Role of phase space structures in collisionless drift wave turbulence and impact on transport modeling

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Abstract.

In fusion plasmas, several mechanisms such as heating, wave-particle interaction etc can drive deviations of distribution function from Maxwelian to form phase space structures. This article discusses the impact of phase space structures in drift wave turbulence on dynamics and transport modeling. The two cases of (i) coherent holes and (ii) incoherent granulations (clusters of correlated resonant particles with finite life time) are treated. Their dynamical impact on driving subcritical instability is analyzed by explicitly calculating the nonlinear growth rate. The role of zonal flows is also addressed. It is explained how phase space structures can be related to transient events and non-diffusive transport, issues in current confinement research.

1. Introduction

Transport modeling is important for understanding the confinement of fusion plasmas. Conventionally, transport flux is modelled using quasilinear theory[1, 2], which represents transport flux in terms of mean fields and transport coefficients. The result typically gives flux in the form of diffusion with diffusivity given by the mixing length formula, $D \sim \gamma/k_\perp^2$. The conventional approach works rather well, and it is claimed that its prediction agrees with experiments[3, 4]. However, there are several phenomena that cannot be addressed by the simple mean field model. Examples include, but not limited to, fast transient event, non-local transport, confinement scaling, etc[5, 6, 7]. In particular, validation study reports underprediction of transport levels by local simulation[8]. These together suggest that quasilinear transport modeling is rather simplified, and that transport modeling beyond quasilinear theory is important to improve the predictability of confinement property of future devices.

There are several attempts to construct transport modeling beyond quasilinear transport models. An example is the study of so-called non-local transport[6, 7, 9, 10, 11, 12]. Here, the effects such as turbulence spreading[13, 14, 15], avalanches[16, 17], etc are addressed. In addition, transport by strongly resonant turbulence is another important issue. Many of the instabilities in fusion plasmas, such as drift waves, collisionless trapped electron modes (CTEM), collisionless trapped ion modes (CTIM), etc are characterized by 1D resonance between particles and modes. As a result, turbulence can have a Kubo number larger than the order of unity[18, 19, 20]. Here the Kubo number is defined as $K \sim \tilde{v}\tau_c/\Delta_c$, where $\tilde{v}$ is the typical strength of turbulent velocity field, τ_c is the correlation time of turbulence (seen by resonant particles), and Δ_c is the correlation length of turbulence. When $K \gtrsim 1$, particle trajectory is deformed from unperturbed orbit, so that quasilinear theory is not applicable. Moreover, when turbulence is characterized by $K \gtrsim 1$ in collisionless plasmas, formation of phase space structures, such as BGK vortex[21], holes[22, 23, 24], clumps[25], granulations[26, 27, 28, 29], etc, can form. See Ref. [30] for illustration of different type of phase space structures, such as coherent holes and incoherent granulation eddys. Indeed, these phase space structures are very efficient in tapping free energy, since they can drive anomalous transport even when waves cannot. So it is an important issue to clarify how phase space structures impact dynamics of turbulence and transport processes[31, 32, 33, 34, 30].

The formation of phase space structures in collisionless plasmas are well known. For example, in unmagnetized plasmas, it is argued that granulations, a cluster of resonant particles correlated via resonance, can form[31, 32, 33]. Once formed, they can exert dynamical friction and drive anomalous resistivity in plasmas. This was confirmed by numerical studies[35, 30]. The simplified models were applied for the chirping of EP modes[25] in fusion plasmas. More recently, Refs.[36, 37] report the impact of phase space structures in driving subcritical instability[38] and abrupt excitation of EGAM.

Phase space structures can also form in magnetized plasmas, or in the family of drift wave turbulence[39, 27, 28]. Once formed, they release free energy stored in the gradient. As demonstrated for trapped ion turbulence[28, 29, 34], trapped ion granulations can give rise to anomalous loss of ion heat and particles. Transport caused by trapped ion granulations cannot be described by quasilinear theory. Rather, transport flux is formulated as Lenard-Balescu flux with dynamical friction, exerted by electrons in the case of trapped ion granulations. Recent studies show that trapped ion granulations couple to flows, such as zonal flows[29] and toroidal flows[40]. The coupling of phase space structures and flows is not only for trapped ion turbulence; coherent drift hole[23] also couples to flows[24]. However, in drift wave turbulence, incoherent granulations are likely to be present. The coupling of drift wave granulations and zonal flows can be useful to experimentally identify drift wave granulations in basic experiments with cylindrical geometry, as discussed later in the paper. Here, the coupling to flows and intermittent excitation of eddys[41] can be used as a trigger to conditionally average the data from Laser-Induced-Fluorescence (LIF) data for the parallel dynamics.

In this paper, we describe the basic feature of phase space structures in drift wave turbulence and its relation to current issues in confinement physics. Here we present a result based on additional analysis, which includes the derivation of the growth rate of granulations in drift wave turbulence from the two point analysis. In particular, we emphasize the dynamical impact of the presence of phase space structures, to drive subcritical instabilities and zonal flows. The connection to current issues in confinement studies are also addressed.

The remaining of the paper is organized as follows. In section 2, we explain models and discuss the applicability and limitations of quasilinear models in the system. In section 3, coherent drift holes (ion hole, more specifically) are introduced and their role of driving subcritical instability is discussed. In section 4, we turn to the case of stochastic granulations and analyze their dynamics by using the 2 point analysis. Here we describe the basic features of ion granulations, including their basic scales and sharp correlation within the resonance broadening scales. Analogy to discreteness effect in thermal equilibrium plasmas is explained. We also discuss how granulations can extract free energy subcritically and interact with zonal flows. Implication on transport modeling is discussed in section 5. Section 6 is summary and discussion. Here we discuss relevant tests by numerical and physical experiments on the ideas presented in the paper.

2. Model and a case study: from linear and quasilinear analysis to phase space structures

In this paper, we consider a cylindrical plasma, with magnetic field in z direction. The direction of inhomogeneity (density gradient) is in the x direction. The correspondence between $z \leftrightarrow \|$ and $(x, y) \leftrightarrow (r, \theta)$ is understood throughout. For simplicity, we treat ions by the drift kinetic equation

$$\partial_t f_i + v_\| \nabla_\| f_i + \frac{c}{B}\hat{z} \times \nabla\phi \cdot \nabla f_i + \frac{e}{m_i} E_\| \frac{\partial f_i}{\partial v_\|} = 0. \tag{1}$$

Electrons are assumed to be close to Boltzmann response

$$\frac{n_e}{n_0} = (1 - i\delta_e)\frac{e\phi}{T_e} \tag{2}$$

where δ_e is the phase shift. Ions and electrons satisfy a quasilineutrality condition via the gyro-kinetic Poisson equation

$$\frac{n_e}{n_0} = \int dv_\| f_i + \rho_s^2 \nabla_\perp^2 \frac{e\phi}{T_e}. \tag{3}$$

Linear analysis gives the dispersion relation of waves (drift wave). Calculating the linear response of Eq.(1) to potential perturbation and integrating over the velocity space, we have $\tilde{n}_{i,e}/n_0 = \chi_{i,e}(e\tilde{\phi}/T_e)$, where the susceptibilities $\chi_{i,e}(\mathbf{k}, \omega)$ are given by

$$\chi_i(\mathbf{k}, \omega) = \frac{\omega_{*e}}{\omega} - i\frac{\pi}{|k_\||}\left[\omega_{*e} + \frac{T_e}{T_i}\omega\right]\langle f_i \rangle|_{\omega/k_\|}, \tag{4}$$

$$\chi_i(\mathbf{k}, \omega) = 1 - i\delta_e. \tag{5}$$

The quasi-neutrality condition is given by

$$\chi(\mathbf{k}, \omega)\frac{e\phi}{T_e} = 0 \tag{6}$$

where

$$\chi(\mathbf{k}, \omega) = 1 + k_\perp^2 \rho_s^2 - \frac{\omega_{*e}}{\omega} + i\mathrm{Im}\chi_e - i\mathrm{Im}\chi_i. \tag{7}$$

Here we note that the susceptibility plays the role of the dispersion function $D(\mathbf{k}, \omega)$ in the conventional mode analysis. The linear dispersion relation is obtained from $\mathrm{Re}\chi(\mathbf{k}, \omega) = 0$, which gives

$$\omega_{\mathbf{k}} = \frac{\omega_{*e}}{1 + k_\perp^2 \rho_s^2}. \tag{8}$$

Thus we have electron drift waves. The growth rate is given by

$$\gamma = -\frac{\mathrm{Im}\chi(\mathbf{k}, \omega)}{\partial\chi(\mathbf{k}, \omega)/\partial\omega|_{\omega_{\mathbf{k}}}} = \frac{\omega_{\mathbf{k}}^2}{\omega_{*e}} \left(\delta_e - \frac{\pi}{|k_\parallel|} \left[\omega_{*e} + \frac{T_e}{T_i}\omega_{\mathbf{k}} \right] \langle f_i \rangle|_{\omega/k_\parallel} \right). \tag{9}$$

Drift waves are destabilized due to the phase shift and stabilized by the ion Landau damping.

Using the model, we can discuss the applicability and limitation of quasilinear approach for transport analysis. Calculating the response δf to potential perturbation from the kinetic equation, we have

$$\delta f_{\mathbf{k}\omega} = \frac{i}{\omega - k_\parallel v_\parallel + i k_\perp^2 D_\perp} \left(-\tilde{v}_{E\times B,\mathbf{k}\omega} \partial_x \langle f \rangle + i k_\parallel \phi_{\mathbf{k}\omega} \frac{e}{m_i} \partial_{v_\parallel} \langle f \rangle \right). \tag{10}$$

Here we have retained $E \times B$ nonlinearity in the response function, which is renormalized into the diffusivity term[33]. While the diffusivity can be a function of velocity through the response function, the diffusivity has the dimension of $[length^2/time]$, which corresponds to the diffusivity in the real space. Substituting this into the flux, we have

$$\langle \tilde{v}_x \delta f \rangle = -D_\perp \partial_x \langle f_i \rangle. \tag{11}$$

Here we have neglected the term $\propto k_\parallel k_\theta$ for simplicity. The diffusivity is

$$D_\perp = \sum_{\mathbf{k}\omega} |\tilde{v}_{E\times B}|_{k\omega}^2 \mathrm{Re} \frac{i}{\omega - k_\parallel v_\parallel + i k_\perp^2 D_\perp}. \tag{12}$$

We can solve for $D_\perp$ for a model spectrum

$$|\tilde{v}_{E\times B}|_{\mathbf{k}\omega}^2 = \frac{|\tilde{v}_{E\times B}|_0^2 \delta(\omega - \omega_{\mathbf{k}})}{(1 + (k_\parallel - k_{\parallel 0})^2/\Delta k_\parallel^2)(1 + (k_\perp - k_{\perp 0})^2/\Delta k_\perp^2)}. \tag{13}$$

By performing the k integral via contour integration, we obtain

$$\frac{k_\perp^2 D_\perp}{\tau_{ac}^{-1}} = \frac{1}{2} \left(\sqrt{1 + \frac{4k_\perp^2 |\tilde{v}_{E\times B}|_0^2}{\tau_{ac}^{-2}}} - 1 \right) \tag{14}$$

where

$$\tau_{ac}^{-1} = \Delta k_\parallel \left| \frac{\partial \omega}{\partial k_\parallel} - \frac{\omega}{k_\parallel} \right| + \left| \frac{\partial \omega}{\partial \mathbf{k}_\perp} \cdot \Delta \mathbf{k}_\perp \right| \tag{15}$$

is the auto-correlation time of the spectrum seen by resonant particles. The result is plotted in Fig.1. In this figure, the diffusivity has distinct feature depending on the strength of the perturbation amplitude. First, when the fluctuation amplitude is small so that $2k_\perp v_{E\times B}/\tau_{ac}^{-1} < 1$, we have $D_\perp \sim \tau_{ac} v_{E\times B}^2$ to the first non-trivial order. The diffusivity scales with the fluctuation amplitude to the second power. This is so-called weak turbulence regime. In this case, the dispersal rate of wave packet is larger than the circulation rate of particles, $\tau_{ac}^{-1} > 2k_\perp v_{E\times B}$. Then the wave packet changes its pattern before trajectory of resonant particles deforms. Unperturbed orbit is a good approximation and the quasilinear theory is applicable. This regime also corresponds to the Kubo number $K < 1$. In order to obtain quantitative estimate for transport coefficient, the mixing length estimate, $v_{E\times B} \sim v_* \sim 1/(k_\perp \tau_{ac})$, is often employed. Note that this corresponds to the case of $K \sim 1$. Thus, conventionally, quantitative estimates of transport are obtained from quasilinear theory by pushing the result to the applicability boundary.

On the other hand, when the amplitude is large so that $2k_\perp v_{E\times B}/\tau_{ac}^{-1} > 1$, we have $D_\perp \sim v_{E\times B}/k_\perp$. This is so-called strong turbulence regime. We also note that the similar scaling is directly obtained from Eq.(12) for strongly resonant particles $\omega - k_\parallel v_\parallel \to 0$. In this regime, the $E \times B$ circulation rate exceeds the packet dispersal rate. Particle trajectory is strongly deformed (the Kubo number $K > 1$), and quasilinear analysis with the linear response is not applicable. Moreover, resonant particles form $E \times B$ eddys in this regime. This process leads to the formation of phase space structures, such as holes or granulations, fluctuations other than normal modes or waves, which impact dynamics and transport. Now we turn to the analysis of these components.

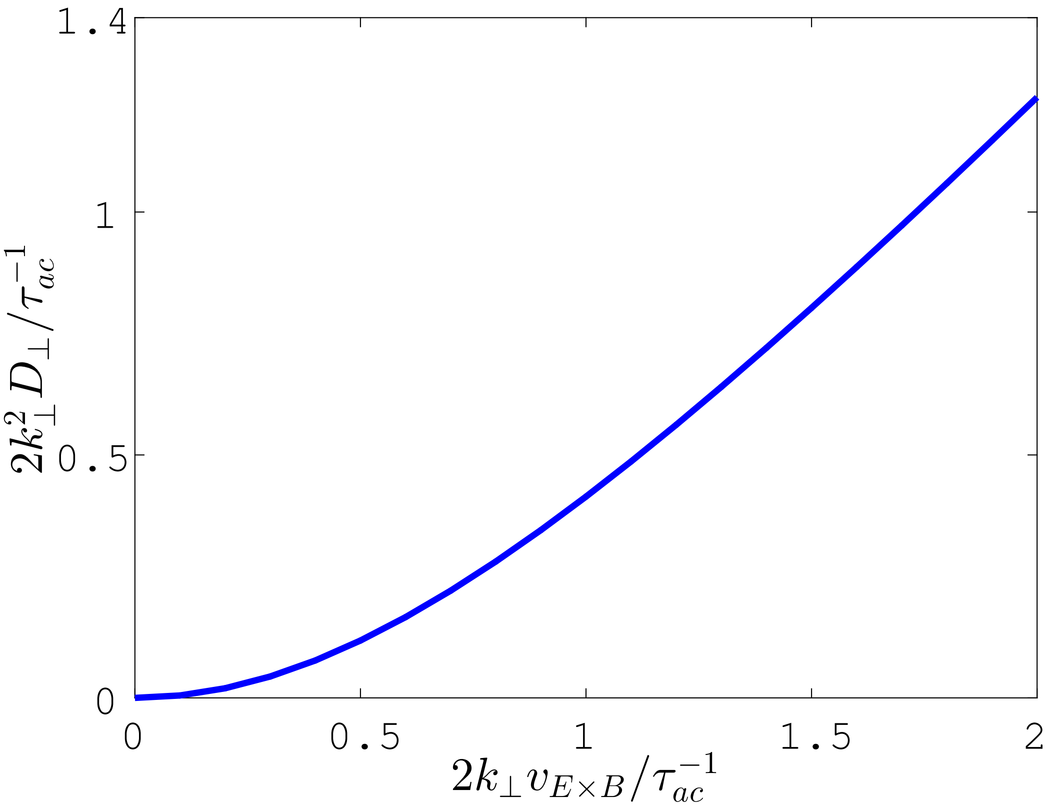


Figure 1. The spatial diffusion coefficient, calculated for a model spectrum, as a function of fluctuation amplitude. For weak amplitude, the diffusivity scales with the $v_{E\times B}^2$. This is so-called weak turbulence regime. On the other hand, for larger amplitude, diffusivity scales as $\propto v_{E\times B}$. This is so-called strong turbulence regime. In this regime, resonant particle trajectory is deformed and the formation of phase space structures (granulations) is likely. The mixing length estimate corresponds to the boundary between the two regions.

3. Holes in drift wave turbulence

When drift wave turbulence is strongly resonant and characterized by a large Kubo number, phase space structures can form. Here, to elucidate the role of these phase space structures in turbulence dynamics, we consider the simplest example of a coherent drift hole, a localized hole structure in phase space density. More specifically, we consider an ion hole. Mathematically, a hole is obtained as a stationary solution of the Vlasov-Poisson systems, with the condition that the entropy of the system is maximized[22, 23]. Their scale is typically given by the turbulence correlation scale for the spatial direction and by the resonance broadening scale in the velocity direction. A schematic picture of a hole localized at $(x_0, v_{\parallel 0})$ is given in Fig.2.

Importantly, once formed, holes can tap free energy and drive subcritical instability. Simply put, this is due to the conservation of the total phase space density. As depicted in Fig.2, we consider a hole at $(x_0, v_{\parallel 0})$ and displace it in phase space. Since phase space density is conserved along trajectory, $df/dt = 0$, the relative depth of the structure grows. Here the projection in x direction is shown. The growth rate of this process can be calculated by following the procedure in [23] as:

$$\gamma_H \sim \tau_c^{-1} \mathrm{Im}\chi_e \mathrm{Im}\chi_i \tag{16}$$

Here dependence of the susceptibility on the wave number and the frequency has been dropped to simplify the notation. The growth process has several distinctive features. First, the hole growth is nonlinear and the growth rate is amplitude dependent $\gamma_H \propto \tau_c^{-1}$. Thus, nonlinear, explosive growth can result. Secondly, dissipation both in ions and electrons act as a trigger for the instability. This is in contrast to the linear growth of drift waves (Eq.(9)) $\gamma_L \propto -\mathrm{Im}\chi_e + \mathrm{Im}\chi_i$. Here the sign convention is $\mathrm{Im}\chi_e < 0$ and $\mathrm{Im}\chi_i < 0$. In the linear growth, only electron dissipation acts as a trigger while ion dissipation (ion Landau damping) introduces stabilization effect. The hole growth is triggered when $\mathrm{Im}\chi_e \mathrm{Im}\chi_i > 0$. A similar feature is obtained for 1D plasmas[42, 38]. The condition is likely satisfied in the presence of free energy in the distribution function. Thus the hole can grow even when linear waves cannot, and drive subcritical instability. Finally, in contrast with hole growth in a 1D plasma, the growth is a synergy of the relaxation in gradients both in $v_\parallel$ and x direction. We elaborate the growth process and

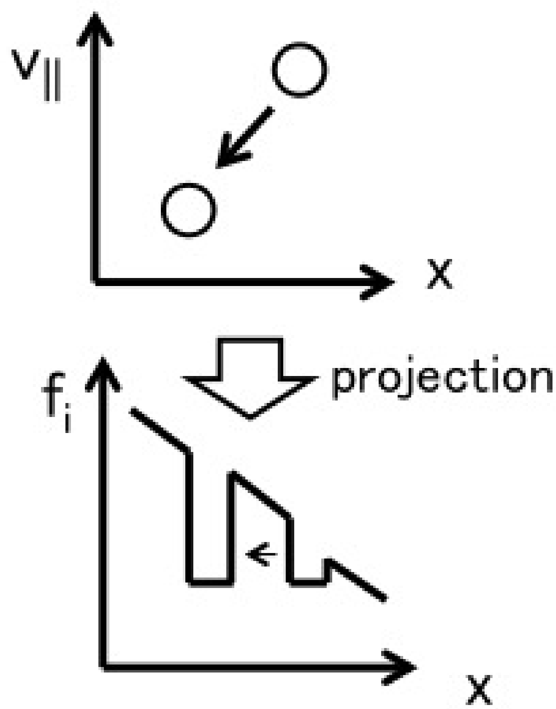


Figure 2. Growth of a single structure. A (hole) structure, initially located at $(x_0, v_{\|0})$, is displaced in phase space and grows, as indicated by the black arrow in the upper picture. The projection into x direction is also plotted (the bottom figure, with the displacement represented by the black arrow). In the displacement, the total phase space density is conserved, since df/dt. In the presence of the background gradients, this leads to the growth of the initial structure.

its significance by considering each case separately.

We start with the description of the displacement in $v_\|$ direction (Fig.3). Here, we consider a linearly stable location in x, so that the set of macroscopic parameters gives $\gamma(\nabla n, \nabla T, ...) < 0$. At this location, we suppose that a process such as heating leads to a quasilinear flattening in $v_\|$ direction and initiates a seed structure. Note that this is quite analogous to the formation of clump-hole pair in Berk-Breizman model[25] or subcritical growth due to the localized structure in 1D Vlasov plasmas[38]. Once created, the structure can be displaced in the $v_\|$ direction. This process can happen

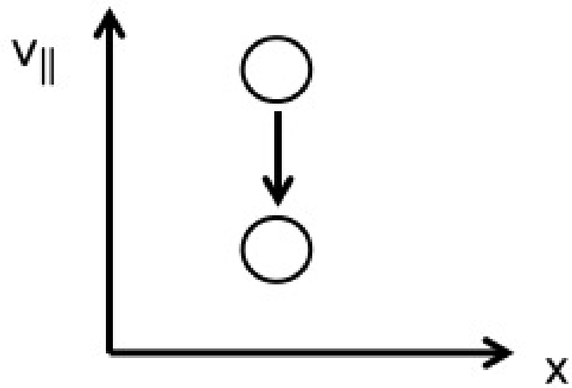


Figure 3. A case of the growth due to the displacement in the $v_\parallel$ direction. This is analogous to the growth in 1D Vlasov plasmas. In order to get displaced, ion structure needs to exchange momentum with electrons. This is allowed in the presence of the electron dissipation δ_e.

when ion holes exchange momentum with electrons irreversibly. This can happen with dissipative coupling to electrons. The resultant growth rate is given[22] by

$$\gamma_H \sim -\tau_c^{-1}\delta_e c_s^2 \left.\frac{\partial f_i}{\partial v_\parallel}\right|_0 . \tag{17}$$

An important feature of this process is that the growth can happen even before the global parameters change to push $\gamma_L > 0$. Then the growth of the ion structure can result without waiting the change of global profiles. Thus the nonlinear, subcritical growth can be a key to understand fast, transient phenomena.

Next, we consider the growth due to the displacement in the spatial direction (Fig.4). During the displacement, quasi-neutrality must be satisfied. For ion structures, this is possible when they can spatially scatter electrons. The finite dissipation δ_e allows ion structures to scatter electrons irreversibly and access to free energy in spatial direction. This leads to the growth of structure, with the growth rate given[23] by

$$\gamma_H \sim -\tau_c^{-1}\delta_e k_y \rho_s c_s \left.\frac{\partial f_i}{\partial x}\right|_0 . \tag{18}$$

Importantly, transport associated with these processes are non-diffusive. This point is

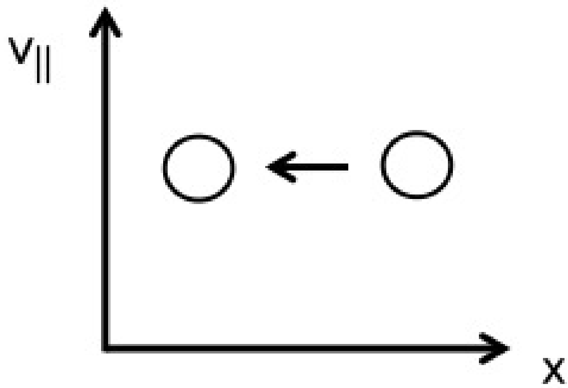


Figure 4. A case of the growth due to the displace,ent in x direction. While accessing to free energy, ion structures must satisfy quasi-neutrality. Then ion structures can access free energy by scattering electrons irreversibly. This is possible in the presence of the finite phase shift in electron δ_e.

further discussed in the context of the impact on transport modeling (in the section 5).

4. Granulations in drift wave turbulence

The above section dealt with coherent holes with effectively infinite life time. In contrast, incoherent granulations, clusters of resonant particles with finite life time, can form in drift wave turbulence. In this section, we formulate their dynamics and discuss basic properties(i.e. scales, correlation, etc), impact on subcritical growth and transport, and a mechanism to drive zonal flows.

4.1. 2 point evolution of phase space density correlation

The dynamics of granulations can be analyzed by calculating the correlation of 2 different points in phase space[33]. As illustrated in Fig. 5, two initially correlated particles tend to move together. Eventually they lose correlation due both to the difference in streaming speed along the magnetic field and to relative $E \times B$ scattering by turbulent field. The dynamics is formulated by calculating the evolution of two point phase space density correlation, or *phasestrophy*[33]:

$$\partial_t \langle \delta f(1) \delta f(2) \rangle + T(1,2) = \mathcal{P}(1,2). \tag{19}$$

Here $\langle ... \rangle$ is the average over the center of mass spatial coordinate, $\mathbf{x}_+ = (\mathbf{x}_1 + \mathbf{x}_2)/2$. Relative coordinates are defined as $v_{\parallel -} \equiv v_{\parallel 1} - v_{\parallel 2}$ and $\mathbf{x}_- = \mathbf{x}_1 - \mathbf{x}_2$. Here

$$T(1,2) = v_{\parallel -} \nabla_{\parallel -} \langle \delta f(1) \delta f(2) \rangle \\ + \nabla_1 \cdot \langle \mathbf{v}_{E \times B}(1) \delta f(1) \delta f(2) \rangle + \nabla_2 \cdot \langle \mathbf{v}_{E \times B}(2) \delta f(1) \delta f(2) \rangle \tag{20}$$

is the triplet term and describes the lifetime of the correlation.

$$\mathcal{P}(1,2) = -\langle \delta f(2) v_{E \times B, x}(1) \rangle \frac{\partial \langle f \rangle}{\partial x} - \frac{q}{m_i} \langle \delta f(2) \tilde{E}_\parallel(1) \rangle \frac{\partial \langle f(1) \rangle}{\partial v_\parallel} + (1 \leftrightarrow 2) \tag{21}$$

is related to the release of free energy and acts as a source. $(1 \leftrightarrow 2)$ denotes the term with the arguments exchanged. We note that the production term takes a typical form of the production of fluctuation energy, namely the product of the fluxes and the driving gradients.

To obtain a more specific form of $T(1,2)$, a closure calculation is necessary. Following the analysis in literature[33, 20], the triplet term is approximated as

$$T(1,2) \cong v_{\parallel -} \nabla_{\parallel -} \langle \delta f(1) \delta f(2) \rangle - \nabla_- \cdot \mathbf{D}_- \cdot \nabla_- \langle \delta f(1) \delta f(2) \rangle. \tag{22}$$

Here $\mathbf{D}_- \cong (\mathbf{k}_0 \cdot \mathbf{x}_-)^2 \mathbf{D}_\perp$, $\mathbf{D}_\perp$ is the diffusivity tensor, and $\mathbf{k}_0$ is the spectrally averaged wave number. For simplicity, we further assume the diffusion tensor is isotropic, namely $\mathbf{D}_\perp = D_\perp(\hat{x}\hat{x} + \hat{y}\hat{y})$ where $D_\perp$ is a scalar diffusivity in the perpendicular direction. Then the triplet term is given by

$$T(1,2) \cong v_{\parallel -} \nabla_{\parallel -} \langle \delta f(1) \delta f(2) \rangle - \left(\frac{\partial}{\partial x_-} D_\perp (k_{0x}^2 x_-^2 + k_{0y}^2 y_-^2 + k_{0z}^2 z_-^2) \frac{\partial}{\partial x_-} \right. \\ \left. + \frac{\partial}{\partial y_-} D_\perp (k_{0x}^2 x_-^2 + k_{0y}^2 y_-^2 + k_{0z}^2 z_-^2) \frac{\partial}{\partial y_-} \right) \langle \delta f(1) \delta f(2) \rangle. \tag{23}$$

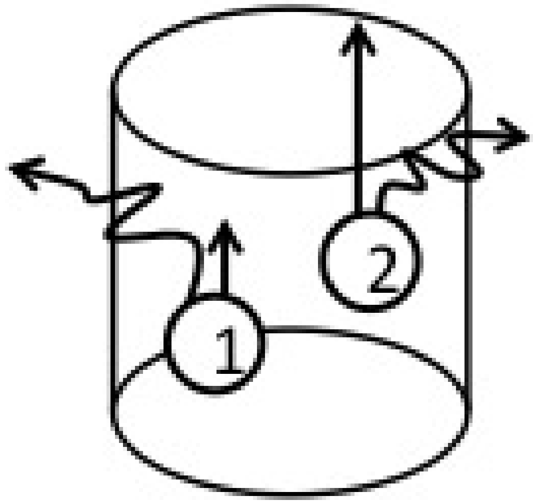


Figure 5. A cartoon of granulations. In drift wave turbulence, resonant particles stream along the field line and are scattered by turbulent fields. Initially separated two resonant particles tend to move together. They lose correlation due to the difference in the streaming speed and due to the relative scattering.

Effective lifetime can be estimated by calculating the evolution of moment of $F(1,2)$, whose dynamics is given by

$$\partial_t F(1,2) + v_{\parallel -}\nabla_{\parallel -}F(1,2) - \left(\frac{\partial}{\partial x_-}D_\perp(k_{0x}^2 x_-^2 + k_{0y}^2 y_-^2 + k_{0z}^2 z_-^2)\frac{\partial}{\partial x_-} + \frac{\partial}{\partial y_-}D_\perp(k_{0x}^2 x_-^2 + k_{0y}^2 y_-^2 + k_{0z}^2 z_-^2)\frac{\partial}{\partial y_-}\right)F(1,2) = 0. \quad (24)$$

The moment is defined by $\langle\langle ...\rangle\rangle \equiv \int d\mathbf{x}_- dv_{\parallel -}(...)F/\int d\mathbf{x}_- dv_{\parallel -}F$. Relevant moments are

$$\partial_t\langle\langle x_-^2\rangle\rangle - 6k_{0x}^2 D_\perp\langle\langle x_-^2\rangle\rangle - 2k_{0y}^2 D_\perp\langle\langle y_-^2\rangle\rangle - 2k_{0z}^2 D_\perp\langle\langle z_-^2\rangle\rangle = 0, \quad (25)$$

$$\partial_t\langle\langle y_-^2\rangle\rangle - 2k_{0x}^2 D_\perp\langle\langle x_-^2\rangle\rangle - 6k_{0y}^2 D_\perp\langle\langle y_-^2\rangle\rangle - 2k_{0z}^2 D_\perp\langle\langle z_-^2\rangle\rangle = 0, \quad (26)$$

$$\partial_t\langle\langle z_-^2\rangle\rangle - \langle\langle v_{\parallel -}z_-\rangle\rangle = 0, \quad (27)$$

$$\partial_t\langle\langle v_{\parallel -}z_-\rangle\rangle\rangle - \langle\langle v_{\parallel -}^2\rangle\rangle = 0, \quad (28)$$

$$\partial_t\langle\langle v_{\parallel -}^2\rangle\rangle = 0. \quad (29)$$

The set of equations is solved with the initial separation $(\mathbf{x}_-, v_{\parallel -})$. When turbulence is isotropic $k_{x0} \sim k_{y0} \sim k_{\perp 0}$, time asymptotic solution is

$$k_{\perp 0}^2 \langle\langle r_-^2 \rangle\rangle \to \left(k_{\perp 0}^2 x_-^2 + k_{\perp 0}^2 y_-^2 + \frac{k_{z0}^2}{2}(z_-^2 + z_- v_{\parallel -}\tau_c + v_{\parallel -}^2 \tau_c^2) \right) e^{t/\tau_c} \tag{30}$$

where $\tau_c^{-1} \equiv 8k_{\perp 0}^2 D_\perp$ is the correlation time. Thus the initially separated 2 points exponentially diverge due to turbulent mixing. When the relative separation becomes comparable to the original correlation length of the turbulent field $k_{\perp 0}^{-1}$, they lose correlation. This gives an effective life time of the correlation as

$$\tau_{cl} = \tau_c \ln \left(k_{\perp 0}^2 x_-^2 + k_{\perp 0}^2 y_-^2 + \frac{k_{z0}^2}{2}(z_-^2 + z_- v_{\parallel -}\tau_c + v_{\parallel -}^2 \tau_c^2) \right)^{-1}. \tag{31}$$

The expression is only valid for the argument of the logarithm smaller than 1. We also note that the typical scale in $v_\parallel$ direction is given by $\Delta v_\parallel \sim 1/(k_{\parallel 0}\tau_c)$. This is the resonance broadening scale. Thus, particles within the resonance broadening scales are correlated together.

4.2. Sharp correlation in phase space

An important feature of granulations is that they have a strong correlation in phase space. At the steady state, the phasestrophy is given by:

$$\langle \delta f(1) \delta f(2) \rangle \cong \tau_{cl}(x_-, y_-, z_-, v_{\parallel -}) \mathcal{P}(v_+). \tag{32}$$

The correlation diverges logarithmically for $\mathbf{x}_- \to 0$, $v_{\parallel -} \to 0$. To elaborate this point further, we have plotted the behavior in Fig.6 for $\mathbf{x}_- = 0$. Note that $\mathcal{P}$ is finite as $1 \to 2$, so its value is normalized to 1 in this figure. The blue curve indicates the total correlation. As the relative separation becomes smaller than the resonance broadening scale, the correlation tends to diverge logarithmically. This cannot be captured by the quasilinear result, as indicated by the red curve. This singular behavior is an indication of the formation of granulations, and necessitates to have $\delta f = \delta f^c + \widetilde{\delta f}$. Here δf^c is the coherent response to the potential fluctuation ϕ, which is included in quasilinear theory. $\widetilde{\delta f}$ is the incoherent part of the fluctuation. It is this $\widetilde{\delta f}$ that is responsible for the diverging behavior of the correlation. Note that the sharp correlation is analogous to that induced by the particle discreteness in plasmas close to thermal equilibrium. Here the effect of the discreteness is extended to the case of turbulent plasmas. Finally, we note that this feature has been tested numerically for simplified 1D Vlasov plasmas.[43, 35] Given the development of computational power and numerical schemes, it may be interesting to revisit this type of study, especially in the context of fusion turbulence, by using modern gyrokinetic simulations. Here, solving gyrokinetic equation gives fluctuation data of distribution function, which can be then used to calculated the correlation. Analysis for reduce trapped ion turbulence (described by the bounce kinetic equation) has been initiated and will be reported in future.

Granulations can be viewed as phase space eddys. In principle, granulations in drift wave turbulence is in 4D phase space, $(\mathbf{x}, v_{-})$. We can slice it into typical 2D planes, as shown in Fig. 7. In the direction parallel to the magnetic field, the dynamics is similar to that of 1D Vlasov plasmas. Phase space eddys in $(z, v_{\parallel})$ plane are analogous to those in 1D Vlasov plasmas. In addition, we also have the direction perpendicular to the magnetic field. In this direction, we can view granulations as $E \times B$ eddys, albeit it arises from resonant particles.

4.3. Dynamics: subcritical growth

Once formed, granulations impact dynamics of collisionless plasmas. Here, as an example, we discuss subcritical instability by granulations.

The growth of incoherent fluctuation can be extracted from the phasestrophy evolution. By setting $\delta f = \delta f^c + \widetilde{\delta f}$, we have $\langle \widetilde{\delta f}(1)\widetilde{\delta f}(2)\rangle = \langle \delta f(1)\delta f(2)\rangle -$

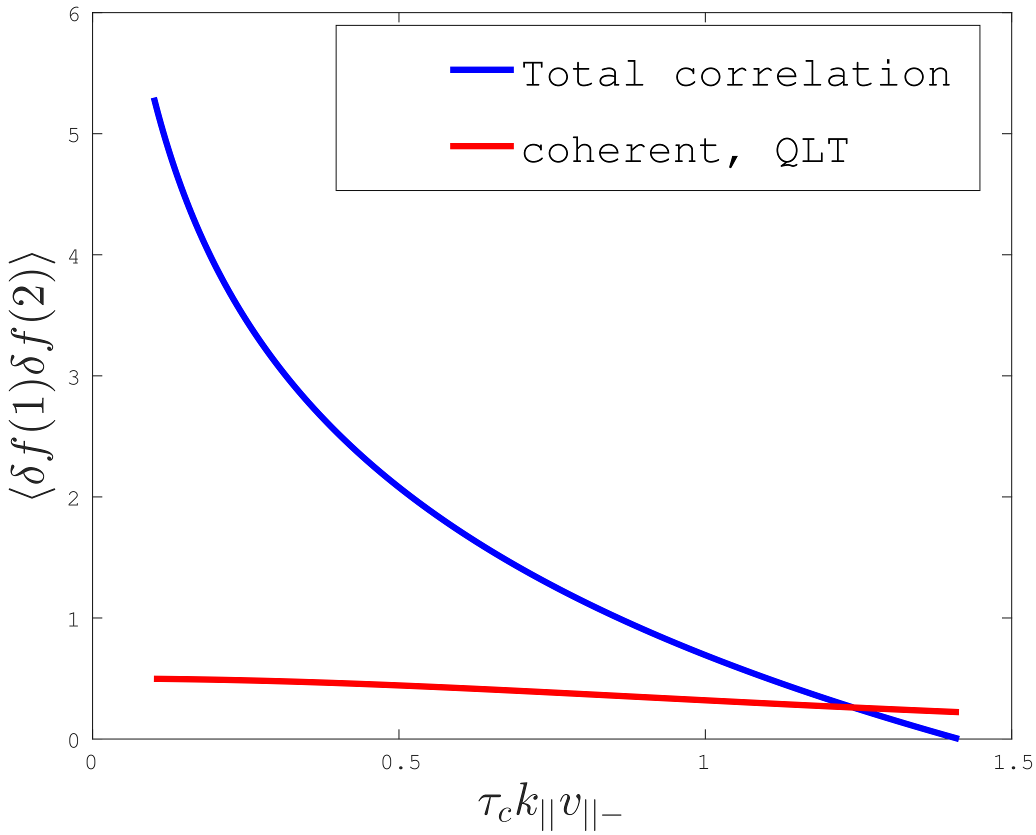


Figure 6. Phase space density correlation, plotted with $\mathbf{x}_- = 0$ and $\Delta v_\parallel$ as a variable. The correlation tends to diverges within the broadening scale, $1/(k_{\parallel 0}\tau_c)$. This behavior cannot be reproduced by the quasilinear theory (red curve). The diverging behavior is due to discreteness effect $\widetilde{\delta f}$, which is induced by granulations

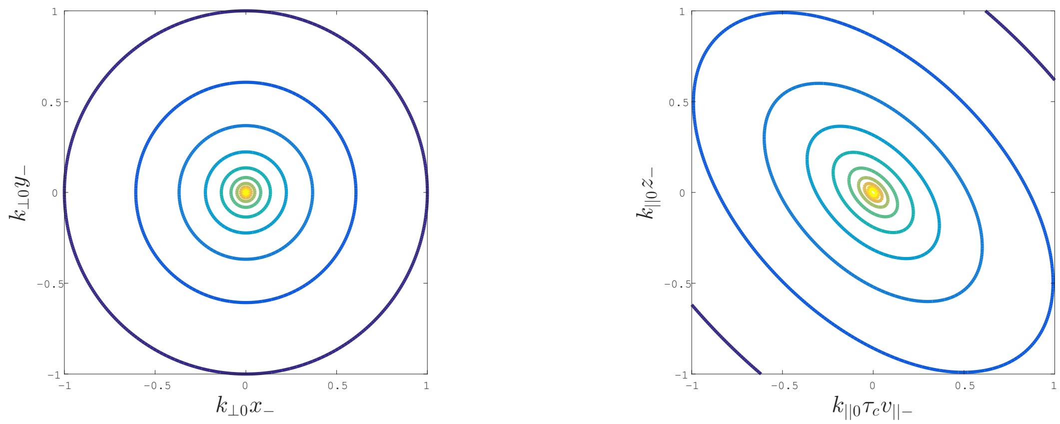
$\langle \delta f^c(1)\delta f^c(2)\rangle - \langle \delta f^c(1)\widetilde{\delta f}(2)\rangle - \langle \widetilde{\delta f}(1)\delta f^c(2)\rangle$. Since[44]

$$\partial_t \langle \delta f^c(1)\delta f^c(2)\rangle + \tau_c^{-1}\langle \delta f^c(1)\delta f^c(2)\rangle_+ = \mathcal{D}, \tag{33}$$

$$\partial_t \langle \delta f^c(1)\widetilde{\delta f}(2)\rangle + \tau_c^{-1}\langle \delta f^c(1)\widetilde{\delta f}(2)\rangle_+ = \mathcal{F}, \tag{34}$$

where $\mathcal{D} = -2\langle \tilde{v}_x \delta f^c\rangle \partial_x \langle f\rangle - 2(e/m_i)\langle \tilde{E}_\parallel \delta f^c\rangle \partial_{v_\parallel}\langle f\rangle$ and $\mathcal{F} = -\langle \tilde{v}_x \widetilde{\delta f}\rangle \partial_x \langle f\rangle - 2(e/m_i)\langle \tilde{E}_\parallel \widetilde{\delta f}\rangle \partial_{v_\parallel}\langle f\rangle$ is the production by the diffusive flux and the dynamical friction respectively, we have

$$\langle \widetilde{\delta f}(1)\widetilde{\delta f}(2)\rangle = \left(\frac{\tau_{cl}}{\gamma_g \tau_{cl} + 1} - \frac{\tau_c}{\gamma_g \tau_c + 1}\right)\mathcal{P}. \tag{35}$$



(a) (b)

Figure 7. Phase space contour of $\langle \delta f(1)\delta f(2)\rangle$ in (x_-, y_-) and $(v_{\|-}, z_-)$. In (a), the correlation is evaluated for the slice of $z_- = 0$ and $v_{\|-} = 0$. In this plane, granulations appear as $E \times B$ eddys, with the typical scale corresponds to the perpendicular correlation length of turbulence. In (b), the correlation is evaluated for the slice of $x_- = y_- = 0$. The correlation is strong for $z_- < k_{\|0}^{-1}$ and $v_{\|-} < 1/(k_{\|0}\tau_c)$. In this plane, granulations are analogous to that of 1D Vlasov plasmas.

Since we are interested in the contribution from small scales, we typically have $\tau_{cl} \gtrsim \tau_c$. With this ordering, we have

$$\langle \widetilde{\delta f}(1)\widetilde{\delta f}(2)\rangle \cong \frac{(1-\tau_c/\tau_{cl})}{(1+\gamma_g\tau_c)^2}\tau_c\mathcal{P}. \tag{36}$$

By taking the velocity integral and solving for the growth rate γ_g, we finally have

$$\gamma_g = \tau_c^{-1}(R-1). \tag{37}$$

Here R is related to drive and is given by

$$R \equiv \sqrt{\frac{\int dv_{\|-}(1-\tau_c/\tau_{cl})\tau_c \int dv_{\|+}\mathcal{P}(v_{\|+})}{\langle(\widetilde{\delta n}/n_0)^2\rangle}}. \tag{38}$$

In order to make further progress, we need a specific form for the production term. Substituting the coherent response δf^c and using the Poisson equation $\chi(e\phi/T_e) = \int dv_\| \widetilde{\delta f}$, we have

$$\mathcal{P}(v_{\|+}) \cong 2\sum_{\mathbf{k}\omega}\frac{|k_\||}{\pi}\frac{\mathrm{Im}\chi_i\mathrm{Im}\chi_e}{|\chi|^2}\int dv_1\langle\widetilde{\delta f}(1)\widetilde{\delta f}(v_{\|+})\rangle_{\mathbf{k}\omega}. \tag{39}$$

Substituting this for the factor R and performing the velocity integral, we finally have

$$R \sim \sqrt{C_0 \frac{\mathrm{Im}\chi_i \mathrm{Im}\chi_e}{|\chi|^2}}. \tag{40}$$

Here $C_0 = (2\sqrt{2}/\pi) \int_0^{1/\sqrt{e}} dx (1 - 1/\ln(x^{-2})) \cong 0.294$. Note that the growth of granulations have several feature in common with that of coherent drift holes; amplitude dependent, electron and ion dissipation as a trigger, etc. Thus, the growth of granulations is also tied to the growth of phase space structures, as depicted in Fig.2.

The difference is that the growth rate $\gamma_g \sim R\tau_c^{-1} - \tau_c^{-1}$ is subtracted by τ_c^{-1}. This factor reflects the fact that granulations have finite life time and typically decorrelate after one circulation time, as fluid eddys. Then the growth rate of granulations is reduced compared to that of holes. This indicates that the growth of granulations may be harder to reproduce than that of coherent holes. Indeed, a recent numerical study for ion acoustic turbulence[30] reports that while initially launched holes subcritically drive fluctuation, a large level of fluctuation amplitude is required in the absence of initial holes to recover subcritical growth. Thus care must be taken when one tries to numerically recover the subcritical growth driven by phase space structures in drift wave turbulence.

5. Coupling to zonal flows

Here we demonstrate that phase space structures can couple to zonal flows. The case of coherent drift hole is treated in Ref.[24]. Here we demonstrate this for incoherent granulations. Simply put, while moving through plasmas, granulations can scatter polarization charge and accelerate flows. This in turn acts as a drag on granulations to conserve momentum. The coupled dynamics can be formulated by investigating the spatial structure of wake produced by granulations via Cerenkov emission. The wake has a spatial structure, with a small scale oscillation and a slow envelope variation. The variation can be characterized by the wave number k_x and the envelope variation ∂_x. The difference in the two scales sets the necessary phase for vorticity flux $\langle \tilde{v}_x \nabla_\perp^2 \phi \rangle \propto k_y k_x \partial_x |\phi|^2$. Note that the latter quantity is Reynolds forcing. The coupled dynamical equation can be obtained by integrating the two point equation over velocity and by retaining the flow coupling through the polarization charge scattering in the production term. This gives

$$\partial_t I = \tau_c^{-1} \left(\frac{\int dv_{\parallel -} \tau_{cl}(v_{\parallel -}) \int dv_{\parallel +} \mathcal{P}_{i,e}}{I} - 1 \right) I$$
$$+ \tau_c^{-1} \int dv_{\parallel -} \tau_{cl}(v_{\parallel -}) \int dv_{\parallel +} \mathcal{P}_{i,pol}. \tag{41}$$

Here $I = \langle (\delta n / n_0)^2 \rangle$ is the turbulence intensity. The first term in the right hand side is related to subcritical growth discussed above. The term $\mathcal{P}_{i,e}$ is due to ion-electron drag. This term is effectively the factor R for granulation growth. The second term is coupling to flows. Explicitly, this term is given by

$$\tau_c^{-1} \int dv_{\parallel -} \tau_{cl}(v_{\parallel -}) \int dv_{\parallel +} \mathcal{P}_{i,pol}$$
$$= 2 \sum_{\mathbf{k}\omega} \left(\omega_{*e} + \frac{T_e}{T_i} \omega \right) \frac{\langle f_i \rangle|_{\omega/k_\parallel}}{\tau_c |k_{\parallel 0}|} \rho_s^2 k_x \partial_x \left| \frac{e\phi_{\mathbf{k}\omega}}{T_e} \right|^2 \tag{42}$$
$$\sim \Delta v_{res} \langle f_i \rangle|_{res} v_{*i} \frac{\partial_x \langle \tilde{v}_x \tilde{v}_y \rangle}{v_{thi}^2}. \tag{43}$$

Thus, collecting all the results, we have the coupled dynamical equation for granulations and flows:

$$\partial_t I = \gamma_{NL} I + \Delta v_{res} \langle f_i \rangle|_{res} v_{*i} \frac{\partial_x \langle \tilde{v}_x \tilde{v}_y \rangle}{v_{thi}^2}, \tag{44}$$

$$\partial_t \langle v_y \rangle = -\partial_x \langle \tilde{v}_x \tilde{v}_y \rangle - \nu \langle v_y \rangle. \tag{45}$$

An important implication is that granulations can carry momentum in poloidal direction. The momentum of granulation is described by $v_{thi}^2 I / (v_{*i} \Delta v_{res} \langle f_i \rangle_{res})$. Note that in the case of drift waves, this quantity corresponds to $k_y N_{\mathbf{k}}$ where $N_{\mathbf{k}}$ is action density. As a result, granulations can exchange momentum with flows to accelerate them. The force exerted by granulations in steady state can be estimated from the dynamical equation, as

$$-\partial_x \langle \tilde{v}_x \tilde{v}_y \rangle \sim -c_s^2 k_{\perp 0} \delta_e c_s^2 I \sim - \left| \frac{c_s^2 I}{\rho_s / \delta_e} \right| . \tag{46}$$

Here the force is in the ion diamagnetic direction. This is plausible, since we have considered ion granulations, which rotate in v_{*i} direction. Ion granulations exchange the momentum to excite flows, which then accelerate flows in v_{*i}. We note that Reynolds force exerted by electron hole is in the electron diamagnetic direction.

The acceleration of zonal flows by phase space structures is an example of flow drive by fluctuation other than drift waves. Oft-invoked mechanisms of zonal flow acceleration is that by drift waves, formulated using wave kinetic approach[45]. However, in turbulent plasmas, fluctuation is supported not only by waves. A wider class of fluctuation, so-called non-mode, such as quasi-modes, eddys, blobs, etc, can be also excited in turbulent plasmas. Indeed, these non-modal fluctuation can accelerate zonal flows. Several experiments report the excitation of zonal flows by blobs[46, 47], eddys, etc. More recently, basic experiments reveal the drive of zonal flows by intermittently excited eddys via advanced data analysis[41]. Helicon plasmas were used in this experiments, thus the eddys are fluid $E \times B$ eddys. In contrast, in collisionless plasmas, such as ECH plasmas or Q-machines, granulations can be excited and accelerate zonal flows. A similar analysis to that in ref[41] can be repeated to characterize the acceleration by $E \times B$ eddys in the perpendicular direction. In addition, we can go one step further to analyze Laser-Induced-Fluorescence (LIF) data for $(z, v_\parallel)$ dynamics. Coupling to zonal flows can be a key to experimentally identify phase space structures (ExB eddys, along with $(z, v_\parallel)$ eddys).

6. Implication on transport modeling

In this section, we discuss the role of phase space structure in the context of transport modeling (Table 1). We start by briefly summarizing the conventional approach for transport analysis using the quasilinear theory. In this approach, the flux $\langle \tilde{v}_r \delta f \rangle$ is calculated by substituting the coherent response $\delta f^c = R\tilde{\phi}$, where R is the response function. Typically this yields the quasilinear diffusive flux, $\langle \tilde{v}_r \delta f \rangle = -D(|\tilde{\phi}|^2)\langle f \rangle'$. We note that in the presence of multiple gradients, the flux can contain non-diffusive components, such as the convective part of particle flux or the residual stress in parallel momentum flux. At this points, transport coefficients are amplitude dependent. The amplitude grows due to linear instability and saturates due to nonlinear interaction. A steady state is achieved when the linear growth balances against the nonlinear damping. In this state, transport coefficients are typically given by the mixing length estimate, $D \sim \gamma / k_\perp^2$, where the growth rate and the wave number are typically estimated from those of the most unstable mode. We note that fluctuation can couple to zonal flows, in which case the level of transport is reduced.

In contrast, in the presence of phase space structures, the nature of transport is different. The difference arises both from the form of the flux and from the fluctuation evolution. First, the flux contains a non-diffusive component. Note that some non-diffusive components can be modeled within the quasilinear framework; however, the origin of the non-diffusive component by phase space structure is physically different. In the case of phase space structure, this arises due to dynamical friction. Let us consider ion granulations as an example (Fig.4). Ion granulations can be viewed as a correlated macro-particle, which can spatially scatter electrons. This induces ion granulation drag

	Conventional	Strongly Resonant Turbulence
Macro/Flux	Quasi-linear flux $-D\langle f\rangle'$ $\rightarrow$ mixing length estimate $D \sim \frac{\gamma_L}{k_\perp^2}$	Lenard-Balescu $-D\langle f\rangle' + F\langle f\rangle$ Dynamical friction Novel path for non-diffusive flux
Micro/ Fluctuation	Mode, waves $\epsilon(\omega, \mathbf{k}) = 0$ linear growth v.s. NL damp. Saturation with mixing length level	Non-mode, structures in phase space (holes/grans.) Nonlinear, subcritical growth $\rightarrow$ Bursty, explosive growth
ZF coupling	Yes, Predator-Prey Amp. dressed by ZF	Yes, Predator-Prey Acceleration by phase space structures

Table 1. Comparison between transport models.

on electrons, with the spatial flux given by:

$$\langle \tilde{v}_x \delta f_i \rangle = \sum_{\mathbf{k}\omega} \frac{k_y \rho_s c_s \delta_e}{|\chi(\mathbf{k},\omega)|^2} \int dv_2 \langle \widetilde{\delta f}_i \widetilde{\delta f}_i(v_2) \rangle_{\mathbf{k}\omega}. \tag{47}$$

Note that the flux is that of ions, while it explicitly depends on the electron phase shift. This is required for ions to couple with electrons. The origin of this flux is quite analogous to Fokker-Planck drag in Lenard-Balescu flux, which arises as a result of inter-species drag. In this case, ion granulations can release free energy by coupling to electrons. As a result, the flux becomes $J = -D\partial_x\langle f\rangle \rightarrow -D\partial_x\langle f\rangle + F\langle f\rangle$ where F is the Fokker-Planck drag. *Thus once formed, granulations can drive non-diffusive transport in plasmas.* Indeed, by accounting for phase space structures, we can have a novel path for non-diffusive transport phenomena. For example, granulations can give rise to intrinsic torque to drive toroidal flows, as demonstrated for trapped ion turbulence[40].

The fluctuation evolution is distinctive in the presence of phase space structures. The evolution is now nonlinear, and the growth rate is amplitude dependent as demonstrated above. By writing the growth rate as $\gamma_0 I^2$, where γ_0 is an amplitude independent growth rate, the fluctuation evolution is

$$\partial_t I = \gamma_0 I^2. \tag{48}$$

This can be integrated to give

$$I = \frac{1}{I_0^{-1} - \gamma_0 t} \tag{49}$$

where I_0 is the initial amplitude at $t = 0$. Thus, a finite time singularity results. An important question is then how it saturates and what is the typical amplitude level set by these phase space structures. We note that if we invoke the mixing length estimate for its amplitude[28], the flux, Reynolds stress, etc by granulations are in the order of those by linearly unstable drift waves. Thus the contribution from granulations cannot be dismissed a priori. However, quantitative evaluation of the absolute amplitude in the presence of phase space structures remains elusive, and we need more investigation. In particular, we need quantitative evaluation of the amplitudes of both unstable waves and granulations, which allows comparison between flux by unstable waves and that by granulations. This is left for future investigation.

7. Summary and Discussion

In this paper, we discussed basic features of phase space structures in drift wave turbulence, such as drift holes and granulations. We have also explained their dynamical impact to drive subcritical instability, zonal flows, and non-diffusive transport, etc.

Relevant results are summarized (Table 2) in the following:

(i) Phase space structures in drift wave turbulence, drift holes and/or granulations, have typical scales $\Delta x \sim \Delta y \sim 1/k_{\perp 0}$, $\Delta z \sim 1/k_{\parallel 0}$, $\Delta v_{\parallel} \sim 1/(k_{\parallel 0}\tau_c)$. Here $k_{\perp 0}^{-1} \sim$ a few ρ_s, $k_{\parallel 0} \sim L_z$ (L_z is the typical length of plasma column), and $\Delta v_{\parallel}$ is the resonance broadening scale. They can be viewed as phase space eddys in $(\mathbf{x}, v_{\parallel})$ with these characteristic scales, as shown in Fig.7. Within these scales, two point correlation of phase space density fluctuation may diverge (Fig.6). Physically, this is due to the discreteness effect induced in turbulent plasmas.

(ii) Coherent drift holes drive subcritical instabilities. Physically, the growth is tied to the growth of phase space structures (Fig.2). The growth can result as a synergy of the release of free energy both in the velocity gradient and the spatial gradient. Nonlinear, explosive growth can result (Eq.(49)).

(iii) The growth rate of granulations is derived. The growth rate is smaller than that of a single structure, as manifested by the factor $R-1$. This reflects the fact that granulations have finite life time and they tend to decorrelate after 1 circulation time. This feature implies that the subcritical growth may not occur with the granulations and that the growth of a single hole may be easier to reproduce in numerical experiments.

(iv) Granulations can drive zonal flows. The coupled dynamics is formulated in Eqs.(44) and (45). Granulations carry poloidal momentum and exchange it to drive zonal flows. The resultant Reynolds force at steady state is given by Eq.(46). Taken together with the case of coherent drift holes, phase space structures in drift wave turbulence can drive zonal flows. This process is an example of zonal flow excitation by non-modal fluctuation, such as blobs, eddys. Here, phase space structures can

	Prediction	Relevant feature
Scale	$\Delta x \sim \Delta y \lesssim k_{0\perp}^{-1}$ $\Delta v_\parallel \lesssim 1/(\tau_c k_{0\parallel})$	Need resolve turbulence scale, resonance broadening. Sharp correlation within the scales.
Subcritical growth	Hole: $\gamma_H \sim \tau_c^{-1} \mathrm{Im}\chi_i \mathrm{Im}\chi_e$ Gran.: $\gamma_g \cong \tau_c^{-1}(R-1)$	Hole growth easier to reproduce. May introduce fast response, non-diffusive transport.
Coupling to zonal flow	Hole: Yes (Ref.[24]) Gran.: Yes (Eq.(44)) Resultant forcing $-\partial_x \langle \tilde{v}_x \tilde{v}_y \rangle \sim \mathrm{sgn}(v_*) \left\| \dfrac{c_s^2 I}{\rho_s/\delta_e} \right\|$	Excitation of ZF by phase space structure (non-mode). Eddys, ZF growth, LIF in collisionless plasmas?

Table 2. Summary of relevant feature and implication.

also drive zonal flows. This coupling can be a key to identify granulations in experiments.

(v) Some of these features suggest that phase space structures can be important to approach current issues in confinement. For example, the subcritical instability driven by holes can describe a fast response of fluctuations. As depicted in Fig.3, holes can drive fluctuation at linearly stable location, before global profiles change. Moreover, granulations drive non-diffusive transport, as discussed in Fig.4, Eq.(47) and Table 1. These features make phase space structures as an attractive candidate behind transient response and non-local transport problems.

Given these theoretical results, a relevant next step in the study of holes and granulations for fusion turbulence may be verification and validation tests by numerical and physical experiments. From numerical perspective, we note that a careful treatment of noise is necessary. As we demonstrated in [30], PIC noise can lead to an artificial growth of fluctuation. Thus, a careful treatment of noise is essential, e.g. by using continuum codes. These seem possible for current gyrokinetic simulation. There are some hints for the formation of granulations and appearance of their effects; a former study[18] reports that Kubo number is larger than unity for CTEM turbulence and non-diffusive transport results. However, in this study, it was argued that this is due to the convective pinch in CTEM turbulence. We argue that further analysis can be performed, such as calculating the phase space density correlation or analyzing the sensitivity of the transport flux to ion dissipation, etc. For this direction, we are currently investigating the effect of granulations in trapped ion turbulence, using a reduced version of the full f gyrokinetic code based on bounce kinetic equation. A detailed feature will be reported in future.

Validation in physical experiments seems challenging. There are some attempts to measure phase space density vortex, by measuring the pulse in potential perturbation or by measuring the dependence of frequency on the fluctuation amplitude, $\omega \sim \omega_b = \sqrt{e\phi/m}$. However, identification of localized structures (eddys, holes, vortex, etc,) in phase space requires further experiments, which require measurement of fluctuating distribution function by LIF etc. A possible approach is to seek for the coupling to zonal flows and use the coupling as a trigger to conditionally average LIF data. Indeed, acceleration of zonal flows by fluid $E \times B$ eddys in collisional helicon plasmas was measured in recent study. We may repeat similar experiments and data analysis for ECH plasmas/Q machines. Here, in addition to measuring $E \times B$ eddys in the perpendicular direction and how they accelerate flows, we can analyze LIF data for $(z, v_\parallel)$ dynamics.

One example where phase space structures can play an important role in fusion plasmas would be the problem of transport in the edge-core coupling region (so-called 'short fall' problem[8]). In these region, local simulation or quasilinear transport models underpredict the level of transport as compared to experiments. A missing element here could be so-called non-local transport effect, including fast transient pulse (such as avalanches etc). For this, holes/granulations can nucleate resonant particles and drive ballistic transport (radially) by dynamical friction. Of course, additional quantitative analysis is required to support this, which we will pursue in future.

In conclusion, granulations and holes (phase space structures) are important to understand turbulence dynamics and transport in collisionless plasma. Phase space structures can be a key to understand unresolved confinement property, such as fast transient event, transport scaling, etc. Given the state of affairs, the topic merits further investigation. Relevant future direction includes investigation of the relative importance of granulations and unstable waves, and in particular, tests by modern gyrokinetic simulation and basic experiments. These will be addressed in future.

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