L-H Transition Threshold Physics at Low Collisionality

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An L→H power threshold scaling including the minimum in $P_{th}(n)$ is discussed, elucidating the impact of inter-species energy transfer on threshold physics. Using a new four-field LH transition model, we study transitions in collisionless, electron heated regimes where the electron-ion coupling is allowed to be completely anomalous, mediated by the fluctuation of $\langle \mathbf{E} \cdot \mathbf{J} \rangle$ work on electrons and ions. New transition scenarios, characterized by the sensitivity of transition evolution to pre-existing L-mode profiles are also considered, using the new model.
Motivation: Physics of L→H Threshold

Present

- Origin of $P_{th}(n)$ minimum?
- Collisional electron-ion coupling (Ryter) especially electron heating
  - $P_{th}$ set only by edge local physics? $a^2/\chi\tau_{equil} \geq 1$? coupling $\iff$ global dependence?
  - Connection to LOC-SOC transition

Looking Ahead

- Collisionless, electron heated plasmas?
  - Coupling anomalous $\langle E \cdot J_{e,i}\rangle$ transfer via fluctuations
  - Shear flow regulation – no collisional drag?
- How does $\nabla P_i|_{\text{edge}}$ rise?

Ryter [1]

Rice[2]
Introduction

- H-mode operation [3, 4, 5, 6] is the regime of choice for good confinement.
- Issues:
  - optimum access to, and efficient sustainability, of H-mode [7, 8]
  - $L \rightarrow H$ transition power threshold, understanding its minimum
  - related problem of hysteresis
- ITER-specific transitions:
  - understand threshold in low collisionality, electron heated regimes
  - (deep) relation to $P_{th \, min}$
- low-collisionality transitions requires model extension beyond collisional $e-i$ coupling
- discussed in this paper:
  - $L \rightarrow H$ power threshold scaling and the origin of $P_{th \, (n)-min}$
  - transitions in collisionless, electron heated regimes
  - new transition scenarios, characterized by sensitivity to pre-existing L-mode profiles
Previous Model

- $k - \epsilon$ for evolution of intensity, shear flow field, $n, T_i, \langle V_{\theta} \rangle$
- shear flow damped by drag
- Separated Species:

$$\frac{\partial T_e}{\partial t} + \text{transport} = Q_e - \text{collisional transfer} - \text{collisionless coupling}$$

$$\frac{\partial T_i}{\partial t} + \text{transport} = Q_i + \text{collisional transfer} + \text{collisionless coupling}$$

- collisionless shear flow damping
- collisionless heating (due shear flow)

$$\gamma_{SF} = \gamma_{visc} \left( \frac{\partial \sqrt{E_0}}{\partial r} \right)^2 + \gamma_{Hvisc} \left( \frac{\partial^2 \sqrt{E_0}}{\partial r^2} \right)^2$$

- $\sqrt{E_0}$ stands for ZF velocity: two contributions come from viscose and hyperviscose ZF damping and corresponding ion heating
Results with Collisional Transfer

- Critical parameter:
  Heating mix

  \[ H_{i/i+e} = \frac{Q_i}{Q_i + Q_e} \equiv H_{mix} \]

- Relating \( H_{mix} \) and \( n \) by monotonic \( H_{mix}(n) \) recovers \( P_{\text{thr}}(n) \) min

- \( P_{\text{thr}}(n) \) minimum recovered only when both \( n \) and \( H_{mix} \) evolved

- 3D curve \( P_{\text{thr}}(n, H_{mix}) \), projected on \( (n, P_{\text{thr}}) \)-plane (top plot) has a minimum
Collisionless, Electron Heated Regimes—Predictions

- Coupling anomalous $\langle J_{i,e} \cdot E \rangle$, not $\propto T_e - T_i$
- Transition mechanism: anomalous $e \rightarrow i$ thermal equilibration front ($e$—cooling front, left Figure)

Transition occurs when $P_i$—front ($i$-heating front, right Figure) approaches edge $\Rightarrow$ triggers increase in ion $\nabla P_i$

New Scenario!, Prediction.
New Model

Motivated by:
- complexity of transitions studied within the previous 6-field model [9, 10]
- strong requirements for capturing sharp fronts

Features:
- 4-field model ($T_{e,i}$, DW, ZF) allows to explore new transition scenarios
- adaptive mesh refinement, high-fidelity collocation scheme

Aimed at:
- understanding low-collisionality transition
- spontaneous transition in the absence of turbulence driven shear flow
- sensitivity of the transition to the pre-existing L-mode density profile
- optimizing access to H-mode
- mapping basins of attraction for different transitions
transitions in collisionless, electron heated regimes where the electron-ion coupling is allowed to be completely anomalous, due to the fluctuation of $\langle \mathbf{E} \cdot \mathbf{J} \rangle$ work on electrons and ions

New transition scenarios, characterized by the sensitivity of transition evolution to pre-existing L-mode profiles are considered (Figure, upper-left corner).
Model New Capabilities and Equations

- the shear flow damping is turbulent, and not just due to collisional drag
- nonlinear flow damping leads to additional turbulent viscous heating of the ions

Equations evolve $T_e, i$, DW intensity, $I$, and ZF velocity $W$:

$$\frac{\partial T_e}{\partial t} = \frac{\partial}{\partial x} \left( \frac{I}{1 + \alpha_t R} + \chi_{neo}^e \right) T_e' - \frac{1}{\tau} (T_e - T_i) + S_e' + \gamma_0 I (\kappa_n + \sigma T_e'/T_e)$$

$$\frac{\partial T_i}{\partial t} = \frac{\partial}{\partial x} \left( \frac{I}{1 + \alpha_t R} + \chi_{neo}^i \right) T_i' + \frac{1}{\tau} (T_e - T_i) + S_i' - \gamma_0 I (\kappa_n + \sigma T_e'/T_e) + \gamma_v |W|^2$$

$$\frac{\partial I}{\partial t} = (\gamma_L - \Delta \omega I - \alpha_0 W^2/2 - \alpha_v R) I + \chi_N (I''^2 + I \cdot I'')$$

$$\frac{\partial W}{\partial t} = \frac{\alpha_0 I W}{1 + \zeta_0 R} - \gamma_c W + \gamma_v (I' W' + IW'')$$
Initial study: suppression factor $R$ obtained for strong thermal $e - i$ coupling ($T_e \approx T_i$) anomalous and collisional (will relax in the next phase)

$$R = \left[ \kappa_n (\kappa_n T + T') \right]^2, \text{ with } T \equiv T_i + T_e.$$  

- notation/units: $T' = \nabla T$, $T/\sqrt{MC_s^2}$, $\kappa_n = L_n^{-1}$, length in min. rad., $a$
- heat sources for electrons and ions (at $x \approx a_{e,i}$)

$$S'_{e,i} = \frac{2S'_{0e,i}}{\sqrt{\pi} D_{e,i}} [\text{erf} ((1 - a_{e,i})/D_{e,i}) + \text{erf} (a_{e,i}/D_{e,i})] \exp \left[ - \left( \frac{x - a_{e,i}}{D_{e,i}} \right)^2 \right]$$

- ITG and CTEM contributions to growth of DW:

$$\gamma_L = \sqrt{T_e} \left[ \gamma_{L0} \Re T_i'/T_i - T'_{i0} - \gamma_{e0} \left( \kappa_n + \sigma T_e'/T_e \right) \right]$$

- shear flow velocity in suppression factor

$$V_E = (c/eBn) p', \quad p = n(T_e + T_i), \quad \langle V_E \rangle' \approx - (c/eB) \kappa_n (\kappa_n T + \kappa_n)$$
Stationary Analytic Solutions

- limit of small $\tau \to 0$, $T_e = T_i + O(\tau) \approx T/2$
- turbulent components sit at their thresholds: $\gamma_v = \chi_N = 0$
- steady state solution for $T_e, i(x)$, $I$ and $W$, obtained from Eqs. on p.10 by setting $\partial_t = 0$ (saturated instabilities for $I$ and $W$)

\[
\left(\frac{\zeta_0 - \alpha_t}{1 + \alpha_t R} R + \chi\right) T' + \frac{\alpha_0}{2\gamma_c} (\chi_i - \chi_e) \Delta T' - S = \text{const}
\]

where $\Delta T = T_i - T_e$, $\partial S/\partial x = -\alpha_0 (S'_e + S'_i) / \gamma_c$ and $\chi = 1 + (\chi^i_{\text{neo}} + \chi^e_{\text{neo}}) \alpha_0/2\gamma_c$

- Assuming $|\Delta T| \ll T$

\[
\left[\chi - \frac{\alpha (\kappa_n T + T')^2}{1 + \omega (\kappa_n T + T')^2}\right] T' = S(x)
\]

(1)

here $\alpha = (\alpha_t - \zeta_0) \kappa_n^2$ and $\omega = \alpha_t \kappa_n^2$, $S(0) = 0$. 

Flux-driven transitions

- sources $S'_{e,i}(x)$ are localized near the origin ($x = 0$, core plasma)
- $S = \text{const}$, for all $x > 0$, flux-driven transitions
- solutions $T = T_e + T_i = T(x)$ depends on five parameters
  - $\chi$, $\alpha$, $\omega$, $\kappa_n$ and $S$.
  - after rescaling: $\kappa_n x \rightarrow x$, $S/\kappa_n \rightarrow S$, $\kappa_n^2\alpha \rightarrow \alpha$, $\kappa_n^2\omega \rightarrow \omega$, obtain simplified bifurcation problem

\[
\left[ \chi - \frac{\alpha (T + T')^2}{1 + \omega (T + T')^2} \right] T' = S = \text{const} \tag{2}
\]

solve for $T$ as a function of $T'$ and three parameters, $a$, $b$, $c$

\[
T = c \sqrt{\frac{T' - a}{b - T'}} - T' \tag{3}
\]

- $a = S/\chi$, $b = S/(\chi - \alpha/\omega)$, $c = (\omega - \alpha/\chi)^{-1/2}$

- resolving above eq. for $T(x) \implies$ solution multiplicity, bifurcation in ($a$, $b$, $c$) parameter space
Phase coexistence and bifurcation diagram

- solution is easily obtained in terms of \( x(T') \)

\[
x(T') = x_0 - \ln |T'| + \frac{c}{b} \sqrt{\frac{T' - a}{b - T'}} + \frac{c(b - a)}{2b\sqrt{ab}} \left[ \tan^{-1} \frac{T' - \sqrt{ab}}{\sqrt{(b - T')(T' - a)}} - \tan^{-1} \frac{T' + \sqrt{ab}}{\sqrt{(b - T')(T' - a)}} \right]
\]

- using new variables

\[
\xi = (T' + T)/c, \quad \delta = (a + T)/c, \quad \beta = (b + T)/c \quad (4)
\]

the phase coexistence domain (green zone, left panel) is bound by two inequalities

\[
\frac{2}{27} \max \left[ 0, \beta \left( \frac{9}{2} - \beta^2 \right) - (\beta^2 - 3)^{3/2} \right] \leq \delta \leq \frac{2}{27} \left[ \beta \left( \frac{9}{2} - \beta^2 \right) + (\beta^2 - 3)^{3/2} \right],
\]

\[
\beta \geq \sqrt{3}
\]
Phase coexistence and bifurcation

- if \((\beta, \delta)\) are outside of the phase-coexistence domain (green) only one solution out of the three possible is real
- for \(\beta > \sqrt{3}\) it corresponds to an H-mode solution (right panel, lower dashed curve)
- for \(\beta \leq \sqrt{3}\) it corresponds to an L-mode solution (lowest real \(T'\) value out of the three solutions with the other two roots becoming complex)
Example of Spontaneous Transition
Spontaneous Transition: Description

- simulation starts from a parabolic temperature profile in L-mode (top two panels, same surface viewed at different angles)
- \( T \) relaxes to a linear profile but DW is generated at the edge (where \( \nabla T \) was initially the strongest) and propagates inward
- \( T \)-profile flattens in the region of active DW but ZF also grows at the edge
- both DW and ZF fronts continue to propagate inward but the DW has also a rear, cancelling front
- it leaves an H-mode state behind with a residual ZF and zero DW
preexisting temperature and density profiles allow for spontaneous LH transition

$\leftarrow \text{DW suppressed over a broad edge region}$

$\uparrow \text{ZF dies out everywhere}$

established H-mode is resilient to a heat pulse

No-Flow Spontaneous Transition
Pulse-Triggered ITB (Work in progress, preliminary results)

- strong DW turbulence and slowly rising ZF
- heat pulse is applied at \( t \simeq 40 \)
- it triggers rapid ZF growth which suppresses the DW in a narrow region
- strong temperature gradient builds up in this region, ITB

run starts from an L-mode which quickly relaxes to a linear temperature profile
Conclusions

- new analytical and numerical 4-field model for describing $L \rightarrow H$ transitions in weakly collisional ITER-related regimes is developed
- new type of transition scenario, which is more sensitive to the pre-existing L-mode structure than to the power variation near the threshold is identified
- dynamical realization of such transitions became possible after an accurate analytic determination of the phase coexistence domain and transition criteria in a multi-dimensional parameter space of the system
- stationary solutions of the model, obtained analytically for that purpose, are also crucial for the code verification
- work studying dynamical evolution of $L \rightarrow H$ transitions numerically is ongoing
References


Y. Ma et al., Nuclear Fusion 52, 023010 (2012).
