Effect of Turbulence Spreading on Subcritical Turbulence in Inhomogeneous Plasmas

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Abstract

The influence of the turbulence spreading is studied on the self-sustained turbulence which is induced by the subcritical instability. It is found that there is a minimum system size that can sustain the self-sustained turbulence, and an analytic formula is derived. The generalization of the Maxwell's construction rule is also derived.

Keywords: turbulence spreading, subcritical turbulence, plasma transport, nonlocal effect, generalized Maxwell's construction, minimum system size

1. Introduction

Substantial progress has been achieved recently in the field of plasma turbulence. One of the key in the theoretical progress, in comparison with the classical view of plasma turbulence,[1] is the recognition of the importance of nonlocal effect of turbulence in inhomogeneous plasmas. The nonlocal interaction has been analyzed from various points of view: One approach is based on the framework of the K-ε model,[2,3] another one treats a long radial transmission of fluctuation energy,[4] and the other approach emphasizes the interaction between fluctuations with different scale lengths.[5] These theoretical approaches have been applied to study the radial profiles of turbulent transport coefficient[6-9] and the transient transport.[10,11] The importance of nonlocal interactions has also been shown by direct nonlinear simulations (DNS).[12-18] In particular, the recent evolution of DNS has encouraged the quantitative test, including the scaling law with respect to control parameters, and stimulated the detailed theoretical analysis on the turbulent spreading.[18-24]

The other key process is the subcritical turbulence. The nonlinear instability mechanisms exist in toroidal plasmas, so that the strong turbulence is sustained even for plasma parameters that predict linear stability for microscopic fluctuations. (See references quoted in refs.25 and 26.) The theoretical progress has allowed to analyze the anomalous transport which is driven by self-sustained turbulence. Experimental observations have suggested a relevance of subcritical turbulence in the toroidal plasmas.[27, 28] Thus, the study of subcritical turbulence under more general circumstances, *e.g.*, including the effects of turbulence spreading, needs to be performed.

Progress on these two directions raises a problem, i.e., the impact of the turbulent spreading on the subcritical turbulence. In this article, we study the influence of the turbulence spreading on the self-sustained turbulence of subcritical instabilities. The stationary solution of the self-sustained turbulence is obtained where the turbulence spreads into strongly stable regions. The reduction of turbulent transport by the radial spreading is analyzed. It is shown that there is a minimum radial plasma extent, in order for a self-sustained turbulence to exist in linearly stable plasmas. A new bifurcation is illustrated. The new phase boundary in parameter space is derived, in which the effect of turbulence spreading is included.

2. Model

We study a one-dimensional model to study the turbulence spreading into stable regions. The turbulence quantities are averaged over the magnetic surface, and a profile in the $x-$ direction is studied. (A slab model is employed, and the $x-$ axis is taken in the radial direction.) We take the case that the plasma has a subcritical instability in the region of $0 \le |x| \le L$, and it is strongly stable in the regions $|x| > L$. (Schematic drawing is given in Fig.1.)

We follow the framework of ref.22, in which a dynamical equation has been formulated in a form as

$$\frac{\partial}{\partial t} I = \Lambda I + \chi_0 \frac{\partial}{\partial x} I^{\alpha} \frac{\partial}{\partial x} I , \qquad (1)$$

where $I = \left|\tilde{\phi}^2\right| / \left|\tilde{\phi}^2_{\rm local}\right|$ is the normalized electrostatic potential fluctuation amplitude, $\left|\tilde{\phi}^2_{\rm local}\right|$ is the level which is given by the local balance of drive and damping $\Lambda = 0$, Λ is the decorrelation rate (including the growth rate and local nonlinear damping rate), and $\chi_0 I^{\alpha}$ is the diffusion coefficient due to a turbulence spreading, in which the dependence on the fluctuation level is represented by use of the index α. (In a weak turbulence limit, $\alpha = 1$ holds, and $\alpha = 1/2$ for strong turbulence limit.[3)]) This model has several limitations. First, the fluctuations, the wavelengths of which are comparable to L, are not included. Second, the incoherent and fluctuating kicks are not taken into account. Noticing that these additional processes may have a substantial influence, we choose the model (1) as a starting point of the analysis. A reciprocal relation between the nonlinear energy cascade in the k -space (which is included in the Λ -term) and the spreading in the real space has been explained in literature. (See refs.2 and 30.)

In the damper region, $|x| > L$, the plasma is stable, and we simply choose

$$\Lambda = -\gamma_{\mathrm{damp}} - \chi_0 k^2 I^{\alpha} < 0 \qquad \text{for } |x| > L \ . \tag{2}$$

where $1/k$ denotes a characteristic scale length of turbulence. In the excitation region, $0 \le |x| \le L$, plasma can be unstable, i.e., Λ can be positive. In the case of subcritical excitation, the dependence of Λ on I is such that

$$\Lambda < 0 \qquad \text{for} \qquad I < I_{\mathrm{c1}} \text{ and } 1 < I \ , \tag{3a}$$

and

$$\Lambda > 0 \qquad \text{for} \qquad I_{\mathrm{c1}} < I < 1 \ , \tag{3b}$$

where I_{c1} is the threshold amplitude of fluctuations for the onset of nonlinear instability. Λ takes the maximum value Λ_0 in the region $I_{\mathrm{c1}} < I < 1$. In a local theory, the solution

$$I = 1 \tag{4}$$

is given for the self-sustained turbulence. (Notice that the normalization of fluctuation is defined such that $I = 1$ holds for local theory.)

The stationary state is given by the equation

$$\chi_0 \frac{\partial}{\partial \mathrm{x}} I^{\alpha} \frac{\partial}{\partial \mathrm{x}} I + \Lambda I = 0 \ . \tag{5}$$

Equation (5) is solved with the boundary condition

$$\frac{\mathrm{d}}{\mathrm{dx}} I = 0 \qquad \text{at } x = 0 \ , \tag{6a}$$

$$I = 0 \qquad \text{at } |x| \to \infty \ . \tag{6b}$$

We introduce a characteristic length

$$\ell = \sqrt{\frac{\chi_0}{(1+\alpha)\Lambda_0}} \ ,$$

(7a)

and the length and Λ are normalized with respect to ℓ and Λ_0 , respectively,

$$\zeta = x/\ell \ , \qquad \hat{L} = L/\ell \ , \qquad \hat{\Lambda} = \Lambda/\Lambda_0 \ . \tag{7b}$$

By use of this normalization, Eq.(5) is rewritten as

$$\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} F + \hat{\Lambda} F^{1/(1+\alpha)} = 0 \ , \tag{8}$$

where a new variable for the fluctuation amplitude is introduced as

$$F = I^{1+\alpha} \ . \tag{9}$$

The flux of turbulence is given by a gradient of F . In Eq.(8), variable F and coefficient $\hat{\Lambda}$ are of the order of unity.

3. Solution

3.1 Solution of the steady state

The solution of Eq.(8) in the unstable layer, $0 \le |\zeta| \le \hat{L}$ (i.e., $0 \le |x| \le L$), is examined. The solution $F(\zeta)$ is symmetric with respect to ζ , and we look for a solution in the domain $0 \le \zeta \le \hat{L}$ with the boundary condition $\mathrm{d}F/\mathrm{d}\zeta = 0$ at $\zeta = 0$. Multiplying $\mathrm{d}F/\mathrm{d}\zeta$ to Eq.(8) and integrating it with respect to ζ once, one has a relation as

$$\frac{\mathrm{d}}{\mathrm{d}\zeta} F = -\sqrt{H(F(0)) - H(F)} \ , \tag{10}$$

where the Sagdeev potential is given as

$$H(F) \equiv 2\int_0^F \mathrm{d}F \, \hat{\Lambda} \, F^{1/(1+\alpha)} \tag{11}$$

and the relation $\mathrm{d}F/\mathrm{d}\zeta = 0$ at $F = F(0)$ is used. The function $H(F)$ is illustrated in Fig.2. Equation (10) is integrated, and the solution $F(\zeta)$ in the domain $0 \le \zeta \le \hat{L}$ is given in an implicit form as

$$\int_F^{F(0)} \frac{\mathrm{d}F}{\sqrt{H(F(0)) - H(F)}} = \zeta \ . \tag{12}$$

This solution Eq.(12) includes an integral constant $F(0)$, which is determined by the continuity condition at $\zeta = \hat{L}$. In the stable region, $\zeta > \hat{L}$, Eq.(8) takes a form

$$\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} F - \left(\frac{\gamma_{\mathrm{damp}}}{\Lambda_0} F^{1/(1+\alpha)} + \frac{\chi_0 k^2}{\Lambda_0} F \right) = 0 \ . \tag{13}$$

Noticing the boundary condition Eq.(6b), i.e., $F \to 0$ as $\zeta \to \infty$, Eq.(13) is integrated once as

$$\left(\frac{\mathrm{d}F}{\mathrm{d}\zeta} \right)^2 = \frac{\gamma_{\mathrm{damp}}}{\Lambda_0} \frac{2+2\alpha}{2+\alpha} F^{(2+\alpha)/(1+\alpha)} + \frac{\chi_0 k^2}{\Lambda_0} F^2 \ . \tag{14a}$$

This gives a relation between $F'(\hat{L})$ (i.e., $\mathrm{d}F/\mathrm{d}\zeta$ at $\hat{\xi} = \hat{L}$) and $F(\hat{L})$ as

$$F'(\hat{L})^2 = \frac{\gamma_{\mathrm{damp}}}{\Lambda_0} \frac{2+2\alpha}{2+\alpha} F(\hat{L})^{(2+\alpha)/(1+\alpha)} + \frac{\chi_0 k^2}{\Lambda_0} F(\hat{L})^2 \ . \tag{14b}$$

The constant $F(0)$ in Eq.(12) is determined to satisfy the condition Eq.(14b) at $\zeta = \hat{L}$.

For obtaining an analytic insight of the problem, we consider the case that the damping in the region $\zeta > \left| \hat{L} \right|$ is strong enough, i.e.,

$$\gamma_{\text{damp}} / \Lambda_0 \to \infty \ . \tag{15}$$

In this limit of strong damping, $\gamma_{\text{damp}} / \Lambda_0 \to \infty$, Eq.(14b) is simplified as

$$F = 0 \ \text{ at } \zeta = \hat{L} \ . \tag{16}$$

(The limit of Eq.(15) does not change the conclusion qualitatively in this article. See appendix for supplementary discussion.) Figure 3 illustrates an example of the self-sustained solution Eq.(12) with Eq.(16), by choosing a model form, which allow a subcritically-excited multiple states,

$$\hat{\Lambda} = 4(1-a)^{-2}\left(-a + (a+1)F - F^2\right) \ , \tag{17}$$

where a is a parameter to denote the magnitude of the linear stability, and the threshold amplitude for the onset of instability is given, in terms of I , as $I_{c1}^{1+\alpha} = a$. (The parameter a is in the ranges $0 < a < 1$, and the linear damping rate is larger for a larger value of a .)

3.2 Influence of turbulence spreading on self-sustained turbulence

The result (12) with Eq.(16) determines the influence of the turbulence spreading on the sustainment of the subcritical turbulence. The dependence on the width of the region $2\hat{L}$ is studied in this subsection. Putting $\zeta = \hat{L}$ into Eq.(12), one has

$$\int_0^{F(0)} \frac{\mathrm{d}F}{\sqrt{H(F(0)) - H(F)}} = \hat{L} \ . \tag{18}$$

From Eq.(18), one sees that there is an upper bound and a lower bound for $F(0)$. From the functional dependence of $H(F)$ shown in Fig.2, $H(F(0))$ must satisfy the condition

$$0 = H(0) \leq H(F(0)) \leq H(1) \ . \tag{19}$$

(If $H(F(0))$ is negative, $F(\hat{L}) = 0$ cannot be satisfied.) The lower bound of $F(0)$ is imposed by the subcritical excitation condition and is given as

$$F(0) \geq F_* \ , \tag{20a}$$

where F_* is defined by the relation

$$H(F_*) = 0 \ . \tag{20b}$$

That is, the value $F(0)$ must satisfy the condition

$$F_* \leq F(0) \leq 1 \ , \tag{21}$$

in order that the nontrivial solution $F(0) > 0$ is allowed for the self-sustained turbulence. The fact that $F(0)$ has a lower bound is different from the case that the turbulence is driven by supercritical instability. In the case of supercritical instability, $F(0)$ continuously reduces to 0 as $\hat{L}$ approaches 0, as has been discussed in ref.22.

Noting the lower and upper bounds, Eq.(21), the influence of the finite length $\hat{L}$ on the turbulence level is studied by analyzing Eq.(18).

First, the case of large $\hat{L}$ is discussed. By definition, $H(F)$ has a local maximum at $F = 1$, which denotes the self-sustained turbulence in a local model. As $\hat{L}$ increases, $F(0)$ approaches to unity. In order to study the situation of $F(0) \sim 1$, $H(F)$ is expanded in the vicinity of $F = 1$ as

$$H(F) = H(1) - \tfrac{1}{2}\left|H''(1)\right|(1-F)^2 + \cdots , \tag{22}$$

($H''(1) = 2\,\Lambda'(1)$, where $H'' = \mathrm{d}^2H/\mathrm{d}F^2$ and $\Lambda' = \mathrm{d}\Lambda/\mathrm{d}F$). Substituting Eq.(22) and $H''(1) = 2\,\Lambda'(1)$ into Eq.(18), one obtains the leading term with respect to $1 - F(0)$ of the left hand side (LHS) of Eq.(18) as

$$\begin{aligned}\int_0^{F(0)} \frac{\mathrm{d}F}{\sqrt{H(F(0)) - H(F)}} &= \frac{1}{2\sqrt{\left|\Lambda'(1)\right|}} \ln\left(\frac{1}{H(1) - H(F(0))}\right) + \cdots \\ &= \frac{1}{2\sqrt{\left|\Lambda'(1)\right|}} \ln \frac{1}{2\left|\Lambda'(1)\right|(1-F(0))^2} + \cdots .\end{aligned} \tag{23}$$

Equations (18) and (23) yield the relation

$$F(0) \simeq 1 - \sqrt{\frac{1}{2\left|\Lambda'(1)\right|}}\, \exp\left(-\sqrt{\left|\Lambda'(1)\right|}\,\hat{L}\right) . \tag{24a}$$

The asymptotic relation for the turbulent intensity I $\left(= F^{1/1+\alpha}\right)$ in the large $\hat{L}$ limit is also given as

$$I(0) \simeq 1 - \frac{1}{1+\alpha}\sqrt{\frac{1}{2\left|\Lambda'(1)\right|}}\, \exp\left(-\sqrt{\left|\Lambda'(1)\right|}\,\hat{L}\right) , \tag{24b}$$

and the turbulent transport coefficient, which has the dependence $\chi = \chi_0 I^{\alpha}$, is given as

$$\chi = \chi_0 \left\{ 1 - \frac{\alpha}{1+\alpha}\sqrt{\frac{1}{2\left|\Lambda'(1)\right|}}\, \exp\left(-\sqrt{\left|\Lambda'(1)\right|}\,\hat{L}\right) \right\} . \tag{24c}$$

In the large $\hat{L}$ limit, one sees that F, I and χ have reduction factors which have exponential decrease with respect to $\hat{L}$.

Next, we show that there is a minimum length of $\hat{L}$ in order that the nontrivial solution $F(0) > 0$ exists. Figure 4 shows $\hat{L}$, which is given by Eq.(18), together with a model Eq.(17), as a function of $F(0)$. The minimum of $\hat{L}$ appears at a value $F(0)$ which satisfies $0 < H(F(0)) < H(1)$. The minimum of $\hat{L}$ indicates the necessary length of the system for the self-sustained turbulence to exist. When $H(F(0))$ becomes smaller than $H(1)$, the estimate Eq.(23) provides

$$\hat{L} \simeq \frac{1}{2\sqrt{\left|\Lambda'(1)\right|}} \ln\left(\frac{1}{H(1)}\right) + \cdots \quad . \tag{25}$$

Alternatively, the limiting values at $H(F(0)) \to 0$ is given from Eq.(18) as

$$\hat{L} \simeq \int_0^{F_*} \frac{\mathrm{d}F}{\sqrt{-H(F)}} \quad . \tag{26}$$

One has an order-of-magnitude of the minimum value of $\hat{L}$ by taking a mean of two limiting formulae, Eqs.(25) and (26) as

$$2\,\hat{L} \geq \int_0^{F_*} \frac{\mathrm{d}F}{\sqrt{-H(F)}} + \frac{1}{2\sqrt{\left|\Lambda'(1)\right|}} \ln\left(\frac{1}{H(1)}\right) + \cdots \quad . \tag{27}$$

This minimum value of $\hat{L}$ introduces a new bifurcation for the self-sustained turbulence, which is caused by the finite width of the unstable region. Figure 5 illustrates transport coefficient at $x = 0$ as a function of $\hat{L}$. A new type of bifurcation is demonstrated. Namely, when $\hat{L}$ approaches to the lower bound, the self-consistent solution disappears.

Third, the result of Eq.(27) provides a generalized Maxwell's construction rule, which has been derived in the absence of the turbulence spreading.[25] In the absence of turbulence spreading (i.e., $\hat{L} \to \infty$), the construction rule that the self-sustained turbulence is realized has been given by

$$H(1) \geq 0 \ , \tag{28}$$

and $H(1) = 0$ predicts the phase boundary. In the presence of the turbulence spreading, i.e., $\hat{L}$ is finite, Eq.(27) yields the condition that $H(1)$ must satisfy as

$$H(1) \geq \exp\left\{ -2\sqrt{\left|\Lambda'(1)\right|}\left(2\,\hat{L} - \int_0^{F_*} \frac{\mathrm{d}F}{\sqrt{-H(F)}} \right) \right\} \ . \tag{29}$$

This result is a generalization of Eq.(28). If one takes the limit of $\hat{L} \to \infty$, Eq.(29) reduces to Eq.(28) in an exponential way. Figure 6 illustrates $H(1)$ as a function of the system size $2\hat{L}$. Below this curve, the subcritical turbulence is not self-sustained. The exponential dependence on $\hat{L}$ is confirmed by the numerical result in Fig.6.

4. Summary and discussion

In this article, we have studied the effects of the turbulence spreading on the subcritically induced self-sustained turbulence. There is a minimum width of subcritically-unstable region, in order to realize the self-sustained turbulence. An analytic formula is derived. This result qualitatively differs from that of supercritical instabilities, which has been studied in ref.22 in detail. The effect of turbulence spreading was also studied in the limit of large system size. In this case, the subcritical and supercritical turbulences show a qualitatively similar dependence. The generalization of the Maxwell's construction rule including the finite size effect is also derived.

The key result associate with the minimum length is also interpreted as follows. This sets a minimal turbulence impulse (or energy), that is, a product of maximal intensity times length, which is required for the turbulence to spread into the stable region. The normalizations of turbulence level and time clarify that the length is the relevant parameter to set the minimal energy of the slug of turbulence which drives the expansion process into stable zone. Thus the critical condition is expressed in terms of the minimum length.

These results show that the combination of the nonlocality and the subcritical excitation mechanism introduces a new understanding of the turbulent transport. The transport coefficient has different parameter dependence form that of the local theory. In addition, the turbulence spreading allows a new type of bifurcation phenomena, which is caused by the critical system size. It should be noted that the subcritical excitation of turbulence has analogy between the subcritical phenomena such as bubble formation. The critical amplitude of turbulence (in local model) corresponds to the critical size for nucleation in the latter problem. (See Chap.18.2 of ref.25.) The spreading of subcritical turbulence will enrich the understanding of bubble formation where spatial motion of nucleus takes place.

One direction of the future study is the dynamical evolution of the turbulence spreading. In the case that the coefficient for the turbulence spreading is constant, i.e., $\alpha = 0$ in Eq.(1), dynamics of the front has been studied in an infinite domain. (See, for instance, §19A of ref.3.) In this case, a ballistic propagation of the front was calculated. It is in proportion to a geometric mean of reaction rate and diffusion time. The front velocity (spreading velocity in this problem) vanishes when the Maxwell's construction rule is satisfied. The extension to the cases with amplitude-dependent diffusivity ($\alpha \neq 0$) attracts attentions. For instance, the ballistic propagation velocity was obtained, with amplitude-dependent diffusivity ($\alpha \neq 0$), in the case of supercritical excitation in ref.29. The analysis of dynamical spreading in subcritical excitations is ongoing and will be reported in future. The other important issue is the comparison with experimental observation. Short wavelength fluctuations were observed in the core plasma of the Enhanced-Reversed Shear mode on TFTR, and the subcritically-excited current-diffusive ballooning mode fluctuation was considered to be a candidate of origin of fluctuations.[30] The identification of the spatial domain where such fluctuations are excited and the examination of the fluctuations near the periphery of this domain wait future intensive studies.

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Appendix:

When Eq.(15) does not hold, Eq.(14b) instead of Eq.(16) is employed for the boundary condition. Even in this case, the presence of minimum length $\hat{L}$ for the self-consistent solution exists. The boundary condition $\left\{F\left(\hat{L}\right), F'\left(\hat{L}\right)\right\}$ is determined by the crossing of two curves Eqs.(10) and (14a). Connecting Eqs.(10) and (14b) at $F = F\left(\hat{L}\right)$, one obtains a relation

$$H\left(F\left(0\right)\right) = H\left(F\left(\hat{L}\right)\right) + \frac{\gamma_{\mathrm{damp}}}{\Lambda_0}\frac{2+2\alpha}{2+\alpha}F\left(\hat{L}\right)^{(2+\alpha)/(1+\alpha)} + \frac{\chi_0 k^2}{\Lambda_0}F\left(\hat{L}\right)^2 \qquad \text{(A.1)}$$

which gives the relation between $F\left(\hat{L}\right)$ and $F\left(0\right)$. We see that the following relation holds:

$$F\left(0\right) > F\left(\hat{L}\right) , \qquad \text{(A.2)}$$

and $F\left(0\right) - F\left(\hat{L}\right)$ is positive definite.

When one examines the right hand side (RHS) of Eq.(A1.) as a function of $F\left(\hat{L}\right)$, there are two cases. One case is that coefficients $\gamma_{\mathrm{damp}}/\Lambda_0$ and $\chi_0 k^2/\Lambda_0$ are so large that the RHS of Eq.(A.1) is a monotonous increasing function of $F(\hat{L})$. (Strong damping case.) In this case, RHS of Eq.(A.1) is positive definite, and

$$F(0) > F_* \tag{A.3}$$

holds. The same argument following Eq.(21) applies. The other case is that the coefficients $\gamma_{\mathrm{damp}}/\Lambda_0$ and $\chi_0 k^2/\Lambda_0$ are small so that RHS of Eq.(A.1) has a minimum. The RHS takes minimum at $F(\hat{L}) = F_{\mathrm{m}}$, where F_{m} satisfies

$$\left(\hat{\Lambda} + \frac{\gamma_{\mathrm{damp}}}{\Lambda_0}\right) + \frac{\chi_0 k^2}{\Lambda_0} F_{\mathrm{m}}^{\alpha/(1+\alpha)} = 0 \ . \tag{A4.}$$

(Note that $\hat{\Lambda}(F_{\mathrm{m}}) < 0$ holds.) In this case, $F(0)$ is bounded as

$$F(0) > F_{**} \ , \tag{A.5}$$

where F_{**} is defined by the relation

$$H(F_{**}) = H(F_{\mathrm{m}}) + \frac{\gamma_{\mathrm{damp}}}{\Lambda_0}\frac{2+2\alpha}{2+\alpha} F_{\mathrm{m}}{}^{(2+\alpha)/(1+\alpha)} + \frac{\chi_0 k^2}{\Lambda_0} F_{\mathrm{m}}{}^2 \ . \tag{A.6}$$

In both cases of the strong damping and weak damping, $F(0)$ is bounded as Eq.(A3) or Eq.(A.5).

Equation.(18) is replaced as

$$\int_{F(\hat{L})}^{F(0)} \frac{\mathrm{d}F}{\sqrt{H(F(0)) - H(F)}} = \hat{L} \ . \tag{A.7}$$

The integral in the LHS of Eq.(A.7) does not vanish. This is because $F(0)$ is finite as is given by Eqs.(A.3) and (A.5), and the range of integral $F(0)-F(\hat{L})$ is finite and positive as is given by Eq.(A.2). From this consideration, it is clear that there is a lower bound for $\hat{L}$

$$\hat{L} > \hat{L}_{\mathrm{c}} \,, \tag{A.8}$$

that allows a non-trivial solution $F(\zeta)$. This result is qualitatively the same as that in the limit of strong damping in §3.2.

References

1) B.B. Kadomtsev, *Plasma Turbulence* (Academic Press, New York, 1965)

2) A. Yoshizawa: Phys. Fluids **27** (1984) 1377

3) A. Yoshizawa, S.-I. Itoh, K. Itoh: *Plasma and Fluid Turbulence* (IOP, England, 2002)

4) P.H. Diamond and T.S. Hahm: Phys. Plasmas **2** (1995) 3640

5) S.-I. Itoh and K. Itoh: Plasma Phys. Control. Fusion **43** (2000) 1055

6) H. Sugama, M. Okamoto, M. Wakatani: Plasma Phys. Contr. Nuclear Fusion Research (Proc. 14th Conf. W¸rzburg 1992) Vol.2, p.353

7) P.H. Diamond, V.B. Lebedev, D.E. Newman *et al.*: Phys. Plasmas **2** (1995) 3685

8) P.H. Diamond *et al.*: Phys. Rev. Lett. **78** (1997) 1472

9) V.B. Lebedev and P.H. Diamond: Phys. Plasmas **4** (1997) 1087

10) T. Kubota, S.-I. Itoh, M. Yagi, K. Itoh: J. Phys. Soc. Jpn. **67** (1998) 3100

11) T. Iwasaki, S. Toda, S.-I. Itoh, M. Yagi, K. Itoh: Nucl. Fusion **39** (1999) 2127

12) X. Garbet *et al.*: Nuclear Fusion **34** (1994) 936

13) R.D. Sydora *et al.*: Plasma Phys. Control. Fusion **38** (1996) A281

14) S. Parker *et al.*: Phys. Plasmas **3** (1996) 1959

15) Y. Kishimoto *et al.* Phys. Plasmas **3** (1996) 1289

16) W.W. Lee and R. Santoro: Phys. Plasmas **4** (1997) 169

17) Z. Lin, S. Ethier, T.S. Hahm *et al.*: Phys. Rev. Lett. **88** (2002) 195004

18) Z. Lin, T.S. Hahm, S. Ethier *et al.*, "*Size scaling of turbulent transport in tokamak plasmas*", Paper TH/1-1, (IAEA, Lyon, 2002).

19) T.S. Hahm: Plasma Phys. Control. Fusion **44** (2002) A87

20) E.J. Kim, P.H. Diamond, M. Malkov *et al.*, Nuclear Fusion **43** (2003) 961

21) Z. Lin and T.S. Hahm: Phys. Plasmas **11** (2004) 1099

22) T.S. Hahm, P.H. Diamond, Z. Lin, K. Itoh, and S.-I. Itoh: Plasma Phys. Contr. Fusion **46** (2004) A323

23) P. H. Diamond, *et al.*: "*Turbulence Propagation and the Assumption of Local Transport: Implications for Transport Barriers*" presented at 9th IAEA Technical Meeting on H-mode Physics and Transport Barriers (San Diego, California, USA, 2003) paper E1.

24) F. Zonca, R. B. White, Liu Chen: Phys. Plasmas **11** (2004) 2488

25) K. Itoh, S.-I. Itoh and A. Fukuyama: *Transport and structural formation in plasmas* (IOP, England, 1999)

26) S.-I. Itoh and N. Kawai, ed.: *Bifurcation Phenomena in Plasmas* (Kyushu Univ., 2002)

27) K. Itoh, S.-I. Itoh and A. Fukuyama: Phys. Rev. Lett. **69** (1992) 1050

28) A. Fukuyama, K. Itoh, S.-I. Itoh, M. Yagi and M. Azumi: Plasma Phys. Control. Fusion **37** (1995) 611

29) O. D. Gurcan, P. H. Diamond, T. S. Hahm, Z. Lin: Phys. Plasmas **12** (2005) 032303.

30) K.-L. Wong *et al.*: Phys. Lett. A **236** (1997) 339

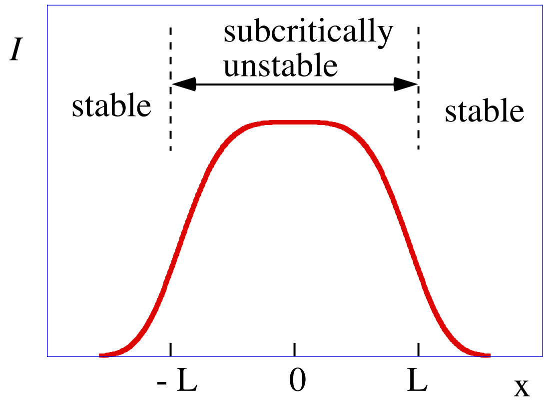

Fig.1 Schematic drawing of the turbulence spreading into stable regions.

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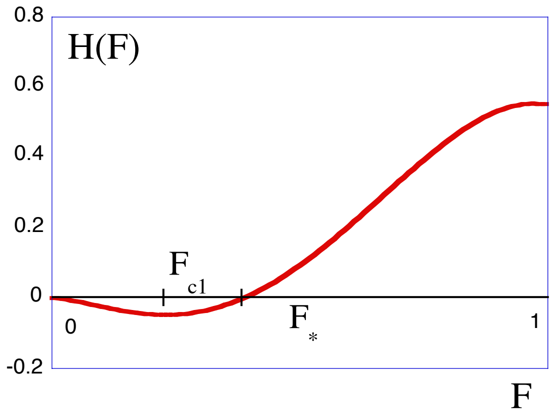

Fig.2 Function $H(F)$. ($F_{c1} = I_{c1}^{1+\alpha}$ denotes the threshold amplitude for nonlinear instability.)

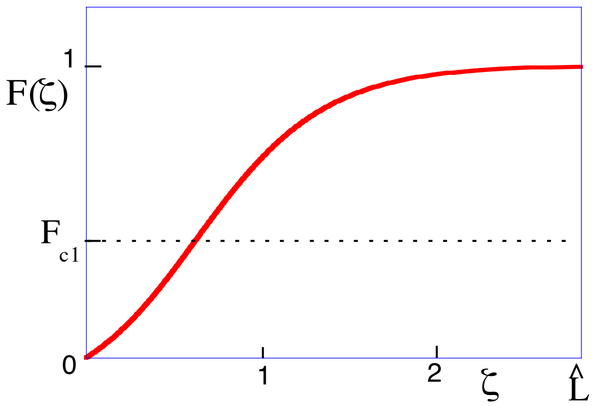

Fig.3 A solution of $F(\zeta)$ in the presence of turbulence spreading. (Parameters are $\hat{L} = 2.83$, $a = 0.4$ and $\alpha = 1$.) $F_{cl} = I_{cl}^{1+\alpha}$

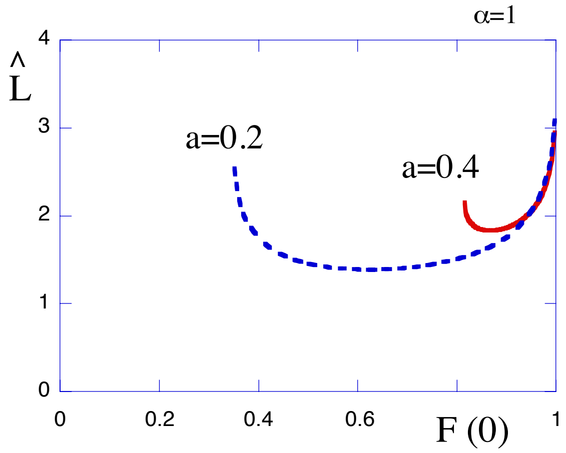

Fig.4 The relation of the system length $\hat{L}$ and $H(F(0))$ for the stronger damping case of $a = 0.4$ (solid line) and weaker one of $a = 0.2$ (dashed line). ($\alpha = 1$)

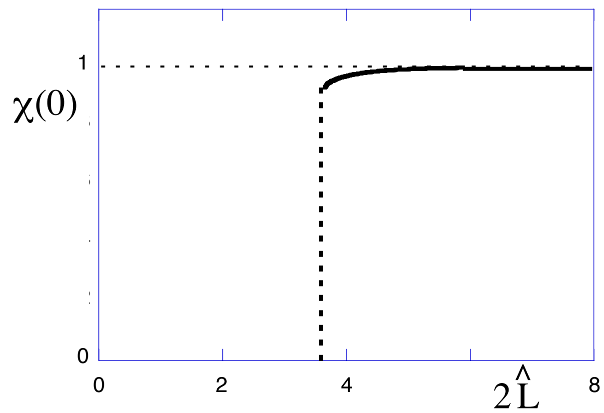

Fig.5 The turbulent conductivity at the center $\chi(0)$ is given as a function of the width of the unstable region $2\hat{L}$. (the case of $a = 0.4$ and $\alpha = 1$.)

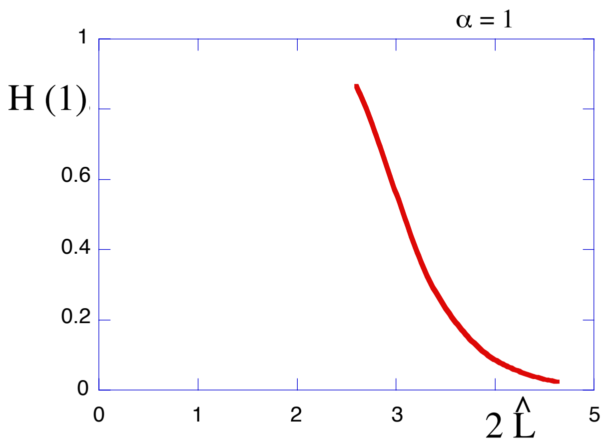

Fig.6 Critical value for the sustainment of subcritical excitation $H(1)$ as a function of the width of the unstable region $2\hat{L}$. Below this curve, the subcritical turbulence is not self-sustained. ($\alpha = 1$)