Physics of SOL Broadening by Turbulence

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Background

- Conventional Wisdom of SOL:

  (cf: Stangeby...)

  – Turbulent Boundary Layer, ala' Blasius, with D due turbulence

  – $\delta \sim (D\tau)^{1/2}, \tau \approx L_c/V_{th}$

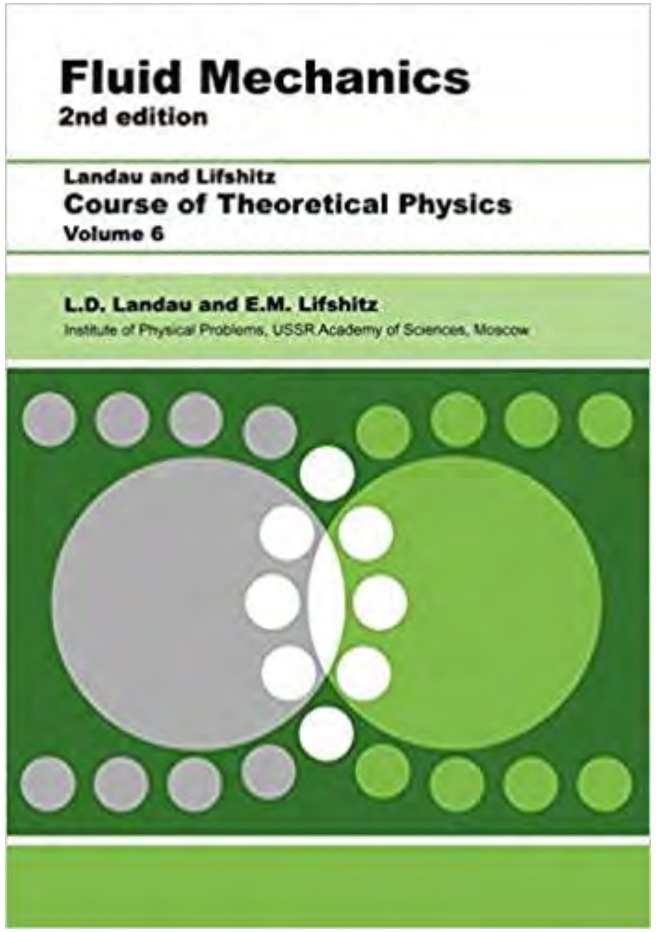
  – $D \leftrightarrow$ local production by SOL instability process

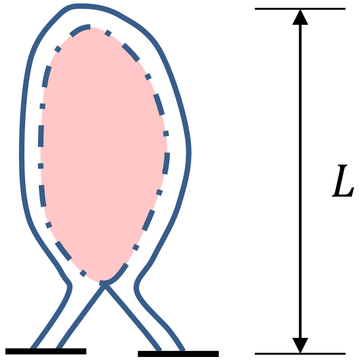
  → familiar approach, D ala' QL

- Features:

  – Open magnetic lines → dwell time τ limited by transit, conduction, ala' Blasius

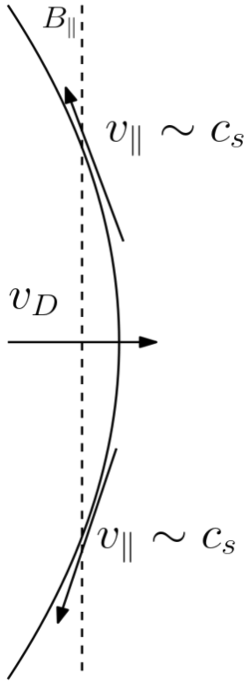
  – Intermittency → "Blobs" etc. Observed. Physics?





Background, cont'd

- But... Heuristic Drift (HD) Model (Goldston +)
  - $V \sim V_{\text{curv}}$, $\tau \sim L_c/V_{thi}$, $\boxed{\lambda \sim \epsilon\, \rho_{\theta i}}$ → SOL width
  - Pathetically small
  - Pessimistic B_θ scaling, yet high I_p for confinement
  - Fits lots of data.... (Brunner '18, Silvagni '20)



- Why does neoclassical work? → ExB shear suppresses SOL modes i.e.

$$\gamma_{\text{interchange}} \sim \frac{c_s}{(R_c\lambda)^{\frac{1}{2}}} - \boxed{\frac{3T_{edge}}{|e|\lambda^2}}$$

shearing ←→ strong λ^{-2} scaling

from: $\frac{c_s}{(R_c\lambda)^{\frac{1}{2}}} - \langle V_E\rangle'$

Background, cont'd

- THE Existential Problem... (Kikuchi, Sonoma TTF):

Desire
- Confinement → H-mode ←→ ExB shear
- Power Handling → broader heat load, etc

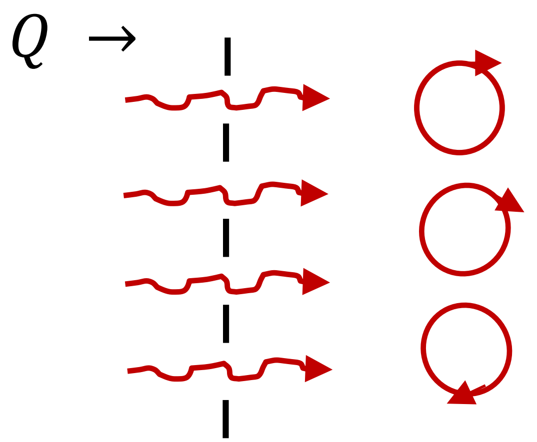
→ <u>Both</u> to be good !

How reconcile? – Pay for power mgmt with confinement ?!

- Spurred:
  - Exploration of turbulent boundary states with improved confinement: Grassy ELM, WPQHM, I-mode, Neg. D ... N.B. What of ITB + L-mode edge?
  - SOL width now key part of the story
  - Simulations, Visualizations (XGC, BOUT...) ~ "Go" to ITER and all be well
- But... What's the Physics ?? <u>How</u> is the SOL broadened?

SOL BL Problem

- SOL Excitation
  - Local production (SOL instabililties)
  - Turbulence energy influx from pedestal
- Key Questions:
  - Local drive vs spreading ratio → Ra
  - Is the SOL usually dominated by turbulence spreading?
  - How far can entrainment penetrate a stable SOL → SOL broadening?
  - Effects ExB shear, role structures ?



local production

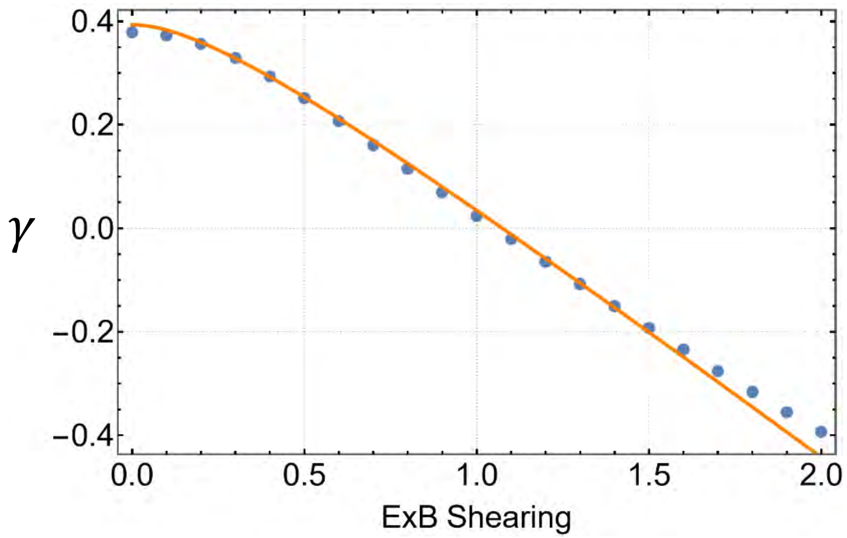
Physics Issues – Part II

[C.f. Chu, P.D., Guo, NF 2022]

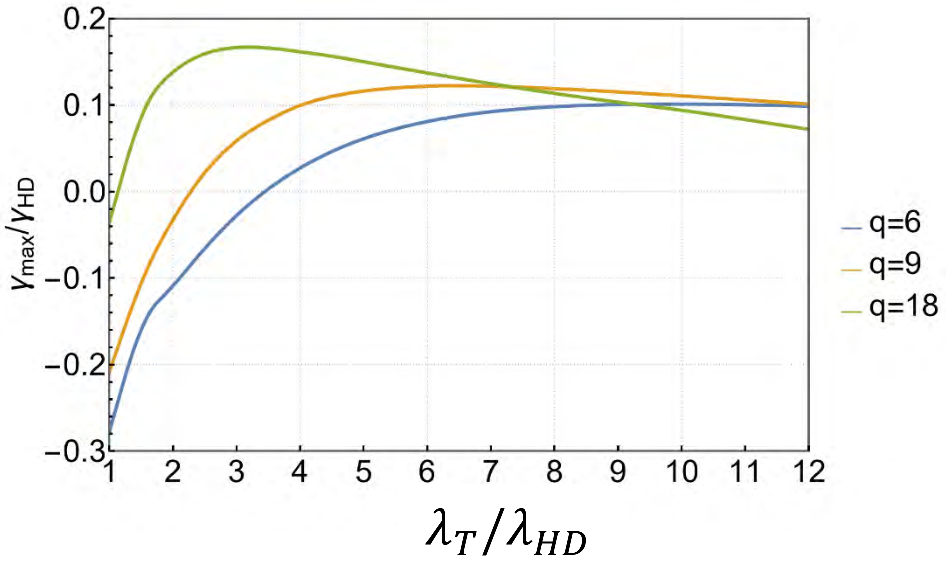
- How calculate SOL width for turbulent pedestal but a locally stable SOL?
  - spreading penetration depth
  - must recover HD in WTT limit
- Scaling and cross-over of λ_q relative HD model
- What is effect/impact of barrier on spreading mechanism?
- Can SOL broadening and good confinement be reconciled ?

Model 1 – Stable SOL – Linear Theory

- Standard drift-interchange with sheath boundary conditions + ExB shear (after Myra + Krash.)



Linear Growth Rate of a specific mode (fixed k_y) v.s. $E \times B$ shear at $q = 5, \beta = 0.001, k_y \cdot \lambda_{HD} = 1.58$.



Maximal Linear Growth Rate of Interchange Mode in the SOL v.s. normalized layer width λ_D/λ_{HD} at different SOL safety factor q (with $\beta = 0.001$)

- Relevant H-mode ExB shear strongly stabilizing $\gamma_{HD} = c_s/(\lambda_{HD} R)^{1/2}$
- Need λ/λ_{HD} well above unity for SOL instability. $V_E' \approx \frac{3T_e}{|e|\lambda^2}$ → layer width sets shear

Model 2 – Two Multiple Adjacent Regions

- "Box Model" – after Z.B. Guo, P.D.

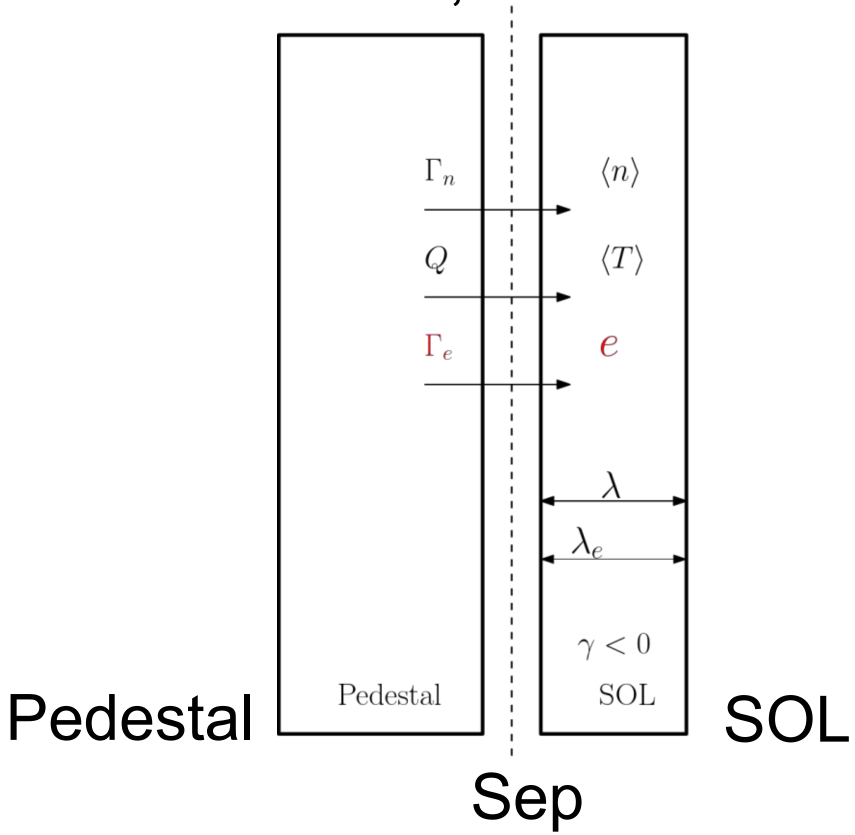


Illustration of Two Box Model: SOL driven by particle flux, heat flux and intensity flux (Γ_e) from the pedestal. The horizontal axis is the radial direction, and vertical axis is the poloidal direction.

- Key Point:
  - Spreading flux from pedestal can enter stable SOL
  - Depth of penetration ➔ extent of SOL broadening

➔ Problem in one of entrainment/penetration

Width of Stable SOL

- Fluid particle: $\frac{dr}{dt} = V_{Dr} + \tilde{V}$

  drift   fluctuating velocity

- Dwell time: $\tau_{\parallel}$

Dwell time $\tau_{\parallel}$
constrains excursion

- $\delta^2 = \langle (\int (V_D + \tilde{V}) dt)(\int (V_D + \tilde{V}) dt) \rangle$

$\langle (\mathrm{step})^2 \rangle$

$$= V_D^2 \tau_{\parallel}^2 + \langle \tilde{V}^2 \rangle \tau_c \tau_{\parallel}$$

correlation time
modest turbulence $\leftrightarrow$ $\tau_c \geq \tau_{\parallel}$

$$= \lambda_{HD}^2 + \varepsilon \tau_{\parallel}^2$$

turbulence energy density

See also
Fokker-Planck analysis
i.e. drift + diffusion

- So $\boxed{\lambda = \left[\lambda_{HD}^2 + \varepsilon \tau_{\parallel}^2\right]^{1/2}}$ → <u>SOL width</u> [Effects add in quadrature]

- How compute ε ? → turbulence energy !

Calculating the SOL Turbulence Energy 1

- $K - \epsilon$ type model, mean field approach (c.f. Gurcan, P.D. '05 et seq)
  - Can treat various NL processes via σ, κ
  - Exploit conservative form model
- $\partial_t \varepsilon = \gamma\varepsilon - \sigma\varepsilon^{1+\kappa} - \partial_x \Gamma_e$

Spreading, turbulence energy flux

NL transfer $\gamma_{NL} \sim \sigma\varepsilon^{\kappa}$

Growth $\gamma < 0$
here contains shear + sheath

- N.B.: No Fickian model of Γ_e employed
- Readily extended to 2D, improved production model, etc.

Calculating the SOL Turbulence Energy 2

- Integrate ε equation $\int_0^{\lambda}$
- Take quantities = layer average
- $\Gamma_{e,0} + \lambda_e \gamma \varepsilon = \lambda_e\, \sigma\, \varepsilon^{1+\kappa}$

Separatrix fluctuation energy flux → Single parameter characterizing spreading

So for $\gamma < 0$,

$$\boxed{\Gamma_{e,0} = \lambda_e |\gamma| \varepsilon + \sigma \lambda_e \varepsilon^{1+\kappa}}$$

λ_e = layer width for ε

$\Gamma_{e,0}$ vs linear + nonlinear damping

Calculating the SOL Turbulence Energy 3

[Mean Field Theory]

- Full system:

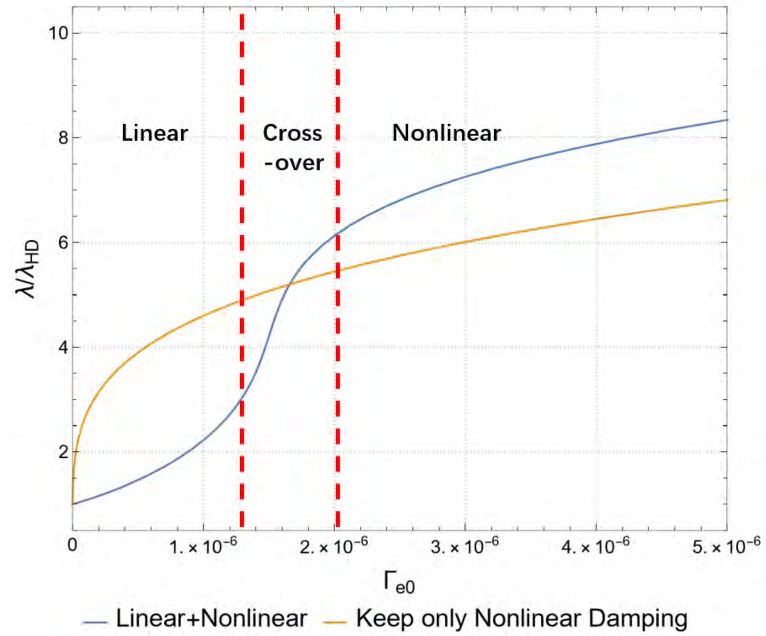
$$\boxed{\begin{aligned}\Gamma_{e,0} &= \lambda_e|\gamma|\varepsilon + \sigma\lambda_e\varepsilon^{1+\kappa} \\ \lambda_e &= \left[\lambda_{HD}^2 + \varepsilon\tau_{\parallel}^2\right]^{1/2}\end{aligned}}$$

Simple model of turbulent SOL broadening

- $\Gamma_{0,e}$ is single control parameter characterizing spreading
- $\tilde{\Gamma}_{0,e}$? Expect $\tilde{\Gamma}_e \sim \Gamma_0$

SOL width Broadening vs $\Gamma_{e,0}$

- SOL width broadens due spreading



(blue) (orange)

λ/λ_{HD} plotted against the intensity flux Γ_{e0} from the pedestal at $q = 4, \beta = 0.001, \kappa = 0.5, \sigma = 0.6$

Variation indicates need for detailed scaling analysis

- Clear decomposition into
  - Weak broadening regime → shear dominated
  - Cross-over regime
  - Strong broadening regime

  ➔ NL damping vs spreading

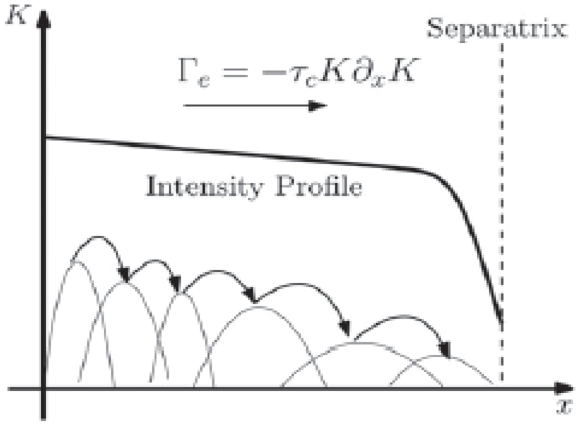
  relevant

- Cross-over for:

  $\langle \tilde{V}^2 \rangle \sim V_D^2$ ➔ cross-over $\Gamma_{0,e}$

- Cross-over for $\tilde{V} \sim O(\epsilon) V_*$

Computing the Turbulence Energy Flux 1



- $\boxed{\text{Need consider pedestal to actually compute } \Gamma_{e,0}}$
- Two elements

Does another trade-off loom?

-- Pedestal Turbulence: Drift wave? Ballooning?

-- Effect of transport barrier ←→ ExB shear layer ➔ barrier permiability!?

- Key Point: shearing limits correlation in turbulent energy flux

i.e. $\Gamma_{e,0} \approx -\tau_c \, I \, \partial_x \, I \approx \tau_c \, I^2 \, / w_{\mathrm{ped}}$ (Hahm, PD +)

ped turbulence intensity

correlation time → strongly sensitive to shearing

N.B. Caveat Emptor re: intensity flux closure !

Computing the Turbulence Energy Flux 2

- Familiar analysis for D → Kubo

$$D = \int_0^\infty d\tau \, \langle V(0)V(\tau)\rangle = \int_0^\infty d\tau \, \sum_k \left|\tilde{V}_k\right|^2 \exp\left[-k_y^2 \omega_s^2 D\tau^3 - k^2 D\tau\right]$$

- Strong shear (relevant) $\boxed{\tau_c = \tau_t^{1/2} \omega_s^{-1/2}}$

$\tau_t \sim 1 \, / \, k\tilde{V}, \quad \omega_s \sim V_E'$

Here, via RFB ➔ $\omega_s = \partial_r \frac{\nabla P_i}{n|e|} \sim \frac{\rho^2}{w_{ped}^2} \, \Omega_{ci}$

- $\tau_c + w_{ped}$ + turbulence intensity in pedestal gives $\Gamma_{e,0} \approx \tau_c I^2 / w_{ped}$
- Need $\Gamma_{e,0} \geq \Gamma_{e,\min} \approx |\gamma| \lambda_{HD}^3 \, \tau_\parallel^{-2}$

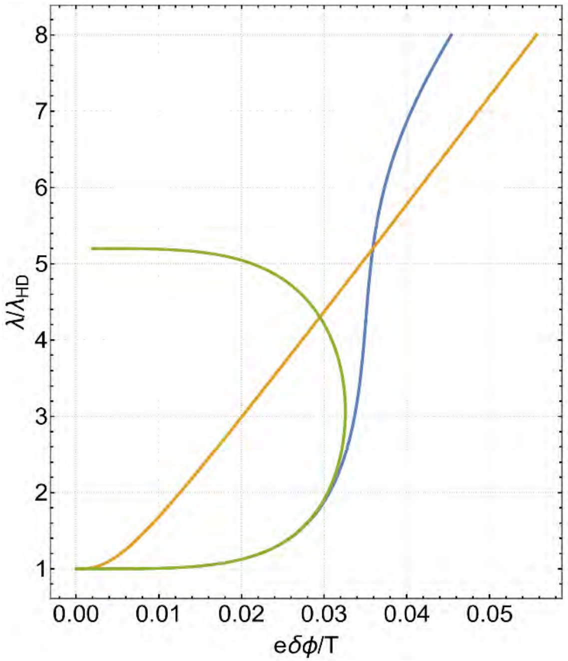
Computing the Turbulence Energy Flux 3

- Pedestal → Drift wave Turbulence
- Necessary turbulence level:
  - Weak Shear $\frac{\delta V}{c_s} \sim \left(\frac{\rho}{R}\right)^{1/2} q^{-1/4}$
  - Strong Shear $\frac{\delta V}{c_s} \sim \left(\frac{\rho}{R}\right)^{1/2} q^{-1/4} \left(\frac{w_{ped}}{\rho}\right)^{-1/8}$

blue – all damping

orange – nonlinear only

green – linear only



➔ λ/λ_{HD} vs $|e|\hat{\phi}/T_e$ in pedestal

➔ ρ/R is key parameter

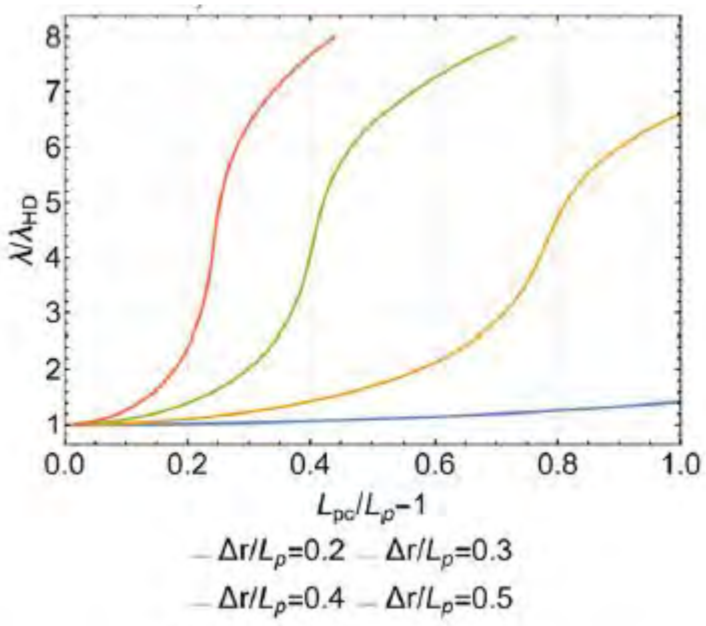
➔ Broadens layer at acceptable fluctuation level

Computing the Turbulence Energy Flux 4

- Pedestal → Ballooning modes ➔ Grassy ELMs
- Necessary relate turbulence to $L_{P,crit} \,/\, L_P \;-1$
- Strong shear:

$$\frac{L_{P_C}}{L_P} - 1 \sim \left(\frac{q\rho}{R}\right)^{\frac{10}{7}} \left(\frac{R}{w_{ped}}\right)^{\frac{16}{7}} \left(\frac{w_{ped}}{\Delta_r}\right)^{\frac{16}{7}} \beta$$

- Supercriticality scales with $\frac{\rho}{R}$, β_t



b) Strong Shear Case

Figure 10. Typical cases for ballooning. The normalized pedestal width λ/λ_{HD} is plotted against supercriticality $L_{pc}/L_p - 1$ at different mode width Δ/L_p.

Computing the Turbulence Energy Flux 5 → Bottom Line

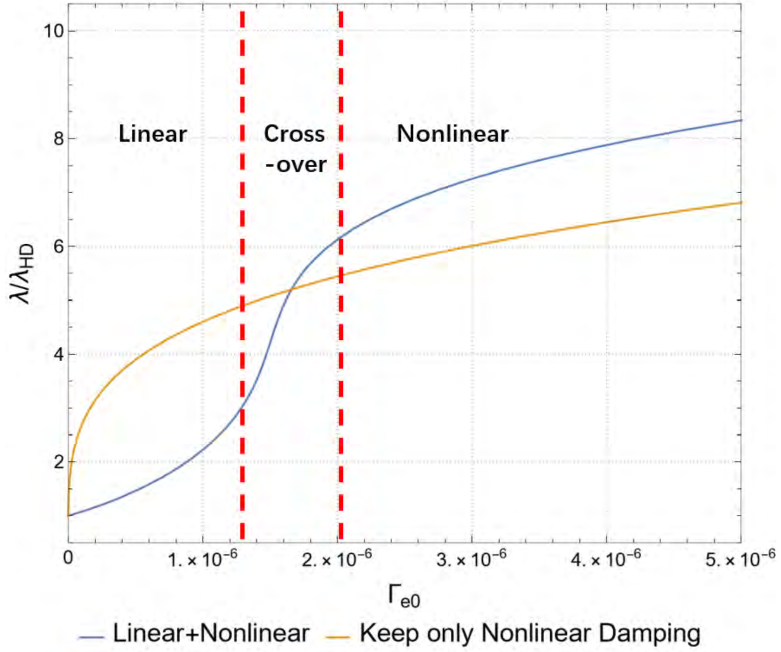
- SOL broadening to $\lambda > \lambda_{HD}$ achieveable at tolerable pedestal fluctuation levels
- DW levels scale ~ $\left(\frac{\rho}{R}\right)^{1/2}$
- Ballooning supercritical scale ~ $\left(\frac{\rho}{R}\right)^{10/7}$ β
- 'Grassy ELM' state promising
- Sensitivity analysis → Cross over ε determined primarily by linear damping (shear). Conclusion ~ insensitive to NL saturation

Partial Summary

- Turbulent scattering broadens stable SOL

$$\lambda = \left(\lambda_{HD}^2 + \varepsilon\tau_{\|}^2\right)^{1/2}$$

- Separatrix turbulence energy flux specifies SOL turbulence drive



$$\Gamma_{0,e} = \lambda_e|\gamma|\varepsilon + \lambda\sigma\varepsilon^{1+\kappa}$$

Broadening increases with $\Gamma_{0,e}$
cross-over for $\langle\tilde{V}^2\rangle \sim V_D^2$

Non-trivial dependence

- $\Gamma_{0,e}$ must overcome shear layer barrier

Yes – can broaden SOL to $\lambda/\lambda_{MHD} > 1$ at tolerable fluctuation levels

Further analysis needed

Broader Messages

- Turbulence spreading is important – even dominant – process in setting SOL width. $\Gamma_{0,e}$ is critical element. $\lambda = \lambda\left(\Gamma_{0,e}, \text{parameters}\right)$
- Production Ratio R_a merits study and characterization
- Spreading is important saturation meachanism for pedestal turbulence
- Simulation should stress calculation and characterization of turbulence energy flux over visualizations and front propagation studies.
- Critical questions include local vs FS avg, channels and barrier interaction, Turbulence 'Avalanches'
- Turbulent pedestal states attractive for head load management

Open Issues

- Quantify $\lambda = \lambda\left(\left.\frac{|e|\hat{\phi}}{T}\right|_{ped}\right)$ dependence
- Structure of Flux-Gradient relation for turbulence energy?
- Phase relation physics for intensity flux? – crucial to ExB shear effects
- Kinetics → $\langle \tilde{V}_r \delta f \delta f \rangle$, Local vs Flux-Surface Average, EM
- SOL Diffusive? → Intermittency('Blob'), Dwell Time ?
- $\boxed{\text{SOL} \rightarrow \text{Pedestal Spreading ?} \leftrightarrow \text{HDL (Goldston) ?}}$

  i.e. Tail wags Dog ? Both wagging ? → Basic simulation, experiment ?

  Counter-propagating pulses ?