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Symmetry Breaking by Parallel Flow Shear:

Dynamics of **Intrinsic Axial Flow** in a Linear Device, and Its Implication for **Intrinsic Rotation** in Tokamaks

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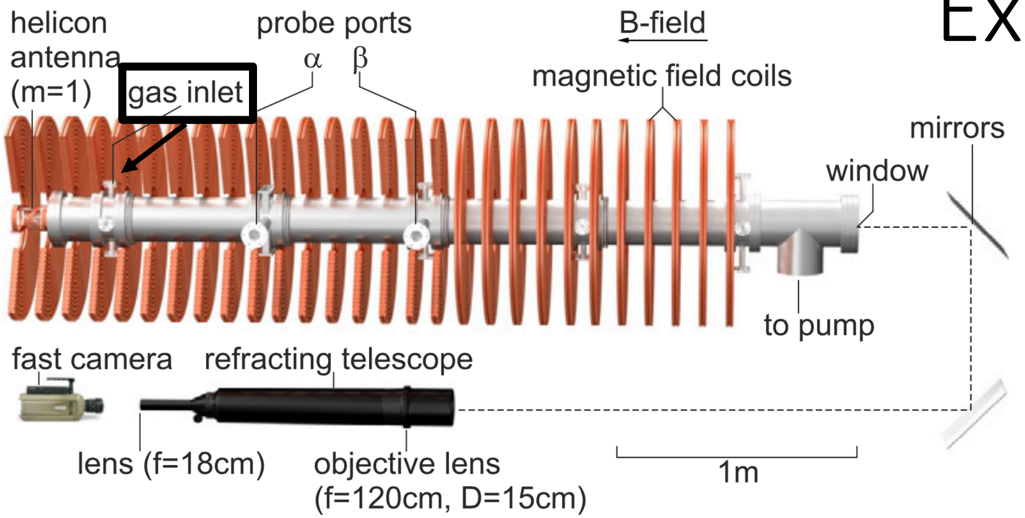
Outline

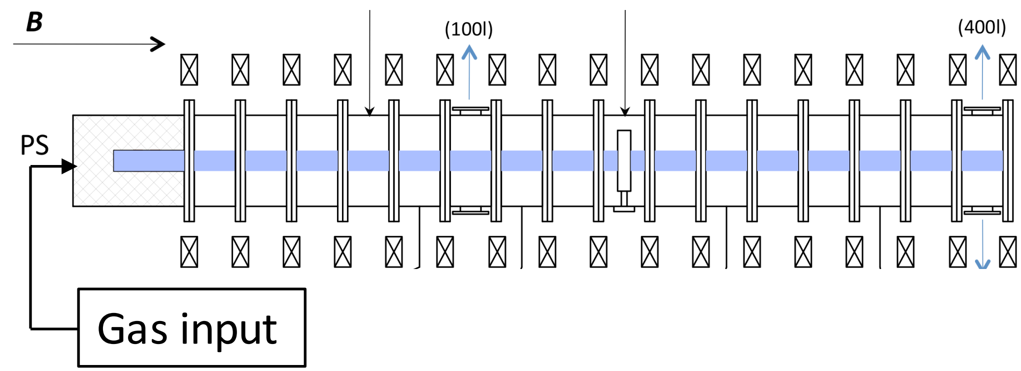
- Motivation
  - Linear Device Configuration and Results: CSDX & PANTA
  - Problem: Origin of axial flow?
- Dynamical Symmetry Breaking Mechanism
  - Introduction to residual stress, problem with applicability of conventional wisdom
  - Dynamical symmetry breaking
  - Residual stress
  - Compare to standard mechanism (negative viscosity vs intrinsic torque)
- Negative Viscosity Phenomena
  - Modulational instability for a test flow shear <-> χ_ϕ vs $|\chi^{Res}|$
  - What stops $\langle v_z \rangle'$ growth? – Parallel Shear Flow Instability (PSFI)
- Flow structure
  - Turbulent pipe flow model: ΔP_z, neutral boundary layer
  - Flow profile: Including PSFI effect → $\langle v_z \rangle'$ structure
- Significance for tokamaks
- Conclusion

Summary

- Existence of intrinsic flow suggested by experiments
- Dynamical symmetry breaking: A test flow shear seeds symmetry breaking → $\delta\langle v_z\rangle'$ feeds back on itself
- $\delta\Pi^{Res} \sim |\chi^{Res}|\delta\langle v_z\rangle'$ induces a negative viscosity increment $|\chi^{Res}|$, total viscosity $\chi_\phi^{\text{tot}} = \chi_\phi - |\chi^{Res}|$
- $\langle v_z\rangle'$ stays under Parallel Shear Flow Instability threshold, total viscosity stays positive
- A synergy of standard residual stress driven by $\nabla T, \nabla \mathrm{P}, \mathrm{etc.}$ and $\delta\Pi^{Res}$ induced negative viscosity increment is implied for tokamaks.

Experiments: configuration





- CSDX[1]
- Gas input from side

  →No axial momentum input

- PANTA[2]
- Gas input from the source end

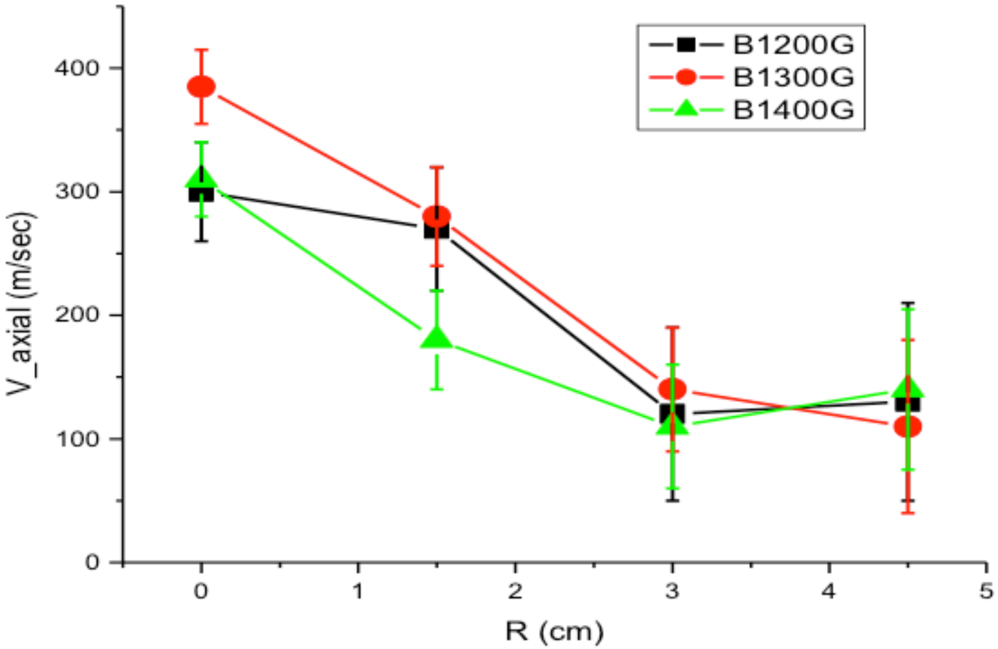
  →Axial momentum input

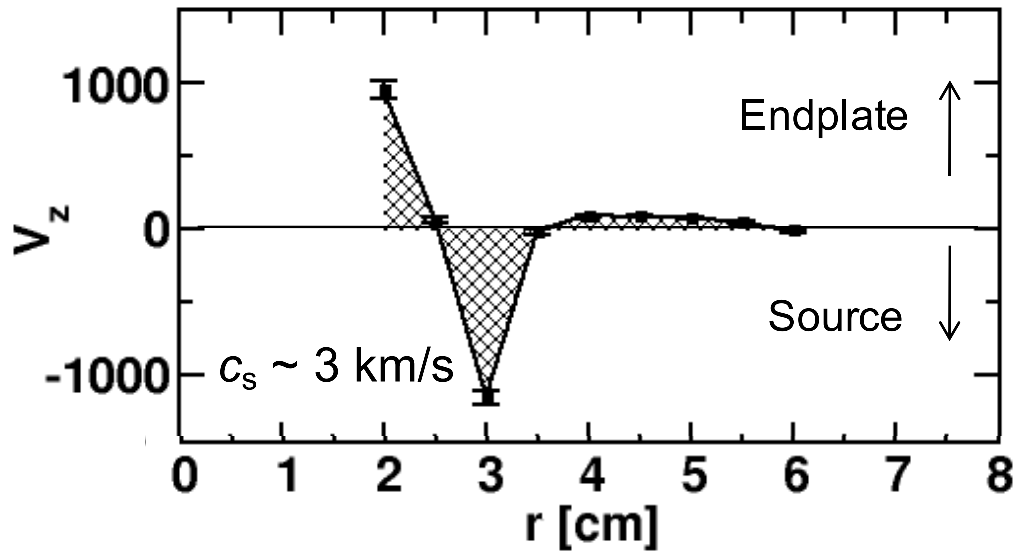
Parameters	CSDX Typical Values	PANTA Typical Values
Source	< 5 kW	3 kW
Pressure	0.1~1.3 Pa	0.1 Pa, 0.4 Pa
B field	Up to 2400 G	900 G
T_e	3~6 eV	3 eV
n_e	0.5~2 x 10^{19} m^3	1 x 10^{19} m^3
T_i	0.3~0.8 eV	---

* [1] S. Thakur et al, Plasma Sources Sci. Technol. **23** (2014) 044006;

[2] T. Kobayashi (2014, Jun). *Parallel flow structure formation by turbulent momentum transport in linear magnetized plasma*. Asia Pacific Transport Working Group, Kyushu University, Japan.

Profile of Axial Flow





- CSDX[1]
- No axial momentum input
- Flow profile steepens as B increases

- PANTA[2]
- Flow reversal
- Input flow: insufficient momentum input

Intrinsic axial flow! ➡ Origin, physics?

* [1] L. Cui (2015, Nov). *Spontaneous Profile Self-Organization in a Simple Realization of Drift-Wave Turbulence*. Invited talk, Session BI3, 57th APS-DPP meeting, Savannah, Georgia.

[2] T. Kobayashi (2014, Jun). *Parallel flow structure formation by turbulent momentum transport in linear magnetized plasma*. Asia Pacific Transport Working Group, Kyushu University, Japan.

Evidence of Intrinsic Flow

Device	Cause of Driven Flow	Evidence of Intrinsic Flow
CSDX	Neutral gas → Ionized & heated during helicon discharge → Ion pressure gradient in axial direction ΔP_z → Driven flow	Flow profile, density profile steepen as ***B*** increases: $\nabla N \uparrow$ → Drift Wave (DW) turbulence ↑ → Turbulent transport of axial momentum ↑ → Intrinsic flow interacts with driven flow → Total flow structure changes
PANTA	Gas thruput into the source end → External momentum source → Drives flow from source to endplate	Flow reversal → Intrinsic flow interacts with driven flow → Global net flow direction: Source → Endplate

Problem

- Axial flows
  - CSDX, PANTA both suggest the existence of intrinsic axial flows
- Questions:
  - (1) What generates the intrinsic flow?
  - (2) How does intrinsic flow interact with driven flow?
- Approach:
  - Intrinsic flow accelerated by $\Pi_{r\parallel}^{Res}$ → Needs symmetry breaking
  - Standard approach

    → Intrinsic torque, $-\partial_r \Pi_{r\parallel}^{Res}$ accelerates flow

    → Does not apply to straight ***B*** fields
  - Here: Dynamical Symmetry Breaking

    → Negative viscosity increment

Standard Approach

- Intrinsic flow is accelerated by the residual piece of the momentum flux:

$$\langle \tilde{v}_r \tilde{v}_\parallel \rangle = -\chi_\phi \frac{d\langle v_\parallel \rangle}{dr} + V_P \langle v_\parallel \rangle + \Pi_{r\parallel}^{Res}$$

  - Ignore momentum pinch V_P
- Correlated by ***B*** field structure, $k_\parallel = k_\theta \frac{x}{L_s}$, $L_s =$ magnetic shear length
- $\Pi_{r\parallel}^{Res} \sim \langle k_\theta k_\parallel \rangle = \sum_k k_\theta k_\parallel |\phi_k|^2 = k_\theta^2 \frac{\langle x \rangle}{L_s}$
  - x : distance from rational surface
- Needs symmetry breaking!

Symmetry Breaking

- Summary of conventional symmetry breaking mechanisms*:

Conventional mechanisms	Key physics
Electric field shear E_r'	Centroid shift → parallel acoustic wave asymmetry → mean $\langle k_\parallel \rangle$
Intensity gradient I'	Spectral dispersion from intensity gradient
Stress from polarization acceleration $\langle \tilde{E}_\parallel \nabla_\perp^2 \tilde{\phi} \rangle$	Guiding center stress from acceleration due to polarization charge
Stress from $\partial_r \langle \tilde{v}_r \tilde{v}_\perp \rangle$ → $\mathrm{B}_\theta \langle J_r \rangle$	$\boldsymbol{J} \times \boldsymbol{B}$ torque from polarization flux

- Do not apply to CSDX or PANTA ← Straight ***B*** fields
- → Dynamical symmetry breaking
  - Drift wave turbulence in presence of axial flow shear
  - $\delta \langle v_z \rangle'$ seeds symmetry breaking

* P.H. Diamond et al, Nucl. Fusion 53 (2013) 104019;
P.H. Diamond et al, Nucl. Fusion 49 (2009) 045002.

Model Equations

- Hasegawa-Wakatani + Axial flow:

$$\frac{D}{Dt}(1 - i\delta - \nabla_\perp^2)\phi + \frac{1}{L_\mathrm{n}}\frac{1}{r}\frac{\partial\phi}{\partial\theta} + \frac{\partial v_z}{\partial z} = 0,$$

$$\frac{D}{Dt}v_z - \langle v_z\rangle'\frac{1}{r}\frac{\partial\phi}{\partial\theta} = -(1 - i\delta)\frac{\partial\phi}{\partial z},$$

Acoustic coupling

- Acoustic coupling
  - Couple axial flow fluctuation to DW
  - Familiar: Convert parallel compression into zonal flow*
- $i\delta$: nonadiabatic electron response → Drift wave instability
- Dispersion relation: $1 + k_\perp^2\rho_\mathrm{s}^2 - i\delta - \frac{\omega_*}{\omega} + \frac{k_\theta k_z \rho_\mathrm{s} c_\mathrm{s}\langle v_z\rangle'}{\omega^2} - (1 - i\delta)\frac{k_z^2 c_\mathrm{s}^2}{\omega^2} = 0$

Drift wave

Symmetry breaking

Ion acoustic wave

* Wang et al, Plasma Phys. Control. Fusion **54** (2012) 095015

Dynamical Symmetry Breaking

- Growth rate ~ frequency shift:

$$\gamma_k \cong \frac{\nu_{\rm ei}}{k_z^2 v_{\rm The}^2}\frac{\omega_*(\omega_*-\omega_k)}{(1+k_\perp^2\rho_s^2)^2}$$

- Frequency:

$$\omega_k \cong \frac{\omega_*}{1+k_\perp^2\rho_{\rm s}^2} - \frac{k_\theta k_z \rho_{\rm s} c_{\rm s}\langle v_z\rangle'}{\omega_*}$$

- Frequency shift ~ Flow shear:

$$\gamma_k \cong \frac{\nu_{\rm ei}}{k_z^2 v_{\rm The}^2}\frac{\omega_*^2}{(1+k_\perp^2\rho_s^2)^2}\left(\frac{k_\perp^2\rho_s^2}{1+k_\perp^2\rho_s^2} + \frac{k_\theta k_z \rho_s c_s\langle v_z\rangle'}{\omega_*^2}\right)$$

- Spectral imbalance:

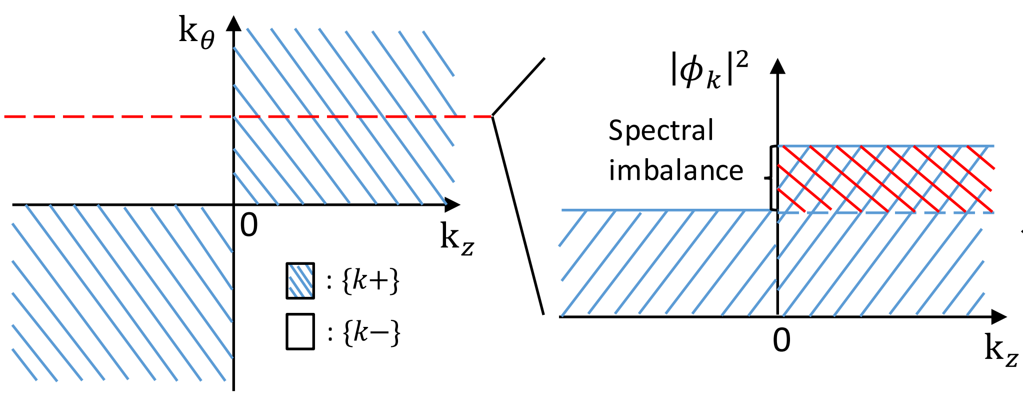
Infinitesimal test axial flow shear, e.g. $\delta\langle v_z\rangle' > 0$

Modes with $k_\theta k_z > 0$ grow faster than other modes,

$$\gamma_k|_{k_\theta k_z>0} > \gamma_k|_{k_\theta k_z<0}$$

Spectral imbalance (Fig. 1)

$$\langle k_\theta k_z\rangle \neq 0$$



$\{k\pm\}$: Domains where modes grow faster/slower

Fig. 1: Spectral imbalance.

Quasilinear Reynolds Stress

- Reynolds stress = diffusive flux + residual stress

$$\langle \tilde{v}_r \tilde{v}_z \rangle = \Re \sum_k \frac{i}{\omega} \left(k_\theta k_z \rho_s c_s - k_\theta^2 \rho_s^2 \langle v_z \rangle' \right) |\phi_k|^2 + \sum_k \frac{\delta}{\omega} k_\theta k_z \rho_s c_s |\phi_k|^2 = -\chi_\phi \langle v_z \rangle' + \Pi_{rz}^{\mathrm{Res}}$$

- Turbulent diffusivity: $\chi_\phi = \sum_k \frac{\nu_{\mathrm{ei}}}{k_z^2 v_{\mathrm{The}}^2} \frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2} k_\theta^2 \rho_s^2 |\phi_k|^2$
- Residual stress: $\Pi_{rz}^{\mathrm{Res}} = \sum_k \left(\frac{\gamma_k}{\omega_k^2} + \frac{\delta}{\omega_k} \right) k_\theta k_z \rho_s c_s |\phi_k|^2$

$$= \sum_k \frac{\nu_{\mathrm{ei}}}{k_z^2 v_{\mathrm{The}}^2} (2 + k_\perp^2 \rho_s^2) \left[\frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2} + \frac{k_\theta k_z \rho_s c_s \langle v_z \rangle'}{\omega_*^2} \right] k_\theta k_z \rho_s c_s |\phi_k|^2$$

- Sum over 2 domains, accounting for the spectral imbalance

$$\Pi_{rz}^{\mathrm{Res}} = \mathrm{Sign}(\delta \langle v_z \rangle') \sum_{\{k+\}} \frac{\nu}{k_\parallel^2 v_{\mathrm{The}}^2} (2 + k_\perp^2 \rho_s^2) \frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2} |k_y k_\parallel| \rho_s c_s \Delta I_k(\delta \langle v_z \rangle')$$

$$\Delta I_k(\delta \langle v_z \rangle') \equiv |\phi_k|^2 \big|_{\{k+\}} - |\phi_k|^2 \big|_{\{k-\}}$$

Spectral imbalance $\sim \delta \langle v_z \rangle'$

Contrast the 2 Stories

	Standard Symmetry Breaking	Dynamical Symmetry Breaking
Free energy source	∇T_i, ∇n, … depending on turbulence type	Only drift wave turbulence so far, ∇n
Symmetry breaker	Radial electric field shear, E_r'; Intensity gradient, $I(x)'$, etc. All tied to magnetic field configuration.	Test axial flow shear, $\delta\langle v_z\rangle'$; No requirement for shear of $\boldsymbol{B}$ structure.
Effect on the flow	Intrinsic torque, $-\partial_r \Pi_{r\parallel}^{Res}$	Negative viscosity, $\delta\Pi_{rz}^{Res} = \lvert\chi^{Res}\rvert\delta\langle v_z\rangle'$
Flow profile	$\langle v_\parallel\rangle' = \dfrac{\Pi_{r\parallel}^{Res}}{\chi_\phi}$	$\langle v_z\rangle' = \dfrac{\text{Flow drive }(\Pi_{rz}^{Res}, \Delta P_z)}{\chi_\phi - \lvert\chi^{Res}\rvert}$
Feedback loop	Open loop: Heat flux → ∇T_i + geometry (magnetic shear) → $\Pi_{r\parallel}^{Res}$ → $\langle v_\parallel\rangle'$	Closed loop: Test flow shear $\delta\langle v_z\rangle'$ → Breaks the symmetry, spectral imbalance → Residual stress Π_{rz}^{Res} → Intrinsic flow, feedback on $\delta\langle v_z\rangle'$ → Test flow shear $\delta\langle v_z\rangle'$

Negative Viscosity Increment

- Calculate negative diffusion $\delta\Pi_{rz}^{Res} \sim |\chi^{Res}|\delta\langle v_z\rangle'$, back-of-envelope style
- Quasilinear residual stress: $\Pi_{rz}^{\mathrm{Res}} \sim \sum_k \frac{\gamma_k}{\omega_k} k_\theta k_z \rho_s c_s N_k$
- $N_k \sim |\phi_k|^2/\omega_k$ wave action density governed by wave kinetic equation:

$$\frac{\partial}{\partial t}N + v_{gr}\frac{\partial}{\partial r}N - \frac{\partial}{\partial r}\left(\omega_k + \mathbf{k}\cdot\mathbf{V}\right)\frac{\partial}{\partial k_r}N = \gamma_k N - \Delta\omega_k \frac{N^2}{N_0}$$

Convection by wave packet — Refraction — Linear growth — Self-interaction

$$\Rightarrow \quad \delta N_k \sim \tau_c \delta\gamma_k N_k \sim \gamma_k^{-1}\delta\gamma_k N_k$$

- Recall: $\gamma_k \cong \frac{\nu_{\mathrm{ei}}}{k_z^2 v_{\mathrm{The}}^2}\frac{\omega_*^2}{(1+k_\perp^2\rho_s^2)^2}\left(\frac{k_\perp^2\rho_s^2}{1+k_\perp^2\rho_s^2} + \frac{k_\theta k_z \rho_s c_s \langle v_z\rangle'}{\omega_*^2}\right) \Rightarrow \delta\gamma_k = \frac{\nu_{\mathrm{ei}}}{k_z^2 v_{\mathrm{The}}^2}\frac{k_\theta k_z \rho_s c_s}{(1+k_\perp^2\rho_s^2)^2}\delta\langle v_z\rangle'$

$$\Rightarrow \quad \delta\Pi_{rz}^{\mathrm{Res}} \sim \frac{\nu_{\mathrm{ei}} L_n^2}{v_{the}^2}\sum_k |\phi_k|^2 \delta\langle v_z\rangle' \sim |\chi^{\mathrm{Res}}|\delta\langle v_z\rangle'$$

Negative Viscosity Increment: cont'd

- Dynamics of a test flow shear

$$\frac{\partial}{\partial t}\delta\langle v_z\rangle' + \frac{\partial^2}{\partial r^2}\left(\delta\Pi_{rz}^{\mathrm{Res}} - \chi_\phi \delta\langle v_z\rangle'\right) = 0$$

$$\delta\Pi_{rz}^{\mathrm{Res}} = \frac{\nu_{\mathrm{ei}} L_n^2}{v_{\mathrm{The}}^2}\sum_k (1+k_\perp^2\rho_s^2)(4+k_\perp^2\rho_s^2)|\phi_k|^2\delta\langle v_z\rangle' = |\chi^{\mathrm{Res}}|\delta\langle v_z\rangle'$$

(From formal calculation)

$$\Rightarrow \quad \frac{\partial}{\partial t}\delta\langle v_z\rangle' - \frac{\partial^2}{\partial r^2}\left(\chi_\phi - |\chi^{\mathrm{Res}}|\right)\delta\langle v_z\rangle' = 0$$

- Negative viscosity increment:

$$|\chi^{\mathrm{Res}}| = \nu_{\mathrm{ei}} L_n^2 / v_{\mathrm{The}}^2 \sum_k (1+k_\perp^2\rho_s^2)(4+k_\perp^2\rho_s^2)|\phi_k|^2$$

- Growth rate of flow shear modulation

$$\gamma_q = -q_r^2(\chi_\phi - |\chi^{\mathrm{Res}}|)$$

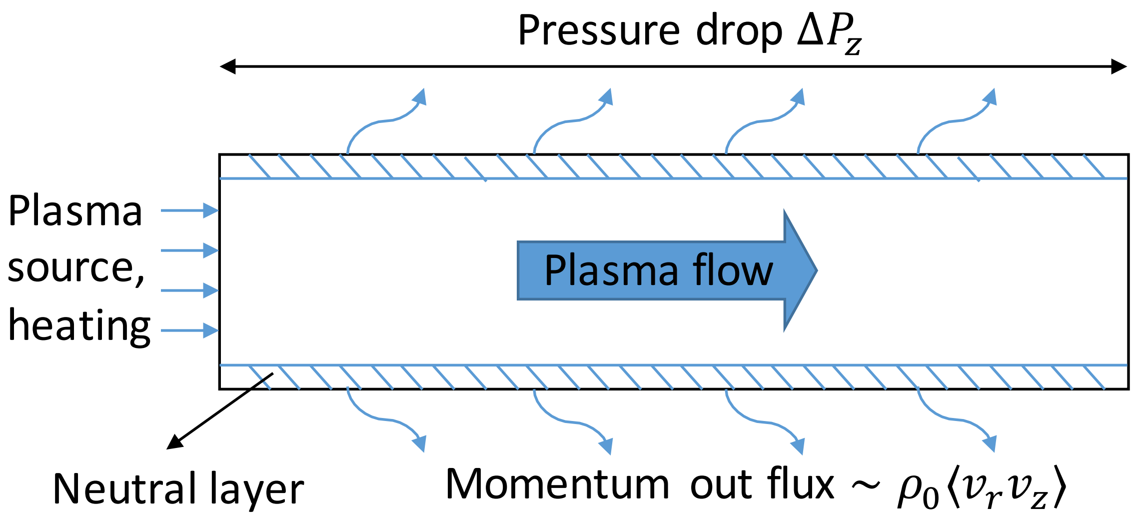
Limits on $\langle v_z \rangle'$

- $\chi_\phi^{\text{tot}} = \chi_\phi - |\chi^{Res}| < 0 \rightarrow \delta\langle v_z\rangle'$ grows, profile steepens, until…
- $\langle v_z \rangle'$ hits Parallel Shear Flow Instability (PSFI) threshold
  - PSFI: recall dispersion relation of the model with adiabatic electrons:
  $$\left(1 + k_\perp^2 \rho_s^2\right) \omega^2 - \omega_* \omega + k_\theta k_z \rho_s c_s \langle v_z \rangle' - k_z^2 c_s^2 = 0$$
  - Unstable $\leftrightarrow$ discriminant $\equiv \omega_*^2 - 4(1 + k_\perp^2 \rho_s^2)(k_\theta k_z \rho_s c_s \langle v_z \rangle' - k_z^2 c_s^2) < 0$
  - $\rightarrow \langle v_z \rangle' > \langle v_z \rangle'_{\text{crit}} \equiv \frac{1}{k_\theta k_z \rho_s c_s}\left[\frac{\omega_*^2}{4(1 + k_\perp^2 \rho_s^2)} + k_z^2 c_s^2\right] \rightarrow$ PSFI
- PSFI turbulence $\rightarrow \chi_\phi^{\text{PSFI}}$ adds on to the ambient χ_ϕ^{tot}

$\rightarrow \chi_\phi^{\text{tot}} = \chi_\phi^{\text{DW}} + \chi_\phi^{\text{PSFI}} \Theta(\langle v_z \rangle' - \langle v_z \rangle'_{crit}) - |\chi^{Res}|$

- χ_ϕ^{PSFI} switched on, $\chi_\phi^{\text{PSFI}} > |\chi^{Res}| \rightarrow \chi_\phi^{\text{tot}} > 0$
- $\langle v_z \rangle'$ stays below PSFI threshold; total viscosity stays positive.
- Similar to nonlinear damping of zonal flow

Flow Structure in Linear Device



	Pipe flow	Plasma flow
Drive	Pressure drop ΔP_z	Ion pressure drop ΔP_z
Boundary condition	No slip	Set by neutral layer
Viscosity	ν	$\chi_\phi - \lvert\chi^{Res}\rvert$

- Idea of Model: Turbulent pipe flow, Prandtl + residual stress
- Prandtl (momentum balance): $\pi R^2 \Delta P_z = 2\pi R L_z \rho_0 \langle v_r v_z \rangle$
- Reynolds stress: $\langle v_r v_z \rangle = -(\chi_\phi - |\chi^{\mathrm{Res}}|)\langle v_z \rangle'$
- → Flow profile: $\langle v_z \rangle' = -\dfrac{R\Delta P_z}{2L_z\rho_0(\chi_\phi - |\chi^{\mathrm{Res}}|)}$

Flow Structure: cont'd

- PSFI → Enhance turbulent diffusion,

$$\chi_\phi^{\mathrm{eff}} = \chi_\phi^{\mathrm{DW}} + \chi_\phi^{\mathrm{PSFI}}\Theta(\langle v_z\rangle' - \langle v_z\rangle'_{crit})$$

- Including PSFI effect:

$$\langle v_z\rangle' = -\frac{R\Delta P_z}{2L_z\rho_0\left[\chi_\phi^{\mathrm{DW}} + \chi_\phi^{\mathrm{PSFI}}\Theta\left(\langle v_z\rangle' - \langle v_z\rangle'_{\mathrm{PSFI}}\right) - |\chi^{\mathrm{Res}}|\right]}$$

- $\chi_\phi^{\mathrm{PSFI}}$ nonlinear in $\langle v_z\rangle'$ → $\chi_\phi^{\mathrm{eff}} > |\chi^{Res}|$ → Profile relaxes
- $\langle v_z\rangle'$ stays below PSFI threshold

Implication for Tokamaks

- Synergy of $\Pi^{Res}(\nabla T, \nabla P, \nabla N)$ and $\delta\Pi^{Res} = |\chi^{Res}|\delta\langle v_z\rangle'$

- DW turbulence, $\boldsymbol{B}$ shear
- Symmetry breaker $(E_r', I(x)', \ldots)$
- → residual stress $\Pi_{r\parallel}^{Res}$

+

- DW turbulence
- Test flow shear
- → negative viscosity $|\chi^{Res}|$

Applicable to electron DW's → CTEM

- Flow profile set by momentum flux balance:

$$\langle v_r v_\parallel \rangle = -\left(\chi_\phi - |\chi^{\text{Res}}|\right)\frac{d\langle v_\parallel\rangle}{dr} + \Pi_{r\parallel}^{\text{Res}} = 0$$

- Enhanced flow profile

$$\frac{d\langle v_\parallel\rangle}{dr} = \frac{\Pi_{r\parallel}^{\text{Res}}}{\chi_\phi - |\chi^{\text{Res}}|}$$

→ Mechanism to enhance intrinsic rotation predictions

Conclusion

- Results from CSDX, PANTA suggest intrinsic axial flow;
- Intrinsic mechanism to generate axial flows and to build up a mean flow profile is introduced:
    - Test flow shear $\delta\langle v_z\rangle'$seeds symmetry breaking and feeds back on itself;
- Different from standard symmetry breaking mechanism:
    - Intrinsic torque $-\partial_r\Pi^{Res}$ driven by $\nabla T, \nabla \mathrm{P}$, ... v.s. Negative viscosity increment $|\chi^{Res}|$ induced by $\delta\Pi^{Res}$;
- Flow structure in a linear device: $\langle v_z\rangle' = -\dfrac{R\Delta P_z}{2L_z\rho_0(\chi_\phi - |\chi^{\mathrm{Res}}|)}$
- Implication for tokamaks:
    - Synergy of $\Pi^{Res}(\nabla T, \nabla \mathrm{P}, \ldots)$ and $\delta\Pi^{Res} = |\chi^{Res}|\delta\langle v_z\rangle'$;
    - Enhanced intrinsic rotation profile: $\dfrac{d\langle v_\parallel\rangle}{dr} = \dfrac{\Pi^{\mathrm{Res}}_{r\parallel}}{\chi_\phi - |\chi^{\mathrm{Res}}|}$