Mode Competition, Saturation Mechanisms and Spatial Patterns in Multi-Scale Turbulence - a Selected Overview

P.H. Diamond

Dept. of Physics, CASS; UCSD

APTWG 2017, Nagoya
G.P. Neko; UCSD

A. Ashourvan; PPPL

P.H. Diamond; UCSD

R. Hajjar; UCSD

Ackn: Lu Wang, Z.B. Guo, C. Holland
Outline

• Preliminaries

• Conceptual Elements
  – Multi-scale problems
  – Mode competition
  – Spatial Patterns

• ‘Multi-scale’ and ‘Multi-step’: towards an improved reduced model

• Some preliminary observations

• Key open questions
Preliminaries

• Why?
  – “Shortfall problem” has resisted efforts for long time [n.b.: problem is controversial]
  – Multi-scale approach \[\begin{cases} \text{Drift - } \rho_i \\ \text{ETG - } \rho_e \end{cases}\] suggested as a road forward (c.f. Holland, et. al. 2014-2017), via GYRO simulations
  – Physics of interactions not elucidated \(\Rightarrow\) prediction is highly problematic
  
  And

  – Physics is interesting!
Conceptual Elements
Multi-Scale Problems

- Interactions of disparate scale populations
  → **Classics:** Langmuir turbulence
  Drift wave – Zonal Flow

- Short
  » Stress
  » rad. pressure

- Large
  » Langmuir collapse is pathway to singularity formation

→ Familiar, though well understood in simplest cases, only
→ Interaction with free energy complicates dynamics
Mode Competition $\rightarrow$ One Complication

$\rightarrow$ Mode competition is some set of $\{\text{modes} \}$ $\text{population}$ competing for available drive/free energy

$\rightarrow$ Analogy from ecology $\rightarrow$ niche overlap (c.f. R. May)

$\rightarrow$ Question is stability of system, distribution of populations

![Diagram](image)
• Populations $N_i$

$$\frac{d}{dt} N_i = N_i \left[ \begin{array}{c} k - \sum_{j=1}^{M} \alpha_{ij} N_j \end{array} \right]$$

• Extends familiar predator-prey idea, where one species accesses free energy, one is symbiotic

• Structure, eigenvalues of competition matrix $\rightarrow$ system state

• Noise strongly affects overlap $w$
Spatial Patterns
(with A. Ashourvan)
Some Questions

I)

Re: Drift-ZF Turbulence

– Impact of ZF well established

– Effectively linear modulation theory developed

But:

– What sets scale of ZF field? \( V_E' \)

– How does modulational instability evolve nonlinearly, saturate

– N.B.: Predator-Prey feedback channel

– Saturation \( \leftrightarrow \) scale connection?

Cf:

- Gurcan, P.D. ’14
- \( DI^2H \), ‘05
Staircase structure

Snapshots of evolving profiles at $t=1$ (non-dimensional time)

Initial conditions: $n = g_0(1 - x), \ u = 0, \ \varepsilon = \varepsilon_0$

Boundary conditions: $n(0,t) = g_0, \ n(1,t) = 0; \ u(0,1;t) = 0; \ \partial_x \varepsilon(0,1;t) = 0$

Structures:
- **Staircase in density profile:**
  - jumps $\rightarrow$ regions of steepening
  - steps $\rightarrow$ regions of flattening
- At the jump locations, turbulent PE is suppressed.
- At the jump locations, vorticity gradient is positive

---

Initial conditions:

$\begin{align*}
n &= g_0(1 - x), & u &= 0, & \varepsilon &= \varepsilon_0 \\
n(0,t) &= g_0, & n(1,t) &= 0; & u(0,1;t) &= 0; & \partial_x \varepsilon(0,1;t) &= 0
\end{align*}$

Boundary conditions:

$\begin{align*}
n &= g_0(1 - x), & u &= 0, & \varepsilon &= \varepsilon_0 \\
n(0,t) &= g_0, & n(1,t) &= 0; & u(0,1;t) &= 0; & \partial_x \varepsilon(0,1;t) &= 0
\end{align*}$

---

Snapshots of evolving profiles at $t=1$ (non-dimensional time)

**Density**

**Vorticity lattices**
**Dynamic Staircases**

- Shear pattern detaches and delocalizes from its initial position of formation.

- Mesoscale shear lattice moves in the up-gradient direction. Shear layers condense and disappear at $x=0$.

- Shear lattice propagation takes place over much longer times. From $t \sim O(10)$ to $t \sim (10^4)$.

- Barriers in density profile move upward in an “Escalator-like” motion.

**Macroscopic Profile Re-structuring**

‘Non-locality’
Time evolution of profiles

(a) Fast merger of micro-scale SC. Formation of meso-SC.
(b) Meso-SC coalesce to barriers
(c) Barriers propagate along gradient, condense at boundaries
(d) Macro-scale stationary profile
Lessons

- **Staircases happen** – Important spatial pattern
  - Staircase is ‘natural upshot’ of modulation in bistable/multi-stable system
  - Bistability is a consequence of mixing scale dependence on gradients, intensity $\leftrightarrow$ define feedback process
  - Bistability effectively locks in inhomogeneous PV mixing required for zonal flow formation
  - Mergers result from accommodation between boundary condition, drive(L), initial secondary instability
  - Staircase is natural extension of quasi-linear modulational instability/predator-prey model $\rightarrow$ couples to transport and b.c. $\leftrightarrow$ simple natural phenomenon
Towards a Reduced Model
Re-visiting Feedback Loops (Minimal)

- Sources and Sinks
  - $Q_e \Rightarrow \nabla T_e$, etc
  - Collisional damping of Z.F.
  - Boundary conditions, outflow

Flux stresses

- Shearing, Scattering
- Shearing, Corrugation
- Inverse cascade
- Inverse cascade
- Stress, Flux
- Stress, Flux
- Flux (?!)
- Short
- Mean
- Long
Observations re: Modelling

• H+D ‘04 key elements:

  ① Straining of ETG scales by D.W.’s
    \[ \nabla_k \cdot D_k^L \cdot \nabla_k \langle N_S \rangle \rightarrow \text{random shearing} \]

  ② ETG stresses negligible, in general

  ③ Noted \( \nabla \langle T \rangle \) modulations by drift wave, but did not develop implication

→ Conclusion that long wavelength activity could regulate short wavelengths, little effect short \( \rightarrow \) long

→ Expect robust large scale activity will suppress short
Other Elements

- **Mode competition** for free energy:

  ![Diagram]

  \[ \nabla T_e \quad \leftrightarrow \quad \begin{cases} Q_{e, DW} \\ Q_{e, ETG} \end{cases} \quad \rightarrow \quad \text{Control to population balance} \]

  - N.B. **Flux driven** simulation ultimately required
  - Spatial scattering (i.e. ‘turbulence spreading’) accompanies straining/shearing

  \[ \partial_t \langle N_s \rangle = \nabla_x \cdot D^L \cdot \nabla_x \langle N_s \rangle + \cdots \]
  
  - Weak nonlocality introduced
Other Elements, cont’d

• **Local** energy transfer, via inverse cascade

\[ \frac{d}{dt} \varepsilon_{ETG} \sim - \left( \frac{\widetilde{V}}{l} \right)_{ETG} \varepsilon_{ETG} \rightarrow \frac{d}{dt} \varepsilon_{ITG} \quad \text{n.b.: Energy conservation!} \]

~ effective noise source for ion scale turbulence

• Small scale \( \rightarrow \) fine scale **envelope**
  
  – Short wavelength envelope field smooth
  
  – Both scales contribute to ZF field
  
  – Enhance inertia due Boltzmann ions is ‘linear’ effect
  
  – Suggests \( \varepsilon_D, \varepsilon_E \) as fluctuation fields
A Simple, Tractable Model

• Aims are understanding/insight

• Primitive equations:
  – Fluid DWs, evolving $\phi, T_e$
  – ETG, evolving $\phi, T_e$
  – $\langle T_e \rangle, \langle \phi \rangle$ couple to both populations

• Reduced model:
  – Evolve $\langle T \rangle, \langle \phi \rangle$ via ion, electron scale vorticity flux, heat flux
  – Evolve $\varepsilon_{DW}, \varepsilon_{ETG}$ envelope fields

$\Rightarrow$ Ultimately 4 fields in $(r, t)$
Elements

• Mean $T$ evolution

$$\partial_t \langle T \rangle = -\partial_r \left\{ \langle \tilde{V}_r \tilde{T} \rangle_D + \langle \tilde{V}_r \tilde{T} \rangle_E \right\} + \chi_0 \partial_r^2 \langle T \rangle + S_0$$

$$\langle \tilde{v} \tilde{T} \rangle_{D,E} \rightarrow \varepsilon_D, \varepsilon_E, \nabla T$$

$$l_D, l_E \rightarrow \text{characteristic scales, evolving}$$

• Key Point:

  – Population induced transports compete to carry heat flux

  – Spreading/spatial scattering $\rightarrow$ weak nonlocality

  – Relative thresholds important
• Mean $\langle V_E \rangle$ evolution

$$
\partial_t \langle V_E \rangle = -\partial_t \left\{ \langle \tilde{V}_r \tilde{V}_\perp \rangle_D + w \langle \tilde{V}_r \tilde{V}_\perp \rangle_E \right\} - \mu_{ef} \langle V_E \rangle
$$

Taylor identity: $-\partial_r \langle \tilde{V}_r \tilde{V}_\perp \rangle \rightarrow \langle \tilde{V}_r \nabla_\perp^2 \tilde{\phi} \rangle$

$$
\langle \tilde{V}_r \nabla_\perp^2 \tilde{\phi} \rangle = -\chi_y \frac{\partial^2}{\partial r^2} \langle V_E \rangle + \Pi_{\text{vort}}^{\text{res}}
$$

$\chi_y \rightarrow f_D \sqrt{E_D} l_D, m \text{ix}$

$\Pi_{\nabla_\perp^2 \phi}^{\text{res}} \rightarrow f_D^{1/2} l_D, m \text{ix} \sqrt{E_D \frac{\Omega_i}{L_T}}$

; Similarly

ETG scales

Weighting for electron inertia correction

Off diagonal, $\nabla T$ driven

See Ashourvan, PD for calculation
• **Drift Wave Energetics:**

\[ \varepsilon_D = \varepsilon_D(r, t) \to \text{DW envelope scale} \]

\[
\partial_t \varepsilon_D + \partial_r \Gamma_{\varepsilon_D} = -Q_{e,turb} \frac{d}{dx} \langle T \rangle - \langle \tilde{V}_r \tilde{V}_\perp \rangle \frac{\partial}{\partial r} \langle V_E \rangle
\]

- \frac{f_D^{1/2} \varepsilon_D^{3/2}}{l_D} + \frac{f_E^{1/2} \varepsilon_E^{3/2}}{l_E} + \frac{d\varepsilon_D}{dt} \bigg|_{\text{cross-scale}}

\[ \Gamma_{\varepsilon_D} = \text{spreading} = -l_D (f_D \varepsilon_D)^{1/2} \partial_r \varepsilon \]

\[ Q_{e,turb} \frac{\partial}{\partial x} \langle T \rangle = \nabla T \text{ relaxation} \leftrightarrow Q_{e,above} \]

\[ \langle \tilde{V}_r \tilde{V}_\perp \rangle \frac{\partial}{\partial r} \langle V_E \rangle \Rightarrow \chi_y \left( \frac{\partial}{\partial r} \langle V_E \rangle \right)^2 + \langle V_E \rangle \Pi_{D,v^2\phi}^{\text{res}} \]
• **ETG Energetics:** \(\varepsilon_E = \varepsilon_E(r, t)\)

\[\Rightarrow \text{ETG envelope scale}\]

\[\partial_t \varepsilon_E + \partial_r \Gamma_E = -Q_{e,E,\text{turb}} \frac{d}{dx} \langle T \rangle - w_E \langle V_r \tilde{V}_\perp \rangle_E \frac{d}{dr} \langle V_E \rangle\]

Energy drain to DW

\[-f_E^{1/2} \frac{\varepsilon_E^{3/2}}{l_E} \quad + \quad \frac{d}{dt} \varepsilon_E \bigg|_{\text{cross scale}} = 0, E\]

\(\Gamma_E = \text{spreading} = -l_E (f_E \varepsilon_E)^{1/2} \partial_r \varepsilon \Rightarrow \text{significant}\)

\[-Q_{e,turb} \frac{\partial}{\partial x} \langle T \rangle = l_{m i x, E} (f_E \varepsilon_E)^{1/2} \left(\frac{d}{dx} \langle T \rangle\right)^2\]

\[w_E \langle \tilde{V}_r \tilde{V}_\perp \rangle \frac{d}{dr} \langle V_E \rangle = w \chi_{y,E} \left(\frac{d}{dr} \langle V_E \rangle\right)^2 + \langle V_E \rangle \Pi_{E,\nabla^2}^{\text{res}}\]

\[\chi_{y,E} = l_E (f_E \varepsilon)^{1/2}, \quad \Pi_{E,\nabla^2}^{\text{res}} = f_D^{1/2} l_E \sqrt{E} \Omega_E / L_T\]
**Characteristic Scales:**

- Need to specify $l_D, l_E$ as basic correlation scales on DW, ETG scales.
- These are modified by shearing, with shear computed self-consistently.
- **Envelope scales →** i.e. $\varepsilon_D, \varepsilon_E$ scales evolved self-consistently with profiles: n.b. $\varepsilon_E$ **not** obviously limited to electron scale.

\[
l_{m,b,D}^2 = l_{0,D}^2 / \left[ 1 + k_D^2 \rho_s^2 \langle V_E \rangle' \tau_{c,D}^2 \right]; \text{ with } \tau_c \text{ hybrid (BDT)}
\]

\[
\rightarrow l_{0,D}^2 / \left[ 1 + \left| V_E' / k_{D} \rho_s k_m \sqrt{\varepsilon_D} \right| \right]; \text{ ETG, D → E}
\]
Cross-Scale Coupling: Nonlocal in scale

- Straining and Scattering of shorts by longs are essential
- Adiabatic theory for shorts $\rightarrow \langle N_E \rangle$:

$$\partial_t \langle N \rangle = \nabla_x \cdot D_x \cdot \nabla_x \langle N \rangle + \nabla_k \cdot D_k \cdot \nabla_k \langle N \rangle$$

$D_x, D_k$ set by DW field

$$D_x = \sum_{k, DW} \langle \vec{V} \vec{V} \rangle_k \tau_{c,k} = \sum_{k, DW} \langle \vec{V} \vec{V} \rangle_k \delta(\omega - k \cdot V_{gr,E})$$

$$D_k = \sum_{k, DW} \left| \frac{\partial}{\partial x} \left( \omega_k \frac{\hat{T}_k}{T} + k \theta \hat{V}_{E,k} \right) \right|^2 \tau_{c,k}$$
Cross-Scale Coupling, cont’d

\[ \frac{d}{dt} \varepsilon_E|_{cross-scale} \] 

\[ = \frac{\partial}{\partial x} D_x[\varepsilon_D] \frac{\partial}{\partial x} \varepsilon_E \]

\[ - \int V_{gr,E} \cdot D_k[\varepsilon_D] \cdot \nabla_k \langle N \rangle_{E} dk \]

For power law spectrum:

\[ \frac{d}{dt} \varepsilon_E|_{cross} \] 

\[ = \text{spreading (above)} + \frac{V_{gr}}{\omega_{ETG}} D_{kr}[\varepsilon_D] \alpha \varepsilon_E \]

n.b.: \( -V_{gr}/\omega < 0 \rightarrow \text{backward wave} \rightarrow \text{straining decrement} \)


→ Spectral structure enters multi-scale problem. Resolution challenges to DNS – link cross-scale interaction spectral structure
So, the model

• 4 equations in $r$, $t$ + mixing lengths (with shearing)

• Calculate $\varepsilon_D(r,t)$, $\varepsilon_E(r,t)$, $\langle T_e(r,t) \rangle$, $\langle V_E(r,t) \rangle$

  n.b. 1 equation beyond Ashourvan, P.D. ’16, ’17

• Control parameters:
  
  – Heat source $\rightarrow$ DW, ETG compete
  
  – Mean flow decrements
  
  – $l_{0,E}$, $l_{0,D}$

• Un-resolved questions

  – ETG stress on DW

  * – Inverse cascade to mean? $\rightarrow$ ZF [Noise + Modulation]
Consideration of system suggests:

- ‘Dimits Shifts’ like state for ETG’s should exist

  - ETG can generate a state of ion scale ZF ?!
  - Regulate DW’s by shearing
  - Lesson: Beware assumptions re: envelope scale
• Transition from low k → ‘Dimits Shift’ regime as \( \langle V_E \rangle_{ext} \) increased

• How are ion ZF’s energized in ETG regime? → inverse cascade as channel?!
Multi-Scale, Multi-Step Staircases

- DW + ZF forms staircase structure (Dif-Pradalier, Ashourvan, P.D. ’10, ’16, ’17 after Dritchel, McIntyre ’08)
- Mechanism is ↔ Envelope scale selection
  - Inhomogeneous mixing(PV) via modulation
  - Feedback on flux

\[ \nabla T_e \]

\[ V_e' \]

- Region of steep $\nabla T_e$ form
- DW suppressed, but ETG ~ insensitive mean $\langle V_e \rangle'$
So

- ETG staircase forming in $\nabla T_e$ jumps!?
  (survive strong DW shears?)

Or

- ETG transport limits staircase formation via feedback on $\nabla T_e$
  (damp ion staircase as $\nabla T$ steepens?)

→

- DW, ETG competition for free energy source is essential!
- Relative thresholds significant
Ongoing and Plans:

• Numerical solution (1+1) of 4 equation model (easily implemented)

• Explore:
  
  – Threshold + flow damping scans
  
  – Spatial patterns – staircase (and its fate)
  
  – Dimits shift regimes, ETG driven ZFs structure
  
  – $Q_e^D, Q_e^E$ branching ratio vs $\nabla T_e$
  
  – Roles of straining, scattering in cross-scale interaction
A New Question re: Old Friend

- How understand interaction between Reynolds stresses and inverse cascade in Zonal Flow formation?
  
  i.e.
  
  - ‘Predator-Prey’ formulations have focused on Reynolds stress
  - R,H retained noise, missed $\nu < 0$

- Noise alone insufficient!

- ‘Diagonal’ part of R.S. / vorticity flux plays essential role!

→ coming attraction ....
Conclusion

• Multi-scale physics still yield new questions for research.

• Yet to confront non-locality, avalanching etc.

• Theory, reduced modelling necessary for understanding large scale simulations.
This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Science, under Award Number DE-FG02-04ER54738.