UC San Diego

Cascades, Spectra, Real Space Structure, Inhomogeneous Mixing and Transport in Active Scalar Turbulence

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Overview

- Models:

Active Scalar Systems with Elasticity

2D MHD ⟷ (analogous) ⟷ 2D CHNS

Why care → Many models in plasmas: ballooning coupling, Hasegawa-Wakatani system, system for drift wave and ITG turbulence -- generalizations of reduced MHD. Magnetic field provides elasticity.

- Roadmap:

2D MHD		2D CHNS
Cascades and spectra in 2D MHD *Known*	→	Cascades and spectra in 2D MHD *?*
Flux expulsion in 2D MHD (passive) *Known*	→	Single eddy mixing in 2D CH Flows *?*
Turbulent transport in 2D MHD ~~Known~~ *Something new*	→	Turbulent transport in 2D CHNS (Future work)

Outline

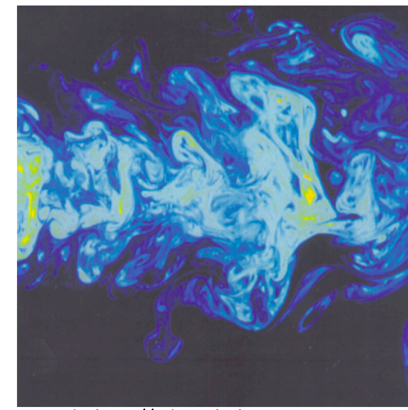
- **Introduction**
  - Active Scalar Systems with Elasticity
    - 2D MHD (MagnetoHydroDynamics)
    - 2D CHNS (Cahn-Hilliard Navier-Stokes)
  - Some Challenges
- Cascades and Spectra in 2D CHNS
- Single Eddy Mixing in 2D Cahn-Hilliard Flow
- Turbulent Transport in 2D MHD
- Conclusions and Future Works

Active Scalar Systems with Elasticity

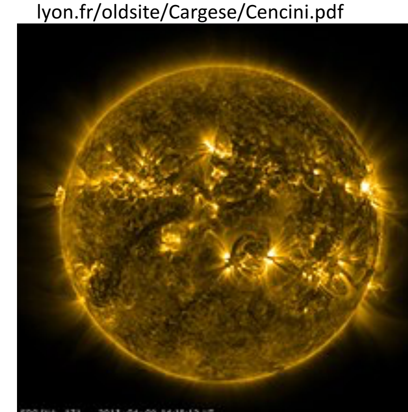
- Most fundamental system exhibiting turbulence: Navier-Stokes Equation

$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{v} + \vec{f}$$

- Passive scalar system: no feedback on fluid motion
  - E.g.: Colorant.
- Active scalar system: with feedback on fluid motion
  - E.g.: MHD, CHNS. Both with Elasticity.



Credit:http://gdr-turbulence.ec-lyon.fr/oldsite/Cargese/Cencini.pdf



Credit:https://en.wikipedia.org/wiki/Magnetohydrodynamics

2D MHD (MagnetoHydroDynamics)

- MHD: describes the macroscopic behavior of plasmas; widely used to model plasmas in Tokamaks, and in astrophysics .
- 2D MHD:

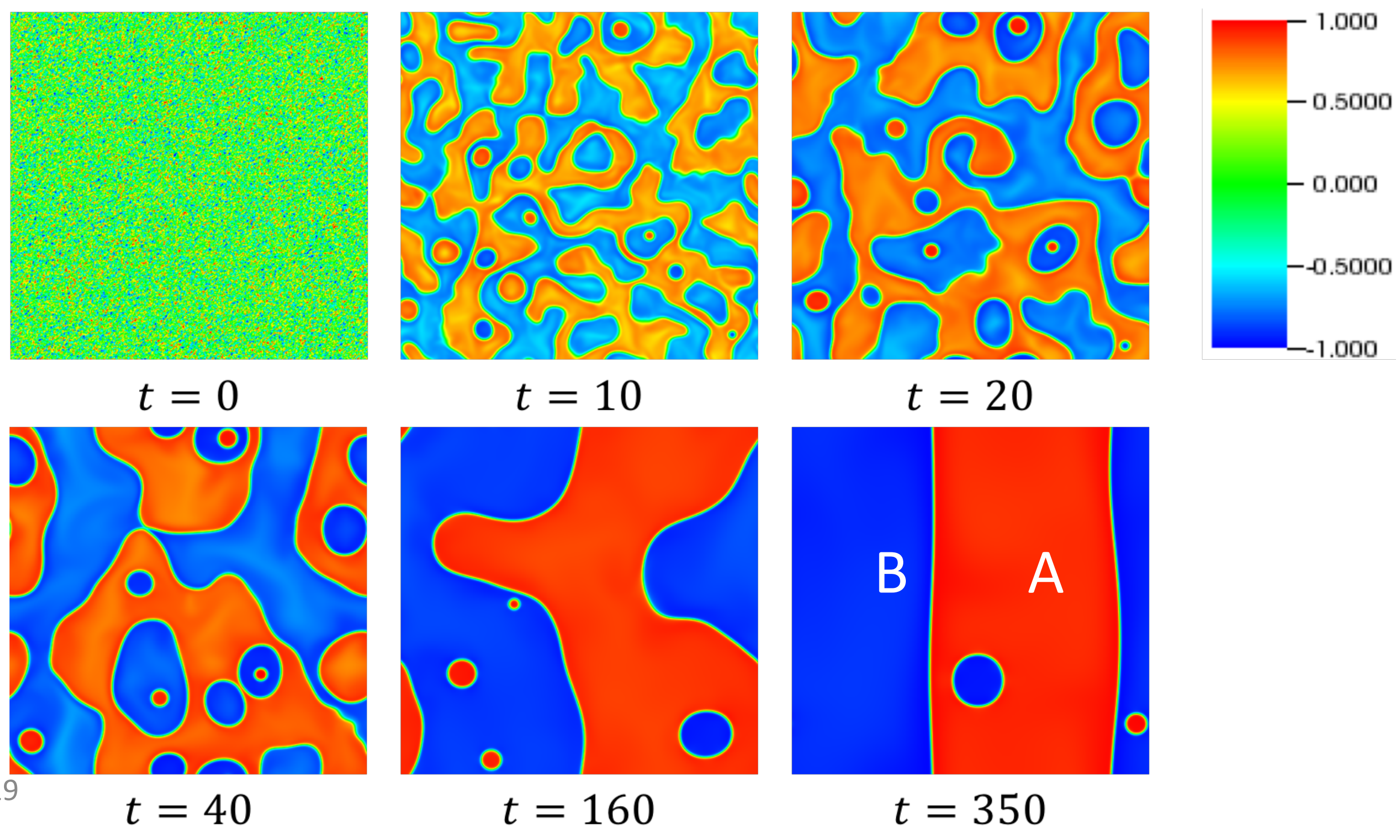
$$\partial_t A + \vec{v} \cdot \nabla A = \eta \nabla^2 A$$

$$\partial_t \omega + \vec{v} \cdot \nabla \omega = \frac{1}{\mu_0 \rho} \vec{B} \cdot \nabla \nabla^2 A + \nu \nabla^2 \omega$$

- 2D MHD closely related to reduced MHD (strong B_0 in z direction in 3D). Important in plasma physics: many models are generalizations of reduced MHD.

2D CHNS (Cahn-Hilliard Navier-Stokes)

- The Cahn-Hilliard Navier-Stokes (CHNS) system describes ***separation of components*** for binary fluid (i.e. ***Spinodal Decomposition***)
- Miscible phase -> Immiscible phase



2D CHNS

- How to describe the system: the concentration field
- $\psi(\vec{r},t) \stackrel{\text{def}}{=} [\rho_A(\vec{r},t) - \rho_B(\vec{r},t)]/\rho$: scalar field
- $\psi \in [-1,1]$
- CHNS equations:

$$\partial_t \psi + \vec{v} \cdot \nabla \psi = D \nabla^2 (-\psi + \psi^3 - \xi^2 \nabla^2 \psi)$$

$$\partial_t \omega + \vec{v} \cdot \nabla \omega = \frac{\xi^2}{\rho} \vec{B}_\psi \cdot \nabla \nabla^2 \psi + \nu \nabla^2 \omega$$

- 2D MHD and 2D CHNS: analogous. Elasticity; elastic wave; conserved quantities; cascades; etc.

Challenges – Dual Cascade

- Some key issues to understanding active scalar turbulence:
  1. the physics of dual (or multiple) cascades;
  2. the nature of “blobby” turbulence;
  3. the effects of negative diffusion/resistivity;
  4. the understanding of turbulent transport.

1. Dual Cascade
   - Physics of dual cascades and constrained relaxation → relative importance, selective decay…
   - Physics of wave-eddy interaction effects on nonlinear transfer (i.e. Alfven effect ←→ Kraichnan)
   - How do dual cascades interact?

Challenges – Blobby Turbulence

2. “Blobby Turbulence”
   - Blobs observed in SOL in Tokamaks.
   - CHNS is a naturally blobby system of turbulence.
   - What makes a blob a blob?
   - What is the role of structure in interaction?
   - How to understand blob coalescence and relation to cascades?

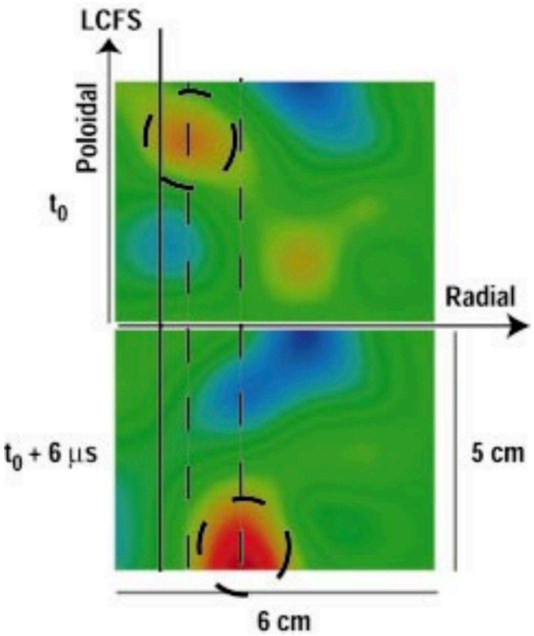


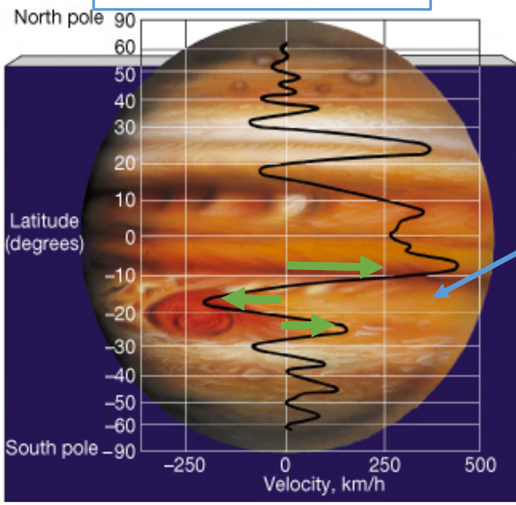
FIG. 4. (Color) Two frames from BES showing 2-D density plots. There is a time difference of 6 μs between frames. Red indicates high density and blue low density. A structure, marked with a dashed circle and shown in both frames, features poloidal and radial motion.

[J. A. Boedo et.al. 2003]

Challenges – Negative Diffusion

3. Zonal flow formation → negative viscosity phenomena
   - ZF can be viewed as a "spinodal decomposition" of momentum.
   - What determines scale?

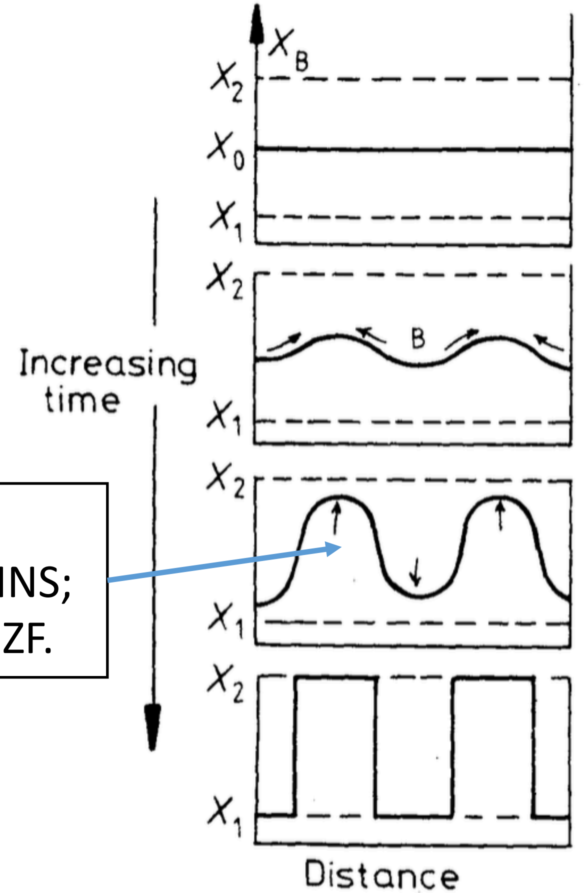
Zonal Flow



http://astronomy.nju.edu.cn/~lixd/GA/AT4/AT411/HTML/AT41102.htm

Arrows:
ψ for CHNS;
flow for ZF.

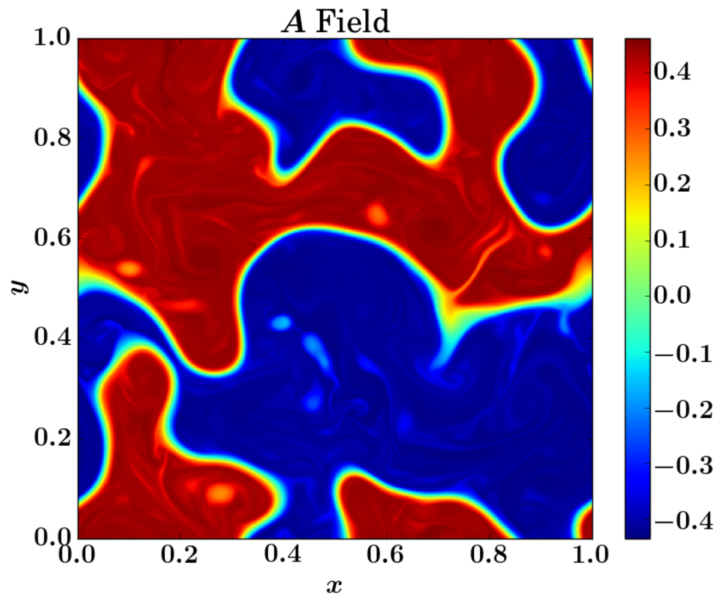
Spinodal Decomposition



[Porter 1981]

Challenges – Turbulent Transport

- 4. Turbulent transport
  - Suppressed in 2D MHD by magnetic field.
  - Previous understandings: mean field theory
  - New observation: blob-and-barrier structure
  - Need new understanding



Outline

- Introduction
- **Cascades and Spectra in 2D CHNS**
  - X. Fan, P. H. Diamond, L. Chacón, and H. Li, Phys. Rev. Fluids **1**, 054403 (2016).
- Single Eddy Mixing in 2D Cahn-Hilliard Flow
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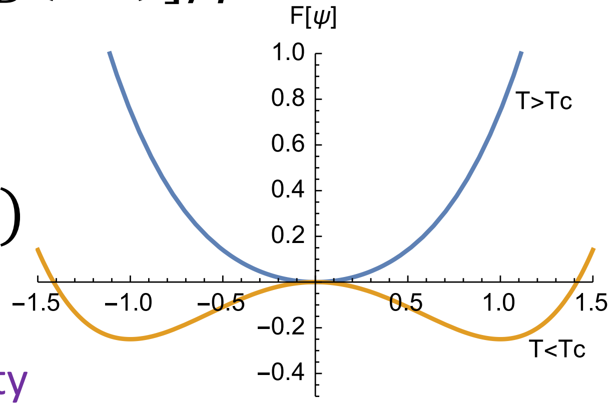
A Brief Derivation of the CHNS Model

- Second order phase transition → Landau Theory.
- Order parameter: $\psi(\vec{r},t) \stackrel{\text{def}}{=} [\rho_A(\vec{r},t) - \rho_B(\vec{r},t)]/\rho$
- Free energy:

$$F(\psi) = \int d\vec{r}\left(\underbrace{\frac{1}{2}C_1\psi^2 + \frac{1}{4}C_2\psi^4}_{\text{Phase Transition}} + \underbrace{\frac{\xi^2}{2}|\nabla\psi|^2}_{\text{Gradient Penalty}}\right)$$



- $C_1(T), C_2(T)$.
- Isothermal $T < T_C$. Set $C_2 = -C_1 = 1$:

$$F(\psi) = \int d\vec{r}\left(-\frac{1}{2}\psi^2 + \frac{1}{4}\psi^4 + \frac{\xi^2}{2}|\nabla\psi|^2\right)$$

A Brief Derivation of the CHNS Model

- Continuity equation: $\frac{d\psi}{dt} + \nabla \cdot \vec{J} = 0.$
- Fick's Law: $\vec{J} = -D\nabla\mu.$
- Chemical potential: $\mu = \frac{\delta F(\psi)}{\delta\psi} = -\psi + \psi^3 - \xi^2\nabla^2\psi.$
- Combining → Cahn Hilliard equation:

$$\frac{d\psi}{dt} = D\nabla^2\mu = D\nabla^2(-\psi + \psi^3 - \xi^2\nabla^2\psi)$$

- $d_t = \partial_t + \vec{v} \cdot \nabla.$
- Surface tension: force in Navier-Stokes equation:

$$\partial_t\vec{v} + \vec{v} \cdot \nabla\vec{v} = -\frac{\nabla p}{\rho} - \psi\nabla\mu + \nu\nabla^2\vec{v}$$

- For incompressible fluid, $\nabla \cdot \vec{v} = 0.$

2D CHNS and 2D MHD

	2D MHD	2D CHNS
Magnetic Potential	A	ψ
Magnetic Field	$\mathbf{B}$	$\mathbf{B}_\psi$
Current	j	j_ψ
Diffusivity	η	D
Interaction strength	$\frac{1}{\mu_0}$	ξ^2

- 2D CHNS Equations:

$$\partial_t \psi + \vec{v}\cdot\nabla\psi = D\nabla^2(-\psi + \psi^3 - \xi^2\nabla^2\psi)$$

$$\partial_t \omega + \vec{v}\cdot\nabla\omega = \frac{\xi^2}{\rho}\vec{B}_\psi\cdot\nabla\nabla^2\psi + \nu\nabla^2\omega$$

$-\psi$: Negative diffusion term
ψ^3: Self nonlinear term
$-\xi^2\nabla^2\psi$: Hyper-diffusion term

With $\vec{v}=\hat{z}\times\nabla\phi$, $\omega = \nabla^2\phi$, $\vec{B}_\psi = \hat{z}\times\nabla\psi$, $j_\psi = \xi^2\nabla^2\psi$. $\psi \in [-1,1]$.

- 2D MHD Equations:

$$\partial_t A + \vec{v}\cdot\nabla A = \eta\nabla^2 A$$

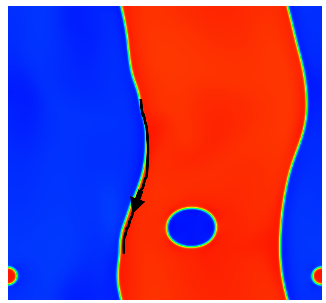
$$\partial_t \omega + \vec{v}\cdot\nabla\omega = \frac{1}{\mu_0\rho}\vec{B}\cdot\nabla\nabla^2 A + \nu\nabla^2\omega$$

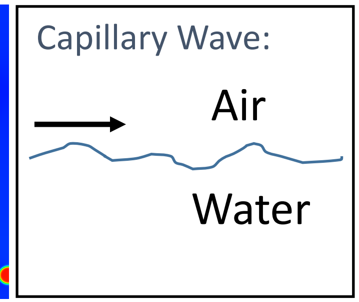
A: Simple diffusion term

With $\vec{v}=\hat{z}\times\nabla\phi$, $\omega = \nabla^2\phi$, $\vec{B} = \hat{z}\times\nabla A$, $j = \frac{1}{\mu_0}\nabla^2 A$

Linear Wave

- CHNS supports linear “elastic” wave:





- Akin to capillary wave at phase interface.
- Propagates ***<u>only</u>*** along the interface of the two fluids, where $|\vec{B}_\psi| = |\nabla\psi| \neq 0$.
- Analogue of Alfven wave in MHD (propagates along B lines).
- Important differences:
  - ➢ $\vec{B}_\psi$ in CHNS is large only in the interfacial regions.
  - ➢ Elastic wave activity does not fill space.

Ideal Quadratic Conserved Quantities

- **2D MHD**

1. Energy

$$E = E^K + E^B = \int (\frac{v^2}{2} + \frac{B^2}{2\mu_0}) d^2x$$

2. Mean Square Magnetic Potential

$$H^A = \int A^2 \, d^2x$$

3. Cross Helicity

$$H^C = \int \vec{v} \cdot \vec{B} d^2x$$

- **2D CHNS**

1. Energy

$$E = E^K + E^B = \int (\frac{v^2}{2} + \frac{\xi^2 B_\psi^2}{2}) d^2x$$

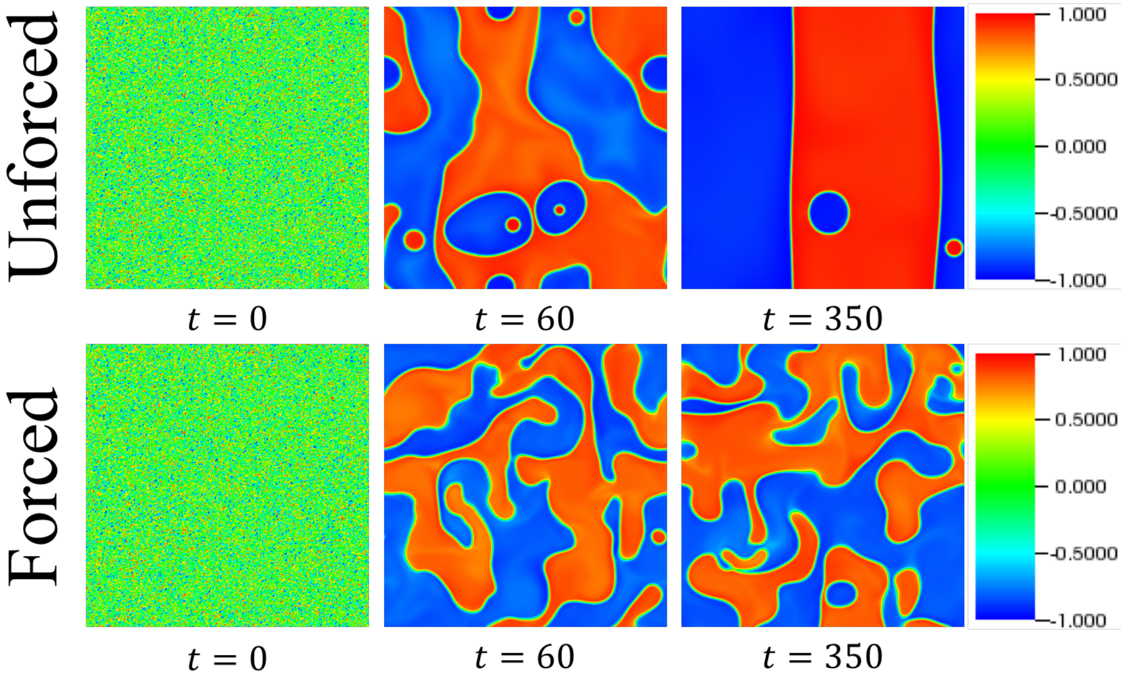
2. Mean Square Concentration

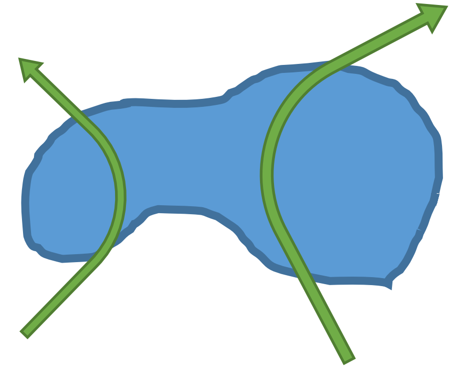
$$H^\psi = \int \psi^2 \, d^2x$$

3. Cross Helicity

$$H^C = \int \vec{v} \cdot \vec{B}_\psi \, d^2x$$

Scales, Ranges, Trends





How big is a raindrop?
- Turbulent straining vs capillarity.
- ρv^2 vs σ/l.

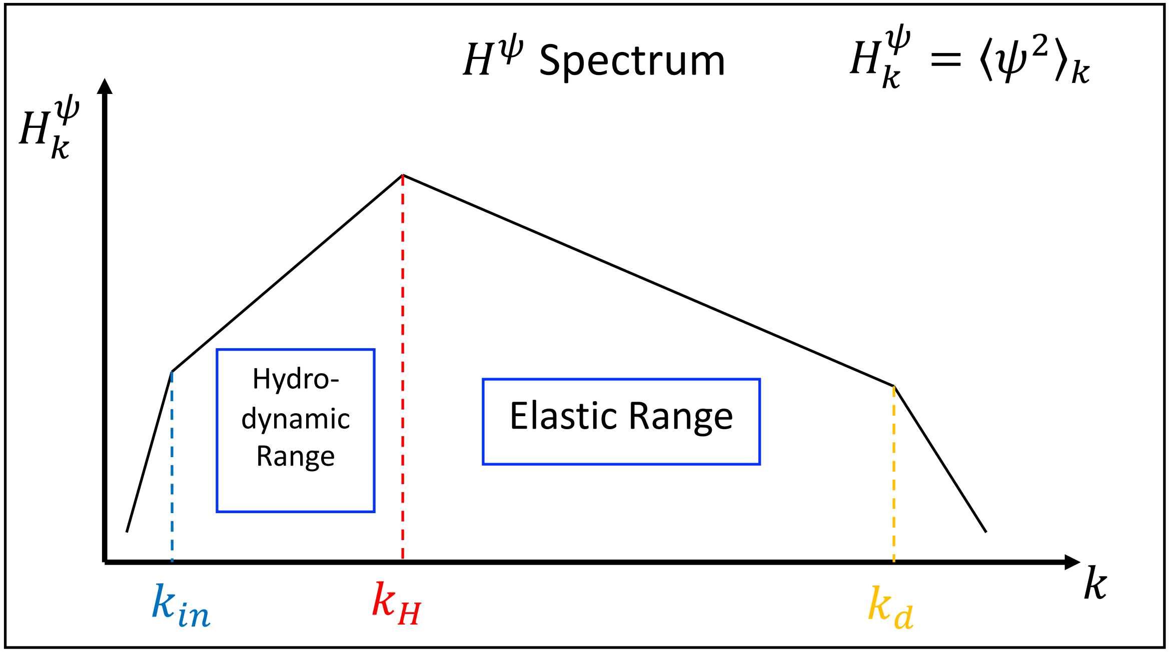
(Hinze 1955)

- Fluid forcing → Fluid straining vs Blob coalescence
- Scale where turbulent straining ~ elastic restoring force (due surface tension): Hinze Scale

$$L_H \sim \left(\frac{\rho}{\xi}\right)^{-1/3} \epsilon_\Omega^{-2/9}$$

Scales, Ranges, Trends

- Elastic range: $L_H < l < L_d$: where elastic effects matter.
- $L_H/L_d \sim (\frac{\rho}{\xi})^{-1/3} \nu^{-1/2} \epsilon_\Omega^{-1/18}$ → Extent of the elastic range
- $L_H \gg L_d$ required for large elastic range → case of interest



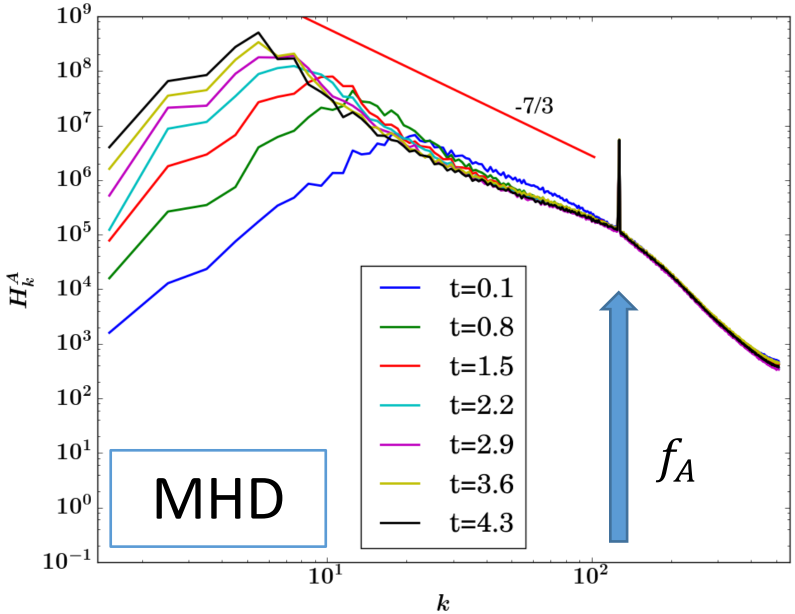
Cascades

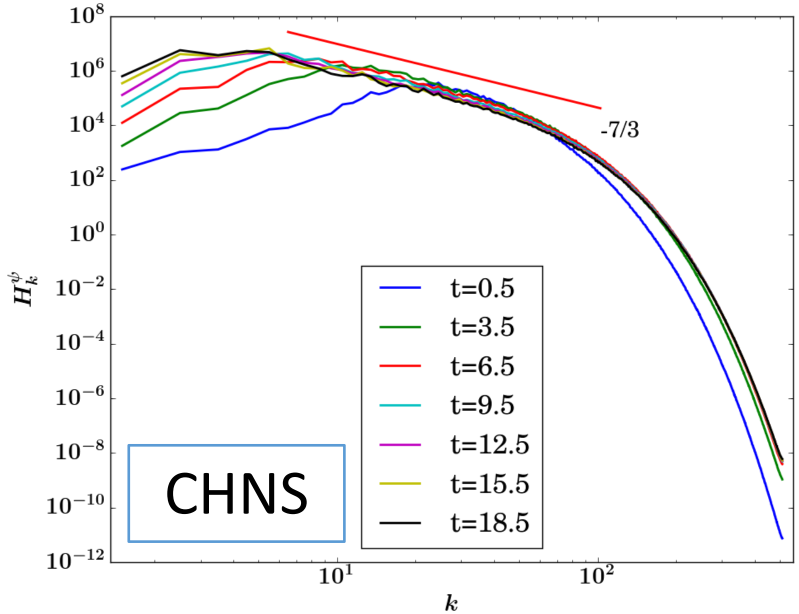
Physics System	Conserved Quantity	Cascade Direction
2D MHD	E_k	Direct
	H_k^A	Inverse
2D CHNS	E_k	Direct
	H_k^ψ	Inverse

- By statistical mechanics studies (absolute equilibrium distributions) → dual cascade:
  - ***Inverse*** cascade of $\langle\psi^2\rangle$
  - ***Forward*** cascade of E
- Blob coalescence in the elastic range of CHNS ←→ flux coalescence in MHD.
- Inverse cascade of $\langle\psi^2\rangle$ is formal expression of blob coalescence process → generate larger scale structures till limited by straining
- Forward cascade of E as usual, as elastic force breaks enstrophy conservation

Power Laws

- $\langle A^2 \rangle$ spectrum: $\langle \psi^2 \rangle$ spectrum:



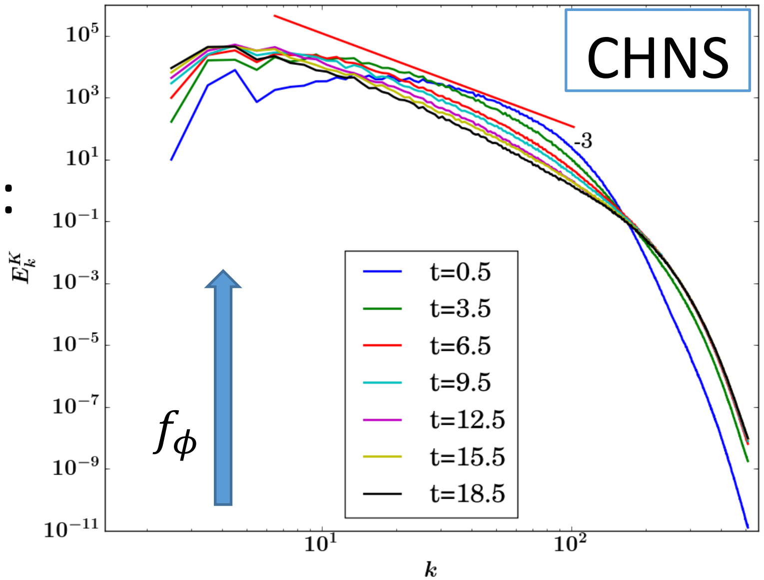


- Both systems exhibit $k^{-7/3}$ spectra.
- Inverse cascade of $\langle \psi^2 \rangle$ exhibits same power law scaling, so long as $L_H \gg L_d$, maintaining elastic range: Robust process.

More Power Laws

- Kinetic energy spectrum (Surprise!):
- 2D CHNS: $E_k^K \sim k^{-3}$;
- 2D MHD: $E_k^K \sim k^{-3/2}$.

!



- The -3 power law:
  - Closer to enstrophy cascade range scaling, in 2D Hydro turbulence.
  - Remarkable departure from expected -3/2 for MHD. ***Why?***
- Why does CHNS ←→ MHD correspondence hold well for $\langle\psi^2\rangle_k \sim \langle A^2\rangle_k \sim k^{-7/3}$, yet break down drastically for energy?
- ***What physics*** underpins this surprise?

Interface Packing Matters!

- Need to understand ***<u>differences</u>***, as well as similarities, between CHNS and MHD problems.

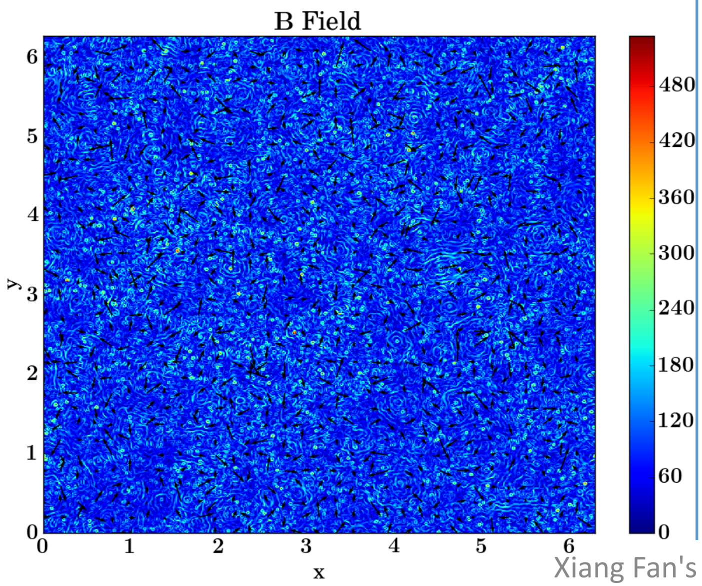
In MHD:

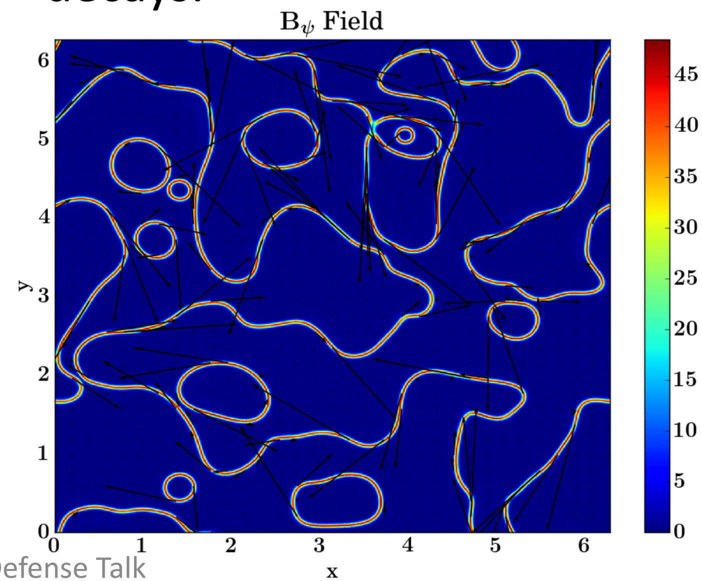
- ➢ Fields pervade system.

In CHNS:

- ➢ Elastic back-reaction is limited to regions of density contrast i.e. $|\vec{B}_\psi| = |\nabla\psi| \neq 0$.
- ➢ As blobs coalesce, interfacial region diminished. 'Active region' of elasticity decays.

MHD



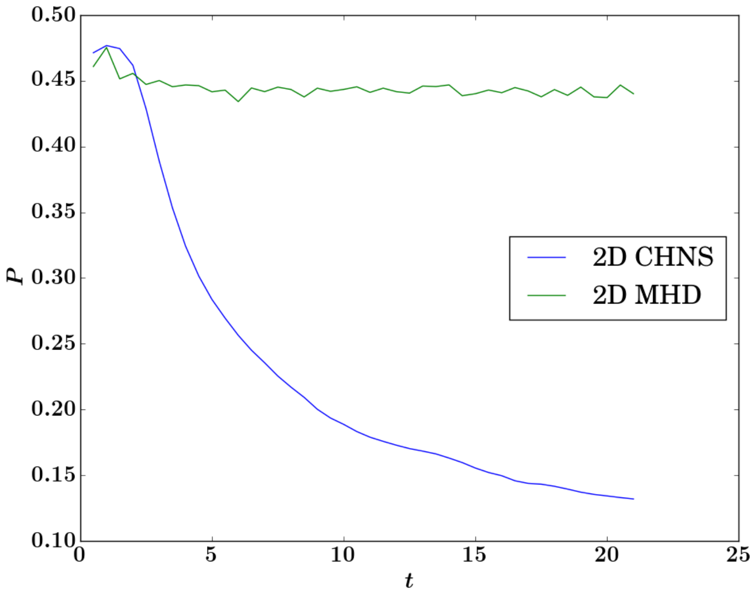


CHNS

Interface Packing Matters!

- Define the ***interface packing fraction*** P:

$$P = \frac{\#\text{ of grid points where } |\vec{B}_\psi| > B_\psi^{rms}}{\#\text{ of total grid points}}$$



- ➢ P for CHNS decays;
- ➢ P for MHD stationary!

- $\partial_t \omega + \vec{v} \cdot \nabla \omega = \frac{\xi^2}{\rho} \vec{B}_\psi \cdot \nabla \nabla^2 \psi + \nu \nabla^2 \omega$: small P → local back reaction is weak.
- Weak back reaction → reduce to 2D hydro

Summary

- Avoid power law tunnel vision!
- ***Real space*** realization of the flow is necessary to understand key dynamics. Track interfaces and packing fraction P.
- One player in dual cascade (i.e. $\langle\psi^2\rangle$) can modify or constrain the dynamics of the other (i.e. E).
- Against conventional wisdom, $\langle\psi^2\rangle$ inverse cascade due to blob coalescence is the robust nonlinear transfer process in CHNS turbulence.

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  - X. Fan, P. H. Diamond, and L. Chacón, Phys. Rev. E **96**, 041101(R) (2017).
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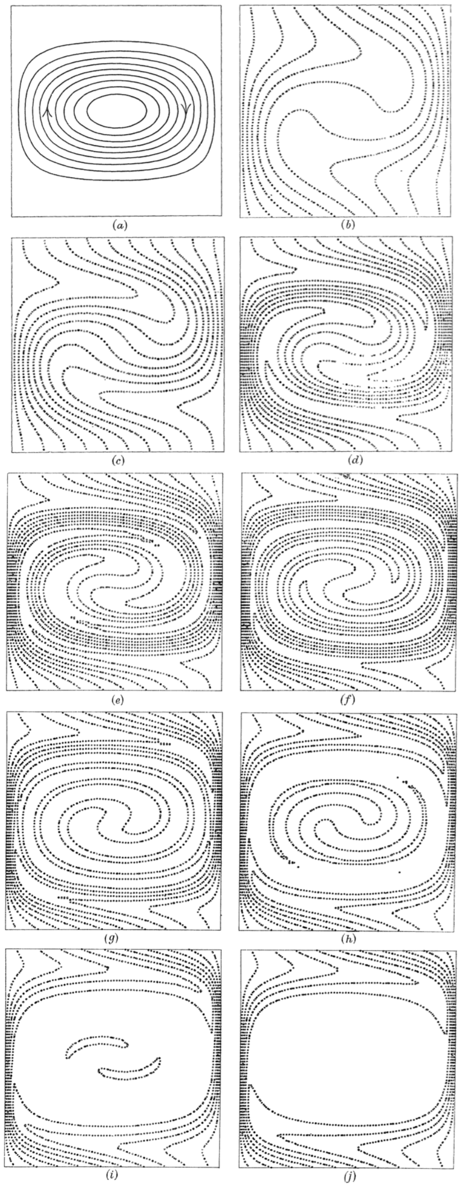
Single Eddy Mixing in 2D MHD: Expulsion

- When a convection eddy is imposed in a weak magnetic field, the magnetic field is expelled and amplified outside the eddy.
- This is called flux expulsion.
- The equation (kinematic, i.e. back reaction is ignored):

$$\partial_t A + \vec{v} \cdot \nabla A = \eta \nabla^2 A$$

- Also relevant to PV homogenization → Zonal Flow



Weiss 1966

Single Eddy Mixing in 2D MHD: Expulsion

- Main results of Weiss 1966 on Expulsion:
  - The final value of $\langle B^2 \rangle$ can be estimated by $\langle B^2\rangle \sim Rm^{1/2} B_0^2$
  - The time for $\langle B^2 \rangle$ to reach a steady state is $\tau \sim Rm^{1/3}\tau_0$

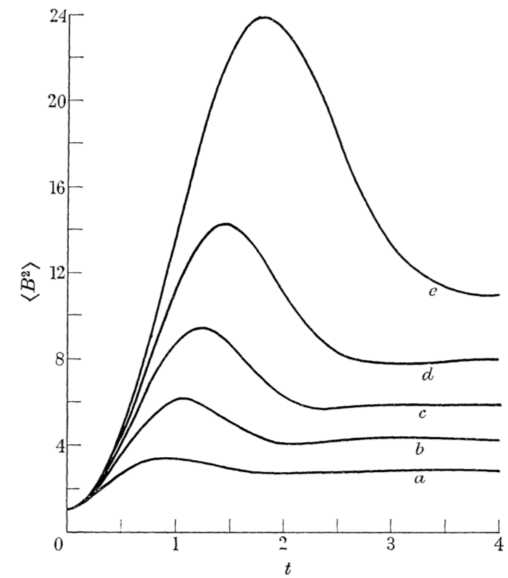


FIGURE 5. Magnetic energy as a function of time. Curves labelled a, b, c, d, e have R_m = 40, 100, 200, 400, 1000 respectively.

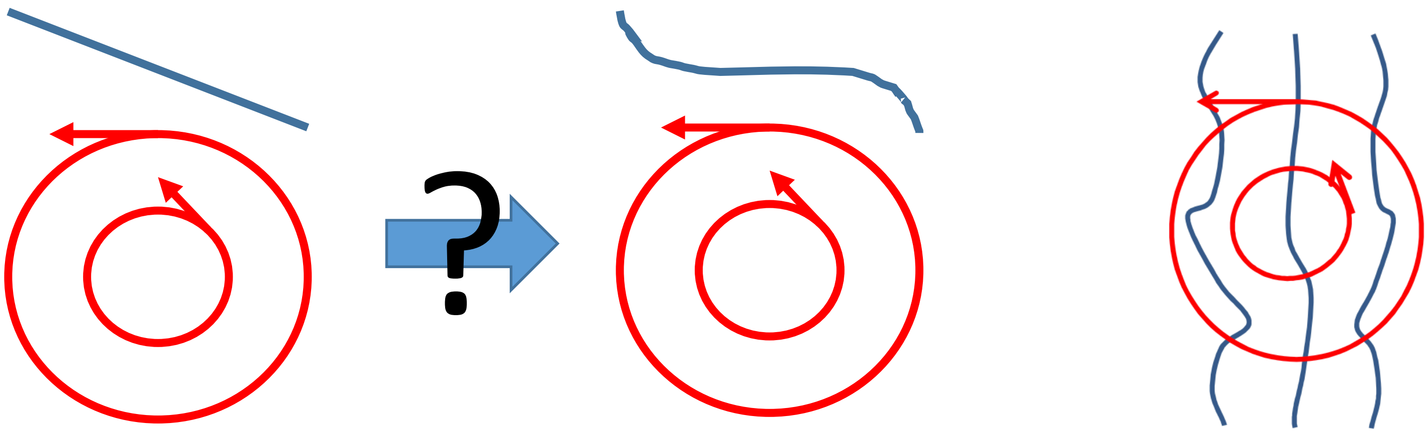
Weiss 1966

- Main results of Rhines and Young 1983 on PV Homogenization:
  - Two stages: rapid and slow
  - Rapid stage: dominated by shear-augmented diffusion, with time scale $\tau_{mix} \sim Pe^{1/3}\tau_0$
  - Slow stage: usual diffusion, with time scale $\tau_{slow} \sim Pe\ \tau_0$

Pe ← → Rm

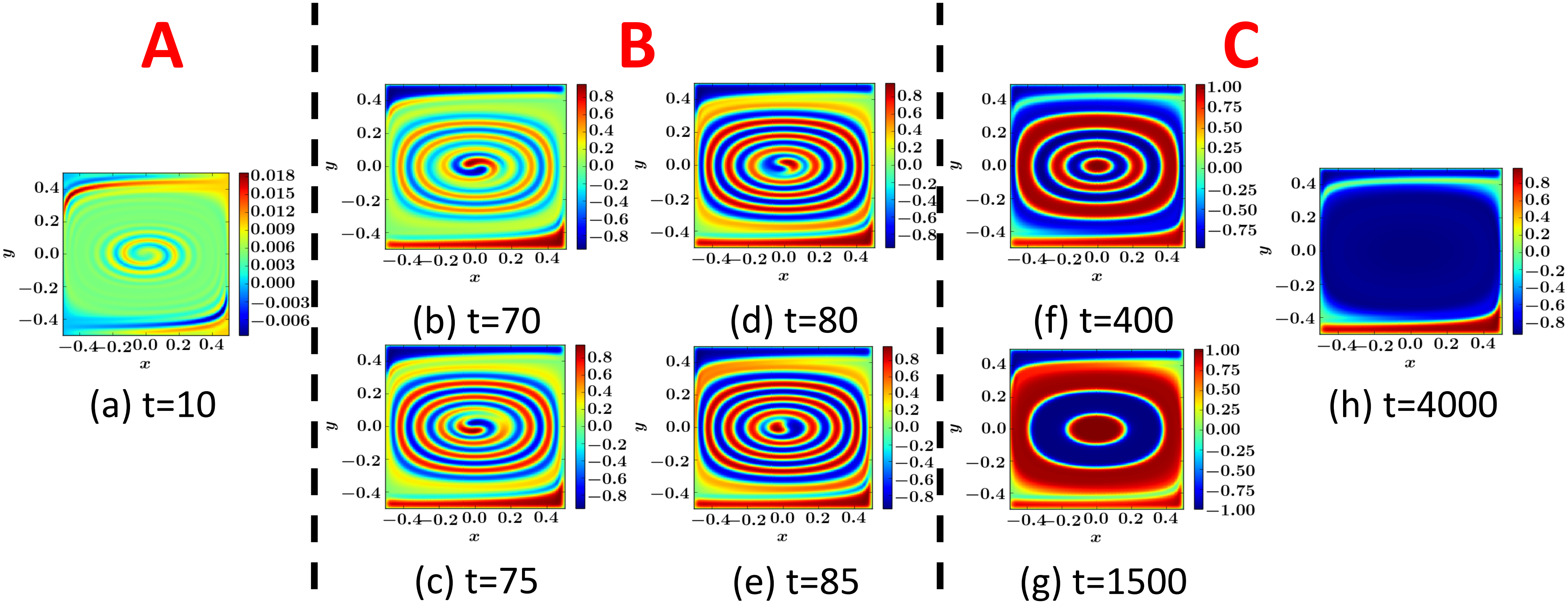
Single Eddy Mixing

- Structures are the key → need understand how a ***single eddy*** interacts with ψ field
- Mixing of $\nabla\psi$ by a single eddy → characteristic time scales?
- Evolution of structure?



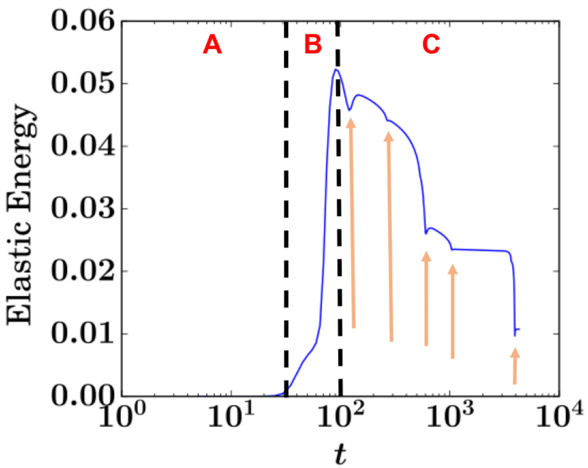
Single Eddy Mixing

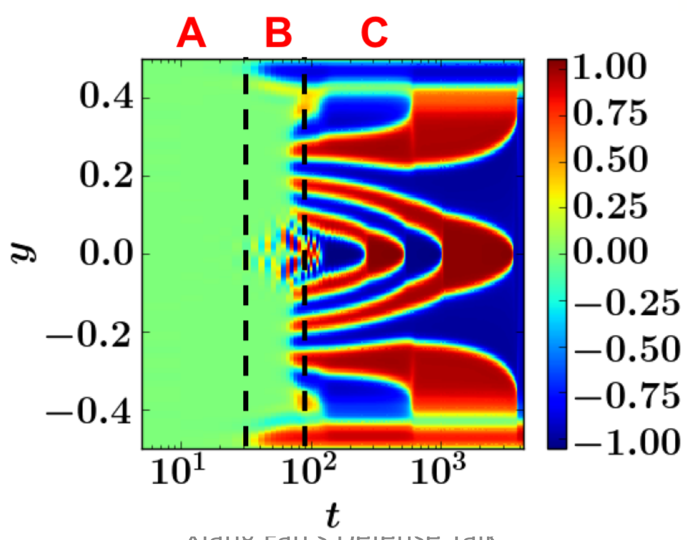
- 3 stages: (A) the *"jelly roll"* stage, (B) the *topological evolution* stage, and (C) the *target pattern* stage.
- Metastable target patterns formed and merge.
- ψ ultimately homogenized in the end.

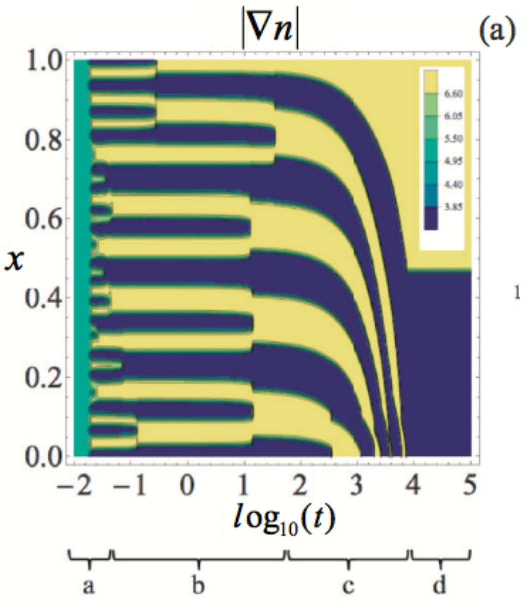


Single Eddy Mixing

- The 3 stages are reflected in the elastic energy plot.
- The target bands mergers are related to the dips in the target pattern stage.
- The bands merge on a time scale long relative to eddy turnover time.
- The band merger process is similar to the step merger in drift-ZF staircases.



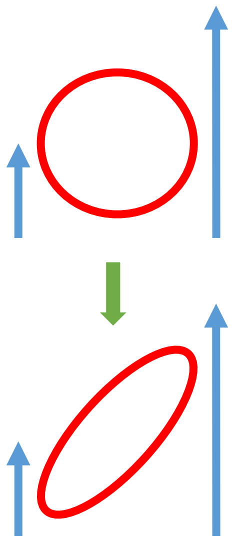




Ashourvan *et.al.* 2016

Time Scales

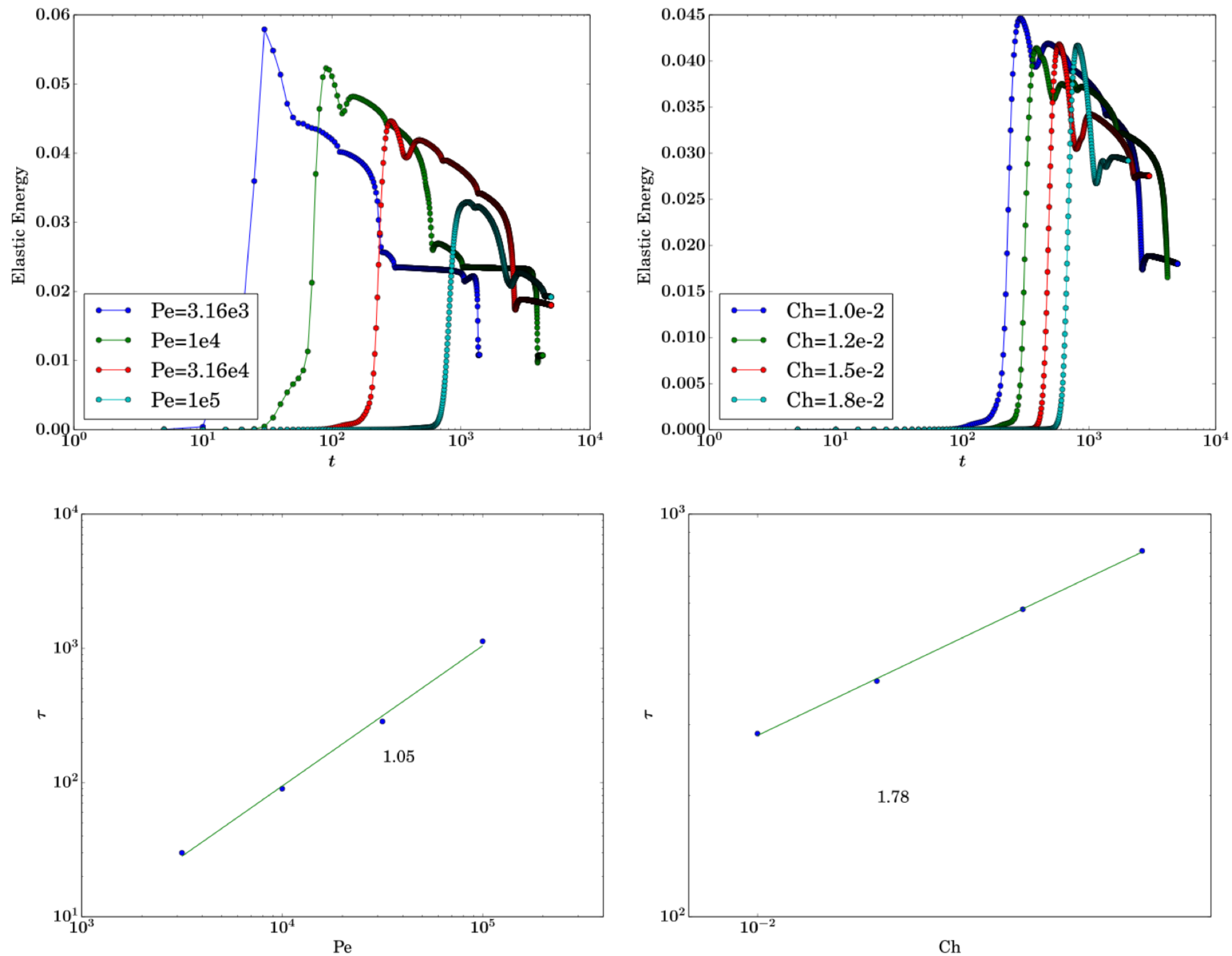
- Analogous to the $Rm^{1/3}$ or $Pe^{1/3}$ time scale in MHD or PV homogenization, the mixing time scale of the shear + dissipation hybrid case is $\tau_{mix} \sim Pe^{1/5} Ch^{-2/5} t_0$.
- Brief derivation:
  - CH equation → $\langle \delta r^4 \rangle \sim D\xi^2 t$
  - Relate δr and δy according to shear s:
    $$\frac{d}{dt}\delta y \sim s\delta r$$
  - So $\langle \delta y^4 \rangle \sim s^4 D \xi^2 t^5$.
  - Note that $Pe \sim L_y v / D$
  - So $\tau_{mix} \sim Pe^{1/5} Ch^{-2/5} t_0$



Time Scales

- Time to reach the maximum elastic energy:

$$\tau_m \sim PeCh^2 t_0$$



Summary

- Even kinematic single eddy mixing can exhibit unexpected nontrivial dynamics.
- 3 stages: (A) the *"jelly roll"* stage, (B) the *topological evolution* stage, and (C) the *target pattern* stage.
- Band merger process occurs on a time scale exponentially long relative to the eddy turnover time.
- Band merger process resembles step merger in drift-ZF staircases.
- Multi time-scale process: the $Pe^{1/5}$ and the Pe^{1} time scales.

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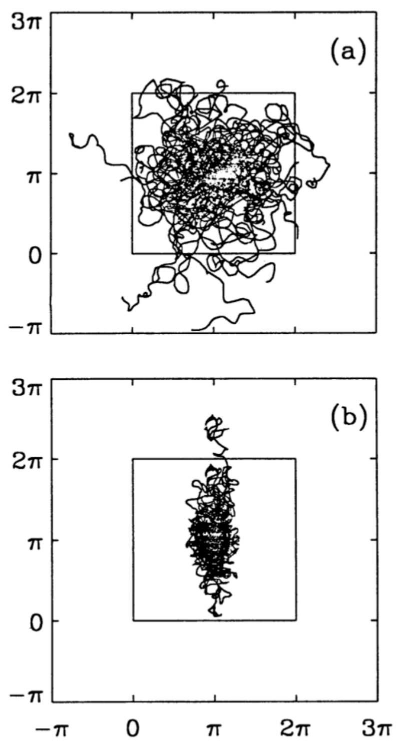
Introduction

- 2D MHD/reduced MHD: fundamental system in plasma physics.
- Turbulent transport: important in fusion studies.
- Kinematic expectation (passive scalar): $\eta_K \sim ul$
- Actual result: turbulent transport is suppressed $\eta_T < \eta_K$
- Conventional wisdom: mean field theory.
- New observation: mean field not applicable in some cases, with greater Rm.

Conventional Wisdom (1)



- [Cattaneo and Vainshtein 1991]: turbulent transport is suppressed even when a weak large scale magnetic field is present.
- Starting point: $\partial_t \langle A^2 \rangle = -2\eta \langle B^2 \rangle$
- Assumptions:
  - Energy equipartition: $\frac{1}{\mu_0 \rho} \langle B^2 \rangle \sim \langle v^2 \rangle$
  - Average B can be estimated by: $|\langle \mathbf{B} \rangle| \sim \sqrt{\langle A^2 \rangle}/L_0$
- Define Mach number as: $M^2 \equiv \langle v^2 \rangle / v_A^2 = \langle v^2 \rangle / (\frac{1}{\mu_0 \rho} \langle \mathbf{B} \rangle^2)$
- Result for suppression stage: $\eta_T \sim \eta M^2$
- Combine with kinematic stage result: $\eta_T \sim \frac{ul}{1 + \mathrm{Rm}/M^2}$
- Lack physics interpretation of the origin of η_T.

Conventional Wisdom (2)

- [Gruzinov and Diamond 1994, 1996] and [Diamond, Hughes, and Kim 2005] derived η_T from dynamics.
- With an external imposed B_0 (i.e. $\frac{\partial \langle A \rangle}{\partial x}$).
- The key of this approach is to calculate the flux $\Gamma_A \equiv \langle v_x A \rangle$
- Standard closure methods yield:

$$\begin{aligned}
\Gamma_A &= \sum_{\mathbf{k}} [v_x(-\mathbf{k})\delta A(\mathbf{k}) - B_x(-\mathbf{k})\delta\phi(\mathbf{k})] \\
&= -\sum_{\mathbf{k}} [\tau_c^{\phi}(\mathbf{k})\langle v^2 \rangle_{\mathbf{k}} - \frac{1}{\mu_0 \rho}\tau_c^{A}(\mathbf{k})\langle B^2 \rangle_{\mathbf{k}}] \frac{\partial \langle A \rangle}{\partial x} \\
&= -\sum_{\mathbf{k}} \tau_c [\langle v^2 \rangle_{\mathbf{k}} - \frac{1}{\mu_0 \rho}\langle B^2 \rangle_{\mathbf{k}}] \frac{\partial \langle A \rangle}{\partial x}
\end{aligned}$$

- Therefore: $\Gamma_A = -\eta_T \frac{\partial \langle A \rangle}{\partial x}$ with $\eta_T = \sum_{\mathbf{k}} \tau_c [\langle v^2 \rangle_{\mathbf{k}} - \frac{1}{\mu_0 \rho}\langle B^2 \rangle_{\mathbf{k}}]$

Conventional Wisdom (2) Cont'd

- Then calculate $\langle B^2 \rangle$ in terms of $\langle v^2 \rangle$. From:

$$\partial_t A + \mathbf{v} \cdot \nabla A = -v_x \frac{\partial \langle A \rangle}{\partial x} + \eta \nabla^2 A$$

- Multiplying by A and sum over all modes:

$$\frac{1}{2}[\partial_t \langle A^2 \rangle + \langle \nabla \cdot \langle \mathbf{v} A^2 \rangle \rangle] = -\Gamma_A \frac{\partial \langle A \rangle}{\partial x} - \eta \langle B^2 \rangle$$

Dropped stationary case   Dropped periodic boundary

- Therefore: $\langle B^2 \rangle = -\frac{\Gamma_A}{\eta} \frac{\partial \langle A \rangle}{\partial x} = \frac{\eta_T}{\eta} B_0^2$
- Define Mach number as: $M^2 \equiv \langle v^2 \rangle / v_{A0}^2 = \langle v^2 \rangle / (\frac{1}{\mu_0 \rho} B_0^2)$
- Result: $\eta_T = \frac{\sum_{\mathbf{k}} \tau_c \langle v^2 \rangle_{\mathbf{k}}}{1 + \mathrm{Rm}/M^2} = \frac{ul}{1 + \mathrm{Rm}/M^2}$
- This theory is not able to describe the system with no B_0, though can be extended.

Simulation Setup

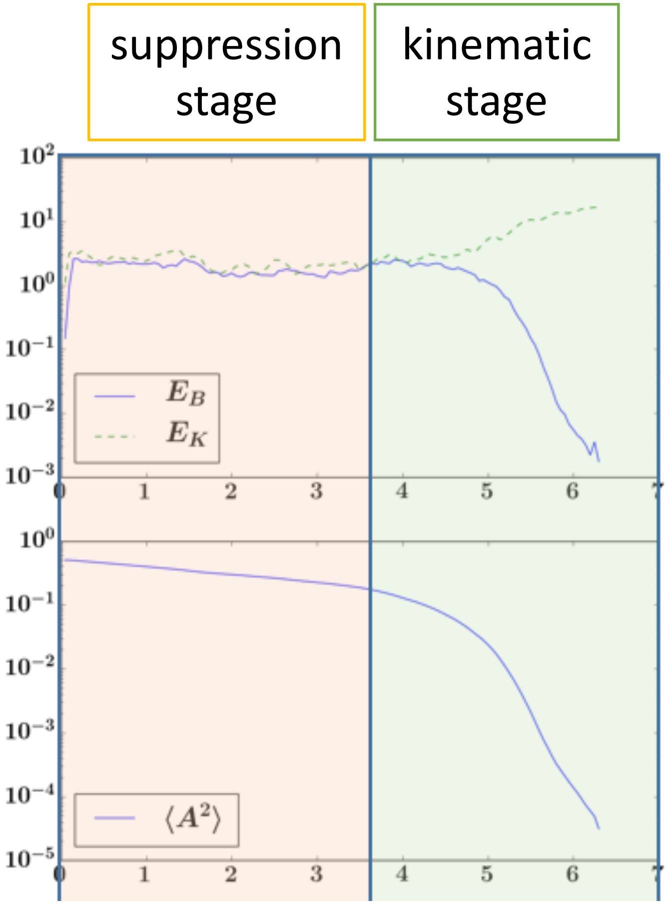
- PIXIE2D: a DNS code solving 2D MHD equations in real space:

$$\partial_t A + \mathbf{v} \cdot \nabla A = \eta \nabla^2 A$$
$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = \frac{1}{\mu_0 \rho} \mathbf{B} \cdot \nabla \nabla^2 A + \nu \nabla^2 \omega + f$$

- 1024^2 resolution.
- External forcing f is isotropic homogeneous.
- Periodic boundary condition.
- Initial conditions:
  - (1) bimodal: $A_I(x,y) = A_0 \cos 2\pi x$
  - (2) unimodal: $A_I(x,y) = A_0 * \begin{cases} -(x-0.25)^3 & 0 <= x < 1/2 \\ (x-0.75)^3 & 1/2 <= x < 1 \end{cases}$

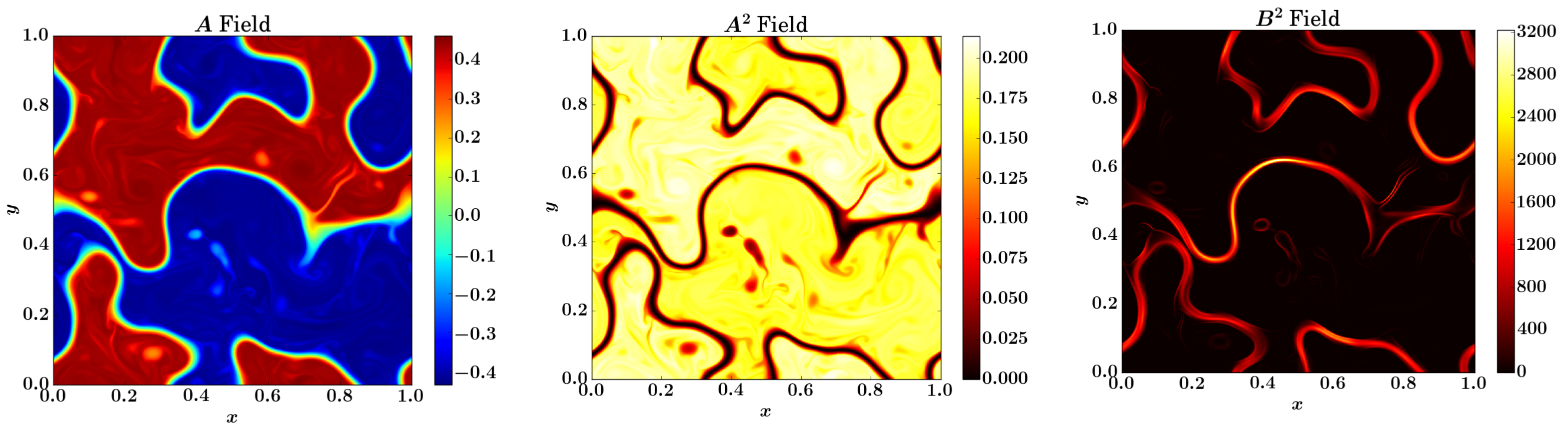
Two Stages

- 1. The suppression stage: the large scale magnetic field is sufficiently strong so that the diffusion is suppressed.
- 2. The kinematic decay stage: the magnetic field is dissipated enough so that the diffusion rate is back to the kinetic rate.
- The suppression is due to the memory provided by the magnetic field.

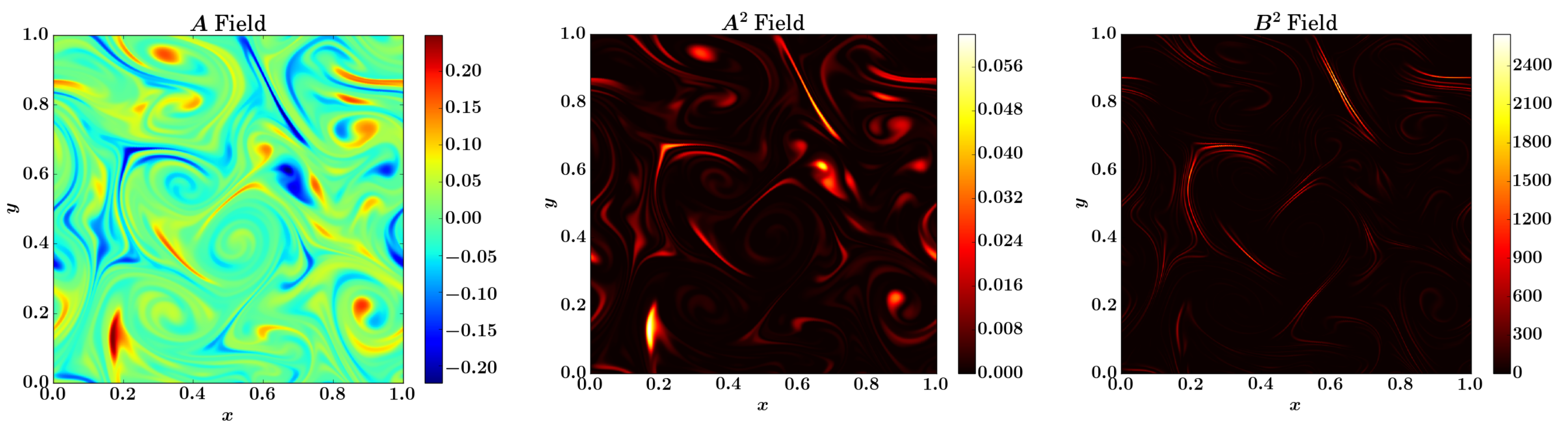


New Observations

- With no imposed B_0, in suppression stage:



- v.s. same run, in kinematic stage (trivial):



New Observations Cont'd

- Nontrivial structure formed in real space in the suppression stage.
- A field is evidently composed of "blobs".
- The low A^2 regions have a clear 1-dimensional shape.
- The high B^2 regions are strongly correlated with low A^2 regions, and also have a 1-dimensional shape.
- We call these 1-dimensional high B^2 regions ``barriers'', because these are the regions where transport is reduced, relative to η_K.

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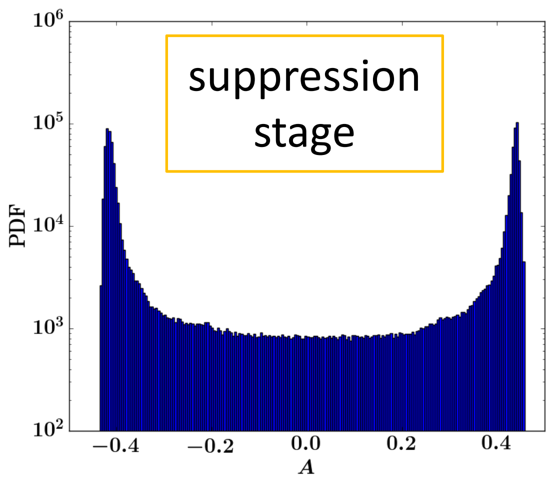
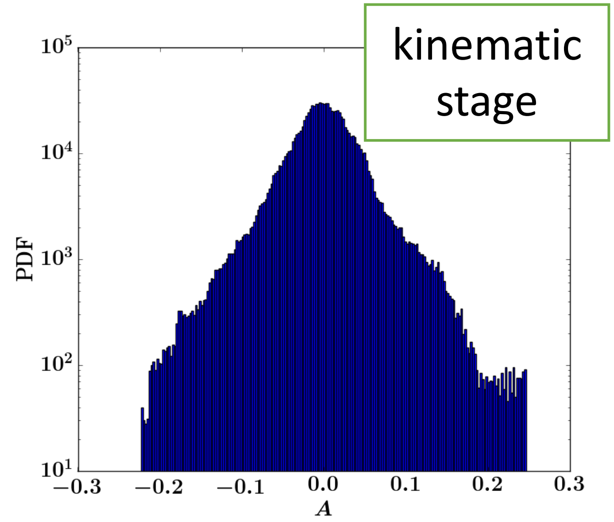
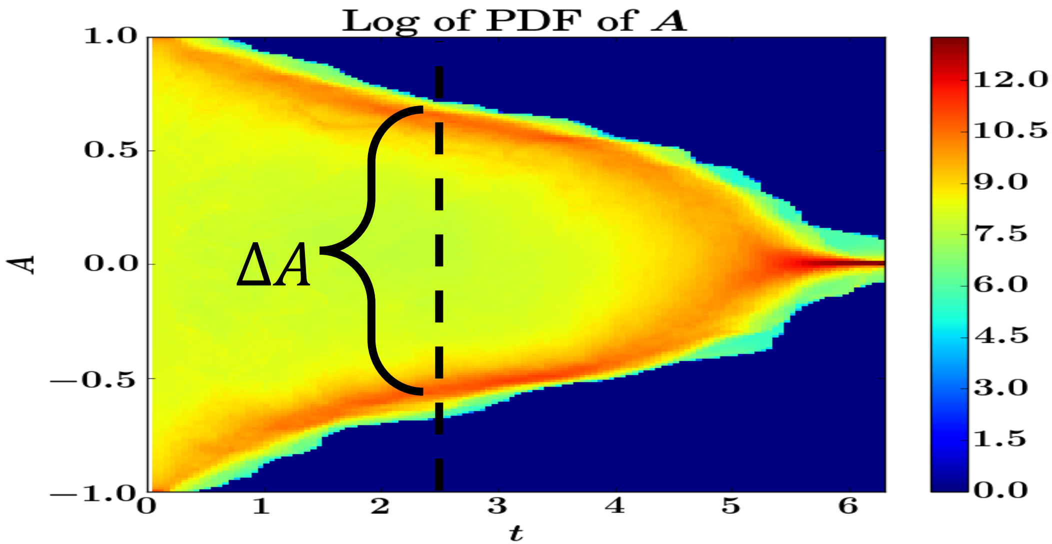
Evolution of PDF of A

- Probability Density Function (PDF) in two stage:





- Time evolution: horizontal “Y”.

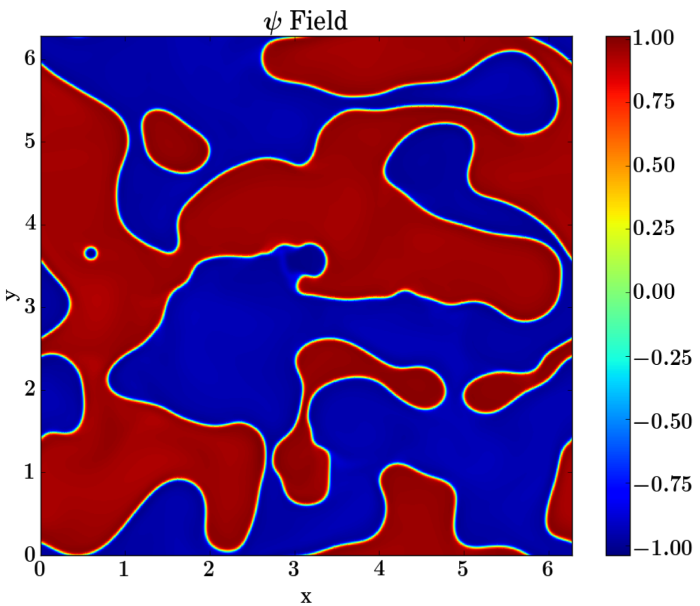


- The PDF changes from double peak to single peak as the system changes from the suppression stage to the kinematic stage.

2D CHNS and 2D MHD

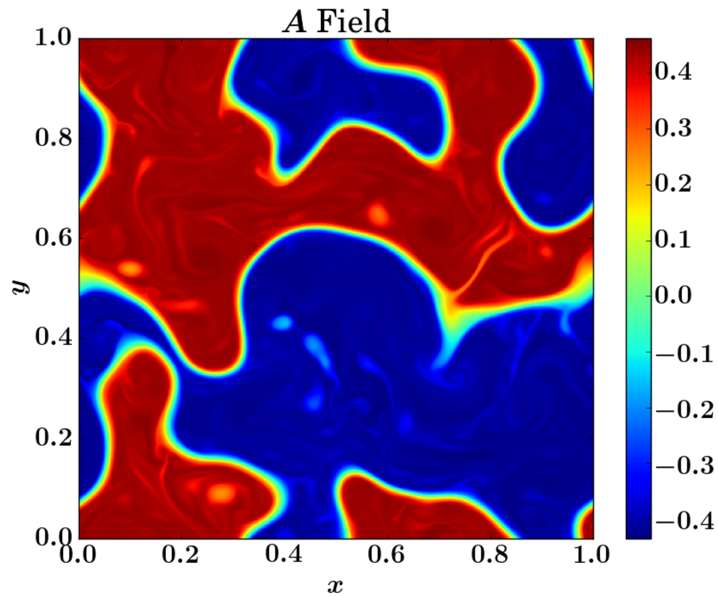
- The A field in 2D MHD in suppression stage is strikingly similar to the ψ field in 2D CHNS (Cahn-Hilliard Navier-Stokes) system:

ψ field in 2D CHNS



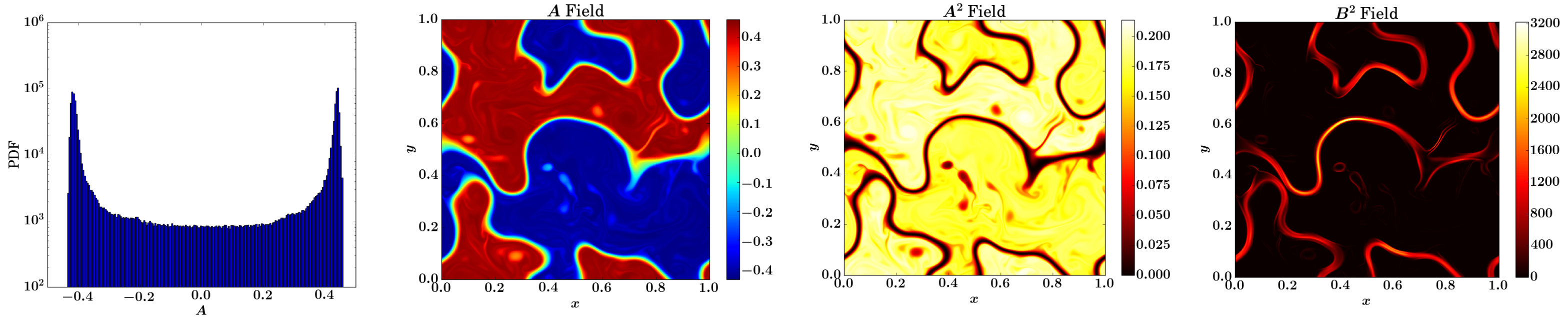
v.s.

A field in 2D MHD

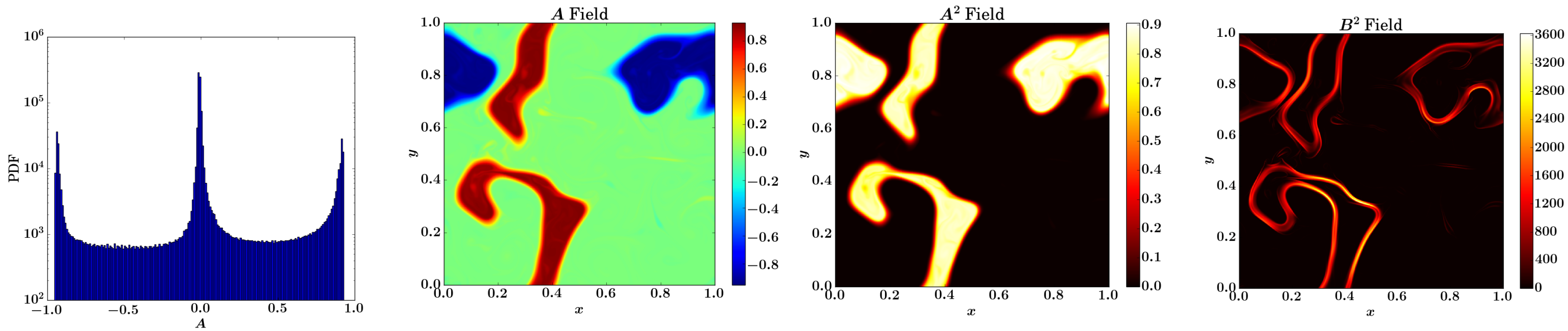


Unimodal Initial Condition

- One may question whether the bimodal PDF feature is purely due to the initial condition. The answer is no.
- Two peaks away from 0 on PDF of A still rise, even if the initial condition is unimodal.



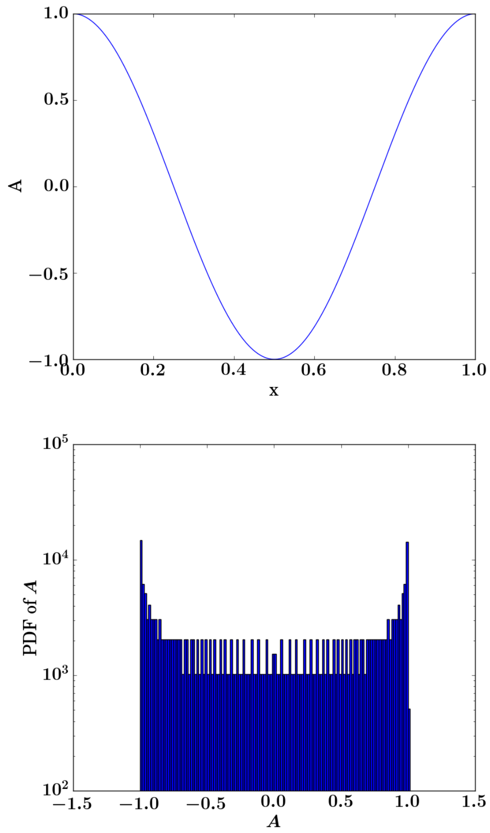
(a1) (a2) (a3) (a4)



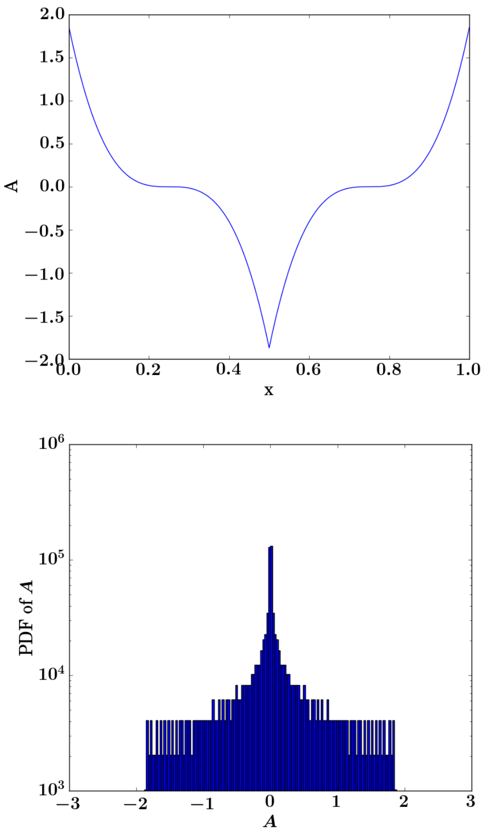
(b1) (b2) (b3) (b4)

Unimodal Initial Condition

Bimodal

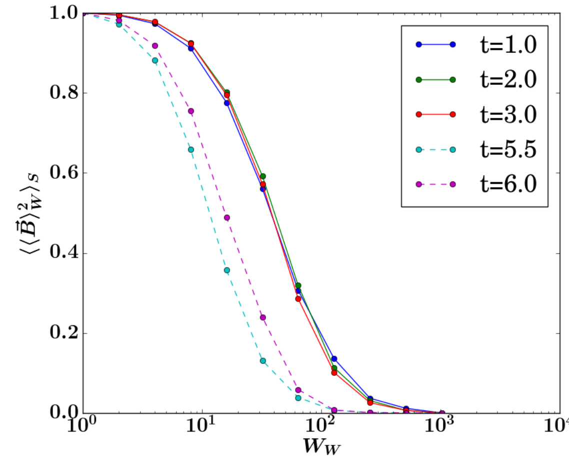


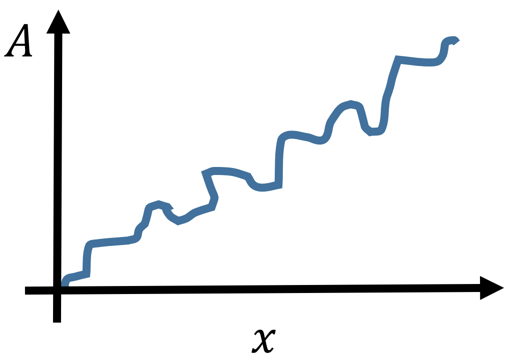
Unimodal



The problem of the mean field $\langle \boldsymbol{B} \rangle$

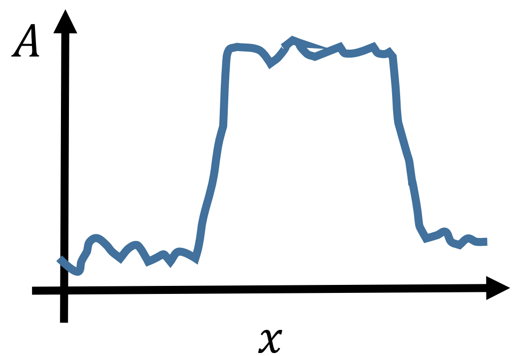
- $\langle B \rangle$ depends on the averaging window.
- With no imposed external field, B is highly intermittent, therefore the $\langle B \rangle$ is not well defined.





$|\langle B \rangle| \sim \sqrt{\langle A^2 \rangle}/L_0$ ✓

v.s.



$\langle B \rangle$ not well defined

Reality

New Understanding

- From $\partial_t \langle A^2 \rangle = -\langle \mathbf{v} A \rangle \cdot \nabla \langle A \rangle - \nabla \cdot \langle \mathbf{v} A^2 \rangle - \eta \langle B^2 \rangle$
- Do not drop 2nd term on RHS. Average taken over an envelope.
- Define diffusion coefficients (closure):

$$\langle \mathbf{v} A \rangle = -\eta_{T1} \nabla \langle A \rangle$$
$$\langle \mathbf{v} A^2 \rangle = -\eta_{T2} \nabla \langle A^2 \rangle$$

- Plugging in: $\partial_t \langle A^2 \rangle = \eta_{T1} (\nabla \langle A \rangle)^2 + \nabla \eta_{T2} \cdot \nabla \langle A^2 \rangle - \eta \langle B^2 \rangle$
- For simplicity: $\langle B^2 \rangle \sim \frac{\eta_T}{\eta} (\langle B \rangle^2 + \langle A^2 \rangle / L_{env}^2)$
- where L_{env} is the envelope size. Scale of $\nabla^2 \langle A^2 \rangle$.
- Define new strength parameter: $M'^2 \equiv \langle v^2 \rangle / (\frac{1}{\mu_0 \rho} \langle A^2 \rangle / L_{env}^2)$
- Result: $\eta_T = \frac{ul}{1 + \mathrm{Rm}/M^2 + \mathrm{Rm}/M'^2} = \frac{ul}{1 + \mathrm{Rm} \frac{1}{\mu_0 \rho} \langle \mathbf{B} \rangle^2 / \langle v^2 \rangle + \mathrm{Rm} \frac{1}{\mu_0 \rho} \langle A^2 \rangle / L_{env}^2 \langle v^2 \rangle}$

New Understanding Cont'd

- Quench is not uniform. Transport coefficient is different in different regions.
- In the regions where magnetic fields are strong, Rm/M^2 is dominant. They are regions of ***barriers***.
- In other regions, i.e. inside blobs, Rm/M'^2 is what remains.
- Summary of important length scales: $l < L_{stir} < L_{env} < L_0$
  - System size L_0
  - Envelope size L_{env}
  - Stirring length scale L_{stir}
  - Turbulence length scale l, here we use Taylor microscale λ
  - Barrier width W

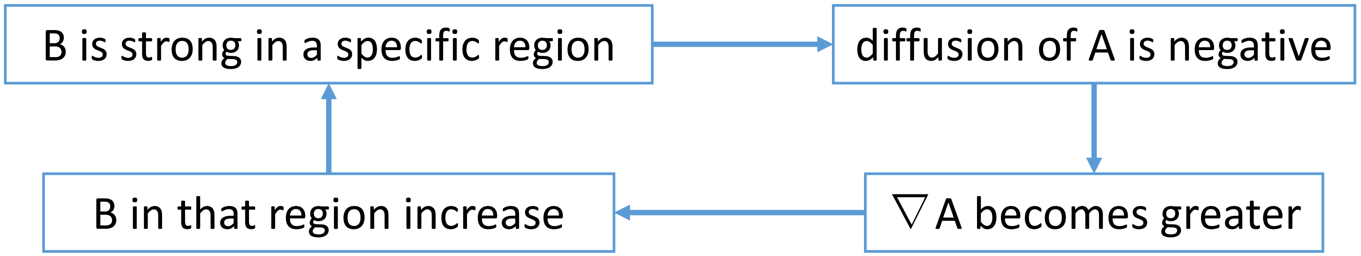
Formation of Barriers

- How do the barriers form?

flux coalescence

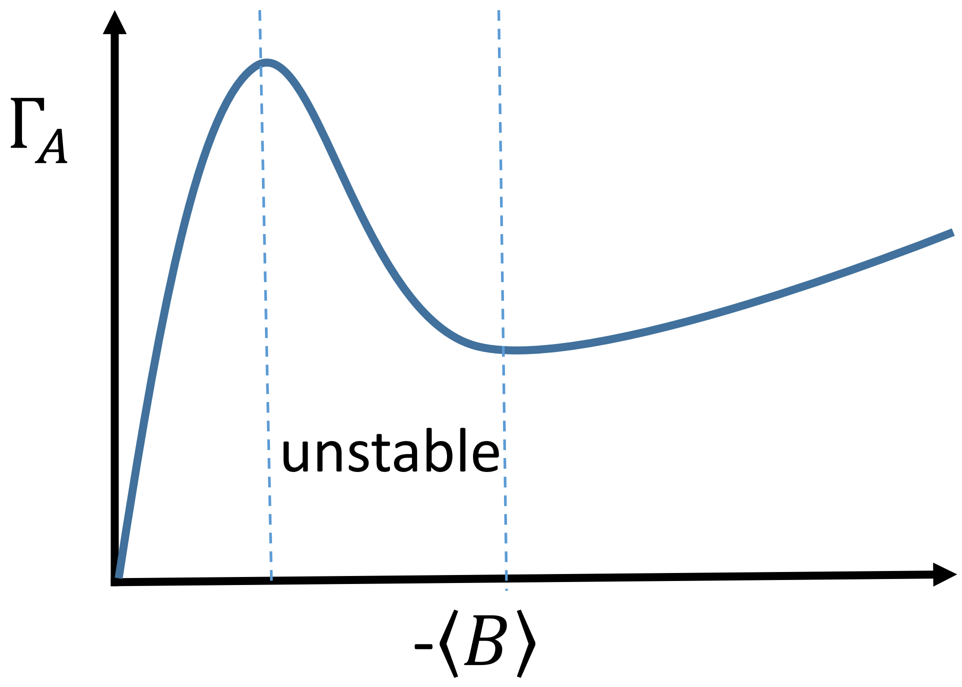
$$\eta_T = \sum_{\mathbf{k}} \tau_c [\langle v^2 \rangle_{\mathbf{k}} - \frac{1}{\mu_0 \rho} \langle B^2 \rangle_{\mathbf{k}}]$$

- From above expression, it is possible for some strong B regions to have negative resistivity, while the resistivity is always positive when averaged over the whole system.
- Positive feedback:



Formation of Barriers Cont'd

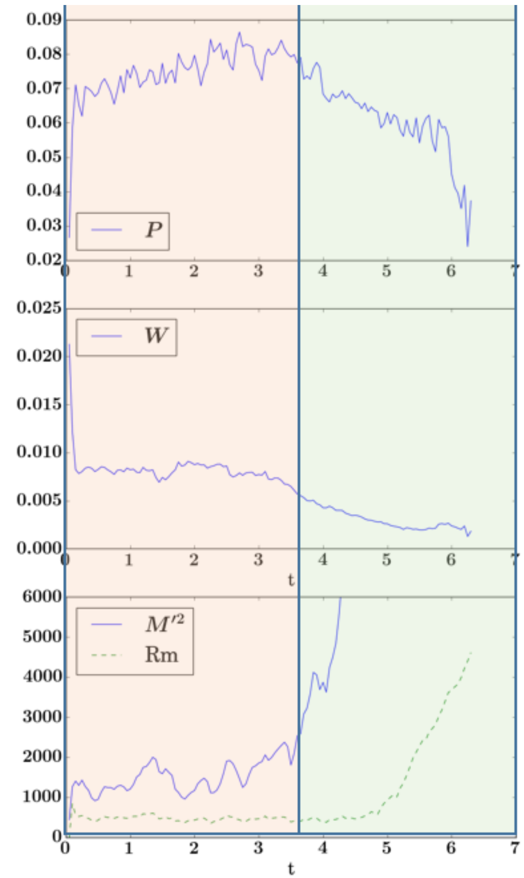
- Negative resistivity leads to barrier formation.
- The S-curve is due to the dependence of B on Γ_A.
- When slope is negative, it is negative resistivity.



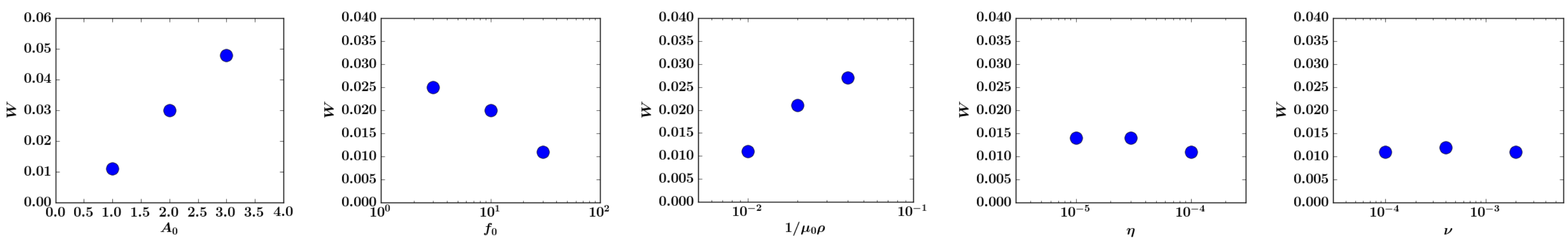
$\Gamma_A = \langle v_x A \rangle$
is the flux

Describing the Barriers

- Time evolution of P and W:



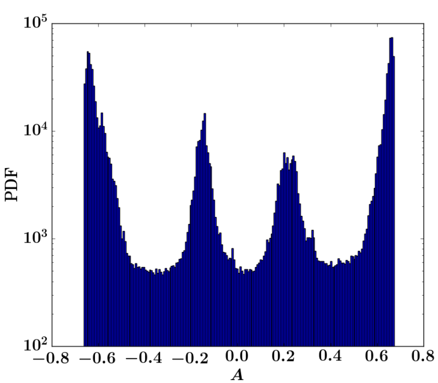
- What determines W:
  - A_0 or $1/\mu_0\rho$ greater, W greater;
  - f_0 greater, W smaller;
  - W not sensitive to η or ν.



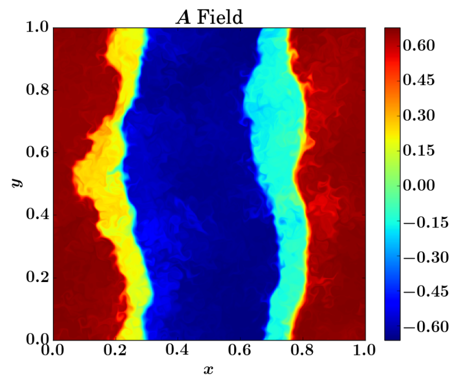
(a) (b) (c) (d) (e)

Staircase

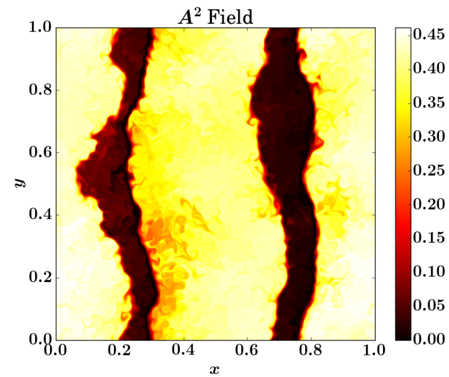
- Staircases emerge spontaneously!
- Initial condition is the usual cos function (bimodal)
- The only major different parameter from runs above is the forcing scale is k=32 (for all runs above k=5).
- Resembles the staircase studies in fusion research.



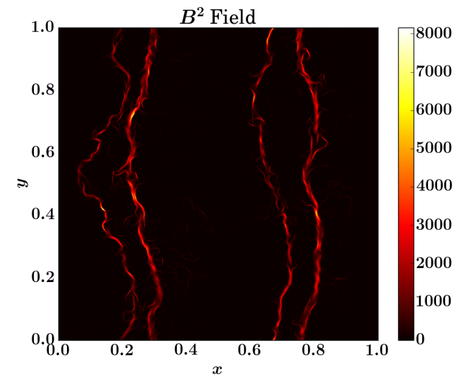
(1)



(2)



(3)



(4)

Summary

- Magnetic fields suppress turbulent diffusion in 2D MHD by: formation of intermittent ***transport barriers***.
- Magnetic structures:
  - Barriers – thin, 1D strong field
  - Blobs – 2D, weak field
- Quench not uniform:

$$\eta_T = \frac{ul}{1 + \mathrm{Rm}\frac{1}{\mu_0\rho}\langle \mathbf{B}\rangle^2/\langle v^2\rangle + \mathrm{Rm}\frac{1}{\mu_0\rho}\langle A^2\rangle/L_{env}^2\langle v^2\rangle}$$

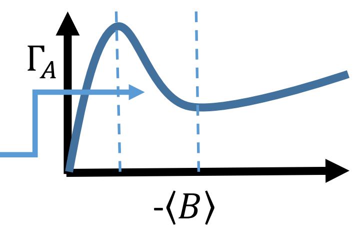
barriers, strong B

blobs, weak B, $\nabla^2\langle A^2\rangle$ remains

- Barriers form due to negative resistivity:

$$\eta_T = \sum_{\mathbf{k}} \tau_c[\langle v^2\rangle_{\mathbf{k}} - \frac{1}{\mu_0\rho}\langle B^2\rangle_{\mathbf{k}}]$$

flux coalescence



- Formation of “magnetic staircases” observed for some i.c.

Outline

- Introduction
- Cascades and Spectra in 2D CHNS
- Single Eddy Mixing in 2D Cahn-Hilliard Flow
- Turbulent Transport in 2D MHD
- **Conclusions and Future Works**

General Conclusions

- Dual (or multiple) cascades can interact with each other, and one can modify another.
- We also show how a length scale, e.g. the Hinze scale in 2D CHNS, emerges from the balance of kinetic energy and elastic energy in blobby turbulence.
- We learn how negative diffusion (flux/blob coalescence) can lead to novel real space structure in a simple system, for example the target pattern.
- Turbulent resistivity can be negative (though for a short time) in a simple system such as 2D MHD. This results in the formation of nontrivial real space structure.
- More generally, we see that studying analogous but different systems can improve our understanding of all of them.

Future Works

- Extension of the transport study in MHD:
  - Numerical verification of the new η_T expression
  - What determines the barrier width and packing fraction
  - Why does layering appear when the forcing scale is small
  - What determines the step width, in the case of layering
  - The transport study may also be extended to 3D MHD ($\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle$ important instead of $\langle A^2 \rangle$). Do barriers regulate magnetic helicity transport in 3D? Implications for α quenching?
- Turbulent transport in CHNS
- Other similar systems can also be studied in this spirit. E.g. Oldroyd-B model for polymer solutions.

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Thank you!