New Results in Negative Viscosity Models for Fusion Plasma Dynamics

-- Zonal Scale Selection, Staircases, Dynamical Symmetry Breaking

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Negative Viscosity Models - Zonal Scale Selection, Staircases and Dynamical Symmetry Breaking

I.) Scales and Staircases

- Theory of zonal flow scale selection and staircase formation
- Model reveals migration and condensation of staircase steps to form macro barrier layers
- Novel mechanism for nonlocality, via 'escalator mode'

II.) Dynamical Symmetry Breaking

- New dynamical symmetry breaking mechanism amplifies toroidal shear flows in electron drift wave turbulence, significant in weak shear plasmas
- Shear amplification enhances residual stress effect on flow profile gradient

Fig: Stages of evolution: a) Micro-steps merge into meso-steps. b) Meso-steps to barriers. c) Barriers condense at boundaries. d) Stationary profile.
Some Questions:

• Key to self-regulation of drift wave turbulence is zonal flow (Diamond et al., 2005).
BUT:
→ Gyro-Bohm breaking observed (McKee, 2006) and related to long range tail (Hennequin, 2015).
→ Zonal flow scale selection and saturation determine degree of GB-breaking. Scale selection-how?
→ ExB staircase is hint as to pattern formation scenario.
→ But previous limitation to simulation precludes understanding. Model needed to step beyond color VG’s.

See: A. Ashourvan, P.H. Diamond
- Submitted, 2016
M. Malkov, P.H. Diamond
- Submitted, 2016

I.) Observation: Coherent ZF Pattern $\iff$ ExB staircase

- ExB flows often observed to self-organize in magnetized plasmas
  eg. mean sheared flows, zonal flows, ...

- ‘ExB staircase’-coherent pattern-is observed  (G. Dif-Pradalier, P.D. et al. Phys. Rev. E. ’10)
  - flux driven, full f simulation
  - Quasi-regular pattern of shear layers and profile corrugations
  - Region of the extent $\Delta \gg \Delta_c$ interspersed by temp. corrugation/ExB jets

$\Rightarrow$ ExB staircases

- Questions:
  - What controls scale selection?
  - How does staircase form and evolve?
  - Nonlinear evolution of modulations?
  - Why coherent?
  - How understand coherent pattern selection in drift wave turbulence?
Beyond Color VG: The Reduced 1D Model

Reduced system is obtained from Hasegawa-Wakatani system for DW

Variables:

\[ u = \partial_x V_y \]  
Zonal shearing field

Reduced density:
\[ \log(N/N_0) = n(x,t) + \tilde{n}(x,y,t), \]
Potential Vorticity (PV):
\[ q = n - u, \]

Vorticity:
\[ \rho_s^2 \nabla^2 (e \varphi / T_e) = u(x,t) + \tilde{u}(x,y,t) \]

Mean field equations:

**Two components**

**Density**
\[ \partial_t n = -\partial_x \Gamma_n + \partial_x [D_c \partial_x n], \quad \Gamma_n = \langle \tilde{v}_x \tilde{n} \rangle = -D_n \partial_x n \]

**Vorticity**
\[ \partial_t u = -\partial_x \Pi_u + \partial_x [\mu_c \partial_x u], \quad \Pi_u = \langle \tilde{v}_x \tilde{u} \rangle = (\chi - D_u) \partial_x n - \partial_x \tilde{v}_x \partial_x u \]

\[ \varepsilon = \frac{1}{2} \langle (\tilde{n} - \tilde{u})^2 \rangle \]
Turbulent Potential Enstrophy (PE):

Turbulence evolution: (Potential Enstrophy)

\[ \partial_t \varepsilon = \partial_x [D_{e \varepsilon} \partial_x \varepsilon] - (\Gamma_n - \Gamma_u) [\partial_x (n - u)] - \varepsilon^{-1} \varepsilon^{3/2} + P \]

Turbulence spreading  
Internal production  
dissipation  
External production \[ \sim \gamma \varepsilon \]

Two fluxes \( \Gamma_n, \Gamma_u \) set model!

\[ \iff \]

Vorticity flux sets flow evolution.
Contains residual stress driven by \( \nabla n \).
What are the Key Points in this model?

- In this model PE conservation is a central feature.
- Mixing of Potential Vorticity (PV) is the fundamental effect regulating the interaction between turbulence and mean fields. Mixing inhomogeneous, via intensity feedback.
- Dimensional and physical arguments used to obtain functional forms for the turbulent diffusion coefficients. From the flux relations for HW system we obtain

\[ D_n \approx l^2 \frac{\varepsilon}{\alpha} \quad \chi \approx c \frac{\varepsilon}{\sqrt{\alpha^2 + a_u u^2}} \]

- Inhomogeneous mixing of PV results in the sharpening of density and vorticity gradients in some regions and weakening them in other regions, leading to shear lattice and density staircase formation.

Key Element: Mixing Scale <-> Tied to Rhines Scale

- \( l = l_0/(1 + l_0^2 [\partial_x (n - u)]^2/\epsilon)^{k/2} = l_0/(1 + l_0^2/l_{Rh}^2)^{k/2} \)
- \( l_{Rh} = \text{Rhines Length} = \sqrt{\epsilon}/|\partial_x q| \)
- From: \( \omega \approx k_{\theta} v_*/(1 + k_1^2 \rho_s^2) \approx l_{Rh} \sqrt{q} \)
- \( D_q \approx l_0^2 \varepsilon^{1/2}/[1 + l_0^2 (\langle q \rangle')^2/\epsilon] \)

PV mixing exhibits quench with \( \nabla q \)

Note: No KH/tertiary. Feedback on gradient drive assures saturation

\( \Rightarrow \) Robust, generic mechanism
Staircase Structure via Modulation

Snapshots of evolving profiles at $t=1$ (non-dimensional time)

Initial conditions: $n = g_0(1-x), \quad u = 0, \quad \varepsilon = \varepsilon_0$

Boundary conditions: $n(0,t) = g_0, \quad n(1,t) = 0; \quad u(0,1;t) = 0; \quad \partial_x \varepsilon(0,1;t) = 0$

Structures:
- Staircase in density profile:
  - jumps $\rightarrow$ regions of steepening
  - steps $\rightarrow$ regions of flattening
- At the jump locations, turbulent PE is suppressed.
- At the jump locations, vorticity gradient is positive
- Staircases are Dynamic

- Barriers in density profile move upward in an “Escalator-like” motion.

- Macroscopic Profile Re-structuring

- Novel Mechanism for ‘Non-locality’ via profiles and boundary conditions.

- Shear pattern detaches and delocalizes from its initial position of formation.

- Mesoscale shear lattice moves in the up-gradient direction. Shear layers condense and disappear at x=0.

- Shear lattice propagation takes place over much longer times. From $t \sim O(10)$ to $t \sim (10^4)$. 
- Profile Evolution: Staircase Coalescence and Condensation to Barrier

(a) Fast merger of micro-scale SC. Formation of meso-SC.

(b) Meso-SC *coalesce* to barriers

(c) Barriers propagate along gradient, condense at boundaries

(d) Macro-scale stationary profile
- Flux driven evolution

Add an external particle flux drive to the density Eq., use its amplitude $\Gamma_0$ as a control parameter to study:

- What profile structure emerges from this dynamics?
- Variation of the macroscopic steady state profiles with $\Gamma_0$. (shearing, density, turbulence, and flux).
- Transport bifurcation of the steady state (macroscopic)
- Particle flux-density gradient landscape.

$$\partial_t n = -\partial_x \Gamma - \partial_x \Gamma_{dr}(x,t) \quad \Rightarrow \text{Source as } \nabla \cdot \Pi_{ex}$$

External particle flux (drive)

$$\Gamma_{dr}(x,t) = \Gamma_0(t) \exp\left[-\frac{x}{\Delta_{dr}}\right]$$

Internal particle flux (turb. + col.)

$$\Gamma = -[D_n(\varepsilon, \partial_x q) + D_{col}] \partial_x n$$
- Transitions to Globally Enhanced Confinement Occur by Staircase Evolution

Steady state solution for the system undergoes a transport bifurcation as the flux drive amplitude $\Gamma_0$ is raised above a threshold $\Gamma_{th}$.

$$\Gamma_1 < \Gamma_{th} < \Gamma_2$$

$\Gamma_0 = \Gamma_1 \rightarrow$ Normal Conf. (NC)
$\Gamma_0 = \Gamma_2 \rightarrow$ Enhanced Conf. (EC)

With NC to EC transition we observe:

- Rise in density level
- Drop in turb. PE and turb. particle flux beyond the barrier position
- Enhancement and sign reversal of vorticity (shearing field)

N.B.: Macro transition occurs via staircase evolution
- Flux Landscape in \((x, \nabla n)\) Forms from Staircase Condensation

Fig: Flux landscape of the local \(\Gamma(x)\) vs \(-\partial_x n\) vs \(x\) for \(g_i = 4.5\). Shades of red are for the enhanced confinement state (EC) and gray scale is for normal confinement state (NC).

**Hysteresis evident in the GLOBAL flux-gradient relation**

In one run from initially flat density profile, \(\Gamma_0\) is adiabatically raised and lowered.

**Forward Transition:**
Abrupt transition from NC to EC (from A to B).

**From B to C:**
Barrier moves to the right with lowering the density gradient.

**Backward Transition:**
Abrupt transition from EC to NC (from C to D). Barrier moves rapidly to the right boundary and disappears.

\[\langle \Gamma \rangle \text{ Global}\]
- Role of Turbulence Spreading?

- Large turbulence spreading wipes out features on smaller spatial scales in the mean field profiles, resulting in the formation of fewer density and vorticity jumps.

- $\beta \to 0$, excessive profile roughness

Initial condition dependence:

- Solutions are not sensitive to initial value of turbulent PE.
- Initial density gradient is the parameter influencing the subsequent evolution in the system.
- At lower viscosity more steps form.
- Width of density jumps grows with the initial density gradient.
Lessons I.)

A) Coherent ZF structures evolve from modulations
   - “Staircase” is ‘natural upshot’ of modulation in bistable/multi-stable system
   - Bistability is a consequence of generic mixing scale dependence on gradients, and intensity $\leftrightarrow$ define feedback process
   - Mergers result from accommodation between boundary condition, drive(L), and initial secondary instability
   - Scale selection for ZF layers is intrinsically global $\Rightarrow$ responds to boundaries $\Rightarrow$ Nonlocality mechanism.

B) ZF Patterns are Dynamic (not previously appreciated)
   - Mergers occur, jumps/steps migrate. B.C.’s, drive all essential.
   - Condensation of mesoscale staircase jumps into macroscopic transport barriers occurs.
   - Global 1st order transition, with macroscopic hysteresis occurs from staircase evolution, condensation.
   - Flux drive + B.C. effectively constrain system states.
II.) Intrinsic Rotation in Weak Shear

- JET: Weak shear AND Rotation $\rightarrow$ Enhanced confinement
- But external torque limited in ITER
- Need understand: **Intrinsic rotation in weak shear regimes**

- Important for:
  - Total effective torque
    \[ \tau = \tau_{ext} + \tau_{intr} \]
  - Contribution to $V'_{E\times B}$

[FIG. 4 (color online). $q_i^{GB}$ vs $R/L_{T_i}$ at $\rho_{tor} = 0.33$ for similar plasmas with different rotation and $s$ values.]

[P. Mantica, PRL, 2011; Rice, PRL, 2013]
Recall: Conventional Wisdom of Intrinsic Rotation

• Self-acceleration by intrinsic torque due to residual stress
  \( \tau_{intr} = -\nabla \cdot \Pi^{Res} \)

\[
\langle \tilde{v}_r \tilde{v} \rangle = -\chi \phi \frac{d\langle v \rangle}{dr} + V_P \langle v \rangle + \Pi^{Res}_r
\]

• Residual stress \( \Pi^{Res}_r \)
  – Driven by turbulence, i.e. \( \Pi^{Res}_r \sim \nabla P, \nabla T, \nabla n_0 \)

\( \Pi^{Res}_r \sim \langle k \theta k \rangle \) etc. requires symmetry breaking in \( k \) space

• Relevance in weak shear dubious!

• Symmetry breaking usually relies on magnetic shear

• Rotation builds up from edge, driven by \( \Pi^{Res}_r \) at edge

[W.X. Wang, PRL, 2009]
Intrinsic Rotation in Weak Shear

• Weak shear \((q' \to 0)\)

• External torque \(\approx 0\)
  • Beneficial for confinement and stability.

• Results
  – GK Simulation: stronger intrinsic rotation at weaker magnetic shear

• Problems:
  – Intrinsic rotation requires symmetry breaking
  – Most involve magnetic shear
  – Conventional symmetry breaking models fail
  – But weak shear
    → non-resonant mode structure!
  – Need re-visit fundamentals of intrinsic torque, absent shear. (J. Li et al., 2016)

Status:

\[
\frac{|V_{\parallel}|}{V_{th}}
\]

[Kwon, NF (2012);
Z.X. Lu, NF&PoP, 2015]
**Intrinsic \( \nabla \langle v_z \rangle \) in Drift Wave Turbulence**

- Axial flow in CSDX:
  - \( \nabla n_0 \) is free energy source
  - \( \langle v_z \rangle' \sim \frac{1}{n_0} \nabla n_0 \)

- Compare:
  - Intrinsic \( \nabla \langle v_z \rangle \) in C-Mod pedestal:
  - \( \Delta \langle v_\phi \rangle \sim \nabla T \)

[Rice, PRL, 2011]
Resolution: Dynamical Symmetry Breaking

Key: $\delta\langle v_z \rangle' \to$ Frequency shift $\to$ Change in $\omega_k - \omega_*$

- **Growth rate <-> frequency shift:**
  $$\omega_k \approx \frac{\omega_*}{1 + k^2 \rho_s^2} \frac{k_\theta k_z \rho_s c_s \langle v_z \rangle'}{\omega_*}$$
  $$\gamma_k = \frac{\nu_{ei}}{k_z^2 \nu_{Te}^2} \frac{\omega_*^2}{(1 + k^2 \rho_s^2)^2} \left( \frac{k^2 \rho_s^2}{1 + k^2 \rho_s^2} + \frac{k_\theta k_z \rho_s c_s \langle v_z \rangle'}{\omega_*} \right)$$

- **Spectral imbalance:**
  - Infinitesimal test axial flow shear, e.g. $\delta\langle v_z \rangle' < 0$
  - Modes with $k_\theta k_z < 0$ grow faster than others
    $$\gamma_k |_{k_\theta k_z < 0} > \gamma_k |_{k_\theta k_z > 0}$$
  - Spectral imbalance in $k_\theta k_z$ space
    $$\langle k_\theta k_z \rangle < 0 \Rightarrow \Pi_{rz}^{Res} \neq 0$$

{\{k\pm\}}: Domains where modes grow faster/slower

Operates through electron growth
Negative Viscosity **Increment**

- Reynolds stress:  \[ \langle \tilde{u}_r \tilde{u}_z \rangle = -\chi \phi \langle v_z \rangle' + \Pi^{Res}_{rz} \]

- Turbulent momentum diffusivity:

\[
\chi = \sum_k \frac{\nu_i \frac{k^2 \rho^2_s}{k^2 v^2_{The}}} \frac{\frac{1}{k^2 \rho^2_s} k^2 \rho^2_s |\phi_k|^2}{1 + k^2 \rho^2_s} \]

- Residual stress \(\rightarrow\) Negative viscosity **increment**

\(\rightarrow\) Mechanism resembles modulational instability: seed + feedback

- \(\delta \Pi^{Res} = |\chi^{Inc}| \delta \langle v_z \rangle'\) [Li et al, PoP, 2016]

\[
\delta \Pi^{Res}_{rz} = \frac{\nu_i L^2_n}{v^2_{The}} \sum_k (1 + k^2 \rho^2_s)(4 + k^2 \rho^2_s) |\phi_k|^2 \delta \langle v_z \rangle'
\]
Modulational Enhancement of $\delta\langle v_z \rangle'$

- $\delta\langle v_z \rangle' \rightarrow \Pi^{Res} \rightarrow \chi^{tot}_{\phi} = \chi_{\phi} - |\chi^{Inc}_{\phi}|$

- Dynamics of $\delta\langle v_z \rangle'$:
  \[
  \frac{\partial}{\partial t} \delta\langle v_z \rangle' + \frac{\partial^2}{\partial r^2} \left( \delta \Pi^{Res}_{rz} - \chi_{\phi} \delta\langle v_z \rangle' \right) = 0
  \]

- Growth rate of flow shear modulation
  \[
  \gamma_q = -q_r^2 \left( \chi_{\phi} - |\chi^{Inc}_{\phi}| \right)
  \]

- $\chi^{tot}_{\phi} < 0 \rightarrow$ Modulational growth of $\delta\langle v_z \rangle'$

- Feedback loop: $\delta\langle v_z \rangle' \rightarrow \Pi^{Res} \rightarrow -|\chi^{Inc}_{\phi}|$
Upper Range of $\langle v_z \rangle'$ Limited by PSFI

- Parallel shear flow instability (PSFI) driven by $\nabla \langle v_z \rangle$, negative compressibility
  
  $$\gamma_k^{PSFI} \approx \sqrt{\frac{k_\theta k_z \rho_s c_s (\langle v_z \rangle' - \langle v_z \rangle'_{crit})}{1 + k_\perp^2 \rho_s^2}}$$
  $$\chi^P_{PSFI} \approx \sum_k |\phi_k|^2 k_\theta^2 \rho_s^2 \frac{4(1 + k_\perp^2 \rho_s^2)^2}{\omega_z^2} \sqrt{\frac{k_\theta k_z \rho_s c_s (\langle v_z \rangle' - \langle v_z \rangle'_{crit})}{1 + k_\perp^2 \rho_s^2}}$$

  $\rightarrow$ Nonlinear in $\nabla \langle v_z \rangle$

- Hit PSFI threshold $\rightarrow \chi^P_{PSFI}$ nonlinear in $\nabla \langle v_z \rangle \rightarrow \chi^{tot}_{\phi} > 0$

- $\delta \langle v_z \rangle' \rightarrow \Pi^{Res} \rightarrow \delta \langle v_z \rangle'$ growth $\Leftarrow$ Saturated by PSFI

$$\chi^{tot}_{\phi} = \chi^{DW}_{\phi} - |\chi^{Inc}_{\phi}| < 0$$

$$\chi^{tot}_{\phi} = \chi^{DW}_{\phi} + \chi^{PSFI}_{\phi} - |\chi^{Inc}_{\phi}| > 0$$

- $\langle v_z \rangle' \rightarrow \Pi^{Res} \rightarrow \delta \langle v_z \rangle'$ growth $\Leftarrow$ Saturated by PSFI
# Comparing Symmetry Breaking Mechanisms

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<th>Dynamical Symmetry Breaking</th>
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<td>$\nabla T_i, \nabla T_e, \nabla n_0, ...$</td>
<td>$\nabla n_0, \nabla T_e$ -- electron drift waves</td>
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<tr>
<td><strong>Symmetry breaker</strong></td>
<td>$E'_r, I(x)', ...$ All tied to magnetic field configuration</td>
<td>Test toroidal flow shear, $\delta \langle v_\phi \rangle'$; No requirement for shear of $B$ structure.</td>
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<td><strong>Effect on flow</strong></td>
<td>Intrinsic torque, $-\partial_r \Pi^\text{Res}_{r</td>
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<td><strong>Flow profile</strong></td>
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<tr>
<td><strong>Feedback loop</strong></td>
<td>Heat flux $\rightarrow \nabla T_i + \text{geometry (magnetic shear)}$</td>
<td>Test flow shear $\delta \langle v_\phi \rangle'$ $\rightarrow$ Spectral imbalance</td>
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Open loop

$\langle v_\parallel \rangle'$ $\rightarrow$ $\Pi^\text{Res}_{r||}$

Intrinsic flow, feedback on $\delta \langle v_\phi \rangle'$

Residual stress $\Pi^\text{Res}_{r\phi} \rightarrow |\chi^\text{Res}_\phi|$
Summary (II.)

• Dynamical symmetry breaking mechanism

• Negative viscosity increment induced by $\Pi^{Res}$

  $- \delta \Pi^{Res} = |\chi_\phi^{Inc}| \delta \langle v_z \rangle'$

  $- \text{Total viscosity: } \chi^{tot}_\phi = \chi_\phi - |\chi_\phi^{Inc}|$

  $- \chi^{tot}_\phi < 0 \rightarrow \text{Modulational growth of } \delta \langle v_z \rangle'$

• Broader lesson for tokamaks

  $- \text{Synergy of } \langle v_\phi \rangle'$ self-amplification and $\Pi^{Res}$

  $- \langle v_\phi \rangle'$ driven by $\tau_{NBI}$, $\Pi^{Res} (\nabla n_0, \nabla T)$

  $- \langle v_\phi \rangle'$ enhanced by $-|\chi_\phi^{Inc}|$