On the Physics of Intrinsic Flow in Plasmas Without Magnetic Shear

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Outline

• Merits of weak magnetic shear + rotation for confinement
• Question: intrinsic rotation with weak shear?
• CSDX: a test bed for intrinsic flows in unsheared magnetic fields
  • Intrinsic axial flow
• New mechanism for intrinsic flows
  • No requirement for magnetic symmetry breaking
  • Builds on electron drift wave turbulence
• Broader lesson for tokamaks
  • Not limited to weak shear regimes
  • Outcome: enhanced $\nabla \langle v_\phi \rangle$
• Future work: flow boundary layer $\leftrightarrow$ ion-neutral coupling
Why “weak shear” profile?

• Long known:
  • weak or reversed shear beneficial for confinement (ERS, NCS, WNS, ...)
    • B.W. Rice (DIII-D), PPCF, 1996;
    • E.J. Synakowski (TFTR), PoP, 1997;
    • M. Yoshida (JT-60U), NF, 2015...

• De-stiffened, ITB-like state observed for “weak shear” (JET)
  • P. Mantica, PRL, 2011;
  • C. Gormazano, PRL, 1998, etc.

• For more, see previous talks by Dr. Turco, Dr. Pan.
Intrinsic Rotation in Weak Shear Profiles

• JET: Weak shear AND Rotation $\rightarrow$ Enhanced confinement

• But external torque limited in ITER

• So need understand: Intrinsic rotation in weak shear regimes

• Important for:
  • Total effective torque
    \[ \tau = \tau_{ext} + \tau_{intr} \]
  • Contribution to $V_{E \times B}$

[P. Mantica, PRL, 2011; Rice, PRL, 2013]

FIG. 4 (color online). $q_i^{GB}$ vs $R/L_{Ti}$ at $\rho_{tor} = 0.33$ for similar plasmas with different rotation and $s$ values.
Intrinsic Rotation in ITB

- Intrinsic rotation: self-accelerated toroidal rotation
- Discovered in JFT-2M, C-Mod (~95°)
- During ITB formation:
  - $\tau_{intr}$ in the core flip sign
  - Build up from edge
- $\tau_{intr} \sim \tau_{ext}$
- Strong coupling between heat transport and momentum transport
- Consistent with $\Delta \langle v_\phi \rangle \sim \nabla T, \nabla P$

[Ida (LHD), PRL, 2013]
Intrinsic Rotation in Weak Shear

• Weak shear ($q' \to 0$)

• External torque $\approx 0$

Intrinsic Rotation?

• Beneficial for confinement and stability. How much?

Status:

• Results
  • GK Simulation: stronger intrinsic rotation at weaker magnetic shear

• Problem
  • Intrinsic rotation requires symmetry breaking
  • Conventional symmetry breaking models fail
  • Most involve magnetic shear
  • But weak shear
    $\to$ non-resonant mode structure!

[Kwon, NF (2012); Z.X. Lu, NF&PoP, 2015]
A conceptual Model of Intrinsic Rotation: Heat Engine

• Car motion vs plasma rotation [Kosuga, PoP, 2010]:

<table>
<thead>
<tr>
<th></th>
<th>Car</th>
<th>Intrinsic Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel</td>
<td>Gas</td>
<td>Heating $\rightarrow \nabla T, \nabla n_0$</td>
</tr>
<tr>
<td>Conversion</td>
<td>Burn</td>
<td>$\nabla T, \nabla n_0$ driven turbulence</td>
</tr>
<tr>
<td>Work</td>
<td>Cylinder</td>
<td>Symmetry breaking</td>
</tr>
<tr>
<td>Result</td>
<td>Wheel rotation</td>
<td>Flow</td>
</tr>
</tbody>
</table>

• Plasma rotation:

Heat flux $\rightarrow$ Turbulence $\rightarrow$ Symmetry breaking

Residual stress $\Pi^{Res}_{\parallel} (\nabla T, \nabla n_0)$

Intrinsic rotation $\langle v_\phi \rangle' \sim \frac{\Pi^{Res}_{\parallel} (\nabla T, \nabla n_0)}{\chi_\phi}$

Usually requires magnetic shear
Details: Conventional Wisdom of Intrinsic Rotation

• Self-acceleration by intrinsic torque due to residual stress
  \( \tau_{intr} = -\nabla \cdot \Pi^{Res} \)
  \[
  \langle \tilde{v}_r \tilde{v}_\parallel \rangle = -\chi \phi \frac{d\langle v_\parallel \rangle}{dr} + V_P \langle v_\parallel \rangle + \Pi^{Res}_{r\parallel}
  \]

• Residual stress \( \Pi^{Res}_{r\parallel} \)
  • Driven by turbulence, i.e. \( \Pi^{Res}_{r\parallel} \sim \nabla P, \nabla T, \nabla n_0 \)

• \( \Pi^{Res}_{r\parallel} \sim \langle k_\theta k_\parallel \rangle \) requires symmetry breaking in \( k \) space

• Symmetry breaking usually relies on magnetic shear

• Rotation builds up from edge, driven by \( \Pi^{Res}_{r\parallel} \) at edge

[W.X. Wang, PRL, 2009]
A Simple Example

• $k_\parallel = k_\theta \frac{x}{L_s} \rightarrow \langle k_\theta k_\parallel \rangle \sim k_\theta^2 \frac{\langle x \rangle}{L_s}$

• $\langle x \rangle$: averaged distance from mode center to rational surface

• $\langle x \rangle$ set, in simple models, by:
  • $E'_r$: centroid shift
  • $I'(x)$: spatial dispersion of envelope

• What of weak shear ($q' \to 0$)?

Intrinsic Parallel Flow in Linear Device

• Controlled Shear Decorrelation Experiment (CSDX)

• Straight, uniform magnetic field in axial direction

• Ideal test bed for studying intrinsic flows in unsheared magnetic fields

Device of CSDX.
More Generally: Why study linear device?

• Correspondence between CSDX and tokamaks:

<table>
<thead>
<tr>
<th>Tokamaks</th>
<th>CSDX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toroidal field structure usually sheared</td>
<td>Uniform axial magnetic field (shear-free)</td>
</tr>
<tr>
<td>Intrinsic toroidal rotation</td>
<td>Intrinsic axial flow</td>
</tr>
<tr>
<td>Rotation boundary condition set by SOL</td>
<td>Axial flow boundary condition set by boundary neutral layer</td>
</tr>
<tr>
<td>L-H transition</td>
<td>Transport bifurcation driven by $\nabla n_0$</td>
</tr>
<tr>
<td>Inward pinch; density peaking</td>
<td>$n_0$ profile steepening; localized net inward particle flux</td>
</tr>
</tbody>
</table>

[Cui, PoP, 2015, 2016]
Observation

• Intrinsic axial flow evident (without momentum input)
• Barrier formed when $B > B_{crit}$
• $\nabla \langle v_z \rangle$ steepening occurs with $\nabla n_0$, $\nabla P_i$ steepening $\rightarrow$ barrier formation

$\langle v_z \rangle$ profile evident, steepens during transition

[Cui, PoP, 2015&2016; TTF talk, 2015]
Intrinsic $\nabla \langle v_z \rangle$ tracks $L_n^{-1}$

- Axial flow in CSDX:
  - $\langle v_z \rangle' \sim \frac{1}{n_0} \nabla n_0$
  - $\nabla n_0$ is free energy source

- Recall
  - Intrinsic rotation in C-Mod:
    - $\Delta \langle v_\phi \rangle \sim \nabla T$ [Rice, PRL, 2011]
Axial Reynolds Power Tracks $L_n^{-1}$

- Total Reynolds power:
  \[ P_{Res}^{tot} = -\int \langle \tilde{v}_r \tilde{v}_z \rangle' \langle v_z \rangle r \, dr \]

- Total power coupled to $\nabla \langle v_z \rangle$ from fluctuations

- Axial Reynolds power rises with $1/L_n$ (for $B < B_{crit}$)

- Evidence for direct connection of fluctuations with intrinsic flow
Theory

• Dynamical Symmetry Breaking

• No requirement on specific magnetic field structure
  → aspects relevant to both weak shear, and standard configurations

• Electron drift waves (CTEM → ITER relevant)
Electron Drift Wave System

• System equations:

\[
\frac{D}{Dt} n_e + \frac{1}{L_n} \frac{1}{r} \frac{\partial \phi}{\partial \theta} + \frac{\partial v_{e,z}}{\partial z} = 0
\]

\[
\frac{D}{Dt} \nabla^2 \phi = \frac{\partial}{\partial z} (v_z - v_{e,z})
\]

\[
\frac{D}{Dt} v_z - \langle v_z \rangle' \frac{1}{r} \frac{\partial \phi}{\partial \theta} = - \frac{\partial n_e}{\partial z}
\]

\[
\left( \frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla \right)
\]

• Non-adiabatic electrons: \( n_e \equiv (1 - i\delta)\phi \)

\[
\delta \equiv \frac{\nu_{ei}(\omega_* - \omega)}{k_z^2 \nu_{The}^2}, \text{ with } 1 < \frac{k_z^2 \nu_{The}^2}{\nu_{ei} \omega} < \infty
\]

• Dispersion relation

\[
1 + k_\perp^2 \rho_s^2 - i\delta - \frac{\omega_*}{\omega} + k_\theta k_z \rho_s c_s \langle v_z \rangle' \frac{1}{\omega^2} - (1 - i\delta) \frac{k_z^2 c_s^2}{\omega^2} = 0
\]
Dynamical Symmetry Breaking

- Growth rate ~ frequency shift:

\[ \omega_k \approx \frac{\omega_*}{1 + k_{\perp}^2 \rho_s^2} - \frac{k_\theta k_z \rho_s c_s \langle \nu_z \rangle'}{\omega_*} \]

\[ \gamma_k \approx \frac{\nu_{ei}}{k_z^2 v_{Te}^2} \left( \frac{\omega_*}{1 + k_{\perp}^2 \rho_s^2} \right) \left( \frac{k_{\perp}^2 \rho_s^2}{1 + k_{\perp}^2 \rho_s^2} \right) + \frac{k_\theta k_z \rho_s c_s \langle \nu_z \rangle'}{\omega_*} \]

- Spectral imbalance:

\[ \delta \langle \nu_z \rangle' < 0 \]

Modes with \( k_\theta k_z < 0 \) grow faster than other modes,

\[ \gamma_k \big|_{k_\theta k_z < 0} > \gamma_k \big|_{k_\theta k_z > 0} \]

Spectral imbalance in \( k_\theta k_z \) space

\[ \langle k_\theta k_z \rangle < 0 \rightarrow \Pi_{\tau Z}^{Res} \neq 0 \]

\{k±\}: Domains where modes grow faster/slower

Spectral imbalance
Negative Viscosity *Increment* 

- Reynolds stress:  $\langle \tilde{v}_r \tilde{v}_z \rangle = -\chi_\phi \langle v_z \rangle' + \Pi_{rz}^{\text{Res}}$

- Turbulent momentum diffusivity:

$$\chi_\phi = \sum_k \frac{\nu_{ei}}{k_z^2 v_{\text{The}}^2} \frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2} k_\parallel^2 \rho_s^2 |\phi_k|^2$$

- Residual stress $\rightarrow$ Negative viscosity *increment*

- $\delta \Pi_{rz}^{\text{Res}} = |\chi_\phi^{\text{Inc}}| \delta \langle v_z \rangle'$ [Li et al, submitted to PoP, 2016]
Modulational Enhancement of $\delta \langle \nu_z \rangle'$

- $\delta \langle \nu_z \rangle' \rightarrow \Pi^{Res} \rightarrow \chi_{\phi}^{tot} = \chi_{\phi} - |\chi_{\phi}^{Inc}|$

- Dynamics of $\delta \langle \nu_z \rangle'$:
  \[
  \frac{\partial}{\partial t} \delta \langle \nu_z \rangle' + \frac{\partial^2}{\partial r^2} \left( \delta \Pi_{rz}^{Res} - \chi_{\phi} \delta \langle \nu_z \rangle' \right) = 0
  \]

- Growth rate of flow shear modulation
  \[
  \gamma_q = -q_r (\chi_{\phi} - |\chi_{\phi}^{Inc}|)
  \]

- $\chi_{\phi}^{tot} < 0 \rightarrow \text{Modulational growth of } \delta \langle \nu_z \rangle'$

- Feedback loop: $\delta \langle \nu_z \rangle' \rightarrow \Pi^{Res} \rightarrow -|\chi_{\phi}^{Inc}|$
Upper Limit of $\langle v_z \rangle'$ Set by PSFI

- Parallel shear flow instability (PSFI)

- Driven by $\nabla \langle v_z \rangle$, negative compressibility (similar to ITG)

\[
\chi^\text{tot}_\phi = \chi^\text{DW}_\phi - \left| \chi^\text{Inc}_\phi \right| < 0
\]

\[
\chi^\text{tot}_\phi = \chi^\text{DW}_\phi + \chi^\text{PSFI}_\phi \Theta(\langle v_z \rangle - \langle v_z \rangle_{\text{crit}}) - \left| \chi^\text{Inc}_\phi \right| > 0
\]
Parallel shear flow instability

• Growth rate and resulting turbulent momentum diffusivity:

\[
\gamma_{k,PSFI} \approx \sqrt{\frac{k_\theta k_z \rho_s c_s \left( \langle v_z \rangle' - \langle v_z \rangle_{crit} \right)}{1 + k^2 \rho_s^2}}
\]

\[
\chi_{\phi,PSFI} \approx \sum_k |\phi_k|^2 k_\theta^2 \rho_s^2 \frac{4(1 + k^2 \rho_s^2)^2}{\omega_*^2} \sqrt{\frac{k_\theta k_z \rho_s c_s \left( \langle v_z \rangle' - \langle v_z \rangle_{crit} \right)}{1 + k^2 \rho_s^2}}
\]

• Hit PSFI threshold \( \rightarrow \chi_{\phi,PSFI} \text{ nonlinear in } \nabla \langle v_z \rangle \rightarrow \chi_{\phi}^{tot} > 0 \)

• \( \delta \langle v_z \rangle' \rightarrow \Pi^{Res} \rightarrow \delta \langle v_z \rangle' \text{ growth } \leftrightarrow \text{ Saturated by PSFI} \)

\[
\chi_{\phi}^{tot} = \chi_{\phi}^{DW} - |\chi_{\phi}^{Inc}| < 0
\]

\[
\chi_{\phi}^{tot} = \chi_{\phi}^{DW} + \chi_{\phi}^{PSFI} - |\chi_{\phi}^{Inc}| > 0
\]
# Comparing Symmetry Breaking Mechanisms

## Standard Symmetry Breaking vs. Dynamical Symmetry Breaking

<table>
<thead>
<tr>
<th></th>
<th>Standard Symmetry Breaking</th>
<th>Dynamical Symmetry Breaking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Free Energy</strong></td>
<td>$\nabla T, \nabla n_0$</td>
<td>$\nabla n_0$</td>
</tr>
<tr>
<td><strong>Symmetry Breaker</strong></td>
<td>$E'_r, I(x)'$, etc.</td>
<td>Test axial flow shear, $\delta \langle v_z \rangle'$; No requirement of $B$ structure.</td>
</tr>
<tr>
<td></td>
<td>Linked to magnetic shear.</td>
<td></td>
</tr>
<tr>
<td><strong>Effect on the Flow</strong></td>
<td>Intrinsic torque, $-\partial_r \Pi^\text{Res}_r$</td>
<td>Negative viscosity increment, $-</td>
</tr>
<tr>
<td><strong>Flow Profile</strong></td>
<td>$\langle v_\phi \rangle' = \frac{\Pi^\text{Res}<em>r}{\chi</em>\phi}$</td>
<td>$\langle v_\phi \rangle' = \frac{R \ast \tau_{\text{ext}} + \Pi^\text{Res}}{\chi_\phi(\nabla n_0, \nabla \langle v_| \rangle) -</td>
</tr>
</tbody>
</table>

**Flow Profile**

- $\langle v_\phi \rangle' = \frac{\Pi^\text{Res}_r}{\chi_\phi}$
- $\langle v_\phi \rangle' = \frac{R \ast \tau_{\text{ext}} + \Pi^\text{Res}}{\chi_\phi(\nabla n_0, \nabla \langle v_\| \rangle) - |\chi_\phi^{\text{Inc}}|}$
Comparison (cont’d)

• Feedback Loop:

Test flow shear $\delta\langle v_z \rangle'$ → Spectral imbalance

Self-amplification

Driven by $\nabla n_0$

Intrinsic flow, feedback on $\delta\langle v_z \rangle' \leftarrow$ Residual stress $\Pi_{rz}^{Res}$

Dynamical Symmetry Breaking
(similar to zonal flow)

Conventional Models

Heat flux → $\nabla T +$ geometry (magnetic shear)

Open loop

$\langle v_{||} \rangle' \leftarrow \Pi_{r||}^{Res}$
Also relevant to Tokamaks

\[ \nabla \langle \nu_\phi \rangle \text{ steepening and } \Pi^{Res} \text{ can act in synergy} \]

\[ \text{Rotation profile gradient enhanced by negative viscosity effect} \]

\[ \langle \nu_\phi \rangle' \sim \frac{\text{Drive}}{\chi_D^{DW} + \chi_D^{PSFI} \Theta(\langle \nu_\parallel \rangle' - \langle \nu_\parallel \rangle'_{\text{crit}}) - |\chi_I^{Inc}|} \]

\[ \text{Drive can be external (} \tau_{\text{NBI}} \text{) or intrinsic (} \Pi^{Res} \text{)} \]
Future Work

• Boundary condition matter

• Flow boundary controlled by neutral particles

• For simple analysis: No-slip boundary condition due to friction

• In tokamaks: Interaction with SOL
Boundary Dynamics Impacts Flow Profile

- Evolution of net axial ion flow:

\[
\frac{\partial}{\partial t} \int_0^R dr \langle v_{i,z} \rangle = \int_0^R dr \frac{\Delta P_i}{\rho_0 L} - \langle \tilde{v}_r \tilde{v}_z \rangle \bigg|_R - \int_{r_b}^R dr \nu_{ni} (\langle v_{i,z} \rangle - \langle v_{n,z} \rangle)
\]

(a) No external source/sink. Net flow = 0.

(b) No-slip at wall \( \rightarrow \) \( v_z \approx 0, \langle \tilde{v}_r \tilde{v}_z \rangle \approx 0 \). Net flow > 0.

(c) \( \Delta P_i \) drive at center, outflux at wall.
Boundary Layer

- Partially ionized $\rightarrow$ Neutral flow within BL
- Neutral flow dynamics (can be solved numerically by BOUT++):

$$\rho_n \left( \frac{\partial \mathbf{v}_n}{\partial t} + \mathbf{v}_n \cdot \nabla \mathbf{v}_n \right) = -\nabla P_n + \rho_n \nu_{ni} \left( \mathbf{v}_i - \mathbf{v}_n \right)$$

- Within neutral layer: $\langle \nu_{i,z} \rangle \approx \langle \nu_{n,z} \rangle$
- Neutral flow within BL sets boundary condition for plasma flows

[Z.H. Wang et al, NF, 2014]
Summary

• We know:
  • Weak shear + rotation $\rightarrow$ beneficial for confinement
  • Intrinsic toroidal rotation exists in weak shear regions

• Question:
  • Intrinsic rotation generation for weak shear $\rightarrow q' \rightarrow 0$?

• New mechanism for intrinsic axial flow generation
  • CSDX: a test bed for intrinsic flows with unsheared magnetic fields
  • No requirement on magnetic shear $\rightarrow$ Broader lesson
Summary (cont’d)

• Results:
  • Dynamical symmetry breaking
  • Negative viscosity increment induced by $\Pi^{Res}$
    • $\delta \Pi^{Res} = |\chi^\text{Inc}_\phi| \delta \langle v_z \rangle'$
    • Total viscosity: $\chi^\text{tot}_\phi = \chi_\phi - |\chi^\text{Inc}_\phi|$
    • $\chi^\text{tot}_\phi < 0 \rightarrow$ Modulational growth of $\delta \langle v_z \rangle'$
  • Flow profile: pipe flow analogy
    • Balance between drive and viscosity
    • In CSDX: $\langle v_z \rangle' \sim \Delta P_i / \chi^\text{tot}_\phi$
Summary (cont’d)

• Broader lesson for tokamaks
  • Synergy of $\langle v_\phi \rangle'$ self-amplification and $\Pi^{Res}$
  • $\langle v_\phi \rangle'$ driven by $\tau_{NBI}$, $\Pi^{Res}(\nabla n_0, \nabla T)$
  • $\langle v_\phi \rangle'$ enhanced by $-|\chi^{Inc}_\phi|$  

• Future work: flow boundary condition
  • Ion-neutral coupling within boundary layer
List of Related Talks/Posters

• April 1\textsuperscript{st}:
  • A. Ashourvon, “On the Structure of the Zonal Shear Layer Field and its Implication for Multi-scale Interactions”

• March 31\textsuperscript{st}:
  • R. Hajjar, “Modelling Transport Bifurcations in the CSDX Linear Device”
  • P. Vaezi, “Nonlinear Simulation of CSDX Including Sheath Physics”

• March 30\textsuperscript{th}:
  • S.C. Thakur, “Spontaneous self-organization from drift wave plasmas to a mixed ITG-drift wave-shear flow system via a transport bifurcation in a linear magnetized plasma device”
  • R. Hong, “Effects of Density Gradient on Axial Flow Structures in a Helicon Linear Plasma Device”
Back-up
Heat Engine

- Efficiency of intrinsic rotation generation by turbulence
- Entropy evolution:

\[
\partial_t S_0 = \int d^3x \left[ n\chi_i \left( \frac{\nabla T}{T} \right)^2 - nK \frac{\langle V_E \rangle'}{v_{thi}^2} \right. \\
\left. + n\chi_\phi \frac{\langle V_\parallel \rangle'}{v_{thi}^2} - n \frac{\Pi_{r\parallel}^{res}}{v_{thi}^2 \chi_\phi} \right].
\]

- Efficiency $\sim \frac{\text{Entropy destruction due to flow generation}}{\text{Net production due to thermal relaxation}}$

\[
e \equiv \frac{\left| \int d^3x P_{flow} \right|}{\int d^3x P_{net}} \quad \Rightarrow \quad e_{IR} \equiv \frac{\int d^3x n(\Pi_{r\parallel}^{res})^2/(v_{thi}^2 \chi_\phi)}{\int d^3x n\chi_i (\nabla T/T)^2}.
\]
Non-resonant mode structure

(a) Coupling between nearby radii

(b) Coupling to distant radii

FIG. 1. Cartoon of the spatial extent of mode-mode interactions (a) among resonant modes only and (b) involving non-resonant modes.

• Non-resonant mode structure [S. Yi, PoP, 2012]
Intrinsic rotation

• Discovered in JFT-2M, Alcator C-Mod plasmas

Steady state toroidal momentum in counter-current injection of NBI is two to three times larger than that in co-current injection. [Ida, JFT-2M, 1995]

Time histories of (impurity) toroidal rotation for a 2.0 MW ICRF heated EDA H-mode plasma [Rice, C-Mod, 2004]
CSDX Parameters

• These experiments were carried out in the Controlled Shear Decorrelation Experiment, which is a 2.8 m long linear helicon plasma device with a source radius 7.5 cm, 1.6 kW RF power input (reflected power less than 30 W), and a gas fill pressure of 3.2 mTorr.
Energy Transfer Ratio

- Axial transfer ratio:
  \[ (R_T)_A = \frac{P^{Re}}{\langle \tilde{v}_Z^2 \rangle} \tau_{ac} \]
- \((R_T)_A\) decreases when \(B > B_{crit}\)
- \(B_{crit} \approx 900G\)
Residual Stress

- Reynolds stress: \[ \langle \tilde{u}_r \tilde{u}_z \rangle = -\chi_\phi \langle v_z \rangle' + \Pi_{rz}^{\text{Res}} \]

- Turbulent diffusivity:
  \[
  \chi_\phi = \sum_k \frac{\nu_{ei}}{k_z^2 v_{The}^2} \frac{k^2 \rho_s^2}{1 + k^2 \rho_s^2} \frac{\kappa^2 \rho_s^2 |\phi_k|^2}{\phi_k} 
  \]

- Residual stress
  \[
  \Pi_{rz}^{\text{Res}} = \operatorname{Sign} (\delta \langle v_z \rangle') \sum_{\{k+\}} \frac{\nu}{k^2 v_{The}^2} (2 + k^2 \rho_s^2) \frac{k^2 \rho_s^2}{1 + k^2 \rho_s^2} |k_y| \rho_s c_s \Delta I_k (\delta \langle v_z \rangle')
  \]

  \[
  \Delta I_k (\delta \langle v_z \rangle') \equiv |\phi_k|^2 \biggr|_{\{k+\}} - |\phi_k|^2 \biggr|_{\{k-\}} \quad \text{Spectral imbalance} \sim \delta \langle v_z \rangle'
  \]
Parallel Shear Flow Instability

• PSFI: recall dispersion relation of the model with adiabatic electrons:

\[(1 + k_{\perp}^2 \rho_s^2) \omega^2 - \omega_\ast \omega + k_\theta k_z \rho_s c_s \langle v_z \rangle' - k_z^2 c_s^2 = 0\]

• Unstable ⇔ discriminant = \[\omega_\ast^2 - 4(1 + k_{\perp}^2 \rho_s^2)(k_\theta k_z \rho_s c_s \langle v_z \rangle' - k_z^2 c_s^2) < 0\]

• \[\langle v_z \rangle' > \langle v_z \rangle'_{\text{crit}} \equiv \frac{1}{k_\theta k_z \rho_s c_s} \left[ \frac{\omega_\ast^2}{4(1 + k_{\perp}^2 \rho_s^2)} + k_z^2 c_s^2 \right] \rightarrow \text{PSFI}\]

• With non-adiabatic electrons

\[\langle v_z \rangle'_{\text{crit}} = \frac{1}{k_\theta k_z \rho_s c_s} \left[ \frac{\omega_\ast^2(1 + k_{\perp}^2 \rho_s^2)}{4[(1 + k_{\perp}^2 \rho_s^2)^2 + \delta^2]} + k_z^2 c_s^2 \right]\]
Parallel Shear Flow Instability

- Negative compressibility

\[
(1 + k_{\perp}^2 \rho_s^2) \omega^2 - \omega^2 \omega + k_\theta k_z \rho_s c_s \langle v_z \rangle' - k_z c_s^2 = 0
\]

\[
\omega^2 - 4(1 + k_{\perp}^2 \rho_s^2)(k_\theta k_z \rho_s c_s \langle v_z \rangle' - k_z c_s^2) < 0
\]

\[
k_\theta k_z \langle v_z \rangle' > 0
\]

\[
\omega^2 \sim \left(1 - \frac{k_\theta \rho_s}{k_z c_s} \langle v_z \rangle'\right) k_z c_s^2
\]

Negative compressibility
Flow Structure

• Pipe flow analogy

Pipe flow:

Pressure drop $\Delta P$

Viscous Sublayer (No-slip Boundary Condition)

Prandtl $\mathbf{v}_G / v_G d \cong 0$ at boundary

CSDX:

Ion pressure drop $\Delta P_i$

Neutral Edge Boundary Layer

Neutral Gas Input $\rho_0 \langle \tilde{v}_r \tilde{v}_z \rangle$
Flow Structure

• With external drive $\Delta P_i$
  
  • Don’t need modulational growth to generate intrinsic flow
  
  • $-|\chi_{\phi}^{Inc}|$ enhances flow gradient

• Momentum balance:

• $\langle v_z \rangle' \sim \frac{\Delta P_i}{\chi_{\phi}^{DW} + \chi_{\phi}^{PSFI} \theta(\langle v_z \rangle' - \langle v_z \rangle'_{\text{crit}}) - |\chi_{\phi}^{Inc}|}$

• $\langle v_z \rangle'$ is kept at or below $\langle v_z \rangle'_{\text{crit}}$ due to $\chi_{\phi}^{PSFI}$
Applications to Tokamaks

• Weak shear

• Initial flow shear (seed)
  \( \Rightarrow \Pi^{Res} \) by dynamical symmetry breaking
  \( \Rightarrow \) Accelerate rotation

• Need modulational growth of \( \delta \langle \nu_\phi \rangle' \)

• OR weak net NBI torque (external) + negative viscosity increment
  \( \Rightarrow \langle \nu_\phi \rangle' \) enhanced by \(-|\chi_\phi^{Inc}|\)