



Intrinsic Axial Flows in CSDX and Dynamical Symmetry Breaking in ITG Turbulence

-- Negative Viscosity Effects and Flow Saturation

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Outline

- Introduction:
 - Why intrinsic rotation + weak shear?
 - Intrinsic flow at zero shear: CSDX experiments; ion features
 - Theory: Dynamical Symmetry Breaking in Collisional Drift Wave turbulence
- Symmetry Breaking in ITG turbulence at zero magnetic shear?
 - PSF-ITG system
 - Symmetry breaking in 3 instability regimes
 - Summary: ∇V_{\parallel} effects on ITG turbulence
- Lesson for tokamaks: interaction with symmetry breaking based on magnetic shear
 - Rotation profiles

Intrinsic Rotation in Weak Shear

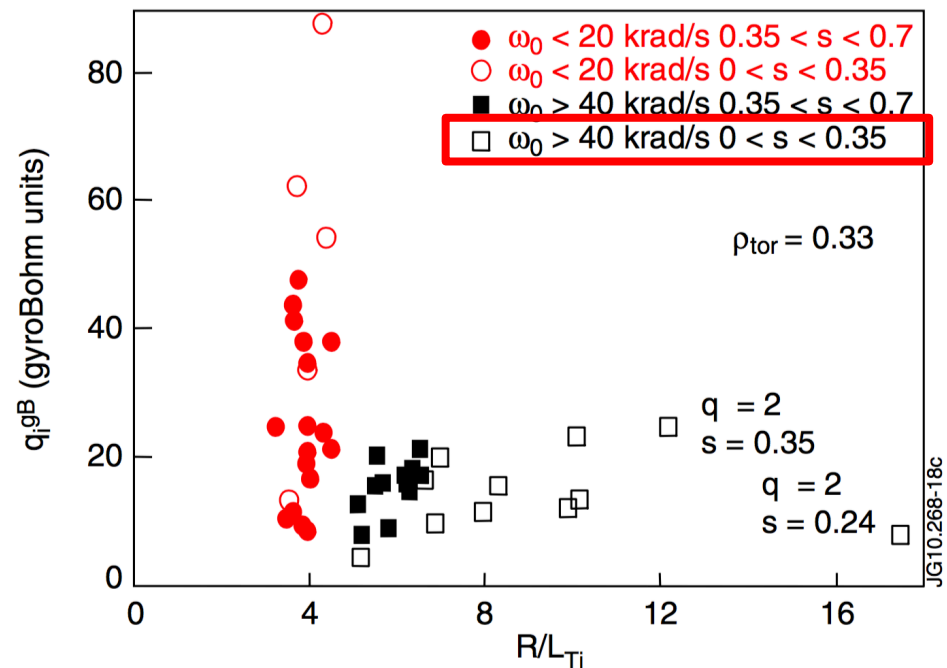
- JET: Weak shear **AND** Rotation → Enhanced confinement
- But external torque limited in ITER
- Need understand: ***Intrinsic rotation in weak shear regimes***

- Important for:

- Total effective torque

$$\tau = \tau_{ext} + \tau_{intr}$$

- Contribution to $V'_{E \times B}$

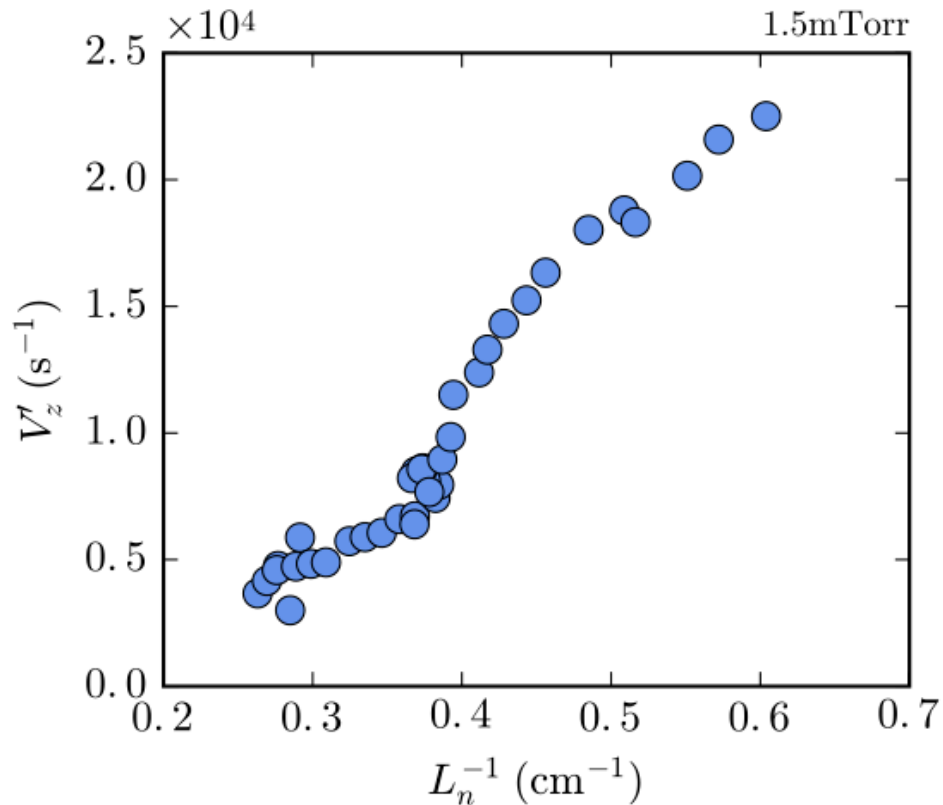


[P. Mantica, PRL, 2011;
Rice, PRL, 2013]

FIG. 4 (color online). q_i^{GB} vs R/L_{Ti} at $\rho_{tor} = 0.33$ for similar plasmas with different rotation and s values.

Intrinsic $\nabla\langle v_z \rangle$ in Drift Wave Turbulence

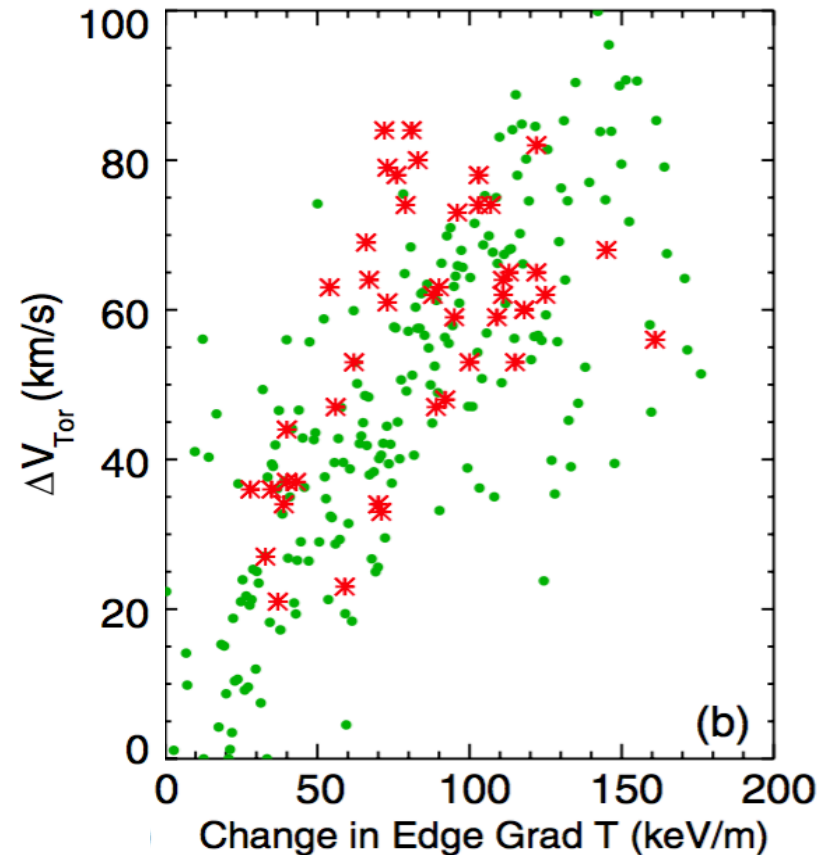
- Axial flow in CSDX:
- ∇n_0 is free energy source
- $\langle v_z \rangle' \sim \frac{1}{n_0} \nabla n_0$



(Zero magnetic shear)

- Compare:
- Intrinsic $\nabla\langle v_z \rangle$ in C-Mod pedestal:
- $\Delta\langle v_\phi \rangle \sim \nabla T$

[Rice, PRL, 2011]



(Standard shear)

Theory of Intrinsic Rotation at Zero Shear

- Intrinsic flow accelerated by residual stress ($\tau_{intra} = -\nabla \cdot \Pi^{Res}$)

$$\langle \tilde{v}_r \tilde{v}_{\parallel} \rangle = -\chi_{\phi} \frac{d\langle v_{\parallel} \rangle}{dr} + V_P \langle v_{\parallel} \rangle + \Pi_{r\parallel}^{Res}$$

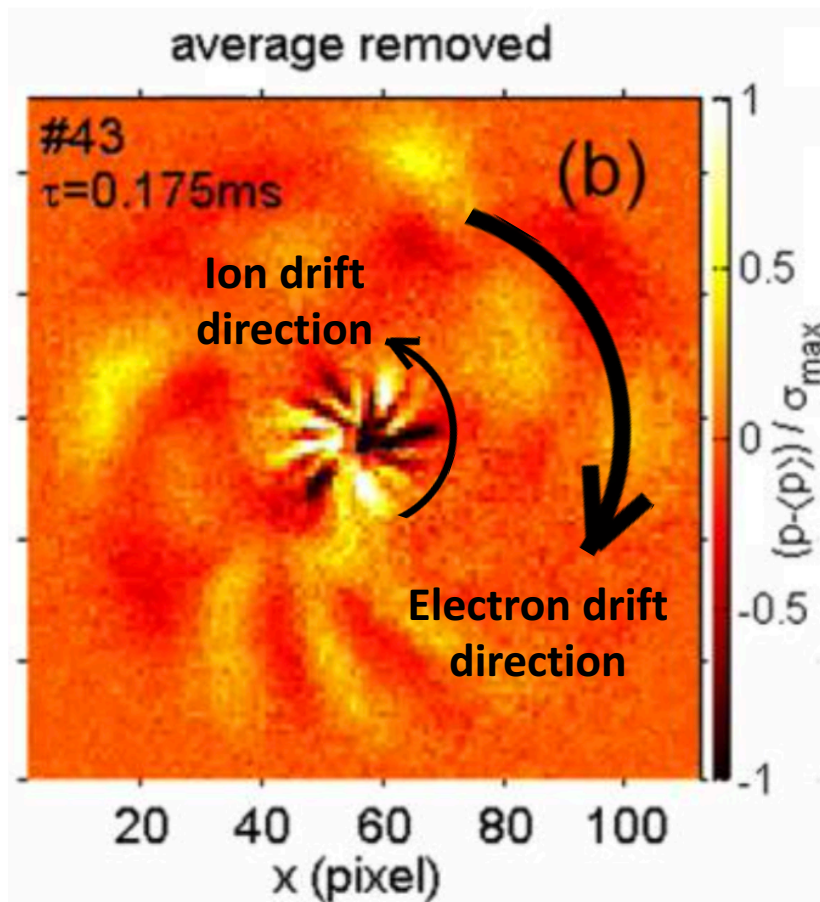
- Residual stress driven by turbulence, i.e. $\Pi_{r\parallel}^{Res} \sim \nabla P, \nabla T, \nabla n_0$
- $\Pi_{r\parallel}^{Res} \sim \langle k_{\theta} k_{\parallel} \rangle$ etc. requires symmetry breaking in $k_{\theta} k_{\parallel}$ space, at ZERO shear
- \rightarrow Dynamical symmetry breaking
- Negative viscosity increment induced by Π^{Res}
 - $\delta \Pi^{Res} = |\chi_{\phi}^{Res}| \delta \langle v_z \rangle' \rightarrow$ Total viscosity: $\chi_{\phi}^{tot} = \chi_{\phi} - |\chi_{\phi}^{Res}|$
 - $\chi_{\phi}^{tot} < 0 \rightarrow$ Modulational growth of $\delta \langle v_z \rangle'$
- Broader lesson for tokamaks
 - Synergy of $\langle v_{\phi} \rangle'$ self-amplification and Π^{Res}
 - $\langle v_{\phi} \rangle'$ driven by $\tau_{NBI}, \Pi^{Res}(\nabla n_0, \nabla T)$, and enhanced by $-|\chi_{\phi}^{Res}|$
 - $\langle v_{\phi} \rangle' \sim \frac{\tau_{NBI} + \Pi^{Res}(\nabla n_0, \nabla T)}{\chi_{\phi} - |\chi_{\phi}^{Res}|}$

Compare Symmetry Breaking Mechanisms

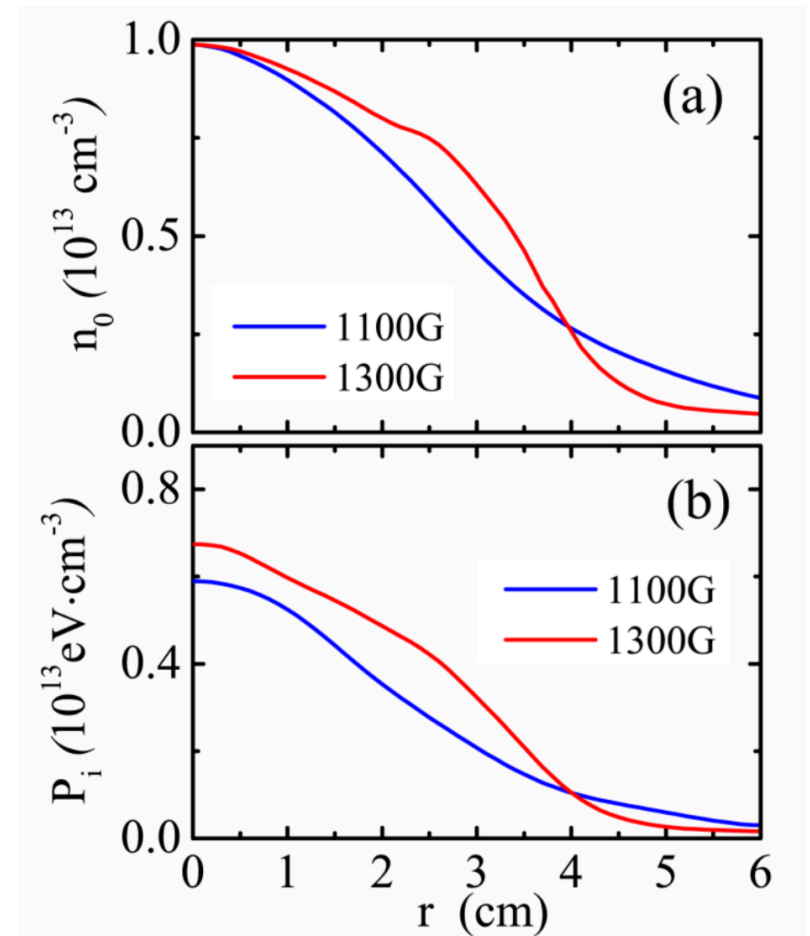
	Standard Symmetry Breaking	Dynamical Symmetry Breaking
Free energy	$\nabla T_i, \nabla T_e, \nabla n_0, \dots$	$\nabla n_0, \nabla T_e$ --electron drift waves
Symmetry breaker	$E_r', I(x)', \dots$ All tied to magnetic field configuration	Test toroidal flow shear, $\delta\langle v_\phi \rangle'$; No requirement for shear of \mathbf{B} structure.
Effect on flow	Intrinsic torque, $-\partial_r \Pi_{r\parallel}^{Res}$	Negative viscosity, $- \chi_\phi^{Res} $ driven by ∇n_0
Flow profile	$\langle v_{\parallel} \rangle' = \frac{\Pi_{r\parallel}^{Res}}{\chi_\phi}$	$\langle v_\phi \rangle' = \frac{\text{Flow drive (e.g. } \Pi_{r\phi}^{Res}, \Delta P_i)}{\chi_\phi (\nabla n_0, \nabla \langle v_\phi \rangle) - \chi_\phi^{Res} }$
Feedback loop		

Ion Features in CSDX

- Mode Coexistence



- T_i profile steepening



Questions

- What happens to ITG turbulence?
 - How does ∇V_{\parallel} affect the ITG turbulence?
 - Does ITG turbulence have dynamical symmetry breaking?
- How does ∇V_{\parallel} induced symmetry breaking interact with symmetry breaking by magnetic shear?

PSF-ITG System

- Fluid model with ion dissipation

$$\frac{d}{dt}(1 - \nabla_{\perp}^2)\phi + \mathbf{v}_E \cdot \frac{\nabla \tilde{n}_0}{n_0} + \nabla_{\parallel} \tilde{v}_{\parallel} = 0,$$

$$\frac{d\tilde{v}_{\parallel}}{dt} + \mathbf{v}_E \cdot \nabla V_{\parallel} = -\nabla_{\parallel} \phi - \nabla_{\parallel} \tilde{p}_i,$$

$$\frac{d\tilde{p}_i}{dt} + \frac{1}{\tau} \mathbf{v}_E \cdot \frac{\nabla P_0}{P_0} + \frac{\Gamma}{\tau} \nabla_{\parallel} \tilde{v}_{\parallel} + \nabla_{\parallel} Q_{\parallel} = 0.$$

- 2 free energy sources: ∇V_{\parallel} and ∇T_i

- Magnetic shear = 0

→ No correlation between parallel and perpendicular directions

→ Simplified geometry (cylindrical)

- Landau damping closure: $Q_{\parallel,k} = -\chi_{\parallel} n_0 i k_{\parallel} \tilde{T}_{i,k}$

(Hammett and Perkins, PRL, 1995) $\chi_{\parallel} = 2\sqrt{2}v_{Thi}/(\sqrt{\pi}|k_{\parallel}|)$

- Dispersion relation:

$$A\Omega^3 - (C_0 - V')\Omega - D = 0$$

$$A \equiv 1 + k_{\perp}^2 \rho_s^2, \quad \Omega \equiv \frac{\omega}{|k_{\parallel} c_s|},$$

$$C_0 \equiv 1 + \frac{1 + k_{\perp}^2 \rho_s^2}{\tau} \Gamma, \quad V' \equiv \frac{k_{\theta} k_{\parallel} \rho_s c_s}{k_{\parallel}^2 c_s^2} \frac{\partial V_{\parallel}}{\partial r},$$

$$D \equiv \frac{\omega_T}{\tau |k_{\parallel} c_s|}, \quad \tau \equiv \frac{T_e}{T_i}$$

- Landau damping effect ignored because

$$\frac{|k_{\parallel} \chi_{\parallel}}{c_s} \sim \frac{v_{Thi}}{c_s} = \frac{1}{\sqrt{\tau}} < 1 \text{ in CSDX}$$

- Criterion for instability: $\Delta \equiv \left(\frac{D}{2A}\right)^2 - \left(\frac{C_0 - V'}{3A}\right)^3 > 0$

∇V_{\parallel} and ∇T_i are coupled nonlinearly



Can be decoupled by limiting relative scale length

$$L_T/L_V \equiv \partial_r \ln T_{i0} / \partial_r \ln V_{\parallel}$$

E.g. consider two extreme cases:

- ITG Instability: $A\Omega^3 - D \approx 0$, $\omega \sim e^{i2\pi/3} (\omega_T k_{\parallel}^2 c_s^2 / \tau A)^{1/3}$
- PSFI (parallel shear flow instability):

$$A\Omega^3 + (V' - C_0)\Omega \approx 0, \quad \omega \sim e^{i\pi/2} \sqrt{V' - C_0/A}$$

Instability Regimes

- Goal: decouple ∇V_{\parallel} and ∇T_i
 - Residual stress $\rightarrow \chi_{\phi}^{Res}$
 - $\chi_{\phi}^{tot} = \chi_{\phi}^{ITG} + \chi_{\phi}^{PSFI} + \chi_{\phi}^{Res}$
- \Rightarrow Flow profile $V'_{\parallel} \sim \Pi_{r_{\parallel}}^{Res} / \chi_{\phi}^{tot}$

- Regimes in ∇V_{\parallel} - ∇T_i space:

(1) Marginally unstable regime: $\Delta \gtrsim 0$

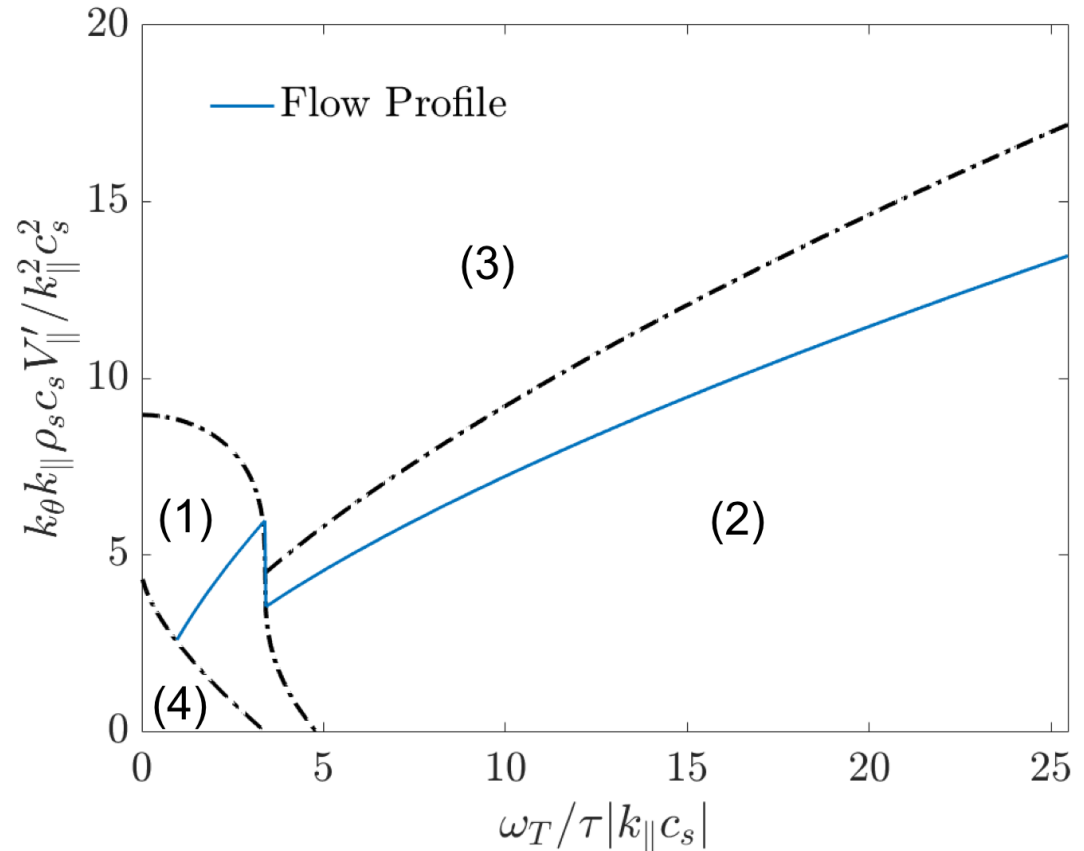
(2) ITG dominant regime

$$\frac{(|k_{\parallel}|L_T)^{2/3}}{|k_{\parallel}|L_V} < \frac{3 c_s}{2^{2/3} V_{\parallel}} \frac{A^{1/3}}{(k_{\theta}\rho_s)^{1/3}\tau^{1/3}}$$

(3) PSFI dominant regime

$$\frac{(|k_{\parallel}|L_T)^{2/3}}{|k_{\parallel}|L_V} > \frac{3 c_s}{2^{2/3} V_{\parallel}} \frac{A^{1/3}}{(k_{\theta}\rho_s)^{1/3}\tau^{1/3}}$$

(4) Stable regime: $\Delta < 0$




Flow profile in different instability regimes. The regimes are identified according to the magnitude of relative scale length $\frac{(|k_{\parallel}|L_T)^{2/3}}{|k_{\parallel}|L_V}$ and magnitude of Δ .

Residual Stress Direction Determined by Mode Phase

- Mode phase θ_k :

- Defined as $\omega = \omega_k + i\gamma_k \equiv |\omega|e^{i\theta_k}$
 - $\rightarrow \theta_k^{ITG} = \frac{2\pi}{3}, \theta_k^{PSFI} = \frac{\pi}{2}$

- Residual stress:

$$\Pi_{r\parallel}^{Res} \approx \Re \sum_k \frac{i}{\omega^2} \frac{\omega_T}{\tau} k_\theta k_\parallel \rho_s c_s |\phi_k|^2$$


- Residual stress due to $\delta V'_{\parallel}$:

Determined by mode phase θ_k : $\Re \frac{i}{\omega^2} \sim \cos\left(\frac{\pi}{2} - 2\theta_k\right)$

$\delta V'_{\parallel} \rightarrow \delta\omega \equiv |\delta\omega|e^{i\delta\theta_k}, \delta\theta_k$: perturbed mode phase

$$\delta\Pi_{r,\parallel}^{Res} \sim -2 \sum_k \cos\left(\frac{\pi}{2} + \delta\theta_k - 3\theta_k\right) \frac{|\delta\omega|}{|\omega|^3} \frac{\omega_T}{\tau} k_\theta k_\parallel |\phi_k|^2$$

$$|\delta\omega| \sim k_\theta k_\parallel \delta V'_{\parallel}$$

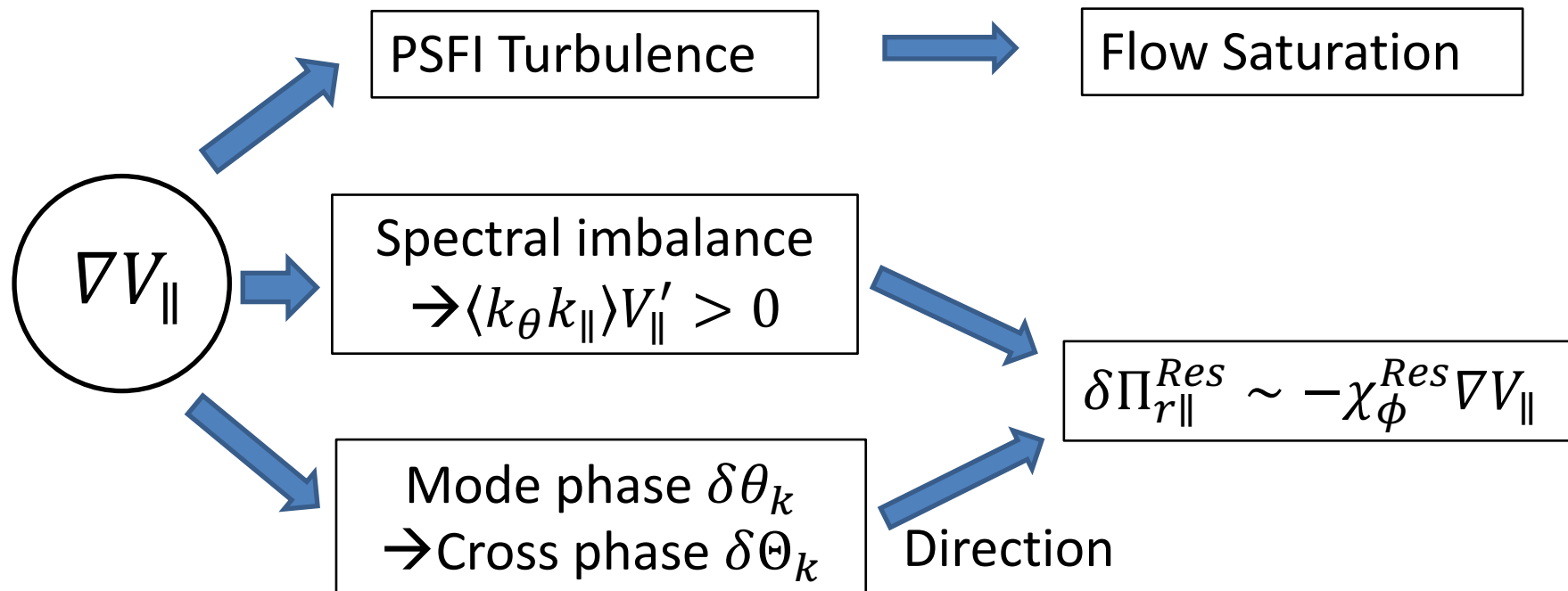
given by $\delta V'_{\parallel}$ induced spectral imbalance



$$\delta\Pi_{r\parallel}^{Res} = -\chi_\phi^{Res} \delta V'_{\parallel}, \text{ with}$$

$$\chi_\phi^{Res} \sim \cos\left(\frac{\pi}{2} + \delta\theta_k - 3\theta_k\right)$$

∇V_{\parallel} Effects on ITG Turbulence



Guideline: Physics of the 3 Regimes

- Marginal regime:
 - Dual perspective: PSFI enhanced by ∇T_{i0} ; ITG enhanced by ∇V_{\parallel}
 - Coexistence of PSFI and ITG turbulence
- ITG regime:
 - Negative viscosity increment induced by $\delta V'_{\parallel} \rightarrow \chi_{\phi}^{Res} < 0$
 - Total viscosity positive $\chi_{\phi}^{tot} = \chi_{\phi} - |\chi_{\phi}^{Res}| > 0$
 - Flow profile enhanced by χ_{ϕ}^{Res} : $V'_{\parallel} \sim \Pi_{r_{\parallel}}^{Res} / (\chi_{\phi} - |\chi_{\phi}^{Res}|)$
- PSFI regime:
 - Flow saturated by PSFI, profile gradient stay at the threshold: $V'_{\parallel} \sim V'_{\parallel, crit}$

	Marginally Unstable	ITG Dominant	PSFI Dominant
Primary Turbulence Drive	∇T_{i0} and ∇V_{\parallel}	∇T_{i0}	∇V_{\parallel}
Mode Phase θ_k	$\lesssim \pi$	$2\pi/3$	$\gtrsim \pi/2$
∇V_{\parallel} Effect on Mode Phase $\delta\theta_k$	$\pi/2$	$\pi/3$	NA
∇V_{\parallel} Induced Symmetry Breaking	$k_{\theta} k_{\parallel} V'_{\parallel} > 0$	$k_{\theta} k_{\parallel} V'_{\parallel} > 0$	$k_{\theta} k_{\parallel} V'_{\parallel} > 0$

Marginal Regime: Flow Profile and Symmetry Breaking

- Weakly unstable ITG turbulence: $\gamma_k \sim \sqrt{\omega_T^2 - \omega_{T,crit}^2}$
- $\omega_{T,crit}^2$ nonlinear in ∇V_{\parallel} \longleftrightarrow $\omega_{T,crit}^2(V'_{\parallel}) = \frac{4\tau^2 k_{\parallel}^2 c_s^2 (C_0 - V')^3}{27A}$

$\rightarrow \Pi_{r\parallel}^{Res}$ and χ_{ϕ} are nonlinear in ∇V_{\parallel} , but $V'_{\parallel} \sim$ their ratio independent on ∇V_{\parallel}

$$\Pi_{r\parallel}^{Res} \cong -\frac{2\sqrt{3}}{3} \sum_k \frac{(2A)^{2/3}}{\tau^{1/3} |k_{\parallel} c_s|^{4/3}} \frac{\sqrt{\omega_T^2 - \omega_{T,crit}^2}}{\omega_T^{2/3}} k_{\theta} k_{\parallel} \rho_s c_s |\phi_k|^2$$

$$\chi_{\phi} \cong \frac{\sqrt{3}}{3} \sum_k \frac{(2A\tau)^{1/3}}{|k_{\parallel} c_s|^{2/3}} \frac{\sqrt{\omega_T^2 - \omega_{T,crit}^2}}{\omega_T^{4/3}} k_{\theta}^2 \rho_s^2 |\phi_k|^2$$



$$|V'_{\parallel}| = \frac{|\Pi_{r\parallel}^{Res}(\nabla V_{\parallel}, \nabla T_{i0})|}{\chi_{\phi}(\nabla V_{\parallel}, \nabla T_{i0})} \sim 2^{4/3} A^{1/3} \left(\frac{\omega_T}{\tau |k_{\parallel} c_s|} \right)^{2/3} \frac{|k_{\parallel} c_s|}{k_{\theta} \rho_s}$$

- Symmetry breaking by ∇V_{\parallel} : $V' = k_{\theta} k_{\parallel} V'_{\parallel} > 0$ lowers $\omega_{T,crit}^2$



ITG more unstable for $k_{\theta} k_{\parallel} V'_{\parallel} > 0$



Spectral imbalance, setting $\langle k_{\theta} k_{\parallel} \rangle V'_{\parallel} > 0$

$$\delta \Pi_{r\parallel}^{Res} = -\chi_{\phi}^{Res} \nabla V_{\parallel}$$

due to spectral imbalance



- Mode phase $\theta_k = \pi - \epsilon$, because $\gamma_k \ll \omega_k$
- ∇V_{\parallel} induced mode phase $\delta\theta_k = \frac{\pi}{2}$
 $\rightarrow \cos\left(\frac{\pi}{2} + \delta\theta_k - 3\theta_k\right) > 0$

$$\chi_{\phi}^{Res} \cong \frac{4^{4/3}}{3^{5/2}} \sum_k \frac{C_0^2}{A^{1/3}} \frac{\tau^{5/3}}{\omega_T^{2/3}} \frac{k_{\theta}^2 \rho_s^2 |k_{\parallel} c_s|^{2/3}}{\sqrt{\omega_T^2 - \omega_{T,crit}^2(0)}} |\phi_k|^2 > 0$$

PSFI and ITG Coexists in Marginal Regime

- ITG turbulence with ∇V_{\parallel} in this regime is equivalent to weakly unstable PSFI turbulence
- $\gamma_k \sim \sqrt{\omega_T^2 - \omega_{T,crit}^2} \Leftrightarrow \gamma_k \sim \sqrt{V' - V'_{crit}}$, with $V'_{crit} = C_0 - \left(\frac{27A\omega_T^2}{4\tau^2 k_{\parallel}^2 c_S^2}\right)^{1/3}$
- $\rightarrow \nabla T_{i0}$ lowers the PSFI threshold



- Dual perspective:
ITG turbulence enhanced by $\nabla V_{\parallel} \Leftrightarrow$ PSFI turbulence enhanced by ∇T_{i0}
- Both PSFI and ITG turbulences exist in marginal regime
- ∇T_{i0} and ∇V_{\parallel} effects are coupled nonlinearly

ITG Regime

- Dominated by ∇T_{i0} , with ∇V_{\parallel} as perturbation

- Growth rate and frequency:

$$\gamma_k \cong \frac{\sqrt{3} \omega_T^{1/3} |k_{\parallel} c_s|^{2/3}}{2 (\tau A)^{1/3}} \left[1 - \left(\frac{\omega_{T,crit}}{2\omega_T} \right)^{2/3} \right]$$

$$\omega_k \cong -\frac{1}{2} \frac{\omega_T^{1/3} |k_{\parallel} c_s|^{2/3}}{(\tau A)^{1/3}} \left[1 + \left(\frac{\omega_{T,crit}}{2\omega_T} \right)^{2/3} \right]$$

$$\omega_T^2 \gg \omega_{T,crit}^2$$



$$\omega \cong e^{i2\pi/3} \frac{\omega_T^{1/3} |k_{\parallel} c_s|^{2/3}}{(\tau A)^{1/3}}$$

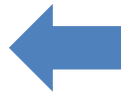
$$\delta\omega = e^{i\pi/3} \left(\frac{\tau}{\omega_T} \right)^{1/3} \frac{k_{\theta} k_{\parallel} \rho_s c_s \delta V'_{\parallel}}{3A^{2/3} |k_{\parallel} c_s|^{2/3}}$$

With $\theta_k = \frac{2\pi}{3}$, $\delta\theta_k = \frac{2\pi}{3}$



$$\chi_{\phi}^{Res} \sim \cos\left(\frac{\pi}{2} + \delta\theta_k - 3\theta_k\right) < 0$$

$$\chi_{\phi}^{Res} = -\frac{\sqrt{3}}{6} \sum_k \frac{(\tau A)^{1/3}}{\omega_T^{1/3} |k_{\parallel} c_s|^{2/3}} k_{\theta}^2 \rho_s^2 |\phi_k|^2$$



- Symmetry breaking by ∇V_{\parallel}
 - Spectral imbalance in $k_{\theta} k_{\parallel}$ space
 - $\langle k_{\theta} k_{\parallel} \rangle V'_{\parallel} > 0$



Negative viscosity induced by residual stress due to perturbed mode phase set by ∇V_{\parallel}

Symmetry Breaking by ∇V_{\parallel} Compared to Drift Wave

- In ITG turbulence, the ∇V_{\parallel} induced spectral imbalance:

- Negative viscosity increment: $\chi_{\phi}^{Res} < 0$

- Total viscosity positive: $\chi_{\phi}^{tot} = \chi_{\phi}^{ITG} - |\chi_{\phi}^{Res}| = \frac{2}{3}\chi_{\phi}^{ITG} > 0$

- Evolution of a test flow shear set by

$$\partial_t \delta V'_{\parallel} = \chi_{\phi}^{tot} \partial_r^2 \delta V'_{\parallel} \rightarrow \gamma_q = -\chi_{\phi}^{tot} q_r^2 < 0$$

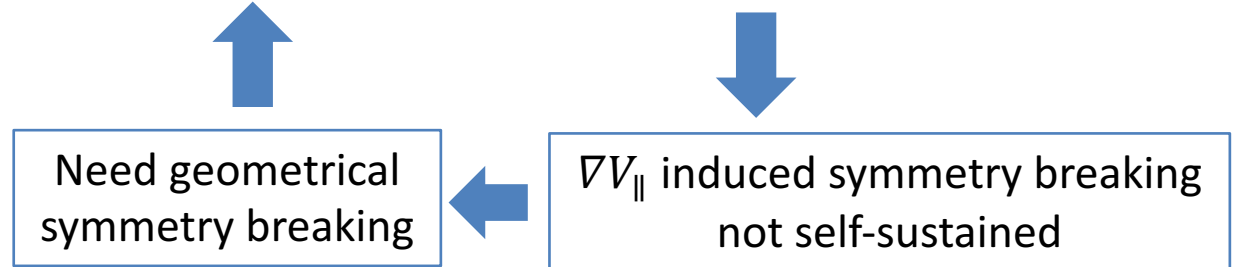
$\rightarrow \delta V'_{\parallel}$ cannot reinforce itself!

	ITG turbulence	Drift Wave turbulence
Direction of correlator	$\langle k_{\theta} k_{\parallel} \rangle V'_{\parallel} > 0$	$\langle k_{\theta} k_{\parallel} \rangle V'_{\parallel} > 0$
Viscosity increment	$\chi_{\phi}^{Res} < 0$	$\chi_{\phi}^{Res} < 0$
Total viscosity	$\chi_{\phi}^{tot} > 0$	χ_{ϕ}^{tot} can be negative
Modulational instability	No	Can exist

Flow Profile in ITG Regime

- ∇V_{\parallel} decoupled from ∇T_{i0}

$$\Pi_{r_{\parallel}}^{Res}(\nabla V_{\parallel}, \nabla T_{i0}) \approx \Pi_{r_{\parallel}}^{Res}(\nabla T_{i0}) + |\chi_{\phi}^{Res}(\nabla T_{i0})| \nabla V_{\parallel}$$



➔ V'_{\parallel} enhanced by $-|\chi_{\phi}^{Res}|$:

$$|V'_{\parallel}| = \frac{|\Pi_{r_{\parallel}}^{Res}(\nabla T_{i0})|}{\chi_{\phi}(\nabla T_{i0}) - |\chi_{\phi}^{Res}(\nabla T_{i0})|} \sim \frac{3}{2} A^{1/3} \left(\frac{\omega_T}{\tau |k_{\parallel} c_s|} \right)^{2/3} \frac{|k_{\parallel}| c_s}{k_{\theta} \rho_s}$$

- $|V'_{\parallel}|$ below PSFI regime threshold

$$|V'_{\parallel, regime}| \sim \frac{3}{2^{2/3}} A^{1/3} \left(\frac{\omega_T}{\tau |k_{\parallel} c_s|} \right)^{2/3} \frac{|k_{\parallel}| c_s}{k_{\theta} \rho_s}$$

PSFI Regime

- Dominated by ∇V_{\parallel} , with ∇T_{i0} as a correction
- Regime threshold is different from PSFI threshold
 - Regime threshold: ∇V_{\parallel} and ∇T_{i0} are well above threshold, and ∇V_{\parallel} is larger than ∇T_{i0}
 - PSFI threshold: ∇V_{\parallel} is large enough to trigger instability, in presence of ∇T_{i0}
 - Consider $|V'_{\parallel,crit}| \ll |V'_{\parallel}| \lesssim |V'_{\parallel,regime}|$
- γ_k nonlinear in ∇V_{\parallel} , $\omega_k \lesssim 0$ due to ∇T_{i0} correction:

$$\gamma_k \cong \frac{|k_{\parallel} c_s|}{\sqrt{A}} \sqrt{V' - C_0}, \quad \omega_k \cong -\frac{\omega_T}{2\tau(V' - C_0)}.$$

- $\rightarrow \Pi_{r_{\parallel}}^{Res}$ and χ_{ϕ} are nonlinear in ∇V_{\parallel}

$$\Pi_{r_{\parallel}}^{Res} \cong -\sum_k \frac{\omega_T^2}{\tau^2} \frac{A^{3/2}}{|k_{\parallel} c_s|^3 (V' - C_0)^{5/2}} k_{\theta} k_{\parallel} \rho_s c_s |\phi_k|^2, \quad \chi_{\phi} = \sum_k \frac{\sqrt{A}}{|k_{\parallel} c_s| \sqrt{V' - C_0}} k_{\theta}^2 \rho_s^2 |\phi_k|^2.$$

- $\rightarrow \nabla V_{\parallel}$ saturated since $|\Pi_{r_{\parallel}}^{Res}|$ drops as ∇V_{\parallel} increases

Flow Profile in PSFI Dominant Regime

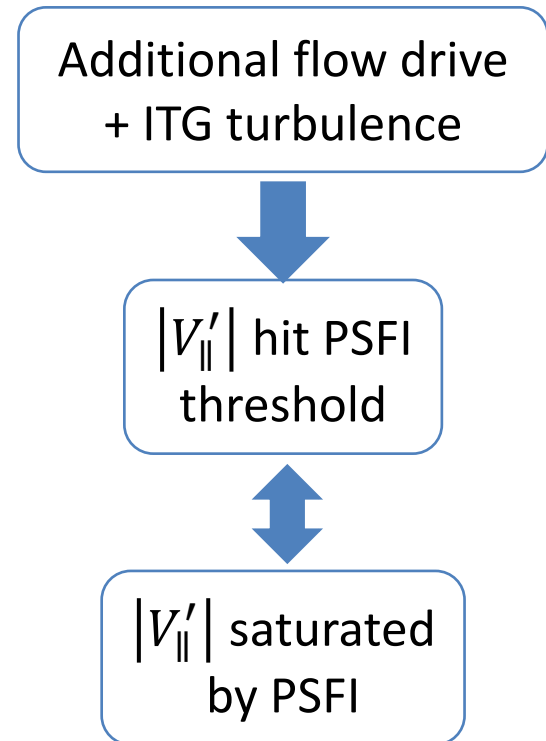
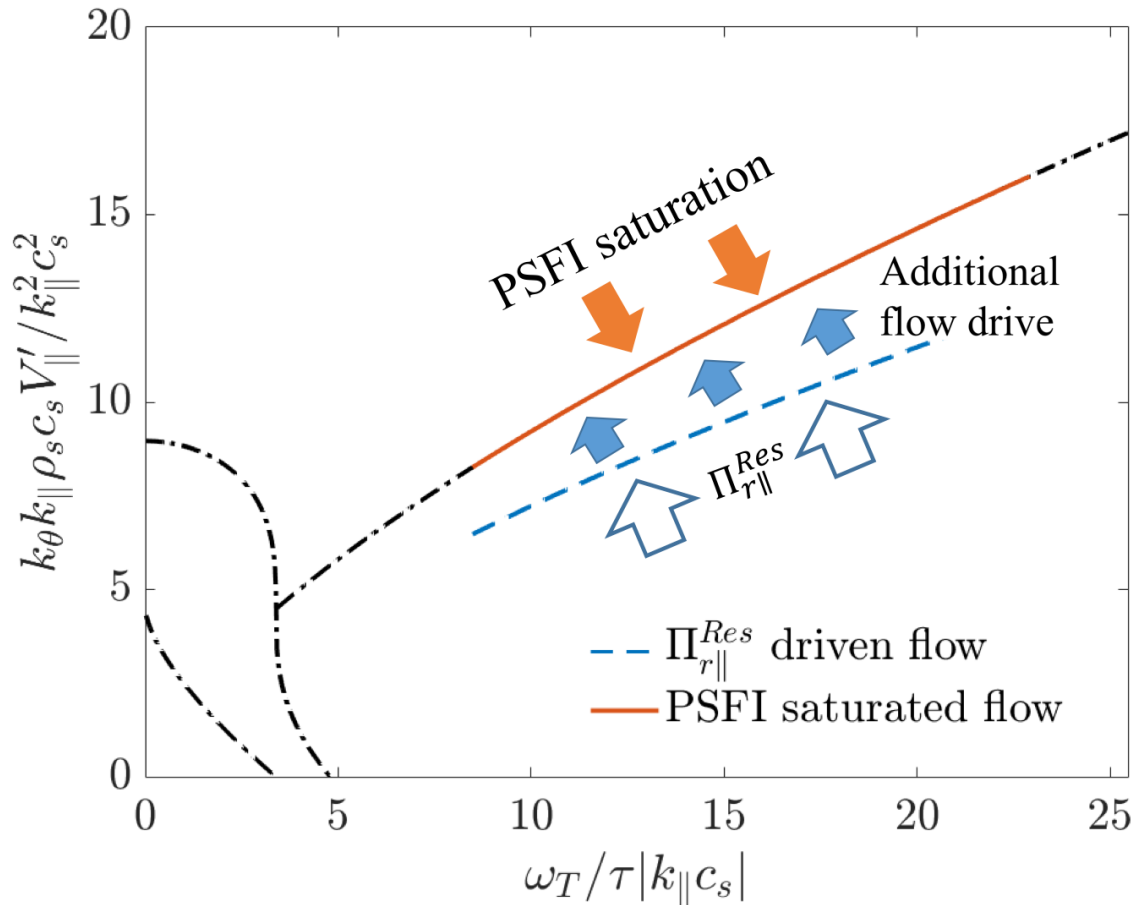
- $|V'_{\parallel}|$ driven by ITG turbulence always below $|V'_{\parallel,regime}|$
 - Additional flow drive can lead to PSFI regime $|V'_{\parallel}| \gtrsim |V'_{\parallel,regime}|$
 - Result: $|V'_{\parallel}|$ saturated by strong PSFI turbulence → $|V'_{\parallel}| \lesssim |V'_{\parallel,regime}|$
- $\Pi_{r\parallel}^{Res}$ and χ_{ϕ} nonlinear in ∇V_{\parallel}

$$|V'_{\parallel}| = \frac{|\Pi_{r\parallel}^{Res}(\nabla T_{i0}, \nabla V_{\parallel})|}{\chi_{\phi}(\nabla T_{i0}, \nabla V_{\parallel})} \sim \frac{A}{(V'_{\parallel})^2} \left(\frac{\omega_T}{\tau |k_{\parallel} c_s|} \right)^2 \left(\frac{|k_{\parallel}| c_s}{k_{\theta} \rho_s} \right)^3$$

$$\rightarrow |V'_{\parallel}| \sim A^{1/3} \left(\frac{\omega_T}{\tau |k_{\parallel} c_s|} \right)^{2/3} \frac{|k_{\parallel}| c_s}{k_{\theta} \rho_s} < |V'_{\parallel,regime}|$$

- $|V'_{\parallel}|$ stays at PSFI threshold due to balance between flow drive and PSFI saturation

$|V'_{\parallel}|$ profile saturated by PSFI



Summary

- ∇V_{\parallel} plays 3 roles in ITG turbulence
 - Drive PSFI \rightarrow saturate V_{\parallel}' profile
 - Symmetry breaking \rightarrow spectral imbalance, $\langle k_{\theta} k_{\parallel} \rangle V_{\parallel}' > 0$
 - Modify mode phase $\rightarrow \chi_{\phi}^{Res} \sim \cos\left(\frac{\pi}{2} + \delta\theta_k - 3\theta_k\right)$
- Interaction between symmetry breaking set by ∇V_{\parallel} and by magnetic shear depends on instability regimes
 - In marginal regime:
 - $\rightarrow \Pi_{r_{\parallel}}^{Res}$ primarily set by geometrical symmetry breaking mechanisms
 - \rightarrow Ratio $\Pi_{r_{\parallel}}^{Res} / \chi_{\phi}$ independent from ∇V_{\parallel}
 - Coexistence of PSFI and ITG turbulence
 - In ITG regime:
 - $\rightarrow \Pi_{r_{\parallel}}^{Res}$ primarily set by geometrical symmetry breaking mechanisms
 - $\rightarrow -|\chi_{\phi}^{Res}|$ enhances V_{\parallel}' profile
 - In PSFI regime:
 - $\rightarrow V_{\parallel}'$ saturated by PSFI $\rightarrow V_{\parallel}'$ stays at the PSFI threshold