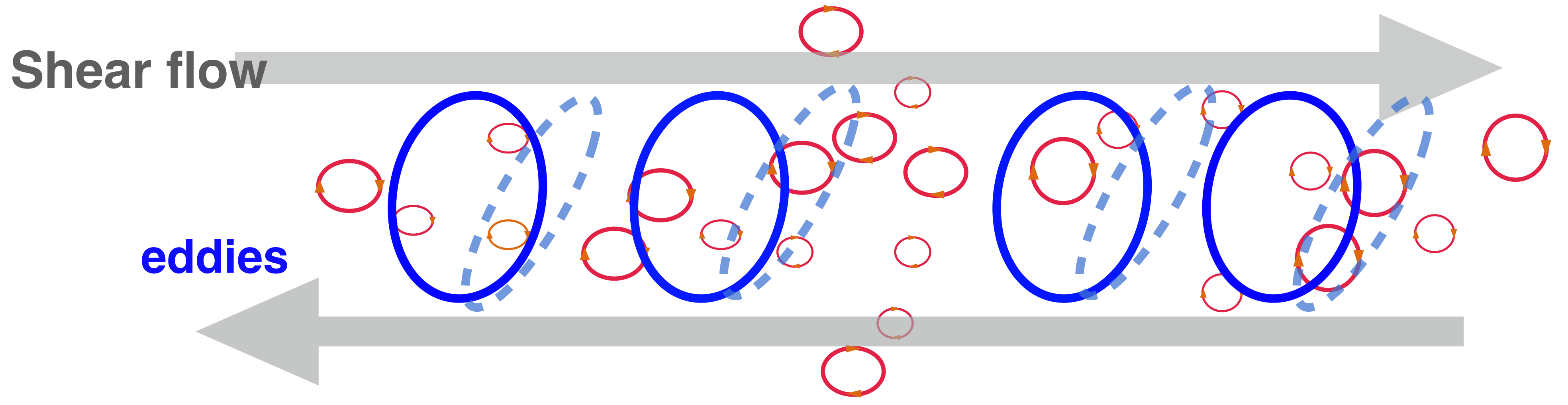
α quantifies the strength of stochastic dephasing.

ITER has smaller ρ_*, leading to a higher α.

The threshold increase linearly, in proportional to α. This is due to stochastic dephasing effect.

Conclusions

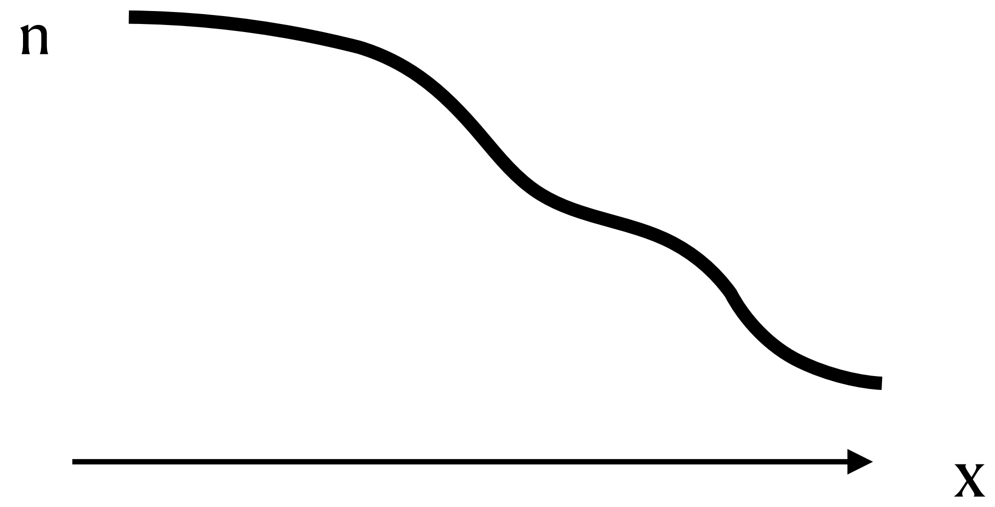
- **Dephasing effect** caused by stochastic fields quenches poloidal Reynolds stress (e.g. $\Delta\omega < Dk_\perp^2$). Here, $D = v_A D_M$.



- Stochastic fields interfere with shear-tilting feedback loop, and prevent the production of zonal flow.
- b^2 shift L-H threshold to higher power, in proportional to $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$. ITER has smaller ρ_*, leading to a higher α.

Future Works

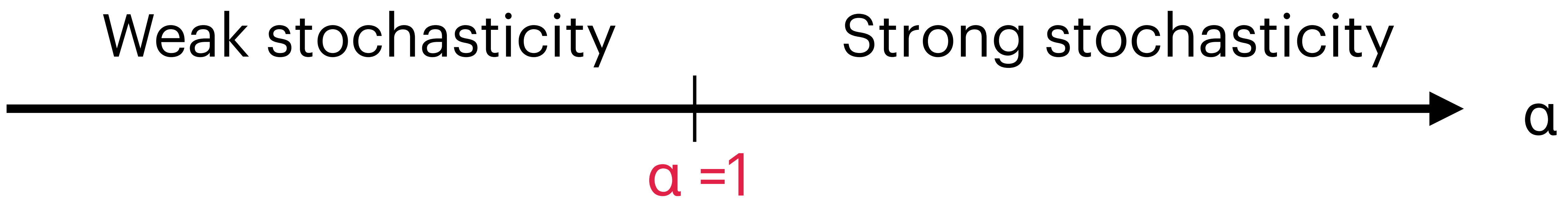
- We study the scale corrugation of staircases in presence of stochastic fields.
- Detailed calculations for symmetry breaking of toroidal residual stress.



Takeaways for Experimentalists

- Reynolds stress suppression due to stochastic dephasing→ generation of zonal flow is suppressed.
  Zonal intensity stays the same but damping occurs due to the stochastic dephasing.

- Stochastic fields broadening effect can be parameterized by α.

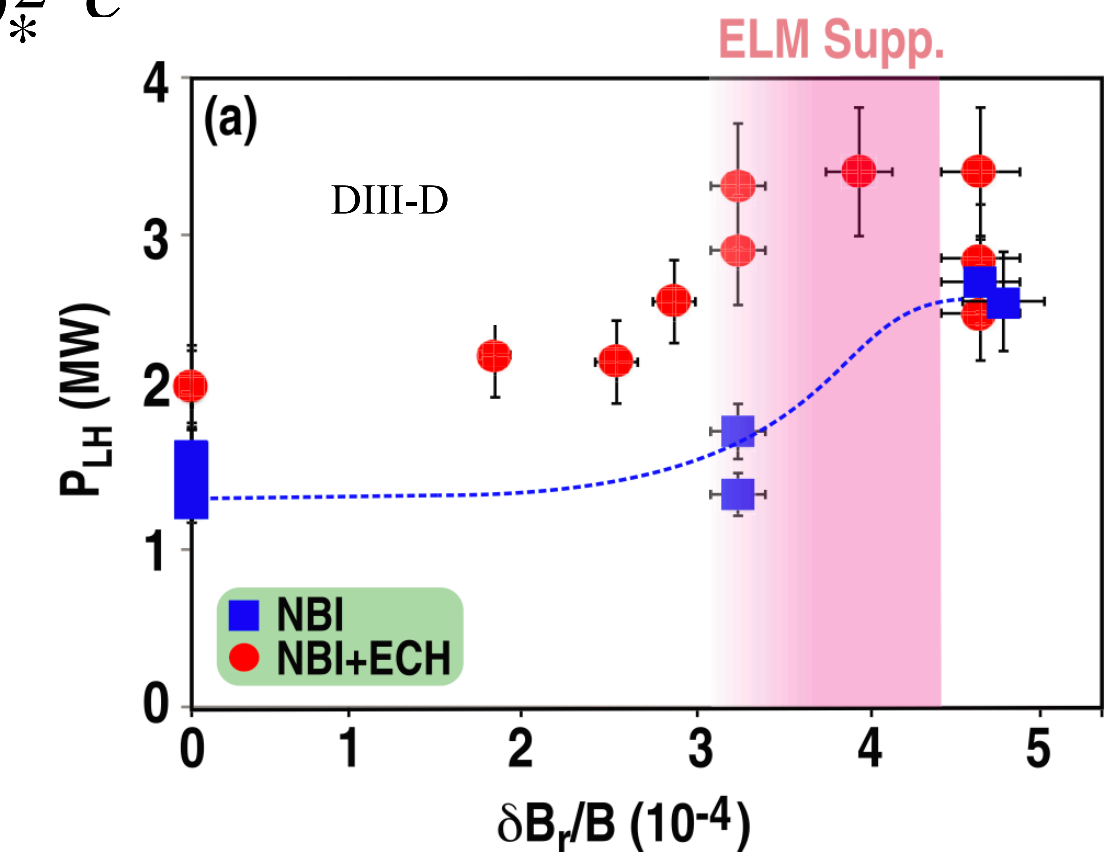


- b^2 shift L-H threshold to higher power, in proportional to $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$.

$$\alpha \propto \frac{1}{\rho_*^2}$$

ρ_* is small $\rightarrow$ $\alpha \uparrow$ (pessimistic)

- Our results predicts the power threshold of L-H transition increases linearly as stochastic magnetic field intensity increases.



(L. Schmitz et al, NF **59** 126010 (2019))

Momentum Transport at the edge of fusion devices—
Drift-wave turbulence

Part II: Ion heat and Parallel Momentum Transport

Ion Heat and Parallel Momentum Transport

Intrinsic rotation and external neutral beam injection are common in experiments in fusion devices.

- Dephasing effect caused by stochastic fields quenches poloidal Reynolds stress—the **mean $E \times B$ shear** is suppressed by this effect. However, observe

$$\frac{\partial}{\partial t}\langle u_{E\times B}\rangle = D_{PV}\frac{\partial}{\partial x}\langle \zeta\rangle - F_{res}\kappa\frac{\partial}{\partial x}\langle p\rangle \longrightarrow \langle E_r\rangle = \frac{\nabla\langle p_i\rangle}{ne} - \langle \mathbf{u}\rangle \times \langle \mathbf{B}\rangle$$

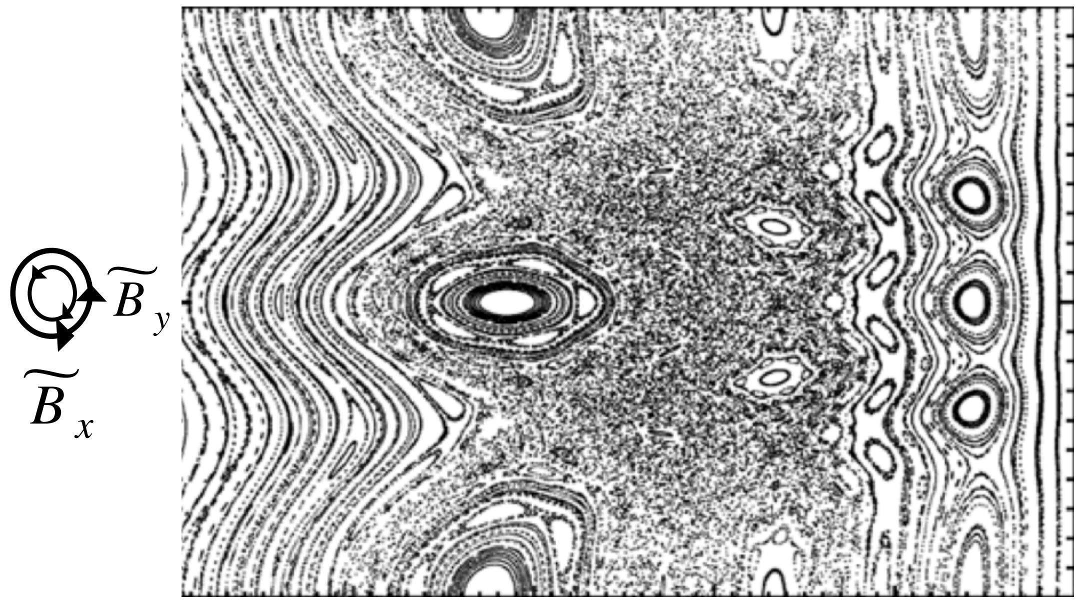
Ion heat/ particle transport

Parallel momentum transport

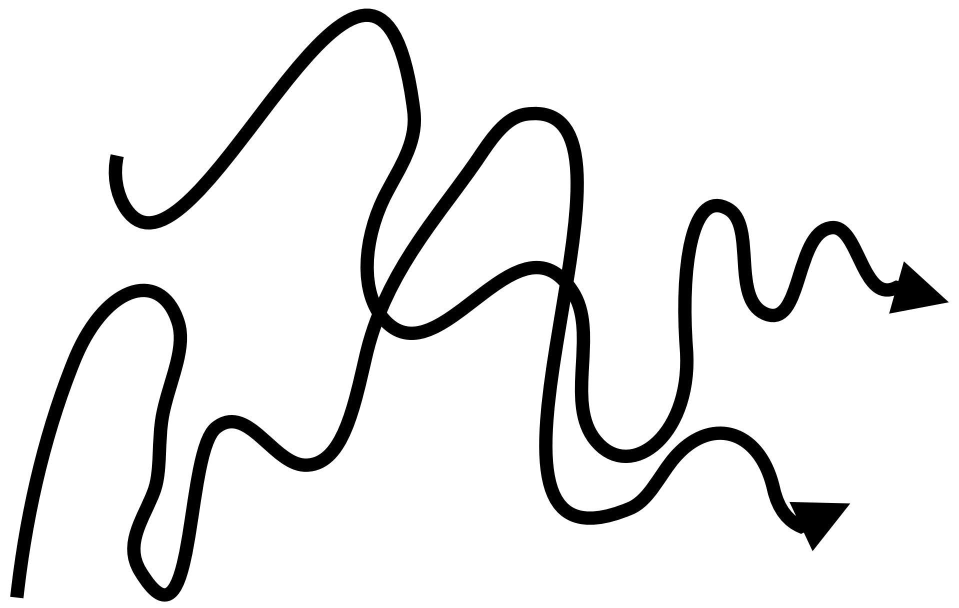
We examine the physics of stochastic fields interaction with the **ion pressure** and the **parallel flow.**

Understanding the physics of ion heat and parallel momentum transport is critical to control the instabilities in fusion devices.

Coexistence of Stochastic Field and Turbulence



Magnetic islands overlapping forms stochastic fields



Strong electrostatic turbulence

Both **stochastic field** and **turbulence** enter the cross-phase $\langle \widetilde{b}\widetilde{p} \rangle$, $\langle \widetilde{b}\widetilde{u}_{\parallel} \rangle$, and hence enter the dephasing mechanism.

In strong turbulence regime, we have faster turbulent scattering timescale:

$$1/\tau_{c,k} \equiv k_{\perp}^{2} D_{T} \gg 1$$

D_T → Turbulent diffusivity

Key question:

How does stochastic fields influence on the response of parallel flow and pressure in strong turbulence regime (faster turbulent scattering timescale)?

We analyze the dephasing effect of stochastic field in strong and weak electrostatic turbulence—how they *together* drives transport.

(Chen et al., PPCF, accepted (2021))

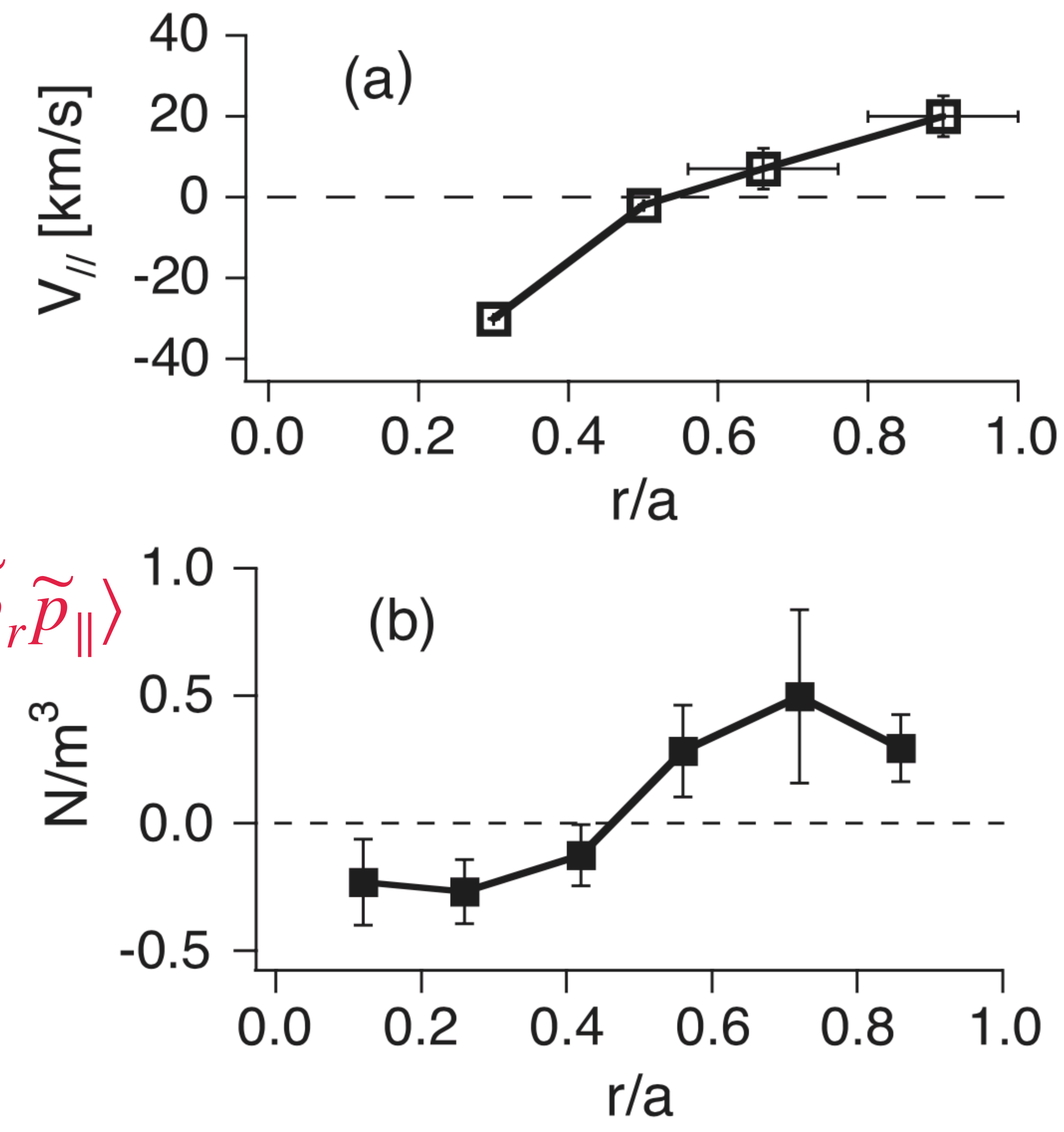
Example

Why we study the Kinetic Stress?

Experimental Result of Madison Symmetric Torus (MST)

$\langle u_\parallel \rangle$

$-\rho \frac{\partial}{\partial r} \langle \widetilde{b}_r \widetilde{p}_\parallel \rangle$



(Ding et al., PRL **110**, 065008 (2013))

Macroscopic parallel flow dynamics.

Nonlinear momentum transport

Microscopic effect measured from the fluctuations of the pressure and the stochastic field.

MST Experimental results: demonstrated the similarity of the kinetic stress to the parallel flow.

Physical Picture of Pressure Response

- We start with the parallel acceleration and pressure equation:

$$\frac{\partial}{\partial t}u_z + (\mathbf{u}\cdot\nabla)u_z = \nabla_z p$$
$$\frac{\partial}{\partial t}p + (\mathbf{u}\cdot\nabla)p = -\gamma p(\nabla_z \cdot \mathbf{u}_z)$$

$$\nabla \rightarrow \nabla_z + b_\perp \nabla_\perp$$
$$\tilde{u}_\perp \cdot \nabla_\perp \equiv \nabla_\perp \cdot \underline{\underline{D}}_T \cdot \nabla_\perp$$

Turbulent fluid diffusivity:

$$D_T \equiv \sum_k |\tilde{u}_{\perp,k}|^2 \tau_{ac}$$

- Local **pressure excess** ($\tilde{b}_x \partial_x \langle p \rangle$) caused by magnetic perturbation is balanced by:

$$0 + (\tilde{u}_\perp \cdot \nabla_\perp)\tilde{u}_z = -\frac{1}{\rho}\frac{\partial}{\partial z}\langle p \rangle - \frac{1}{\rho}\tilde{b}_x \frac{\partial}{\partial x}\langle p \rangle$$

pressure excess $$\frac{1}{\rho}\tilde{b}_x \partial_x \langle p \rangle = \underbrace{D_T \nabla_\perp^2 \tilde{u}_z}_{(a)} - \underbrace{\frac{1}{\rho}\nabla_z \tilde{p}}_{(b)}$$

(a) (Strong turbulence regime) ... by parallel flow perturbation, which is damped by turbulent viscosity.

(b) (Weak turbulence regime) ... by parallel pressure gradient. → Finn et al., PoP **4**, 1152 (1992)

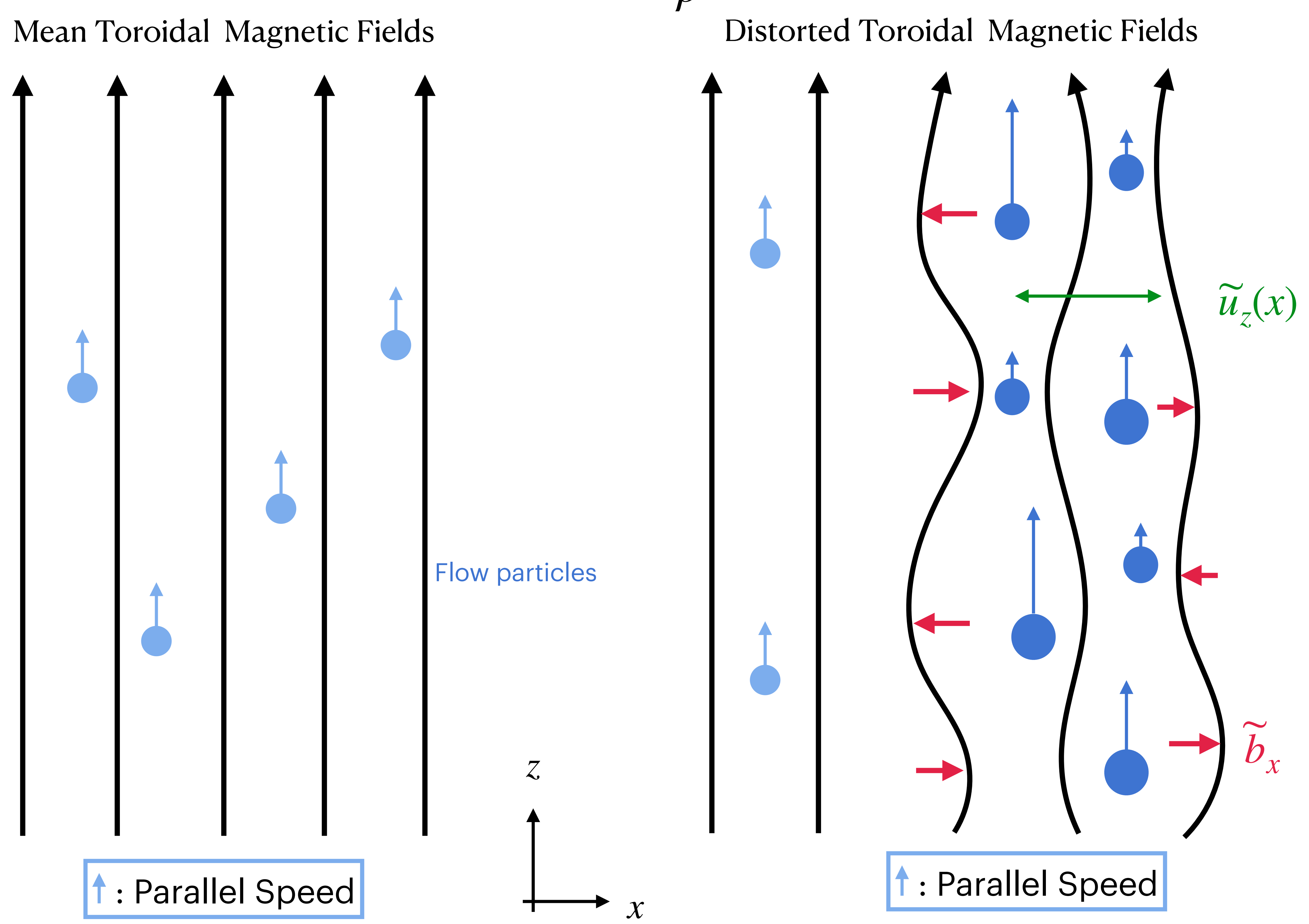
Physical Picture of Pressure Response

Local **pressure excess** ($\widetilde{b}_r \partial_r \langle p \rangle$) caused by magnetic perturbation is balanced by: ($u_{\|}$ response in the same way)

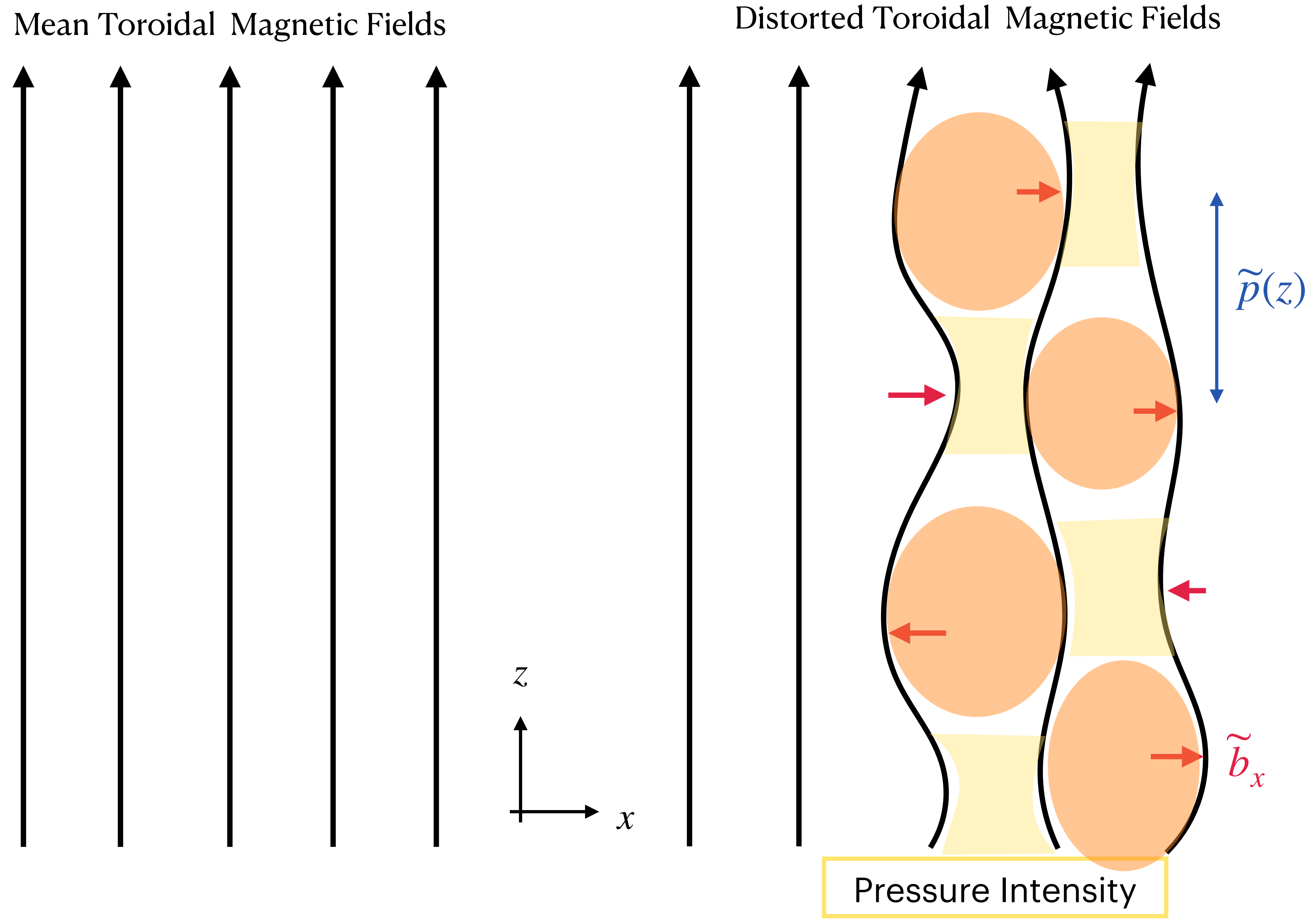
... by parallel flow perturbation, which is damped by turbulent viscosity.

... by parallel pressure gradient. → Finn et al., PoP **4**, 1152 (1992)

1 Strong Turbulence: $\widetilde{b}_r \nabla_r \frac{\langle p \rangle}{\rho} \simeq D_T \nabla_\perp^2 \widetilde{u}_z$



2 Weak Turbulence: $\widetilde{b}_r \nabla_r \langle p \rangle \simeq -\nabla_z \widetilde{p}$ $(\mathbf{B} \cdot \nabla p = 0)$.



Rate of turbulent (i.e. viscous) mixing $D_T / l_\perp^2$ > other rate: turbulent viscosity will dissipate the parallel flow.

Only strong turbulent cases are relevant!

Rate of sound propagation $c_s / l_{\|}$ > other rate: pressure gradient builds up parallelly.

Kinetic Stress and Compressible Energy Flux

- Mean field equation for parallel flow and the pressure equation:

$$\frac{\partial}{\partial t}\langle u_z\rangle + \frac{\partial}{\partial x}\langle \tilde{u}_x\tilde{u}_z\rangle = -\frac{1}{\rho}\frac{\partial}{\partial x}\langle \tilde{b}_x\tilde{p}\rangle \equiv -\frac{\partial}{\partial x}K,$$ where the **kinetic stress** $K \equiv \frac{1}{\rho}\langle \tilde{b}_x\tilde{p}\rangle$

$$\frac{\partial}{\partial t}\langle p\rangle + \frac{\partial}{\partial x}\langle \tilde{u}_x\tilde{p}\rangle = -\rho c_s^2\frac{\partial}{\partial x}\langle \tilde{b}_x\tilde{u}_z\rangle \equiv -\frac{\partial}{\partial x}H,$$ where the **compressible heat flux** $H \equiv \rho c_s^2\langle \tilde{b}_x\tilde{u}_z\rangle$ (or, ion heat density flux)

- Perturbed equation with **Riemann variables** $f_\pm \equiv \tilde{u}_{z,k\omega} \pm \frac{\tilde{p}_{k\omega}}{\rho c_s}$:

The propagator $1/(k_\perp^4 D_T^2 + k_z^2 c_s^2)$ contains the turbulent mixing ($k_\perp^4 D_T^2$) and the magnetic shear effect ($k_z^2 c_s^2$).

Magnetic shear Effect

$$k_z^2 c_s^2 = \left(\frac{k_y x}{L_s}\right)^2 c_s^2$$

Magnetic shear

$$\hat{s} = \frac{r_0}{q}\frac{dq}{dr}$$

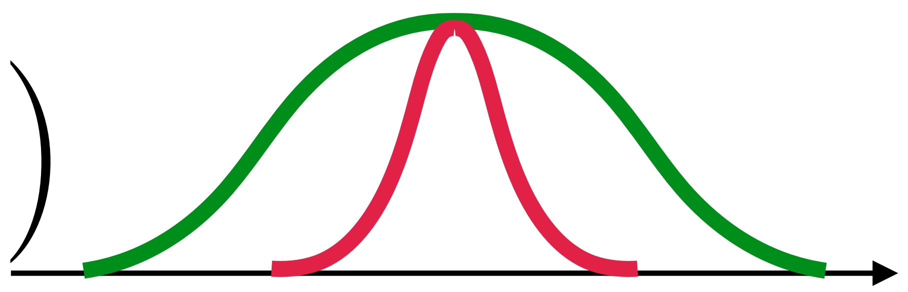
Summation:

$$\sum_{k_y k_z} = \int dm \int dn = \int dm \int dx \frac{|m|}{q^2} q' = r_0 \int dk_y \int dx \frac{|k_y|}{q}\hat{s}$$

$$\langle \tilde{b}_x\tilde{p}\rangle = \int dk_y \frac{k_y r_0^2 q'}{q^2}\int dx\,|\tilde{b}_{x,k}|^2 \frac{\tau_{c,k}}{1+(x/x_s)^2}\left(-\rho c_s^2\frac{\partial}{\partial x}\langle u_z\rangle\right) \quad (1)$$

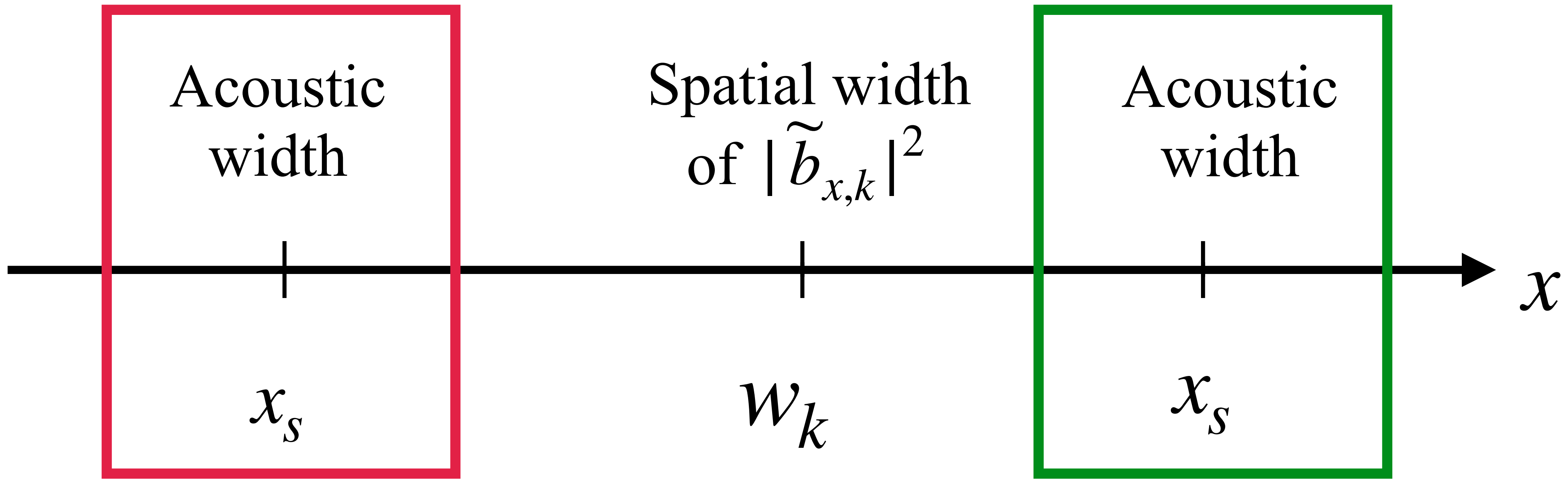
$$+\int dk_y \frac{k_y r_0^2 q'}{q^2}\int dx\,|\tilde{b}_{x,k}|^2 \frac{1}{(k_\perp^2 D_T)^2 + k_z^2 c_s^2}\left(ik_z c_s^2\frac{\partial}{\partial x}\langle p\rangle\right) \quad (2)$$


x


x

Scales

We consider length scales:



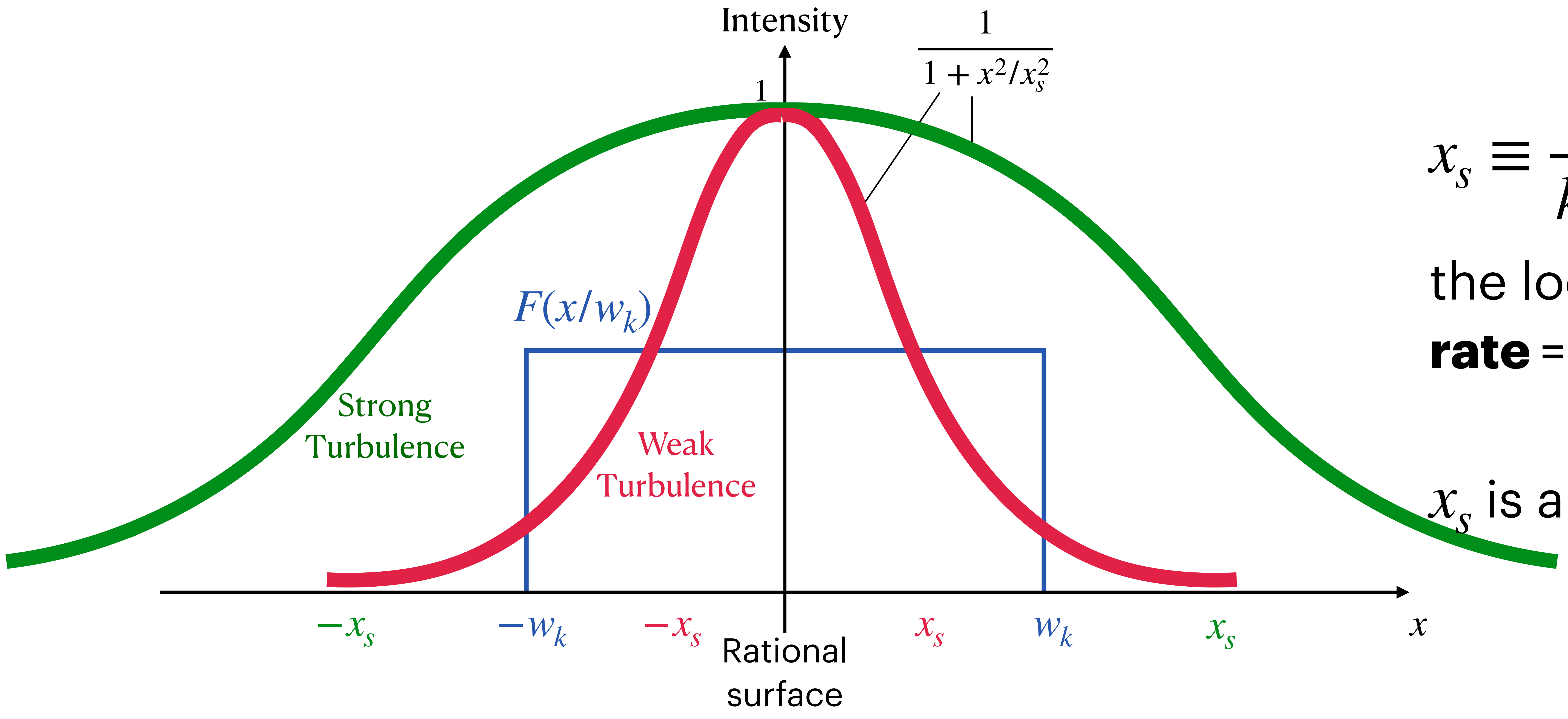
Weak turbulence regime

Strong turbulence regime

Dimensionless parameter

$$\lambda \equiv \frac{x_s}{w_k},$$

defines the competition between the stochastic-field and turbulent effect.



$x_s \equiv \frac{L_s}{k_y c_s \tau_{c,k}}$ is the **acoustic width**—x_s defines the the location where the **parallel acoustic streaming rate** = **decorrelation rate.**

x_s is analogous to ion Landau resonant point.

Results—Strong Turbulence Regime

In strong turbulence ($k_\perp^2 D_T \gg k_z c_s$ or $\lambda > 1$):

$$K \equiv \frac{1}{\rho}\langle \tilde{b}_x \tilde{p} \rangle \simeq -D_{st}\frac{\partial}{\partial x}\langle u_z \rangle$$, where

$$D_{st} = D_{st}(x) = \sum_{k_y k_z} |\tilde{b}_{x,k}|^2 \frac{c_s^2}{k_\perp^2 D_T}$$

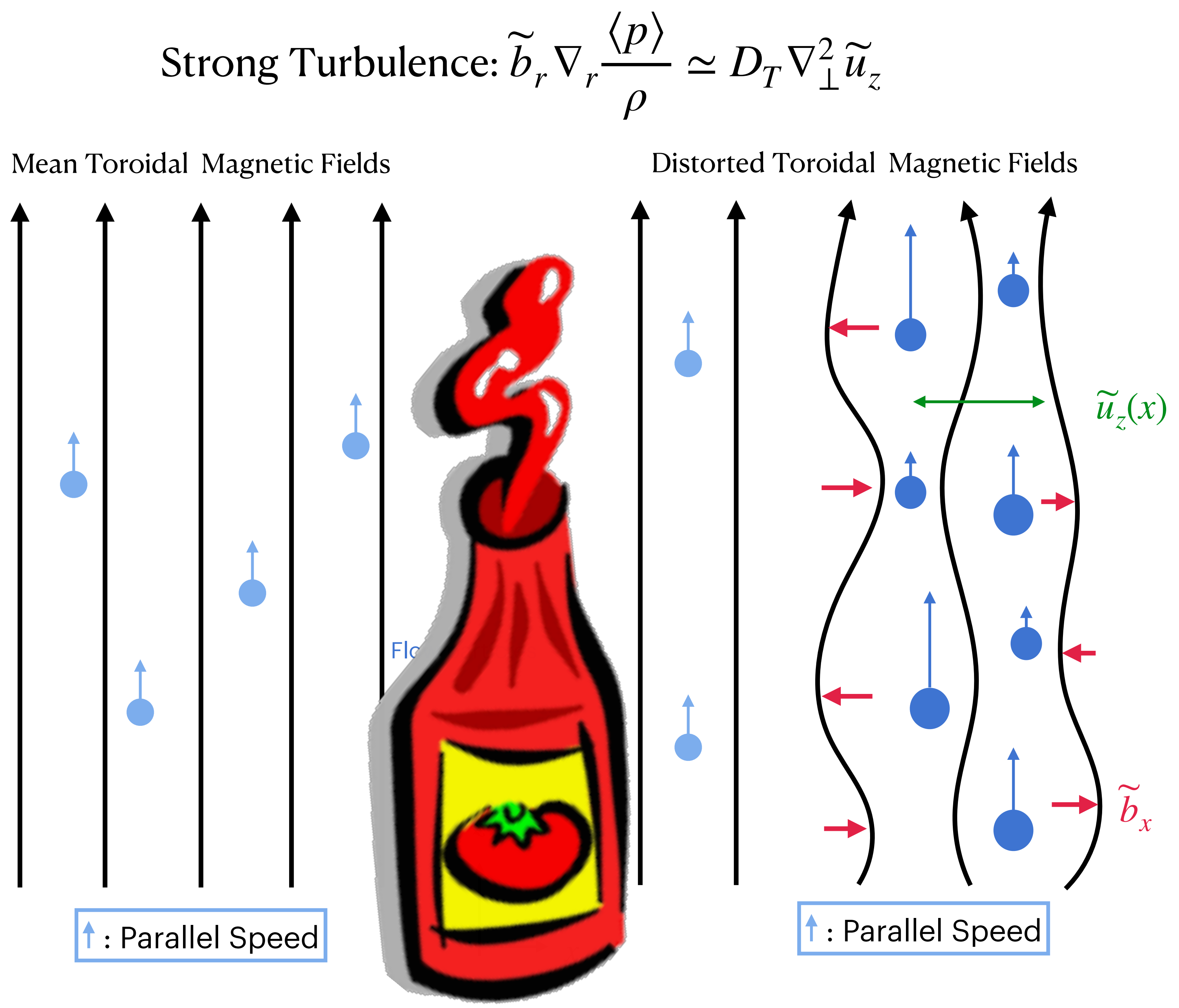
Stochastic B field

Turbulent scattering decorrelation rate

Turbulent fluid diffusivity $D_T \equiv \sum_k |\tilde{u}_{\perp,k}|^2 \tau_{ac}$

D_{st} : the **hybrid turbulent diffusivity**—explain how the kinetic stress is scattered by stochastic B fields and turbulence.

$$H \equiv \rho c_s^2 \langle \tilde{b}_x \tilde{u}_z \rangle \simeq -D_{st}\frac{\partial}{\partial x}\langle p \rangle$$



Pressure gradient $\partial\langle p \rangle/\partial x$ due to $\tilde{b}$ is balanced by turbulent mixing of parallel flow $\nabla_\perp^2 \tilde{u}_z$.

The pressure gradient in presence of tilted B lines balances with the *hybrid* turbulent diffusion.

Stochastic–Field Effect on Electron Particle Flux

Consider electron density evolution:

$$\frac{\partial \langle n_e \rangle}{\partial t} - \frac{\partial}{\partial x}\frac{\langle \tilde{b}_x \tilde{J}_{z,e} \rangle}{|e|} = 0$$

+

Ampère's Law

$$-\nabla_\perp^2 A_z = \mu_0 (J_{z,e} + J_{z,i})$$

→

$$\frac{\partial \langle n_e \rangle}{\partial t} = -\frac{1}{\mu_0 |e|}\frac{\partial}{\partial x}\langle \tilde{b}_x \nabla_\perp^2 \tilde{A}_{z,e} \rangle - n_{0,i}\frac{\partial}{\partial x}\langle \tilde{b}_x \tilde{u}_{z,i} \rangle$$

Total current contribution

Ion current contribution

$$\langle \tilde{b}_x \tilde{u}_z \rangle = -D_M \frac{\partial}{\partial x}\langle u_z \rangle$$

$$D_M \equiv \sum_{k_y k_z} |\tilde{b}_{x,k}|^2 \tau_{d,k} c_s$$

Dispersal timescale of an acoustic wave packet along the stochastic magnetic field

→

$$\frac{\partial \langle n_e \rangle}{\partial t} = -\frac{\partial}{\partial x}\Gamma_{e,s}$$

Electron particle flux

$$= \frac{\partial}{\partial x}\frac{B_0}{\mu_0 |e|}\frac{\partial}{\partial x}\langle \tilde{b}_x \tilde{b}_y \rangle + \frac{\partial}{\partial x} n_0 D_M \frac{\partial}{\partial x}\langle u_{z,i} \rangle$$

Familiar div. Maxwell stress

Ion flow along the tilted B line

Stochastic lines and parallel ion flow gradient drives a net electron particle flux, in additional to the Maxwell force contribution.

Conclusions

- We calculate the ***explicit form*** of the stochastic-field-induced transports—kinetic stress K and the compressive energy flux H—have different mechanisms in presence of strong/weak electrostatic turbulence.
- In practice, ***only* strong turbulent cases** ($k_\perp^2 D_T \gg k_z c_s$ or $\lambda > 1$) **are relevant**. We found mean parallel flow and mean pressure are driven via the ***hybrid diffusivity*** that involves effect of stochastic field and turbulent scattering:

(Chen et al. PPCF **64**, 015006, 2022)

$$D_{st} = D_{st}(x) = \sum_{k_y k_z} |\tilde{b}_{x,k}|^2 \frac{c_s^2}{k_\perp^2 D_T}$$

Stochastic B field

Turbulent scattering decorrelation rate

Future Works

- Magnetic drift—effect of stochastic field and turbulence upon geodesic acoustic modes.
- One should include the effect of $\langle \tilde{b}\,\tilde{\phi} \rangle \neq 0$ in the future. (Cao & Diamond PPCF **64** 035016, 2022)
- Relevant problems: cosmic ray acceleration and propagation.

Summary of My Research

(Chen et al. ApJ **892** (1), 2020)

a. β-plane MHD: We obtain the momentum transport problem of the solar tachocline is due to the highly disordered magnetic field decorrelation effect.

b. We found Stochastic fields form a **fractal, elastic network**.

c. We show the stochastic-field induced dephasing at the mean field intensity lower than that for the fully Alfvénization.

(Chen et al. PoP **28**, 042301, 2021)

d. Poloidal momentum transport in stochastic B-field: We obtain the suppression of PV diffusivity and the shear-eddy tilting feedback loop.

e. Calculate power threshold increment for L-H transition. Intrinsic Rotation in presence of stochastic fields.

(Chen et al. PPCF **64**, 015006, 2022)

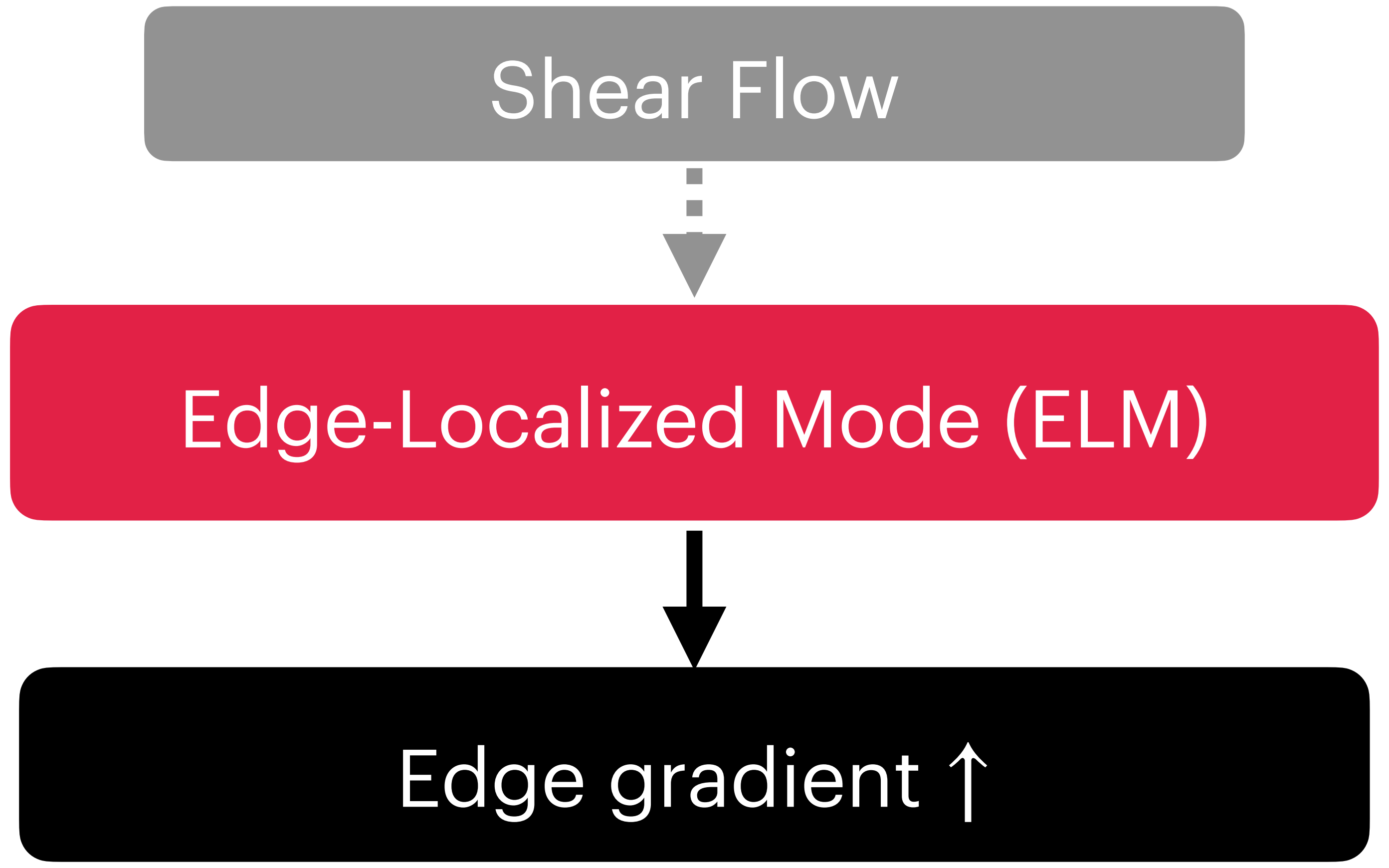
f. Parallel and ion heat transport in stochastic B-field: We found that in strong turbulence regime, the mean flow is driven by stochastic-turbulent scattering.

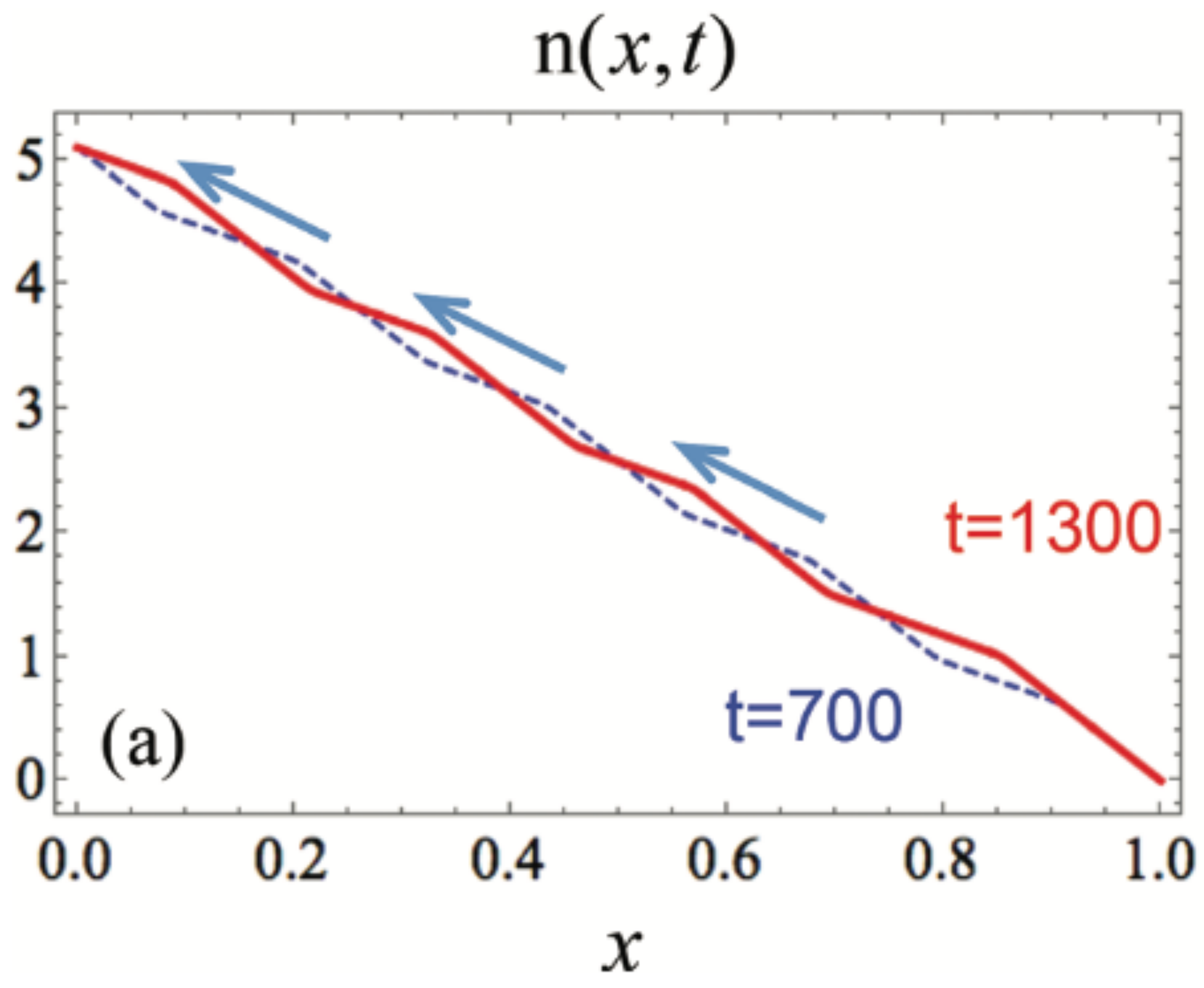
g. We calculate the ***explicit form*** of the stochastic-field-induced transports which have different mechanisms in presence of strong/weak electrostatic turbulence.

Future Work

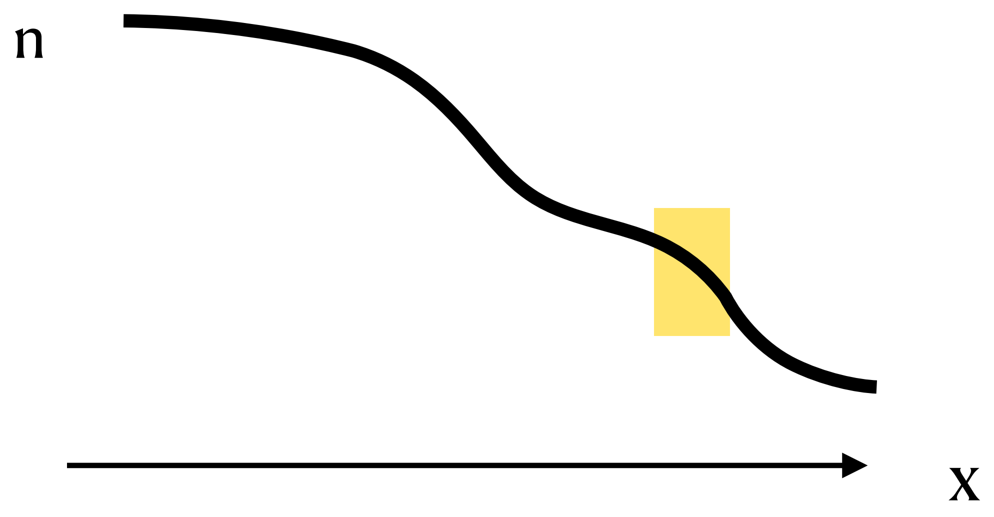
Staircase and Mixing length in presence of stochastic fields

Fate of Spatial structure of zonal flow?

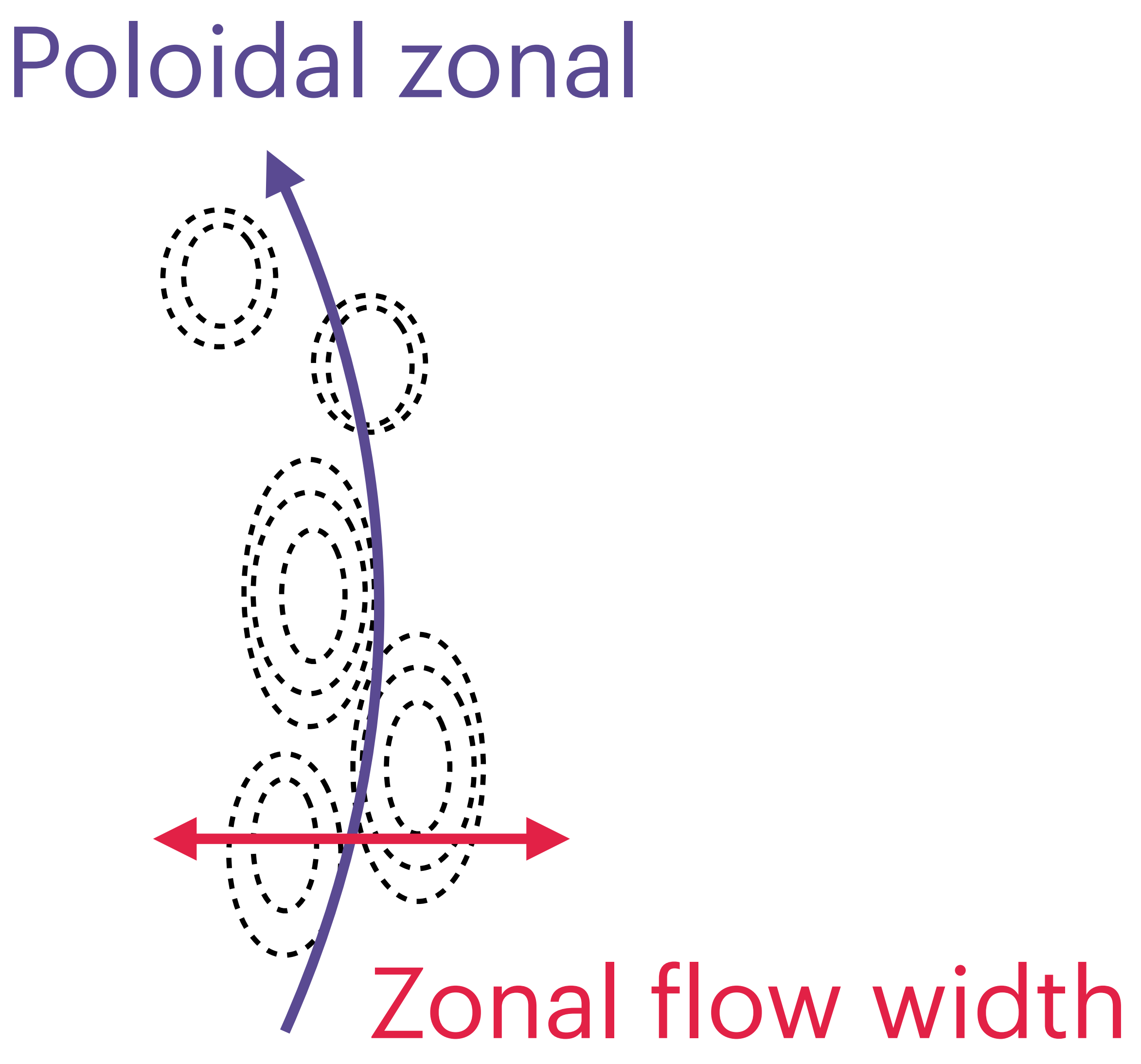




Ashourvan & Diamond, PoP **24,** 012305 (2017)



Density corrugation



Zonal flow width is related to corrugation length.

We are interested in zonal flow width in presence of stochastic fields.

The mixing length (l_{mix}) depends on **two scales:**

- Driving scale: l_0
- Rhines scale: $l_{RH} = \dfrac{\sqrt{\epsilon}}{|\partial_x q|}$

➡ mixing scale: $l_{mix} = \dfrac{l_0}{(1 + l_0^2(\partial_x q)^2/\epsilon)^{\kappa/2}} = \dfrac{l_0}{(1 + l_0^2/l_{RH}^2)^{\kappa/2}}$

Main effect of diffusivity D_n and χ

For α_{DW} (a measurement of the resistive diffusion rate in the parallel direction) > 1 in H-W regime:

Density diffusivity:

$$D_n \simeq \frac{l_{mix}^2 \epsilon}{\alpha_{DW}}$$

Resistive diffusion rate:

$$\alpha_{DW} = \frac{k_{\parallel}^2 v_{the}^2}{\nu}$$

+

Stochastic Fields Effect

$$k_{\parallel} = \underline{k} \cdot \underline{\hat{b}_0} \simeq \frac{1}{Rq} + \underline{b}_{\perp} \cdot \underline{k}_{\perp} \simeq \frac{1}{Rq} + \frac{b_{\perp}}{l_{mix}}$$

$$D_n \simeq \frac{l_{mix}^2 \epsilon \nu / v_{the}^2}{(\frac{1}{Rq})^2 + (\frac{b}{l_{mix}})^2}$$

Same for χ (or D_{PV} in this case).

Competition btw $\frac{1}{Rq}$ v.s. $\frac{b_{\perp}}{l_{mix}}$ gives $Ku_{mag} = bRq/l_{mix}$ → $Ku_{mag} = Ku_{mag}(l_{mix})$

The mixing length is not likely affected by b^2.

A change of scale selection or staircase corrugation requires $Ku_{mag} \geq 1$.

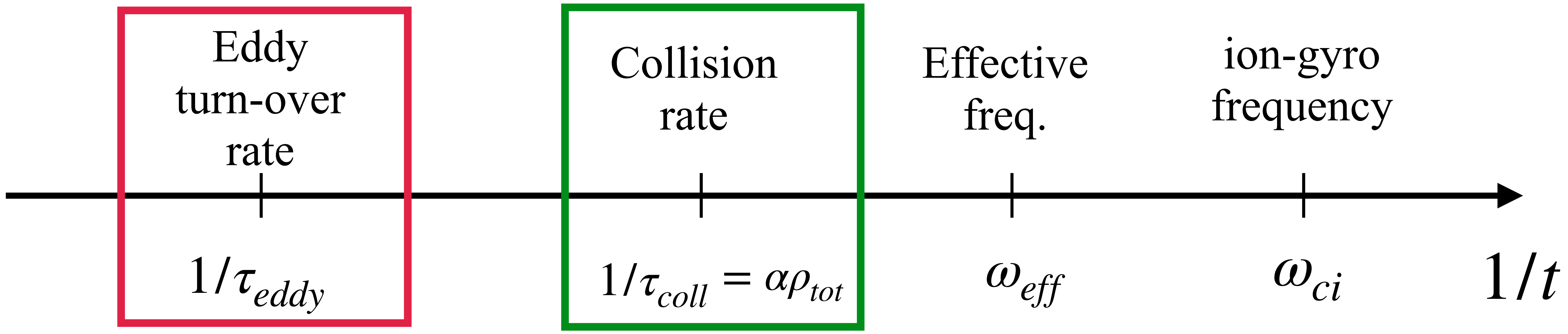
Future Work

Neutrals and Drift-Rossby-Alfvén turbulence:
Ambipolar Diffusion

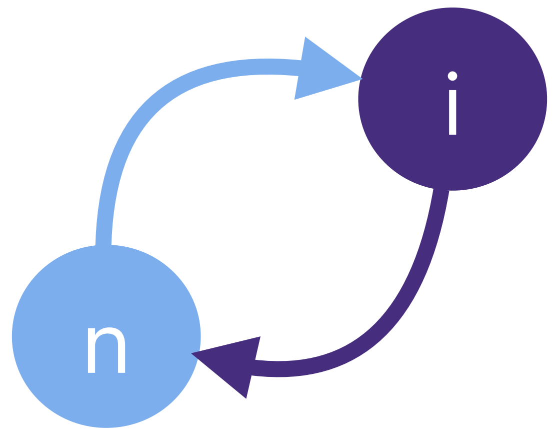
Important Time Scales

- We consider rates:

$$\gamma = \frac{(\alpha\rho_i)\omega_{eff}^2}{\omega + i\eta_{eff}k^2} \cdot \tau_{eddy} \cdot \frac{1}{\omega_{ci}}$$



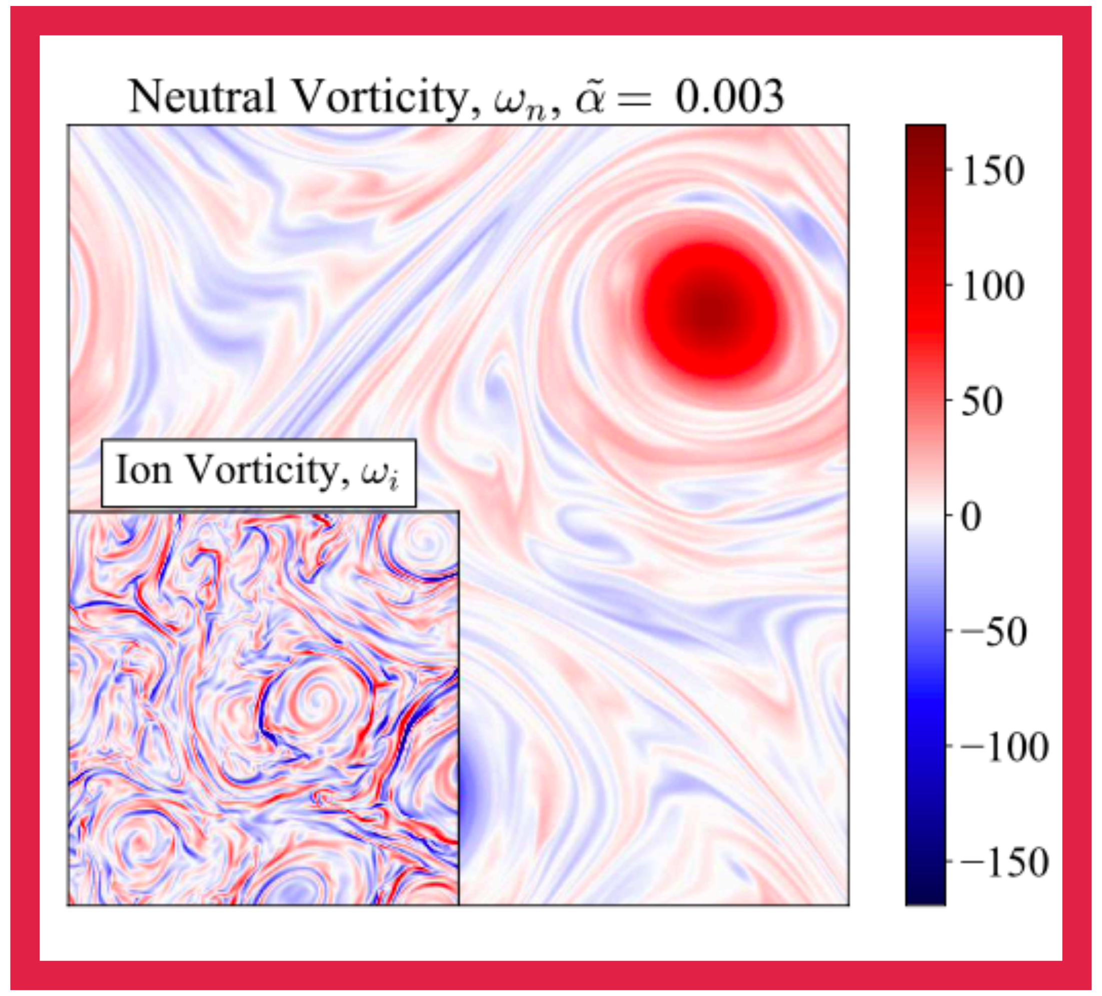
- A simplified coupling parameter: $\tilde{\alpha} \equiv \frac{\tau_{eddy}}{\tau_{coll}} = \frac{l_\perp \rho_{tot}\alpha}{\tilde{u}}$ → Strength of coupling



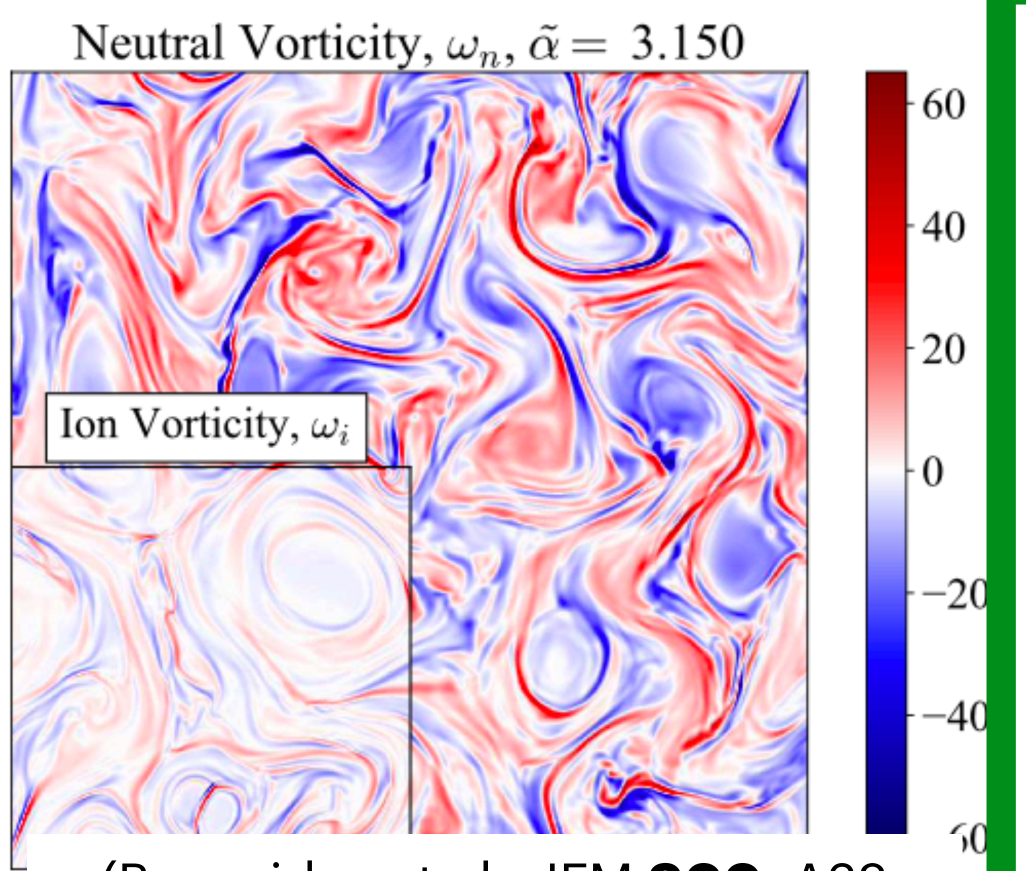
- Dynamics of neutrals and ions:

Weak coupling:

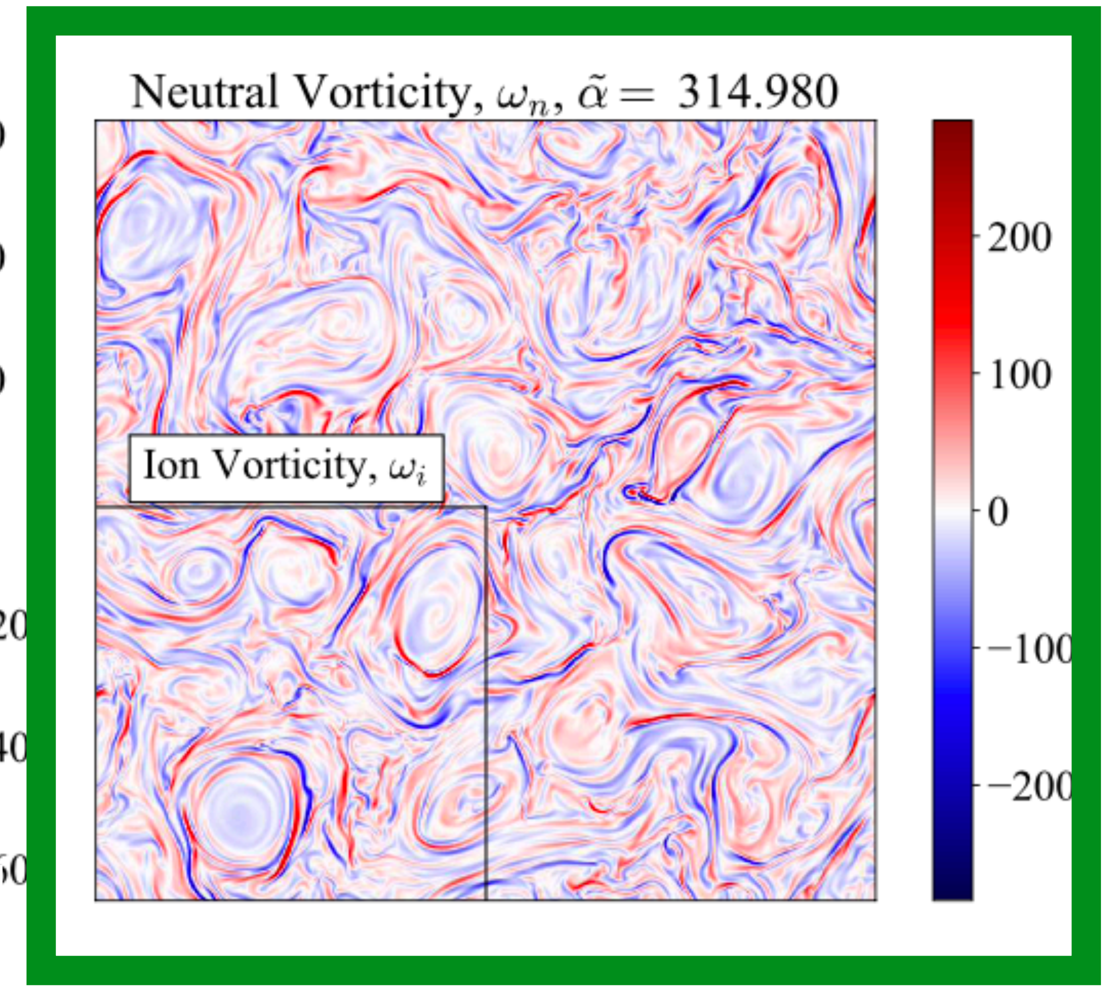
Neutrals and ions not well-coupled —> Two fluid.



small α



(Benavides et al., JFM **900**, A28 (2020))



large α

Strong coupling:

Neutrals couple to ions and behaves as **one MHD fluid**.

Ambipolar Conclusions

- We study Non-trivial neutral effect on DW-ZF turbulence: Ambipolar diffusion
- We derive the **key parameter γ** for the Drift-Alfvén + Neutral effect:

$$\gamma = \frac{(\alpha\rho_i)\omega_{eff}^2}{\omega + i\eta_{eff}k^2} \cdot \tau_{eddy} \cdot \frac{1}{\omega_{ci}}$$

- We study the Drift-Alfvén wave with neutrals and found—
  In strong coupling regime: **one MHD fluid**.
  In weak coupling regime: **two fluid.**
- **Modified Zel'dovich Theorem**. We derive the key parameter that **regulates** Maxwell and Reynolds stress competition:

$$\langle \widetilde{B}^2 \rangle \propto B_0^2/\eta \text{ (original)}$$

$$\langle \widetilde{B}^2 \rangle \propto B_0/A^{1/2} \text{ (ambipolar effect)}$$

Future Works

- Calculate $\widetilde{B}$ evolution for arbitrary $\gamma \rightarrow$ Pouquet + neutrals (clarify asymptotic regime).
- Study the physics of neutral entrainment.

Thank you!

Backup Slides

Derivation of Magnetic Diffusivity

Vorticity equation: $(\frac{\partial}{\partial t} - \mathbf{u}\cdot\nabla)\nabla^2\phi - v_A(\cdot\nabla_\parallel + b_{st,\perp}\cdot\nabla_\perp)J_\parallel = 0$

$$\begin{cases} 0^{\text{th}}\ \text{order}: v_A\frac{\partial}{\partial z}J_{0,z} = 0 \\ 1^{\text{st}}\ \text{order}: (\frac{\partial}{\partial t} - \langle u_y\rangle\frac{\partial}{\partial y})\nabla^2\tilde{\phi} - v_A(\nabla_\parallel + b_{st,\perp}\cdot\nabla_\perp)\tilde{J}_\parallel = 0 \end{cases}$$

Curly bracket : $\{\ \} = \int_{-\infty}^{+\infty} d\tau$

$$\{\frac{i}{-b_{st,\perp}k_\perp}\} = \int_{-\infty}^{+\infty} d\tau\{e^{ib_{st,\perp}k_\perp\int_0^\tau d\tau'}\} = \int_{-\infty}^{+\infty} d\tau e^{-\underline{k}_i\underline{\underline{D}}_{M,ij}\underline{k}_j\tau}$$

$$\int_0^{+\infty} d\tau = \int_0^{+\infty}\frac{dl}{|v_A|}$$

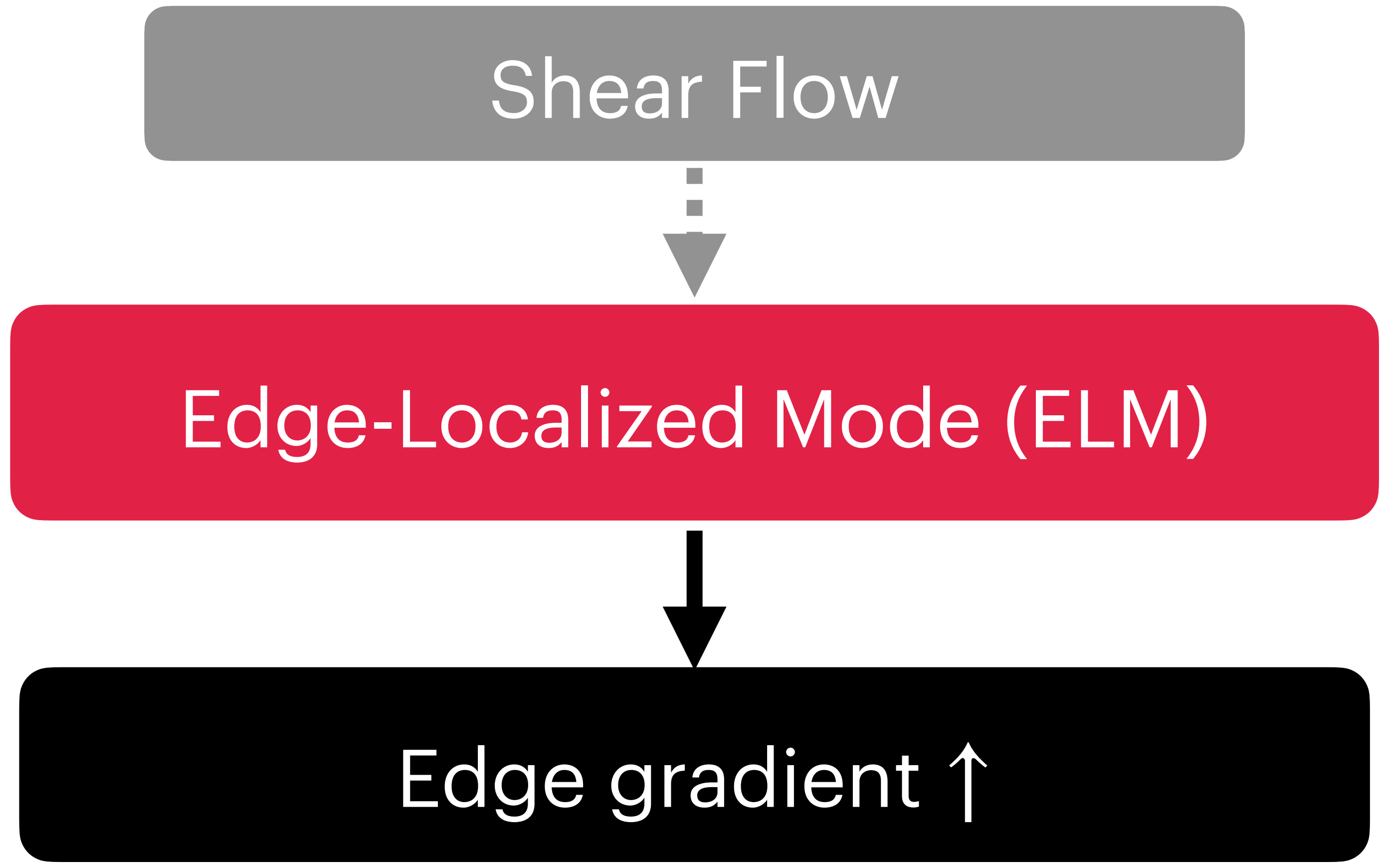
dl is along magnetic fields

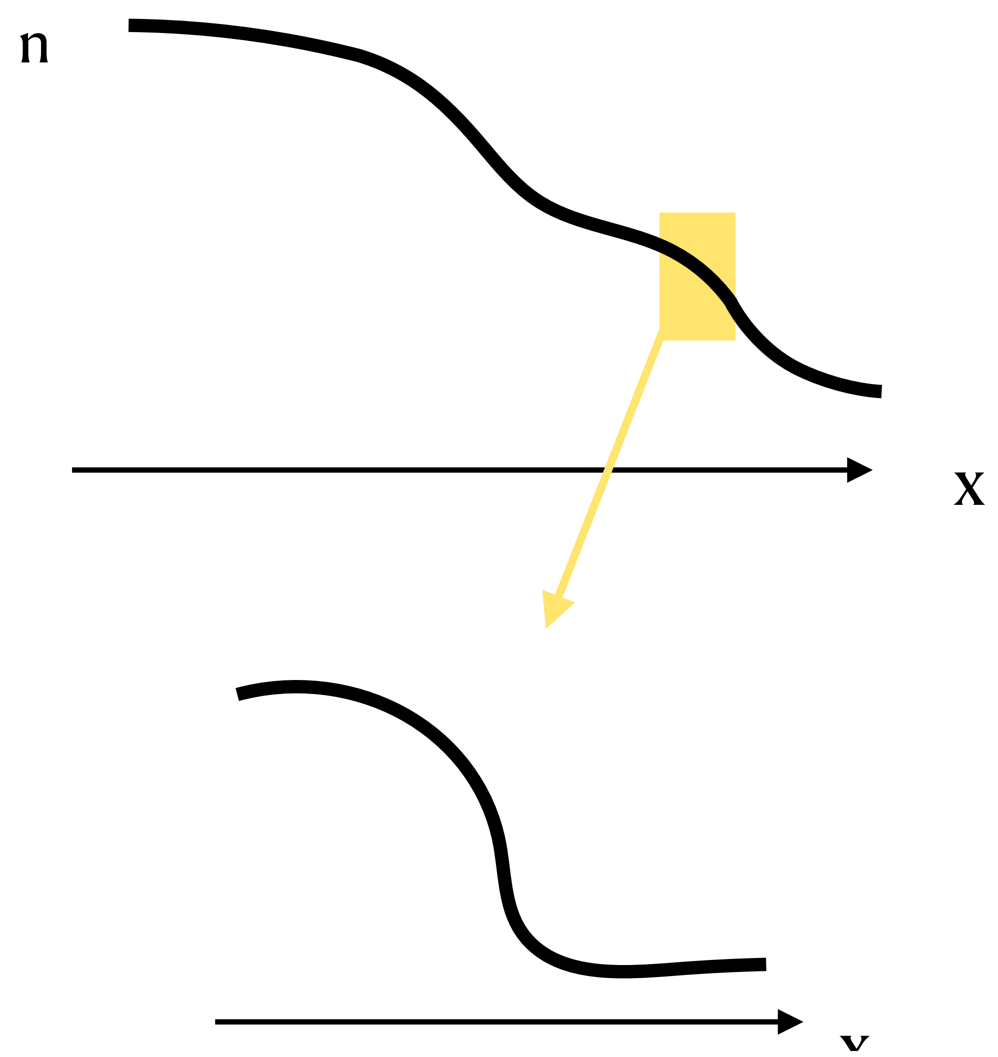
Characteristic velocity of $b_{st,\perp}$ (parallel wave packet transit timescale)

$$D \equiv v_A D_M = v_A\sum_k \frac{B^2_{st,k}}{B_0^2}\pi\delta(k_z) \propto v_A\frac{1}{v_A^2}v_A B_{st}^2$$

Diffusivity D is independent of B_0 .

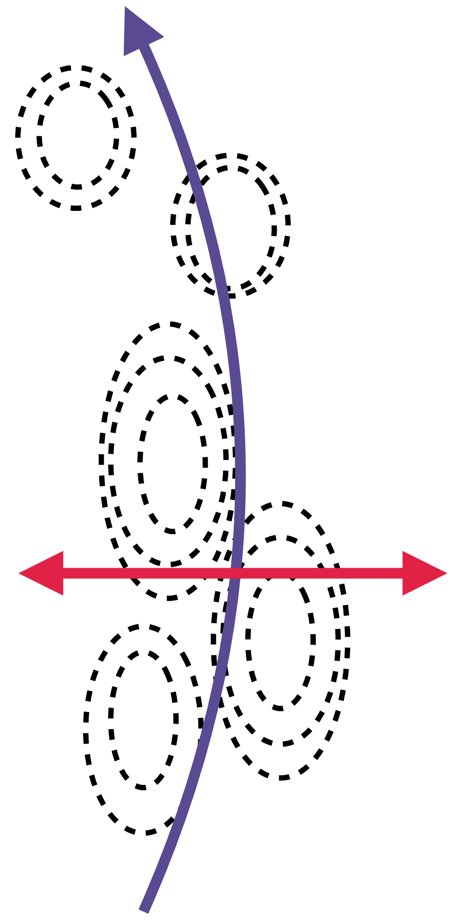
Fate of Spatial structure of zonal flow?





Density corrugation

Poloidal zonal



Zonal flow width

Zonal flow width is related to corrugation length.

We are interested in zonal flow width in presence of stochastic fields.

Backup Slides

Layering Structure—Mixing Length Model

A mixing length model for layering:

- Reduce evolution equations (based on H-W model).
- Energy and Potential entropy (PE) conserved.

Density: $\frac{\partial}{\partial t}\langle n\rangle = \frac{\partial}{\partial x}\left(D_n \frac{\partial \langle n\rangle}{\partial x}\right) + D_c \frac{\partial^2}{\partial x^2}\langle n\rangle$

turb. particle diffusion

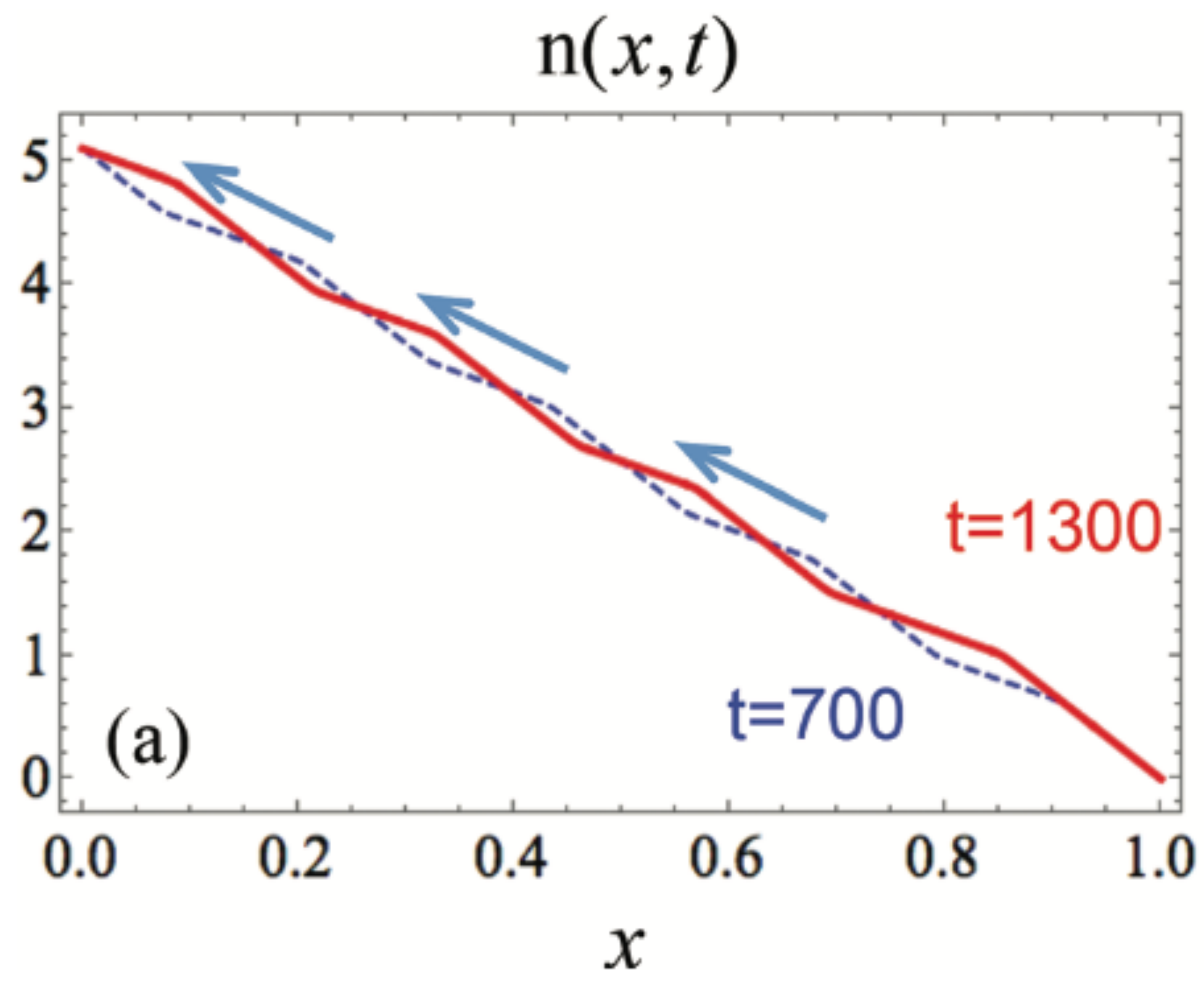
Potential Vorticity: $\frac{\partial}{\partial t}\langle \zeta\rangle = \frac{\partial}{\partial x}\left((D_n - \chi) \frac{\partial \langle n\rangle}{\partial x}\right) + \chi \frac{\partial^2}{\partial x^2}\langle \zeta\rangle + \mu_c \frac{\partial^2}{\partial x^2}\langle \zeta\rangle$

residual stress; turb. Viscous diffusion

Turbulent potential Enstrophy: $\frac{\partial}{\partial t}\epsilon = \frac{\partial}{\partial x}\left(D_\epsilon \frac{\partial \epsilon}{\partial x}\right) + \chi[\frac{\partial (n-\zeta)}{\partial x}]^2 - \epsilon_c^{-1/2}\epsilon^{3/2} + P$

PE diffusion; mean-turb PE Coupling; PE Dissipation

- n : density
- ζ : potential vorticity
- ϵ : turbulent PE $\quad \epsilon \equiv (\delta n - \delta\zeta)^2/2$
- D_n : turbulent particle diffusivity
- χ : turbulent vorticity
- P : production



Ashourvan & Diamond, PoP **24,** 012305 (2017)

Density corrugation forms staircase-like structure.

Scale Selection

The mixing length (l_{mix}) depends on **two scales:**

- Driving scale: l_0
- Rhines scale: $l_{RH} = \dfrac{\sqrt{\epsilon}}{|\partial_x q|}$

➡ mixing scale: $l_{mix} = \dfrac{l_0}{(1 + l_0^2 (\partial_x q)^2/\epsilon)^{\kappa/2}} = \boxed{\dfrac{l_0}{(1 + l_0^2/l_{RH}^2)^{\kappa/2}}}$

$$\begin{cases} \text{Strong mixing } (l_{RH} > l_0): & l_{mix} \simeq l_0 \text{ (Weak mean PV gradient)} \\ \text{Weak mixing } (l_0 > l_{RH}): & l_{mix} \simeq l_0^{1-\kappa} l_{RH}^{\kappa} \text{ (Strong PV gradient)} \end{cases}$$

l_{mix} (hybrid length scale) sets the scale of zonal flow.

What is the effect of stochastic fields on staircases?

Backup Slides

Main effect of diffusivity D_n and χ

For α_{DW} (a measurement of the resistive diffusion rate in the parallel direction) > 1 in H-W regime:

Density diffusivity:

$$D_n \simeq \frac{l_{mix}^2 \epsilon}{\alpha_{DW}}$$

Resistive diffusion rate:

$$\alpha_{DW} = \frac{k_{\parallel}^2 v_{the}^2}{\nu}$$

+

Stochastic Fields Effect

$$k_{\parallel} = \underline{k} \cdot \underline{\hat{b}_0} \simeq \frac{1}{Rq} + \underline{b_{\perp}} \cdot \underline{k_{\perp}} \simeq \frac{1}{Rq} + \frac{b_{\perp}}{l_{mix}}$$

→

$$D_n \simeq \frac{l_{mix}^2 \epsilon \nu / v_{the}^2}{(\frac{1}{Rq})^2 + (\frac{b}{l_{mix}})^2}$$

Same for χ (or D_{PV} in this case).

Competition btw $\frac{1}{Rq}$ v.s. $\frac{b_{\perp}}{l_{mix}}$ gives $Ku_{mag} = bRq/l_{mix}$ → $Ku_{mag} = Ku_{mag}(l_{mix})$

The mixing length is not likely affected by b^2.

A change of scale selection or staircase corrugation requires $Ku_{mag} \geq 1$.

Backup Slides

Why we study neutrals in fusion devices?

Neutrals influx

↓

Charge exchange damping or friction

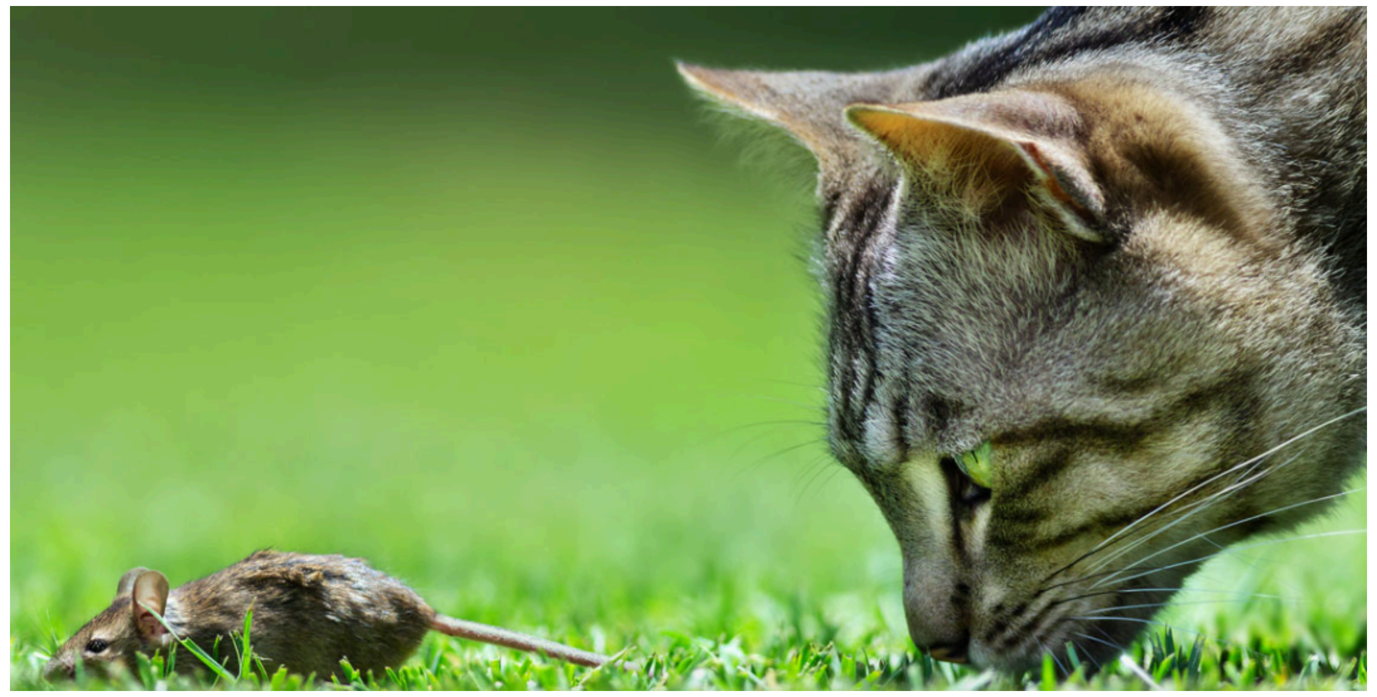
↓

Predator prey model:

predator: zonal flow
prey: turbulence

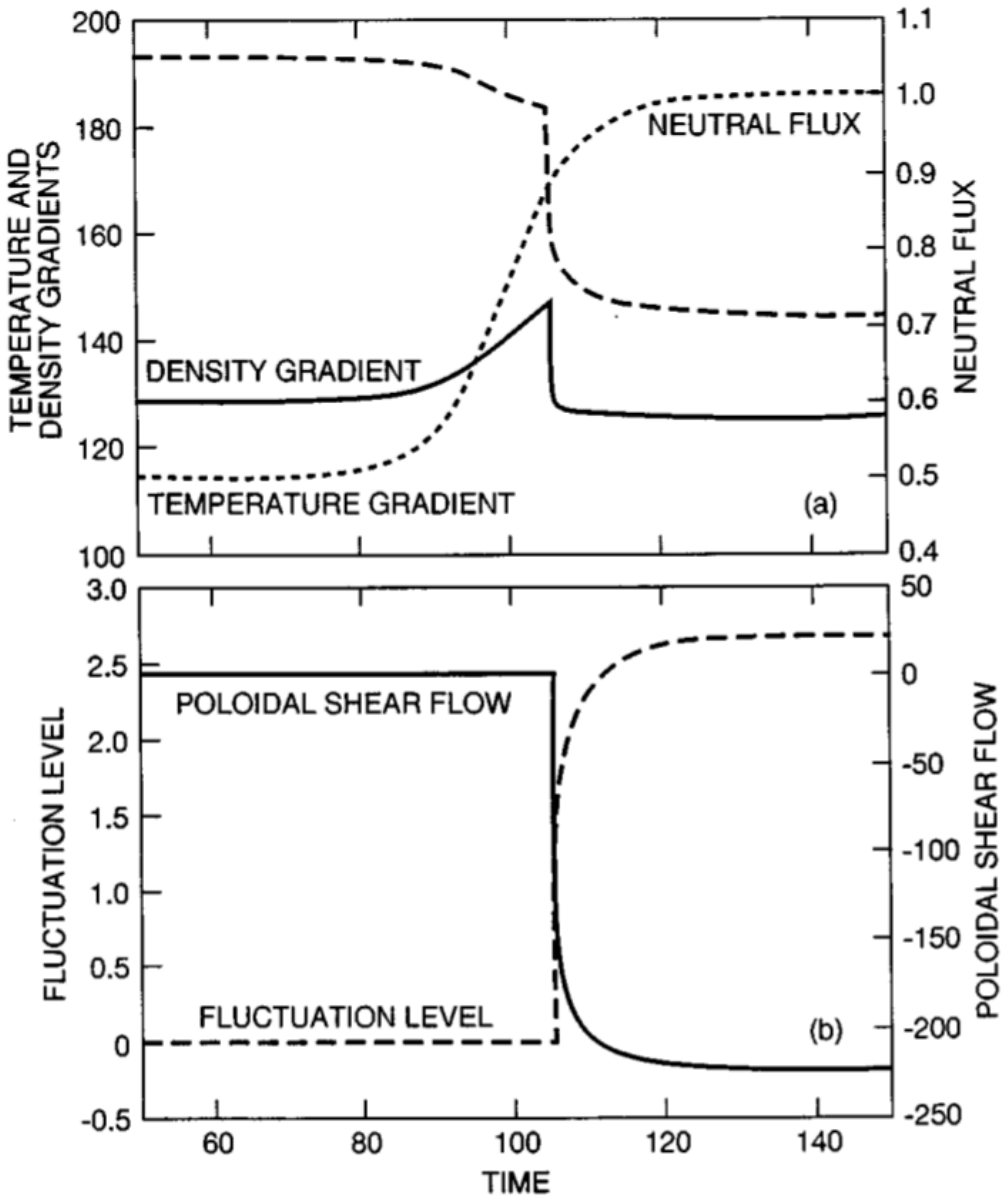
↓

Increase the flow shear threshold



Conventional wisdom:

1. Neutrals damp the poloidal (zonal) flow, increase fluctuation level.
2. Increase effective fluctuation energy.



(Carreras et al., PoP **3**, 4106 (1996))

The neutral density is important in studies of L-H transition power threshold in fusion device.

Non-trivial: 1. E-M effect →Ambipolar diffusion (classic in Astrophys). ✓
2. Entrainment of neutral particles.

Drift–Rossby Waves and Zonal Flow

(Simplest possible model)

- Evolution of zonal flow is from the competition btw the Reynolds and Maxwell Stress:

Zonal flow evolution:

$$\frac{\partial}{\partial t}\langle u_y\rangle = -\frac{\partial}{\partial x}\left(\langle \tilde{u}_x \tilde{u}_y\rangle - \frac{\langle \widetilde{B}_x \widetilde{B}_y\rangle}{\mu_0 \rho}\right)$$

Reynolds stress ⟷ Maxwell stress

Competition

Induction equation:

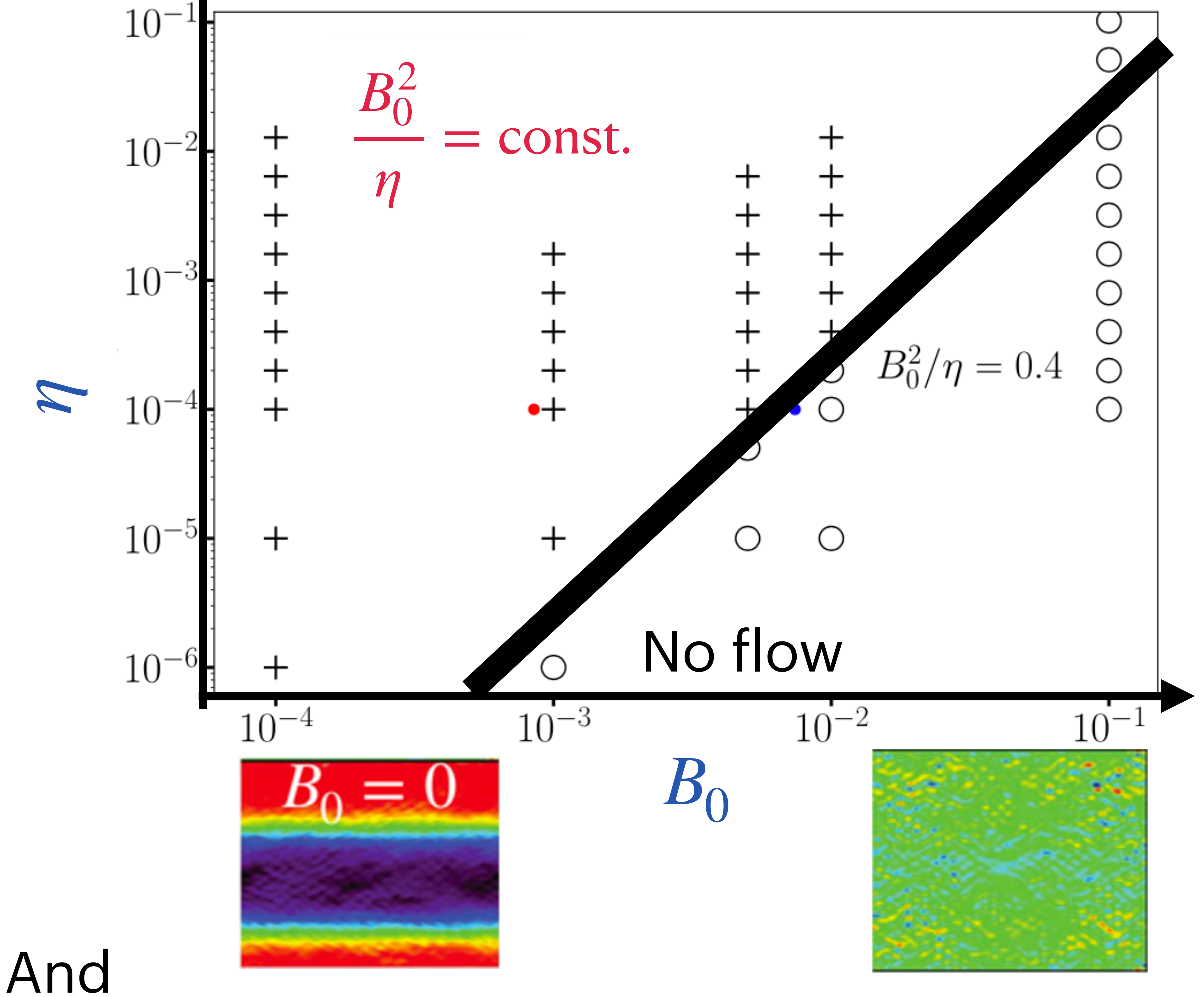
$$\frac{\partial \widetilde{\mathbf{B}}}{\partial t} = \nabla \times (\widetilde{\mathbf{u}_\mathrm{i}} \times \mathbf{B}) + \eta \nabla^2 \widetilde{\mathbf{B}}$$

η: increase the zonal flow

B_0: suppress the zonal flow

$\rightarrow \frac{B_0^2}{\eta}$: regulates Mag. And Reynolds stress competition

(Chen et al., PoP, 28, 032306 (2021))



→Zel'dovich Theorem

What if we add neutrals? Neutrals will enter this competition and have a **production** on zonal flow.

Multi-Fluid Model

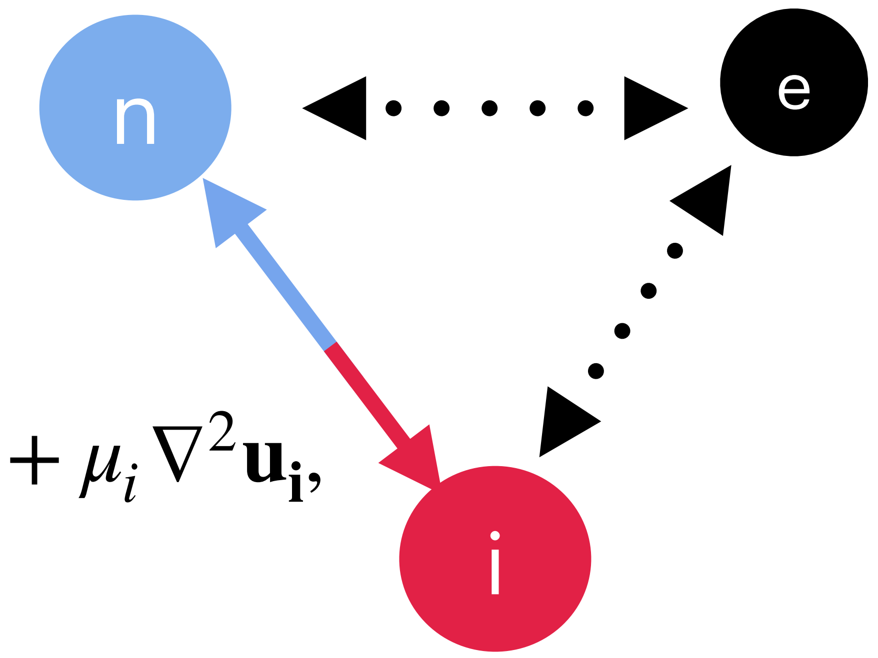
- Equation of motion of three species on **β-plane** (rel. to planetary system):

n: $$\rho_n\left(\frac{\partial}{\partial t}+\mathbf{u_n}\cdot\nabla\right)\mathbf{u_n}=-\nabla p_n^*-\underbrace{\rho_n\nu_{ni}\left(\mathbf{u_n}-\mathbf{u_i}\right)}_{\text{i-n drag}}-\underbrace{\rho_n\nu_{ne}\left(\mathbf{u_n}-\mathbf{u_e}\right)}_{\text{e-n drag}}-\underbrace{2\rho_n\mathbf{\Omega}\times\mathbf{u_n}}_{\text{Coriolis force}}+\mu_n\nabla^2\mathbf{u_n}$$

e: $$0=\rho_e\left(\frac{\partial}{\partial t}+\mathbf{u_e}\cdot\nabla\right)\mathbf{u_e}=-\nabla p_e^*-\underbrace{\rho_e\nu_{ei}\left(\mathbf{u_e}-\mathbf{u_i}\right)}_{\text{i-e drag}}-\underbrace{\rho_e\nu_{en}\left(\mathbf{u_e}-\mathbf{u_n}\right)}_{\text{n-e drag}}-ne(\mathbf{E}+\mathbf{u}_e\times\mathbf{B})+\mu_e\nabla^2\mathbf{u_e}$$

i: $$\rho_i\left(\frac{\partial}{\partial t}+\mathbf{u_i}\cdot\nabla\right)\mathbf{u_i}=-\nabla p_i^*-\underbrace{\rho_i\nu_{in}\left(\mathbf{u_i}-\mathbf{u_n}\right)}_{\text{n-i drag}}-\underbrace{\rho_i\nu_{ie}\left(\mathbf{u_i}-\mathbf{u_e}\right)}_{\text{n-e drag}}+ne(\mathbf{E}+\mathbf{u}_i\times\mathbf{B})-\underbrace{2\rho_i\mathbf{\Omega}\times\mathbf{u_i}}_{\text{Coriolis force}}+\mu_i\nabla^2\mathbf{u_i},$$

Ignore coupling btw e-i and e-n $m_e\nu_{en}\ll m_i\nu_{in}$



Strength of coupling

$$\rho_i\nu_{in}=\rho_n\nu_{ni}\equiv\alpha\rho_i\rho_n$$

- Ion and neutral field equation:

$$\rho_i\left(\frac{\partial}{\partial t}+\mathbf{u_i}\cdot\nabla\right)\mathbf{u_i}=-\nabla p_i^*+\frac{1}{\mu_0}(\mathbf{B}\cdot\nabla)\mathbf{B}+\rho_i\nu_{in}\left(\mathbf{u_n}-\mathbf{u_i}\right)-2\rho_i\mathbf{\Omega}\times\mathbf{u_i}+\mu_i\nabla^2\mathbf{u_i}$$

$$\rho_n\left(\frac{\partial}{\partial t}+\mathbf{u_n}\cdot\nabla\right)\mathbf{u_n}=-\nabla p_n^*+\rho_n\nu_{ni}\left(\mathbf{u_i}-\mathbf{u_n}\right)-2\rho_n\mathbf{\Omega}\times\mathbf{u_n}+\mu_n\nabla^2\mathbf{u_n}$$

In high coupling regime, ion has force balance:

$$\mathbf{J}\times\mathbf{B}+\mathrm{drag}=0$$

So the neutrals will also feel the $\mathbf{J}\times\mathbf{B}$ force.

Electromagnetism: Ambipolar Diffusion

JxB force acts on ions

↓

Ambipolar Diffusion

Magnetic diffusion: $\eta \rightarrow \eta + \eta_{am}$

Damps the pert. magnetic field $\widetilde{B}$

↓

The Maxwell Stress decreases

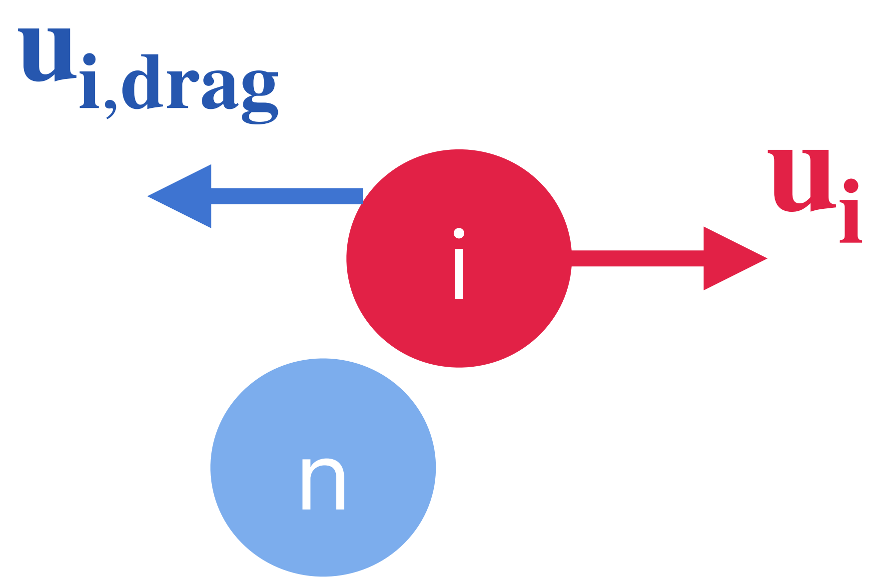
↓

Increase zonal flow and damps drift-wave turbulence

Ohm's Law:

$$\mathbf{E} = \eta \mathbf{J} - \mathbf{u_i} \times \mathbf{B} - \frac{m_e \nu_{en}}{e}(\mathbf{u_i} - \mathbf{u_n}),$$

JxB due to the ion-neutral coupling:



$$\mathbf{u_{i,drag}} = \mathbf{u_i} - \mathbf{u_n} = \frac{\mathbf{J} \times \mathbf{B}}{\alpha \rho_i \rho_n}$$

B field frozen into the neutrals, but now the magnetic diffusion is **ambipolar diffusion** η_{am}.

Key question:

How does the **ambipolar diffusion** (non-linear B-diffusivity from neutral effect) affect the zonal flow? What's the effect on magnetic perturbation $\widetilde{B}$.

Backup Slides

Linear Modes of Coupled System

- In the linear theory, we obtain scalar equations:

Vorticity equation:

$$\rho_i\left(\frac{\partial}{\partial t}+\langle\mathbf{u_i}\rangle\cdot\nabla\right)\tilde{\zeta}_i=-\rho_i\tilde{u}_{i,y}\frac{\partial}{\partial y}\langle\zeta_i\rangle+B_{0,x}\frac{\partial\tilde{J}_z}{\partial x}+\tilde{B}_y\frac{\partial\langle J_z\rangle}{\partial y}+\alpha\rho_i\rho_n\left(\tilde{\zeta}_n-\tilde{\zeta}_i\right)+\mu_i\nabla^2\tilde{\zeta}_i.$$

$$\rho_n\left(\frac{\partial}{\partial t}+\langle\mathbf{u_n}\rangle\cdot\nabla\right)\tilde{\zeta}_n=-\rho_n\tilde{u}_{n,y}\frac{\partial}{\partial y}\langle\zeta_n\rangle+\alpha\rho_i\rho_n\left(\tilde{\zeta}_i-\tilde{\zeta}_n\right)+\mu_n\nabla^2\tilde{\zeta}_n$$

Induction equation:

$$\tilde{A}_z=\frac{-\tilde{\zeta}_n}{k^2}\frac{B_0k_x}{\omega+i\eta k^2+i\frac{B_0^2}{\mu_0\alpha\rho_i\rho_n}k^2}$$

Coupling effect

$$\eta_{eff,k}\equiv\eta+\frac{B_0^2}{\mu_0\alpha\rho_i\rho_n}$$

$$\tilde{\zeta}_{n,k}=\frac{D_n}{D}=\frac{S_1\rho_i(-i\omega_i+\nu_ik^2)+\alpha\rho_i\rho_n(S_1+S_2)}{\rho_i\rho_n(-i\omega_i+\nu_ik^2)(-i\omega_n+\nu_nk^2)+\alpha\rho_i\rho_n\left(\rho_i(-i\omega_i+\nu_ik^2)+\rho_n(-i\omega_n+\nu_nk^2)\right)+\frac{i\alpha\rho_i\rho_n}{\mu_0}\frac{B_0^2k_x^2}{\omega+i\eta_{eff}k^2}}$$

- We obtain a critical dimensionless parameter:

$$\gamma=\frac{(\alpha\rho_i)\omega_{eff}^2}{\omega+i\eta_{eff}k^2}\cdot\tau_{eddy}\cdot\frac{1}{\omega_{ci}}$$

→ Important Time scales

$$\omega_{eff}^2==\equiv\frac{B_0^2k_x^2}{\omega+i\eta_{eff}k^2}$$

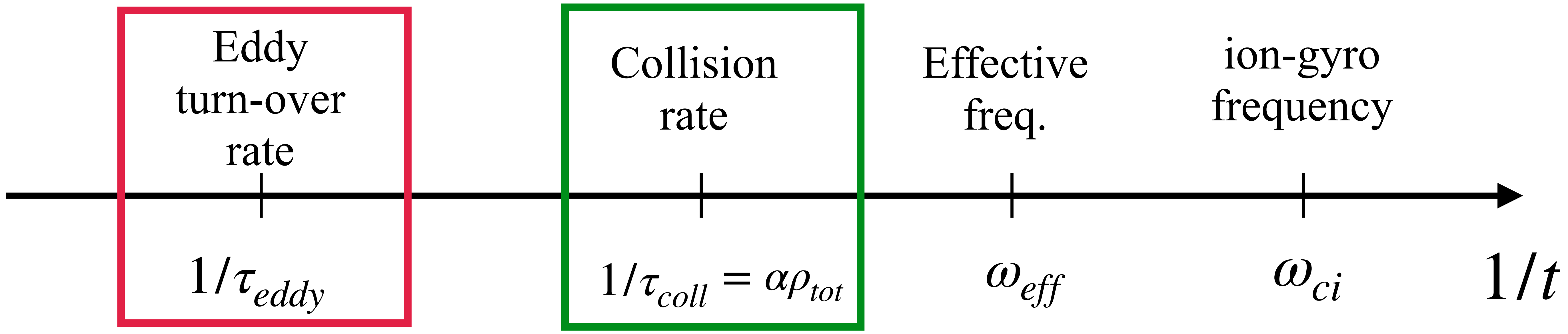
$$\begin{cases}\gamma\ll 1 & \text{weak coupling regime}\\ \gamma\gg 1 & \text{strong coupling regime}\end{cases}$$

Backup Slides

Important Time Scales

- We consider rates:

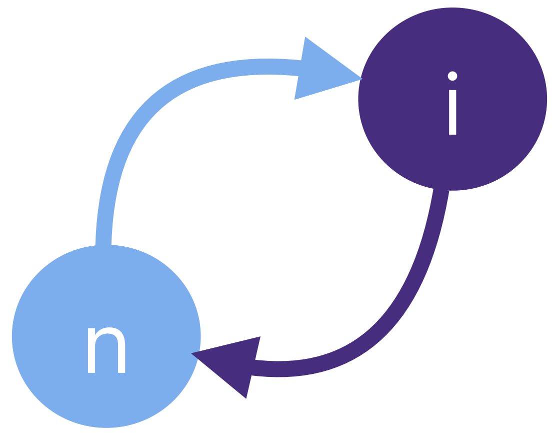
$$\gamma = \frac{(\alpha\rho_i)\omega_{eff}^2}{\omega + i\eta_{eff}k^2} \cdot \tau_{eddy} \cdot \frac{1}{\omega_{ci}}$$



- A simplified coupling parameter:

$$\tilde{\alpha} \equiv \frac{\tau_{eddy}}{\tau_{coll}} = \frac{l_\perp \rho_{tot}\alpha}{\tilde{u}}$$

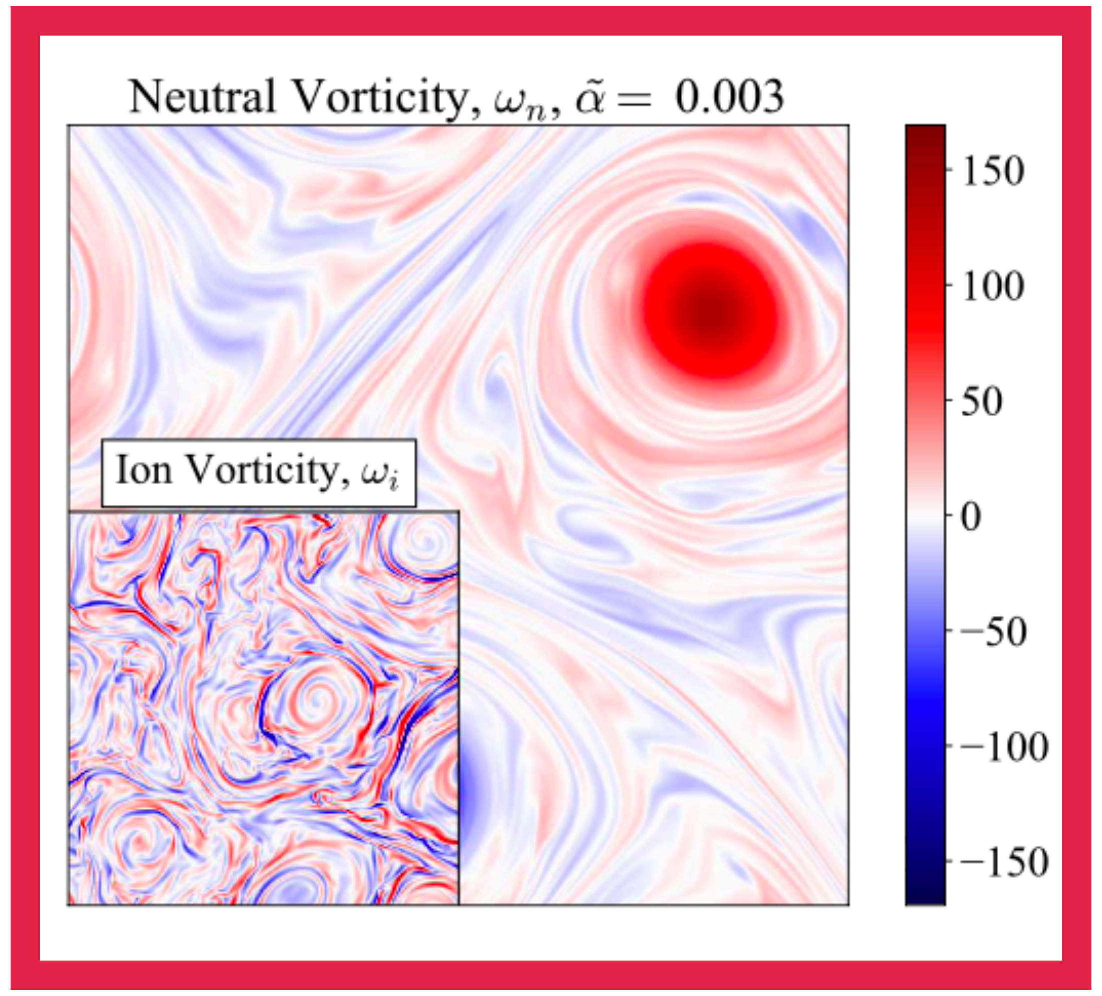
→ Strength of coupling



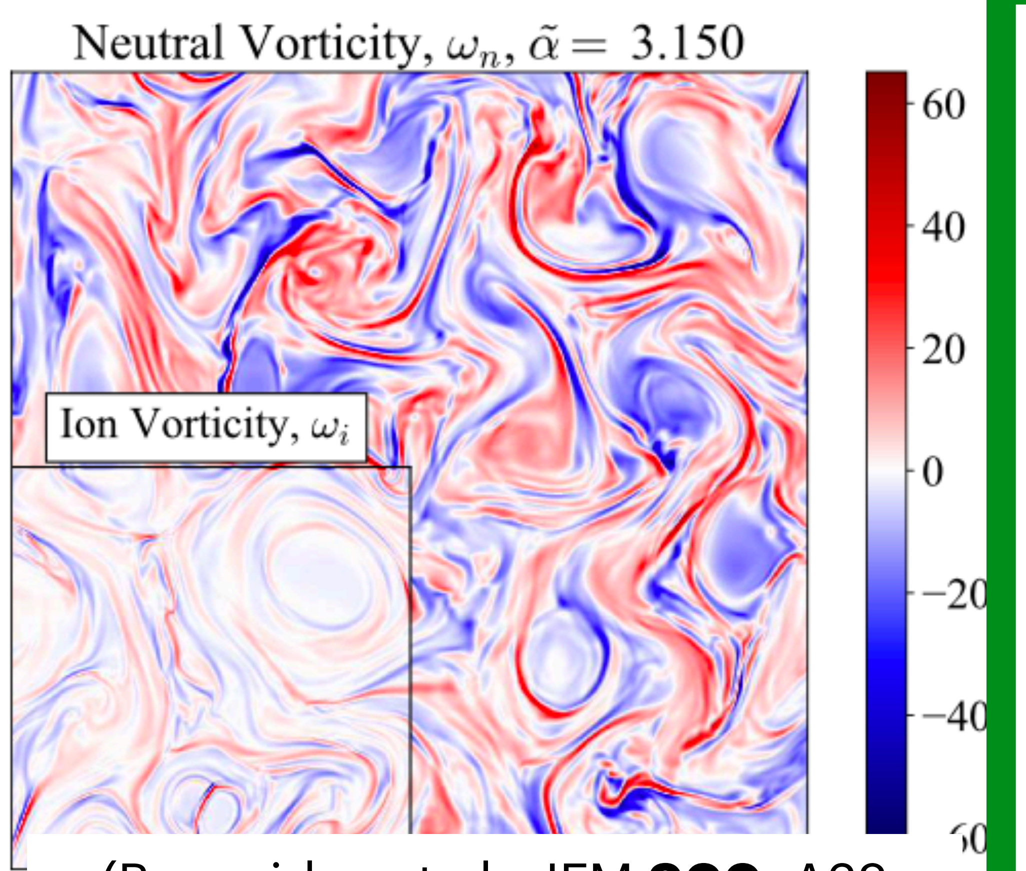
- Dynamics of neutrals and ions:

Weak coupling:

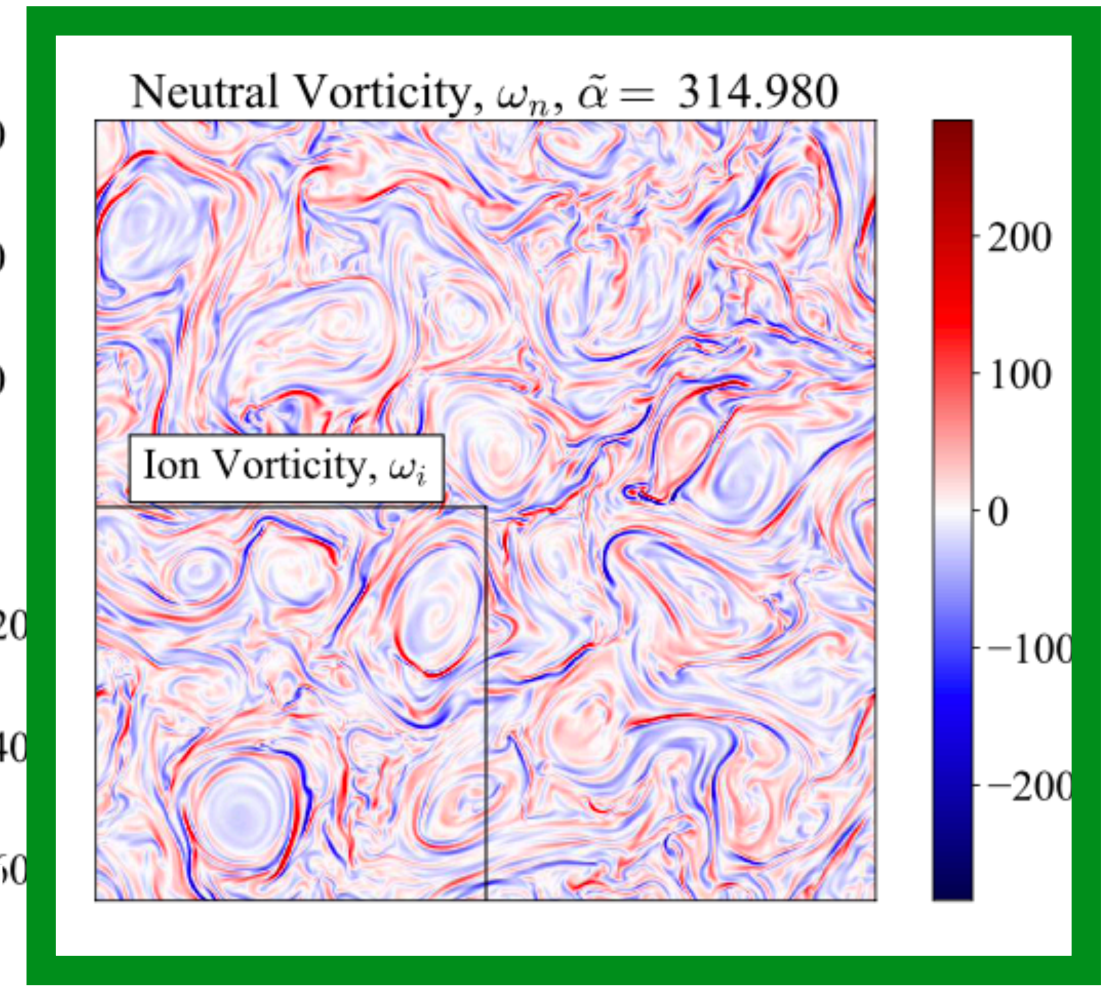
Neutrals and ions not well-coupled —> Two fluid.



small α



(Benavides et al., JFM **900**, A28 (2020))



large α

Strong coupling:

Neutrals couple to ions and behaves as **one MHD fluid**.

Deriving Mag. Potential Equation

- The induction equation, and consider **strong coupling regime**: $\alpha\rho_i \gg 1/\tau_{eddy}$:

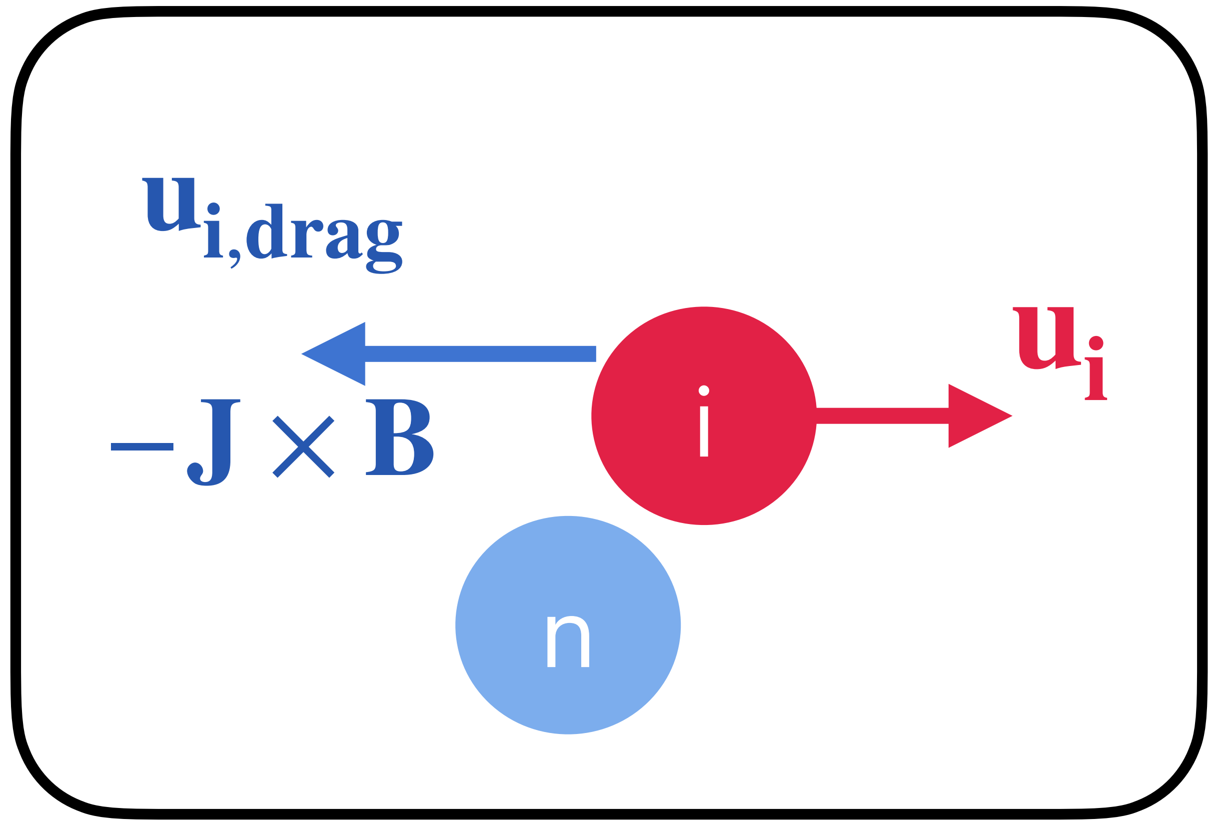
JxB-drag balance on ion: $-\mathbf{J}\times\mathbf{B} = \mathbf{f_{d,i}}$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u_i}\times\mathbf{B}) + \eta\nabla^2\mathbf{B}\,.$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u_n}\times\mathbf{B}) + \nabla\times(\frac{\mathbf{J}\times\mathbf{B}}{\alpha\rho_i\rho_n}\times\mathbf{B}) + \eta\nabla^2\mathbf{B},$$

$$\mathbf{u_i} = \mathbf{u_n} - \frac{2\mathbf{u}_{i,drag}}{1+\rho_i/\rho_n}$$

$$\mathbf{u_i} = \mathbf{u_n} + \frac{\mathbf{J}\times\mathbf{B}}{\alpha\rho_i\rho_n}$$



- The **magnetic potential** equation becomes

$\eta \rightarrow \eta + \underline{\underline{\eta_{am}}}$

Resistivity correction due to the ion-neutral drag

$$\frac{\partial A_z}{\partial t} + (\mathbf{u_n}\cdot\nabla)A_z = \frac{1}{\mu_0\alpha\rho_i\rho_n}\underline{\nabla}\cdot\left[\underline{\underline{\eta_{am}}}\cdot\underline{\nabla}A_z\right] + \eta\nabla^2 A_z + C$$

$$= \frac{1}{\mu_0\alpha\rho_i\rho_n}\begin{pmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y}\end{pmatrix}\begin{pmatrix}\mathscr{F} & \mathscr{G}\\ \mathscr{H} & \mathscr{I}\end{pmatrix}\begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\end{pmatrix}A_z + \eta\nabla^2 A_z + C$$

Ambipolar diffusion tensor for A:

$$\underline{\underline{\eta_{am}}} = \begin{pmatrix}\mathscr{F} & \mathscr{G}\\ \mathscr{H} & \mathscr{I}\end{pmatrix} = \begin{pmatrix}\frac{1}{2}(B_y^2 - B_x^2) & -B_xB_y\\ -B_xB_y & \frac{1}{2}(B_x^2 - B_y^2)\end{pmatrix}$$

Additional terms:

$$C = \frac{1}{\mu_0\alpha\rho_i\rho_n}\left[\frac{-B^2}{2}\nabla^2 A_z + B_x^2\frac{\partial^2}{\partial x^2}A_z + B_y^2\frac{\partial^2}{\partial y^2}A_z + 2B_xB_y\frac{\partial^2}{\partial x\partial y}A_z\right]$$

Results — Two-Fluid Rossby-Alfvén Wave

By solving the linear equations for ions, neutrals, and magnetic potential, we have

- Strong coupling regime γ>>1 $(1/\tau_{eddy} \ll \alpha\rho_i \ll \omega_{eff} \ll \omega_{ci})$:

$$(\omega - \omega_R + i\nu k^2)(\omega + i\eta_{eff}k^2) = \frac{\rho_i}{\rho_{tot}} \frac{B_0^2 k_x^2}{\omega + i\eta_{eff}k^2} \qquad \eta_{eff,k} \equiv \eta + \frac{B_0^2}{\mu_0 \alpha \rho_i \rho_n}$$

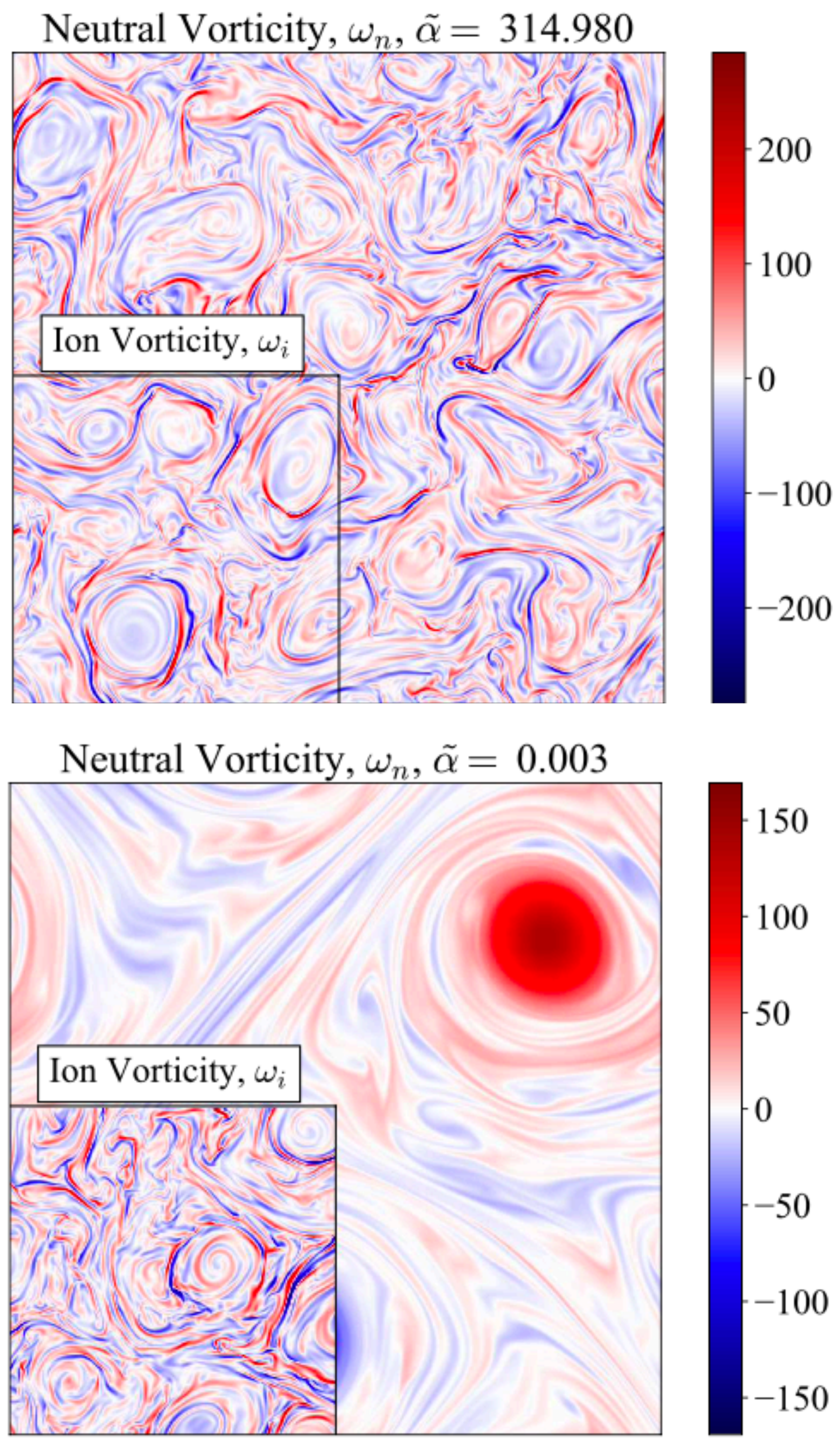
In strong collision regime, the ions and neutrals are strongly coupled and behave like **single MHD fluid**.

But contains **ion-neutral coupling effect**.

- Weak coupling regime γ<<1 $(\alpha\rho_i \ll 1/\tau_{eddy} \ll \omega_{eff} \ll \omega_{ci})$:

$$(\omega_i - \omega_R + i\nu k^2 + i\alpha\rho_n)(\omega_n - \omega_R + i\nu k^2 + i\alpha\rho_i) = -\alpha^2\rho_i\rho_n$$

Ions and neutrals evolve separately but have a weak mutual drag on each other.



(Benavides et al., JFM **900**, A28 (2020))

Backup Slides

How to Determine Maxwell Stress?

- Zeldovich Theorem:

Magnetic diffusion η damps the pert. magnetic field $\widetilde{B}$ →

$$Rm \simeq \frac{\langle \widetilde{B}^2 \rangle}{B_0^2} = \frac{\langle \widetilde{u}^2 \rangle \tau_c}{\eta}$$

$$\langle \widetilde{B}^2 \rangle = \langle \widetilde{u}^2 \rangle \tau_c \frac{B_0^2}{\eta} \propto \frac{B_0^2}{\eta}$$

enhance

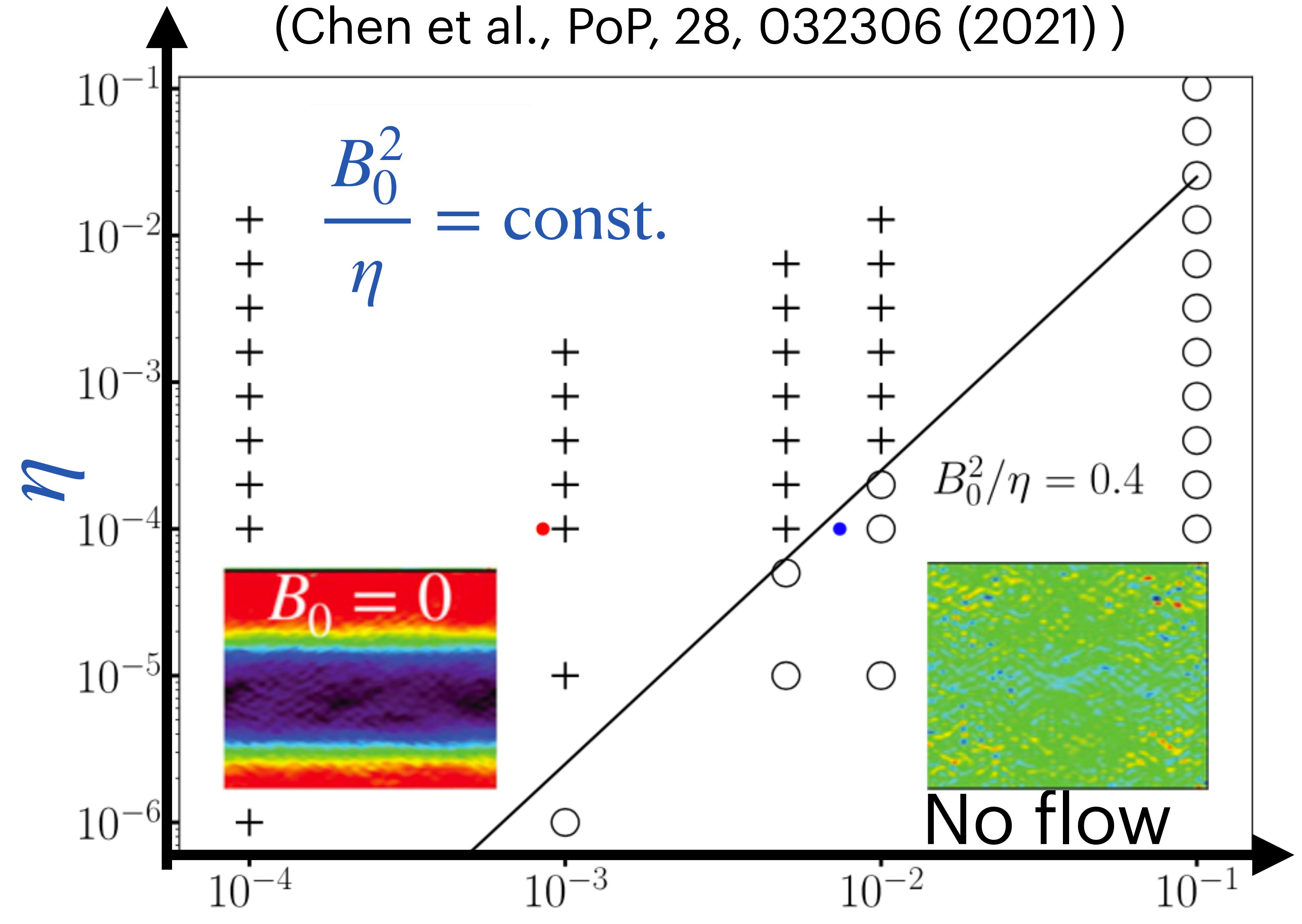
suppress

B_0^2/η is a regulator in the competition btw Reynolds and Maxwell stress competition.

Evolution of zonal flow →

$$\frac{\partial \langle u_y \rangle}{\partial t} = -\frac{\partial}{\partial x}\left(\langle \widetilde{u}_x \widetilde{u}_y \rangle - \frac{\langle \widetilde{B}_x \widetilde{B}_y \rangle}{\mu_0 \rho} \right)$$

(Chen et al., PoP, 28, 032306 (2021))



Reynolds and Maxwell Stress competition

Mean diffusivity η damps while B_0^2 increase the Maxwell stress $\langle \widetilde{B}_x \widetilde{B}_y \rangle$. And hence B_0^2/η regulates the growth of zonal flow.

What is the effect of neutrals on the Zel'dovich Theorem?

Modified Zel'dovich Theorem

- Without neutrals, magnetic stress is proportional to B_0^2/η :

$$\langle \widetilde{B}^2 \rangle = \langle \widetilde{u}^2 \rangle \tau_c \frac{B_0^2}{\eta} \propto \frac{B_0^2}{\eta}$$

- Now, consider the neutral-ion drag effect:

$$\eta \rightarrow \eta + \eta_{am} \qquad Rm \rightarrow Rm + Rm_{am}$$

$$Rm_{tot} = Rm + Rm_{am} = \frac{\langle \widetilde{B}^2 \rangle}{B_0^2} + \frac{\langle \widetilde{B}^2 \rangle}{\eta \mu_0 \alpha \rho_i \rho_n} \frac{\langle \widetilde{B}^2 \rangle}{B_0^2}$$

Assumption: $\langle \widetilde{B}^4 \rangle \simeq \langle \widetilde{B}^2 \rangle \langle \widetilde{B}^2 \rangle$

$$\eta \nabla^2 \mathbf{B} \quad \text{v.s} \quad \nabla \times \left(\frac{\mathbf{J} \times \mathbf{B}}{\alpha \rho_i \rho_n} \times \mathbf{B}\right)$$

Change in Scaling

	Maxwell Stress Intensity
No Neutrals	$\langle \widetilde{B}^2 \rangle \propto B_0^2/\eta$
Ambipolar diffusion effect	$\langle \widetilde{B}^2 \rangle \propto B_0/A^{1/2}$

...alters the competition

- In the limit where ambipolar diffusion dominated ($Rm \ll Rm_{am}$) we have:

$$Rm_{tot} \simeq Rm_{am} = \frac{\langle \widetilde{B}^2 \rangle}{\eta \mu_0 \alpha \rho_i \rho_n} \frac{\langle \widetilde{B}^2 \rangle}{B_0^2} = \frac{\langle \widetilde{u}^2 \rangle \tau_c}{\eta}$$

Maxwell Stress intensity: $\langle \widetilde{B}^2 \rangle = \left(\langle \widetilde{u}^2 \rangle \tau_c \mu_0 \rho_i \frac{B_0^2}{A} \right)^{1/2} \propto \frac{B_0}{A^{1/2}}$

$$A \equiv \frac{1}{\alpha \rho_n} = \frac{1}{\nu_{in}}$$

The magnetic stress effect on zonal flow generation with neutrals is less sensitive to B_0 .

Conclusions

- We study Non-trivial neutral effect on DW-ZF turbulence: Ambipolar diffusion
- We derive the **key parameter γ** for the Drift-Alfvén + Neutral effect:

$$\gamma = \frac{(\alpha\rho_i)\omega_{eff}^2}{\omega + i\eta_{eff}k^2} \cdot \tau_{eddy} \cdot \frac{1}{\omega_{ci}}$$

- We study the Drift-Alfvén wave with neutrals and found—
  In strong coupling regime: **one MHD fluid**.
  In weak coupling regime: **two fluid.**
- **Modified Zel'dovich Theorem**. We derive the key parameter that **regulates** Maxwell and Reynolds stress competition:

$$\langle \widetilde{B}^2 \rangle \propto B_0^2/\eta \text{ (original)}$$

$$\langle \widetilde{B}^2 \rangle \propto B_0/A^{1/2} \text{ (ambipolar effect)}$$

Future Works

- Calculate $\widetilde{B}$ evolution for arbitrary $\gamma \rightarrow$ Pouquet + neutrals (clarify asymptotic regime).
- Study the physics of neutral entrainment.