A Model for Staircases in Drift Wave Turbulence → Beyond BLY…

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BLY = Balmforth, Llwellyn-Smith, Young

This research was supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, under Award Number DEFG02-04ER54738.

Outline

- Prologue
- What really is – and is not – in the model?
- Beyond BLY – Issues, Buried Bodies and Flux-Driven Dynamics
- Where to Next?

Basic Results: W.X. Guo, previous talk

Some Thoughts

- BLY, et. seq. is a <u>model</u>
- “All models are wrong, some models are useful” – George Box
- BLY definitely is useful !

But also:

- “Some models are too good to be true. Other models are too true to be good.” – Anon.

The Bounty of BLY, for Drift Wave Systems

* • A. Ashourvan, P.D. – Phys. Rev. E. Rap. Comm. (2016), PoP (2017)

  ➔ Hasegawa-Wakatani drift wave turbulence

• M. Malkov, P.D. – Phys. Rev. Fluids (2019)

  ➔ QG/β –plane

* • W.X. Guo, P.D., Hughes et. al. – PPCF (2019)

  ➔ H-W Drift Wave Turbulence

What is in the model?

Basic Equations ↔ Hasegawa-Wakatani (life beyond CHM)

$$\frac{d}{dt}\nabla_\perp^2\phi + \chi_{\parallel e}\nabla_\parallel^2(\phi - n) = \mu\nabla_\perp^2\nabla_\perp^2\phi$$

$$\frac{d}{dt}n + \chi_{\parallel e}\nabla_\parallel^2(\phi - n) = D_0\nabla_\perp^2 n$$

$$\frac{d}{dt} = \partial_t + \nabla\phi \times \hat{z} \cdot \nabla \qquad n = \langle n(x)\rangle + \tilde{n} \qquad \nabla_\perp^2\phi = \langle\nabla_\perp^2\phi(x)\rangle + \nabla_\perp^2\tilde{\phi}$$

- PV $q = n - \nabla_\perp^2\phi$ conserved! , to μ , D_0
- $\chi_\parallel \neq 0 \rightarrow \langle\tilde{v}_r\tilde{n}\rangle \neq 0$ 'negative dissipation mechanism' ➔ drift instability (Sagdeev, et. al.)

  $\omega \leq \omega_{*e} \rightarrow \langle\tilde{v}_r\tilde{n}\rangle > 0$

  shear
- ZF ➔ $k_\parallel = 0$ $n \leftrightarrow \nabla_\perp^2\phi$ PV exchange
- ZF $\rightarrow \langle\tilde{v}_r\nabla^2\tilde{\phi}\rangle \rightarrow$ Reynolds force

  Corrugation $\rightarrow \langle\tilde{v}\tilde{n}\rangle \rightarrow$ particle flux

$\langle\tilde{n}\nabla^2\tilde{\phi}\rangle$?

c.f. singh, P.D. 2021

‘Bistable’ Mixing – A Simple Mechanism

- Mean field model with 2 mixing scales (after BLY 1998)
- So, for H-W:
- Density: $$\frac{\partial}{\partial t}\langle n\rangle = \frac{\partial}{\partial x}\left(D_n \frac{\partial \langle n\rangle}{\partial x}\right) + D_c \frac{\partial^2 \langle n\rangle}{\partial x^2}$$
- Vorticity: $$\frac{\partial}{\partial t}\langle u\rangle = \frac{\partial}{\partial x}\left[(D_n - \chi)\frac{\partial \langle n\rangle}{\partial x}\right] + \chi \frac{\partial^2 \langle u\rangle}{\partial x^2} + \mu_c \frac{\partial^2 \langle u\rangle}{\partial x^2},$$

simple mixing + 2 length scale
→ staircase

- Enstrophy(intensity): $$\frac{\partial}{\partial t}\varepsilon = \frac{\partial}{\partial x}\left(D_\varepsilon \frac{\partial \varepsilon}{\partial x}\right) + \chi\left[\frac{\partial \langle n-u\rangle}{\partial x}\right]^2 - \varepsilon_c^{-1/2}\varepsilon^{3/2} + \gamma_\varepsilon \varepsilon.$$ → includes ~~crude~~ crude turbulence spreading model
- $D, \chi \sim \tilde{V} l_{mix}$

$$\boxed{l_{\text{mix}} = \frac{l_0}{\left(1 + l_0^2 [\partial_x (n-u)]^2 / \varepsilon\right)^{\kappa/2}},}$$

l_0 → excitation scale (drive)
l_R → Rhines scale (emergent)
ω_{MM} vs $\Delta\omega$ - can be generalized

two scales!

- Scale cross-over → ‘transport bifurcation’
- $l_0/l_R < 1$ → strong mixing (eddys)
- $l_0/l_R > 1$ → weak mixing (~~eddys~~) (waves) → sharpening feedback
- Is this ~ equivalent to ‘two-fluid’ mixing length model (E.A. Spiegel)

How, Why ?

- PV is mixed → natural for 'mixing length model' exploits conserved phase space density
- Potential Enstrophy is natural formulation – $\langle \delta f^2 \rangle$ for intensity → conserved
- Beyond BLY → 2 mean fields $\langle n \rangle$, $\langle \nabla^2 \phi \rangle$ + ε – fluctuation potential enstrophy

  → exchange and couplings
- Reynolds work and particle flux couple mean and fluctuations
- Nonlinear damping ↔ forward enstrophy cascade
- $D_n, \chi \rightarrow$ turbulent transport coefficients are fundamental
- Glorified '$k - \epsilon$ model'

How, Why ? Cont'd

- $l_{mix} > \rho_s$ $\rightarrow$ simplifies inversion $(\nabla^2\phi \rightarrow V)$
- Dissipative DW ~ adiabatic regime: $k_\parallel^2 V_{the}^2/\nu \gg \omega$

  $D_n \approx \tilde{v}^2/\alpha \sim \epsilon l^2/\alpha$ $\longrightarrow$ $\langle v_r \tilde{n}\rangle$ phase fixed by α!

  Major simplification $\longrightarrow$ <u>solid</u>, where applicable

  $\chi \sim D_n$ (non-resonant diffusion)
- $\langle \tilde{v}_r \nabla^2\phi\rangle = -\chi\partial_x\langle\nabla^2\phi\rangle + \Pi_{resid}[\nabla n]$

  $\langle\nabla^2\phi\rangle =$ <u>shear</u> χ on
- $\langle \tilde{v}_r \tilde{q}^2\rangle$ $\rightarrow$ $-l^2\epsilon^{1/2}\partial_x\varepsilon$ spreading, entrainment, SOFT

How, Why ? Cont'd

- D_n, χ regulate P.E. exchange between mean, fluctuations ➔ key role in model

- Mixing Length: $l_{mix} = \frac{l_0}{\left[1+\frac{l_0^2[\partial_x(n-u)]^2}{\epsilon}\right]^{\kappa/2}} = \frac{l_0}{1+\left(l_0^2\ /\ l_{Rh}^2\right)^{\kappa/2}}$

  Physics: "Rossby Wave Elasticity'

  i.e. $D \sim \frac{\langle\tilde{v}^2\rangle}{\Delta\omega} \rightarrow \langle\tilde{v}^2\rangle\frac{\Delta\omega}{\omega_r^2+(\Delta\omega)^2} \approx \langle\tilde{v}_r^2\rangle\frac{\Delta\omega}{\omega_r^2}$ for $\Delta\omega < \omega_r$

  → waves enhance memory

  → $\omega_r \sim \nabla\langle q\rangle$ → nonlinear Γ_{PV} vs $\langle q\rangle$ → S-curve

- Soft point: κ → suppression exponent

  $\kappa = 1$ doesn't always work

  Rigorous bound, from fundamental equations?

Beyond BLY

- **Issues, Buried Bodies**

  and Flux-Driven Systems

N.B. In some cases, body parts visible above ground…

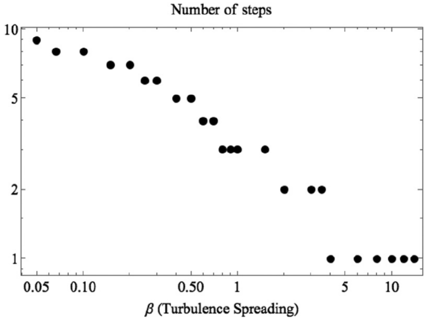
Spreading/Entrainment

- Spreading/entrainment effect on P.E. is unconstrained, beyond $\nabla \cdot \Gamma_q$ structure

  Contrast: $D_n,\ \chi$ Follow standard $k - \epsilon$ model CRUDE !

- How robust is staircase to effects of entrainment, avalanching… ?

- $D_\varepsilon \rightarrow \beta\, l^2\, \epsilon^{1/2}$

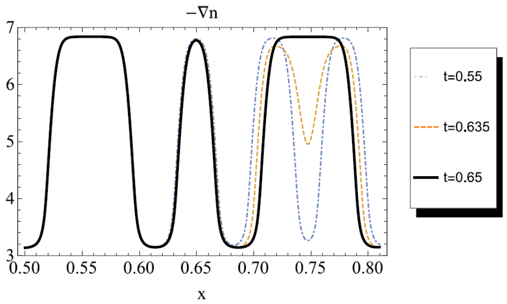


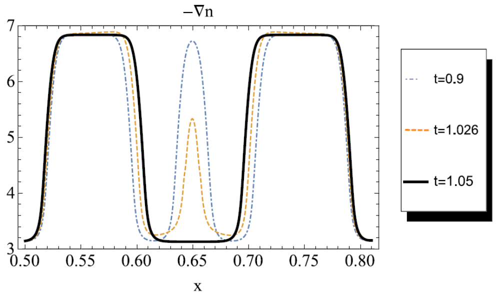
Entrainment has significant effect on S.C. structure

Large β $\rightarrow$ wash out S.C.

- Important !

Mergers Happen !





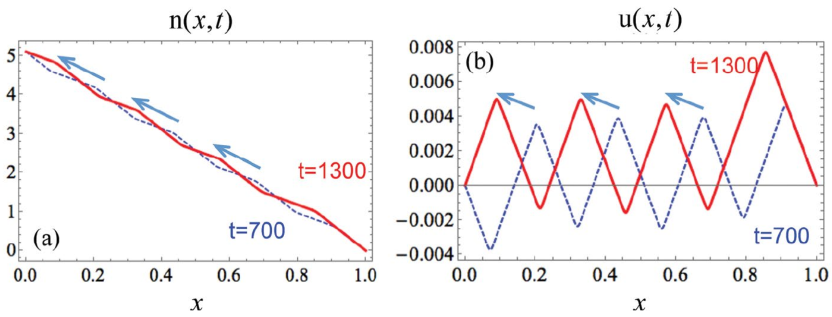
- 'Type-II' merger (c.f. Balmforth, in 'Interfaces')
- 'Type-I' (motion) mergers also observed

➔ Staircase coarsens….

➔ Obvious TBD:

- – Interplay/Competition of Spreading and Mergers?
- – Scan coarsening time vs β, merger rate vs increments in β

Staircases and Dynamics ! (Globally)



- B.C. Neumann LHS, Dirichlet RHS.. (ala' sandpile) → asymmetry
- 'Escalator Modes'
- 'Shear Migration'

appear. Cause, Consequence?

➔ "Non-locality" → c.f. next week (Yan, P.D.)

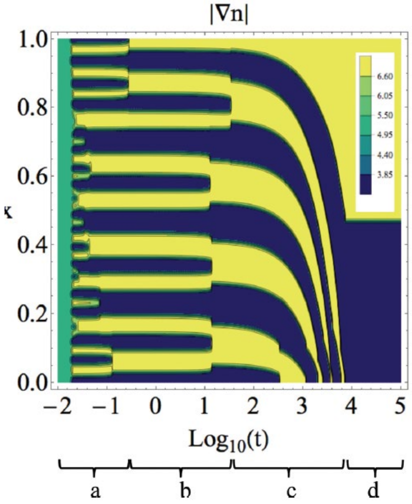
- Needs further study…

➔ Credible model must address staircase dynamics

Dynamics is ~ local (mergers) and global (above)

Dynamic Staircases, Cont'd

- Steps and barriers observed to condense to outer boundary



Is this a way to understand L→H transition?

Ashourvan, P.D. (2016)

- Collapse of staircase into macroscopic barriers?
- Need quantify!

Flux Driven Studies

- MFE problems are almost always flux-driven, with source and sink. Not addressed in BLY ’98.

- For conservative drive:

Collisional transport (‘neoclassical’)

Drive (conservative)

$$\partial_t n = \partial_x D_n \partial_x n + D_c \partial_x^2 n - \partial_x \Gamma_{dr}(x)$$

$$\Gamma_{dr}(x) = \Gamma_0 \exp[-x/\Delta_{dr}]$$

strength

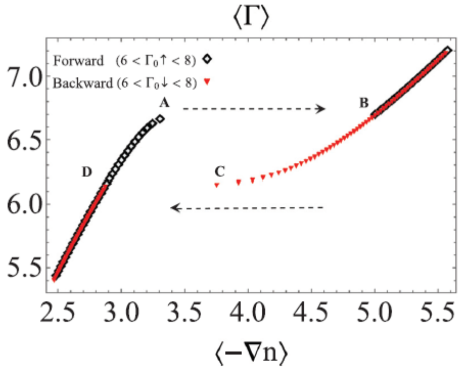
Profile of deposition

$D_n = l^2 \varepsilon / \alpha$ as before

- Now address global confinement dynamics

Global Bifurcation in Staircase

- Average $\langle\Gamma\rangle$ vs $\langle\nabla n\rangle$ plot shows global transport bifurcation and hysteresis



S-curve once more,
with feeling !

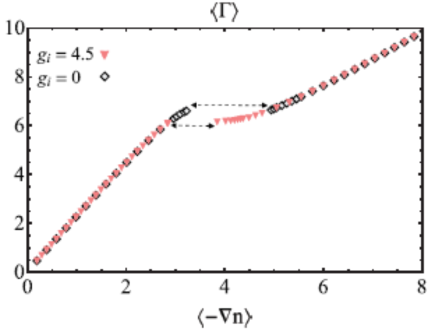
- Global confinement bifurcation, in staircase state

- Regional weightings l_0 , l_{Rh}.   Good confinement, l_{Rh} dominates

- Merits of staircase state ?!   Compare to single barrier ?!

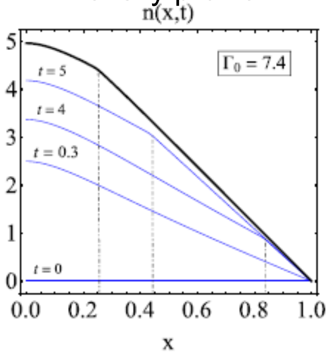
Global Bifurcation, Cont'd

~ Steady State



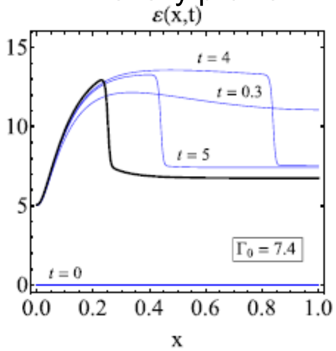
Final state $\langle \Gamma \rangle$ vs $\langle \nabla n \rangle$

Density profile



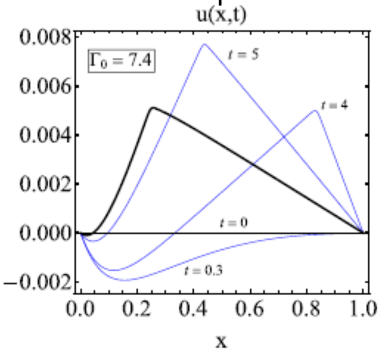
Profile steepens

Intensity profile



Intensity drops

Shear profile



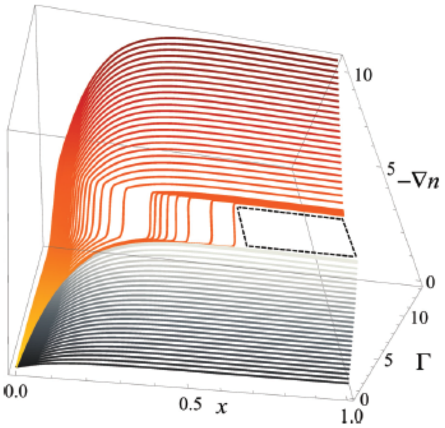
Shear

Global and Local ↔ Flux Landscape

Flux Landscape ↔ family of S-curve

Red → enhanced confinement

Grey → normal confinement



- See also (shameful advertising)
  - P.D., V.B. Lebedev, el. al., PRL ’97
  - Lebedev, P.D., Phys. Plasmas ’98 (barrier propagation)

Where to next?

N.B. Recall –

“Some models are too good to be true.

Other models are too true to be good.”

New Applications – 'Stress Test' the Model

N.B. BLY already 'flogged thru the fleet', but…

- Stochastic field effects: Samantha Chen, P.D. $Ku_b(l_{mix}) > 1$

- Thermal Rossby / ITG → PV conservation broken (buoyancy)

  …. → $\langle \tilde{v}_r \tilde{T} \rangle$ - phase ! ➔ New Twist

- Multi-scale: DW + ETG (GDP + P.D.)

Theory-Enhance Model (but not too complicated!!)

- NL noise – incoherent mode coupling. How represent in M.L.T. ?

  n.b. inhomogeneous mixing – inhomogeneous noise !?

  c.f.: R. Singh, P.D. – PPCF 2021

  B. Farrell, et. al. – 'critical opalescence'

- Dressed parcels – two component model (E. Spiegel, D. Gough "On taking mixing length theory seriously")

  i.e. 'slug' + waves

  ➔ akin dressed test particle model (plasma)

  ➔ implicit in $l_0, \; l_{Rh}$ BLY-type model ?

But what is the gain ?

- Exploit Relation to Wave Kinetics (Vlasov Eqn. for parcel)

  $N = \omega E_W \approx \Omega$ for zonal symmetry (Dubrulle + Nazarenko)

  Potential enstrophy

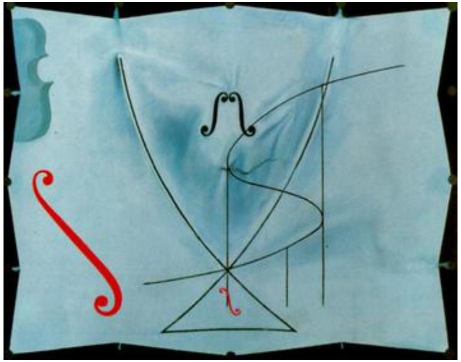
  WKE — stochastic: PD et. al. '05

  coherent: Kaw, Garbet

- Easy to propose extensions, but may jeopardize the simplicity and clarity of BLY '98

Concluding Thoughts

- Problem of layering evolves along a winding road with many jumps, rather like the S-curve…



- So, keep in mind the adventures of:

The Vice-Admiral:

William Bligh; F.R.S., R.N.