Secondary instabilities of large scale flow and magnetic field in the electromagnetic short wavelength drift-Alfvén wave turbulence

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It is shown that the short wavelength ($k_\perp^2\rho_i^2>1$, $k_\perp$ is the characteristic wave vector, and ρ_i is the ion Larmor radius) electromagnetic drift wave turbulence in typical conditions is unstable with respect to the excitation of large scale perturbations of the plasma flow and magnetic field. It is found that the generation of zonal flow is reduced (compared to the long wavelength ion temperature gradient turbulence) due to the Boltzmann nature of the ion response. Magnetic fluctuations further reduce zonal flow drive due to competition of the Reynolds and Maxwell stresses. It is shown that secondary magnetic field structures may be generated in the electromagnetic drift wave turbulence thus leading to the increased levels of the electron energy transport. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1500394]

I. INTRODUCTION

In this paper we investigate secondary instabilities that may occur in the short wavelength, $k_\perp^2\rho_i^2\geqslant 1$ electromagnetic turbulence. We are interested primarily in the regime $\omega \leqslant k_\parallel v_e$ where the electromagnetic effects become important for moderate values of plasma pressure $\beta\geqslant m_e/m_i$ and the mode becomes essentially a short-wavelength version of an Alfvén wave. Note that in this regime the electron response is non-Boltzmann due to the contribution of the perturbed vector potential to the parallel electron momentum balance. A short wavelength Alfvén wave can be destabilized by the electron temperature gradient[1] and thus is similar to the electron temperature gradient (ETG) mode. Though somewhat similar to the ETG generalized for a finite beta, electromagnetic effects considered in this paper are significantly different, in particular due to the low frequency (adiabatic) limit. Our interest in this particular regime ($\omega\leqslant k_\parallel v_e$) is motivated by earlier studies of these modes[2–6] which suggest a possibility of the spontaneous generation of small scale magnetic structures. In this work we show that secondary instabilities in the short wavelength electromagnetic turbulence may lead to the secondary generation of the magnetic field leading to structures somewhat similar to those in Refs. 2–6. Such coherent magnetic structures generated on a larger length scale (compared to the scale length of the background turbulence) may significantly increase electron energy transport. It is worth noting that due to high electron parallel conductivity, for the modes considered here, the electron temperature perturbations are essentially governed by fluctuations of the magnetic vector potential, due to the condition $\mathbf{B}\cdot\nabla T=0$.

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Recently, it has been realized that generation of the large scale shear flow (zonal flow) may play an important role in self-regulation of the drift wave turbulence.[7–9] Such spontaneously excited large scale flow occurs as a result of the intrinsic nonambipolarity of the radial plasma current (in other words due to the radial momentum flux[10–12]). In the electromagnetic case, the effect of magnetic fluctuations leads to a reduction of the zonal flow generation due a partial cancellation of the Reynolds stress by the Maxwell stress.[12–15] Besides a reduction of the zonal flow drive, the electromagnetic turbulence may generate secondary magnetic field via a sort of fast dynamo mechanism.[15] In this paper we investigate competing effects of the Reynolds and Maxwell stresses for the short wavelength turbulence and show that besides the reduction of the zonal flow drive, magnetic fluctuations may even be more important as a source of the secondary magnetic field structures. It should be mentioned here that for short wavelength turbulence the nature of the Reynolds stress changes: it is due to the electron rather than ion dynamics as for the long wavelength fluctuations.

Coupling of slow large scale perturbations and modulations of fast small scale background turbulence naturally suggest the wave kinetic equation as a tool for these studies.[7] Generally the wave kinetic equation is written as a conservation law for the wave quanta density usually taken as the wave action. It was noted however[16–19] that for the drift wave-zonal flow systems the form of the conserved quantity may be different from the standard definition of the wave action (defined as a ratio of the wave energy to the wave eigen-frequency). Presumably, this occurs due to the destruction of the standard canonical variables in the presence of the inhomogeneous mean flow. It was shown[18] that the particular form of the conserved quantity (generalized wave action in-

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variant) is defined by the form of the matrix coefficient describing the coupling of the small and large scale fluctuations. Such an approach allows one to formulate a wave kinetic equation and corresponding generalized wave action invariant even for the case of a two-dimensional ideal Euler fluid without waves.[17] For the electrostatic drift wave turbulence the structure of the generalized wave action invariants was investigated in Ref. 18 (see also Ref. 19). To study the evolution of the zonal flows, in this work we employ two approaches. The first approach (Sec. IV) is a direct perturbative calculation of the growth rate of the zonal flow within the weak turbulence approximation. In the second approach (Sec. V), we derive a generalized wave action invariant for (predominantly) magnetic turbulence modulated in the presence of zonal flow, and then use a wave action equation to obtain the growth rate of the zonal flow. We show here that both methods give the same result for the zonal flow growth rate. In the present paper, we also investigate how the form of the generalized wave action is affected by slow magnetic fluctuations and derive the generalized wave action for the electromagnetic drift wave turbulence in the presence of the slowly varying background magnetic field (Sec. VI). The latter wave kinetic equation is used in Sec. VII to study instability of large scale magnetic structures.

II. SHORT WAVELENGTH ALFVÉN MODE

The electron density balance equation is

$$\frac{\partial}{\partial t}n+\mathbf{v}_E\cdot\nabla n_0-\frac{1}{e}\nabla_\parallel J-\frac{n_0}{\omega_{ce}}\nabla\cdot\mathbf{b}\times\frac{d_0}{dt}\mathbf{v}_E=0. \tag{1}$$

While the continuity equation is independent of the $\omega/k_\parallel v_e$ parameter, the form of the electron momentum balance is sensitive to the value of $\omega/k_\parallel v_e$. In general, for arbitrary $\omega/k_\parallel v_e$, wave–particle interaction effects become important, which requires kinetic consideration. However in the limit $\omega \ll k_\parallel v_{\mathrm{th}}$ a simple perturbative treatment is possible. In this limit the electron momentum balance equation takes the form

$$0=-enE_\parallel - T_e\nabla_\parallel n. \tag{2}$$

Here we have

$$E_\parallel=-\nabla_\parallel\phi-\frac{1}{c}\frac{\partial\psi}{\partial t}, \tag{3}$$

$$\nabla_\parallel=\frac{\partial}{\partial z}-\frac{1}{B_0}\hat{\mathbf{z}}\cdot\nabla\psi\times\nabla. \tag{4}$$

The energy balance equation in this regime is replaced by

$$\nabla_\parallel T=0, \tag{5}$$

so that temperature perturbations are effectively decoupled due to high thermal conductivity along the total magnetic field. The effects of the collisionless wave electron interaction can be added into Eqs. (2) and (5) through kinetic closures. Note that, in contrast to the usual ETG mode, electron inertia is omitted from the parallel Ohm's law and the electron temperature is constant along the field lines. Note also that curvature effects are omitted.

Equations (1)–(3) are coupled through Ampère's law,

$$J=-\frac{c}{4\pi}\nabla^2\psi, \tag{6}$$

and the quasineutrality equation. The ion density response is taken as

$$n=-\frac{e\phi}{T_i}n_0=-\frac{e\phi}{T_e}\tau n_0, \tag{7}$$

where $\tau=T_e/T_i$.

Linearizing Eqs. (6) and (7) we obtain the following dispersion equation:

$$(\tau\omega+\omega_{*e})(\omega-\omega_{*e})=(1+\tau)k_\parallel^2 v_{\mathrm{A}}^2 k_\perp^2\rho_i^2. \tag{8}$$

Here $v_{*e}=-cT_e n_0'/(en_0B_0)$, $L_n^{-1}=n_0^{-1}\partial n_0/\partial r$, $\omega_{*e}=k_y v_{*e}$. The dispersion equation (8) smoothly matches to the Alfvén wave branch at $k_\perp^2\rho_i^2 \geqslant 1$. So it is called a short wavelength Alfvén mode. It follows from (8) that there exist two modes, $\omega\simeq-\omega_{*e}/\tau$ and $\omega\simeq\omega_{*e}$, that are coupled via electromagnetic effects due to the term on the right hand side of (8). From Eqs. (9) and (10) in Sec. III, it follows that $\omega\simeq-\omega_{*e}/\tau$ is an electrostatic, while $\omega\simeq\omega_{*e}$ is predominantly an electromagnetic branch. The latter branch was investigated earlier[2–6] as a possible source of magnetic turbulence, and it was shown that it can be destabilized by the temperature gradient effects (η_e).[1] It is interesting to note that within the approach of small scale magnetic island, it is ultimately temperature gradient effects that are responsible for the mode excitation.[2–6] Therefore, to some extent, it is similar to the standard ETG mode.

III. REYNOLDS AND MAXWELL STRESS TENSORS

Basic nonlinear equations for short wavelength electromagnetic fluctuations can be obtained from (1) to (7)

$$\frac{\partial}{\partial t}\frac{e\phi}{T_e}-v_{*e}\frac{\partial}{\partial y}\frac{e\phi}{T_e}-\frac{c}{4\pi en_0}\frac{\partial}{\partial z}\nabla^2\psi + \frac{c}{4\pi en_0B_0}\hat{\mathbf{z}}\cdot\nabla\psi\times\nabla\nabla^2\psi-\rho_e^2\frac{d_0}{dt}\nabla_\perp^2\frac{e\phi}{T_e}=0, \tag{9}$$

$$\frac{\partial}{\partial t}\psi+v_{*e}\frac{\partial}{\partial y}\psi+2c\frac{\partial}{\partial z}\psi-\frac{2c}{B_0}\hat{\mathbf{z}}\cdot\nabla\psi\times\nabla\phi=0, \tag{10}$$

where $\hat{\mathbf{z}}=\mathbf{B}_0/B_0$ is the unit vector along the magnetic field, $\tau=1$.

There are two sources of zonal flow generation in Eq. (9). One is due to the $\mathbf{V}_E\cdot\nabla\nabla^2\phi$ nonlinearity from the electron polarization current [the last term in Eq. (9)], and the other one is due to the $\hat{\mathbf{z}}\cdot\nabla\psi\times\nabla\nabla^2\psi$ nonlinearity from the total gradient of the parallel current. Ultimately the latter term is due to the Lorentz force, $\mathbf{J}\times\mathbf{B}$. The first term can be identified as the Reynolds stress tensor,[10–12] while the second one is the Maxwell stress tensor. We assume here that the shear flow has a radial scale length $k_x^{-1}<\rho_e$, so that the plasma response to the zonal flow component is Boltzmann. Then the $\mathbf{V}_E\cdot\nabla n$ nonlinearity does not appear in the continuity equation. As a result, in the electrostatic approximation, the modification of the background turbulence by the large scale mode occurs in the higher order (due to the term

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$\overline{\mathbf{V}}_E\cdot\nabla\nabla^2\tilde{\phi}$, where $\overline{\phi}$ and $\tilde{\phi}$ represent large scale and small scale components). On the contrary, when magnetic fluctuations are present, the large scale field affects the background modes in the lowest order, via $\hat{\mathbf{z}}\cdot\nabla\tilde{\psi}\times\nabla\overline{\phi}$ term in Ohm's law ($\tilde{\psi}$ is small scale component of the magnetic vector potential), thus indicating the relative importance of magnetic fluctuations for the short wavelength turbulence.

The equation describing the mean electrostatic potential is

$$\frac{\partial}{\partial t}\frac{e\overline{\phi}}{T_e}+R^{\psi}-R^{\phi}=0, \tag{11}$$

where the Reynolds and Maxwell stress are

$$R^{\psi}\equiv\frac{c}{4\pi e n_0 B_0}M^{\psi}=\frac{c}{4\pi e n_0 B_0}\hat{\mathbf{z}}\cdot\nabla\psi\times\nabla\nabla^2\psi, \tag{12}$$

$$R^{\phi}\equiv\rho_e^2\frac{ce}{B_0 T}M^{\phi}=\rho_e^2\frac{ce}{B_0 T}\hat{\mathbf{z}}\cdot\nabla\phi\times\nabla\nabla^2\phi. \tag{13}$$

Note that for short wavelength turbulence mean electrostatic potential $\overline{\phi}$ is generated [as given by Eq. (11)] contrary to the case of long wavelength drift wave turbulence where the mean vorticity is generated, so that the evolution equation for the potential has a form $\partial\nabla_\perp^2\overline{\phi}=S$, where S is a source term.[7,14,20] It indicates that the mean flow induced by the long wavelenth turbulence instrinsically has higher shear compared to the short wavelength case.

By employing the dispersion equation (8) we obtain for the ratio of amplitudes of magnetic and electrostatic fluctuations

$$\frac{M^{\psi}}{M^{\phi}}\simeq\frac{|\psi^2|}{|\phi^2|}=\frac{2}{k_\perp^2c^2/\omega_{pe}^2}\frac{\omega+\omega_{*e}}{\omega-\omega_{*e}}. \tag{14}$$

Correspondingly, we have for the ratio of the Maxwell to the Reynolds stress

$$S\equiv\frac{R^{\psi}}{R^{\phi}}=\frac{2}{k_\perp^2\rho_e^2}\frac{\omega+\omega_{*e}}{\omega-\omega_{*e}}. \tag{15}$$

This expression allows us to conveniently compare relative contrubutions of the electrostatic (Reynolds) and magnetic (Maxwell) fluctuations. Similarly to the long wavelength case, Reynolds and Maxwell stresses have the same structure in terms of the electrostatic and magnetic potentials, see Eqs. (12) and (13). Equation (8) indicates that in our case there exists two branches, one is essentially electromagnetic, rotating in the electron direction, $\omega>0$, and the other branch is electrostatic, $\omega<0$, rotating in the ion direction ($\omega_{*e}>0$). As follows from (15), the Maxwell stress prevails for electromagnetic modes moving in the electron direction. Effect of the magnetic fluctuations is also increasing for the longer wavelengths, $k_\perp^2c^2/\omega_{pe}^2<1$. Note that such competition between the Reynolds and Maxwell stresses is somewhat different from the long wavelength electromagnetic turbulence[13,14,21] where a partial cancellation in the total zonal flow drive occurs due to the Alfvénization of the turbulence; such cancellation becomes exact for pure Alfvén wave turbulence $\omega=k_\parallel v_A$ which is force free. One should also remember that, as noted previously in the short wavelength case, the Reynolds stress occurs from the electron rather than ion polarization current.

IV. LARGE SCALE FLOW INSTABILITY

To describe the dynamics of a large-scale plasma flow we employ a multiple scale expansion thus assuming that there is a sufficient spectral gap separating large-scale and small-scale motions. The electrostatic and magnetic potential are represented as a sum of fluctuating and mean quantities

$$\phi(\mathbf{X},\mathbf{x},T,t)=\overline{\phi}(\mathbf{X},T)+\tilde{\phi}(\mathbf{X},\mathbf{x},T,t), \tag{16}$$

$$\psi(\mathbf{X},\mathbf{x},T,t)=\overline{\psi}(\mathbf{X},T)+\tilde{\psi}(\mathbf{X},\mathbf{x},T,t), \tag{17}$$

where ϵ is a formal small parameter of the scale separation. In this section we consider only the effect of the mean potential $\overline{\phi}(\mathbf{X},T)$, neglecting the large scale magnetic potential $\overline{\psi}(\mathbf{X},T)$.

Using the Fourier transformation the Maxwell force can be written in the form

$$\begin{aligned}M^{\psi}&=\int d\omega_1 d\omega_2 d^2k_1 d^2k_2\hat{\mathbf{z}}\cdot\mathbf{k}_1\times\mathbf{k}_2k_2^2\tilde{\psi}_{k_1}\tilde{\psi}_{k_2}\\&\quad\times\exp(-i(\omega_1+\omega_2)t+i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{x})\\&=\frac{1}{2}\int d\omega_1 d\omega_2 d^2k_1 d^2k_2\hat{\mathbf{z}}\cdot\mathbf{k}_1\times\mathbf{k}_2(k_2^2-k_1^2)\tilde{\psi}_{k_1}\tilde{\psi}_{k_2}\\&\quad\times\exp(-i(\omega_1+\omega_2)t+i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{x}).\end{aligned} \tag{18}$$

It is convenient to make the Fourier transform of (18) with respect to the slow variables $\mathbf{X}$ and T so that

$$M(\mathbf{X},T)=\int M_{q,\Omega}\exp(-i\Omega T+i\mathbf{q}\cdot\mathbf{X})d^2q\,d\Omega. \tag{19}$$

Then, using $\mathbf{q}\ll(\mathbf{k}_1,\mathbf{k}_2)$ for "slow" wave vector $\mathbf{q}$ and $\Omega\ll(\omega_1,\omega_2)$ for "slow" frequency Ω we obtain

$$\begin{aligned}M_{q,\Omega}&=-\int d\omega_1 d^2k_1(\hat{\mathbf{z}}\cdot\mathbf{k}_1\times\mathbf{q})(\mathbf{q}\cdot k_1)\\&\quad\times\langle\tilde{\psi}_{\mathbf{k}_1,\omega_1}\tilde{\psi}_{\mathbf{q}-\mathbf{k}_1,\Omega-\omega_1}\rangle.\end{aligned} \tag{20}$$

We treat the effect of the mean plasma flow perturbatively, so that the fluctuating electrostatic potential and pressure are represented

$$\tilde{\psi}=\hat{\psi}^{(0)}+\hat{\psi}^{(1)}. \tag{21}$$

Here, $\hat{\psi}^{(0)}(\mathbf{x},t)$ is associated with a "free" turbulence which is assumed isotropic and homogeneous in the absence of mean flow, and $\hat{\psi}^{(1)}(\mathbf{X},T,\mathbf{x},t)$ is the first-order modifications of fluctuations due to the mean flow $\overline{\phi}(\mathbf{X},T)$. The statistical average $\langle\tilde{\psi}_{\mathbf{k}_1,\omega_1}\tilde{\psi}_{\mathbf{q}-\mathbf{k}_1,\Omega-\omega_1}\rangle$ is calculated in quasilinear approximation

$$\begin{aligned}\mathcal{L}&\equiv\langle\tilde{\psi}_{\mathbf{k}_1,\omega_1}\tilde{\psi}_{\mathbf{q}-\mathbf{k}_1,\Omega-\omega_1}\rangle\\&=\langle\hat{\psi}^{(0)}_{\mathbf{k}_1,\omega_1}\hat{\psi}^{(1)}_{\mathbf{q}-\mathbf{k}_1,\Omega-\omega_1}\rangle+\langle\hat{\psi}^{(1)}_{\mathbf{k}_1,\omega_1}\hat{\psi}^{(0)}_{\mathbf{q}-\mathbf{k}_1,\Omega-\omega_1}\rangle.\end{aligned} \tag{22}$$

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The first-order response field $\hat{\psi}^{(1)}(\mathbf{X},\mathbf{x},t,T)$ is determined by the convection term $\overline{\mathbf{V}}_E \nabla \hat{\psi}^{(0)}$ in the momentum balance equation (10). Performing the Fourier transformation of (10) in time and space we obtain

$$-i(\omega-\omega_{*k})\hat{\psi}^{(1)}_{\mathbf{k},\omega}+2ick_z\hat{\phi}^{(1)}_{\mathbf{k},\omega} = \int d^2p\, d\omega' C_{\mathbf{p},\mathbf{k}-\mathbf{p}}\overline{\phi}_{\mathbf{p},\omega'}\hat{\psi}^{(0)}_{\mathbf{k}-\mathbf{p},\omega-\omega'}, \tag{23}$$

$$-i(\omega+\omega_{*k})\frac{e\hat{\phi}^{(1)}_{\mathbf{k},\omega}}{T}+\frac{ck_zk_\perp^2}{4\pi en_0}\hat{\psi}^{(1)}_{\mathbf{k},\omega}=0, \tag{24}$$

where the coupling coefficient is

$$C_{\mathbf{k}_1,\mathbf{k}_2}=\frac{2c}{B_0}\hat{\mathbf{z}}\cdot\mathbf{k}_1\times\mathbf{k}_2. \tag{25}$$

Combining (23)–(25) we obtain

$$-i\frac{(\omega-\omega_k^+)(\omega-\omega_k^-)}{\omega+\omega_{*k}}\hat{\psi}^{(1)}_{\mathbf{k},\omega} = \int d^2p\, d\omega' C_{\mathbf{p},\mathbf{k}-\mathbf{p}}\overline{\phi}_{\mathbf{p},\omega'}\hat{\psi}^{(0)}_{\mathbf{k}-\mathbf{p},\omega-\omega'}, \tag{26}$$

$$\hat{\psi}^{(1)}_{\mathbf{k}_1,\omega_1}=i\frac{\omega_1+\omega_{*k}}{(\omega_1-\omega_k^+)(\omega_1-\omega_k^-)} \times\int d^2p\, d\omega' C_{\mathbf{p},\mathbf{k}-\mathbf{p}}\overline{\phi}_{\mathbf{p},\omega'}\hat{\psi}^{(0)}_{\mathbf{k}_1-\mathbf{p},\omega_1-\omega'}. \tag{27}$$

Similarly, we find

$$\hat{\psi}^{(1)}_{q-\mathbf{k}_1,\boldsymbol{\Omega}-\omega_1}=i\frac{\Omega-\omega_1+\omega_{*(q-k)}}{(\Omega-\omega_1-\omega^+_{q-k})(\Omega-\omega_1-\omega^-_{q-k})} \times\int d^2p\, d\omega' C_{\mathbf{p},q-\mathbf{k}_1-\mathbf{p}} \times\overline{\phi}_{\mathbf{p},\omega'}\hat{\psi}^{(0)}_{q-\mathbf{k}_1-\mathbf{p},\Omega-\omega_1-\omega'}. \tag{28}$$

Using (27) and (28) we obtain

$$\mathcal{L}=i\frac{\omega_1+\omega_{*k}}{(\omega_1-\omega_k^+)(\omega_1-\omega_k^-)}C_{\mathbf{q},\mathbf{k}_1}\overline{\phi}_{\mathbf{q},\Omega}I_{k_1-q,\omega_1-\Omega} \tag{29}$$

$$+i\frac{\Omega-\omega_1+\omega_{*(q-k)}}{(\Omega-\omega_1-\omega^+_{q-k})(\Omega-\omega_1-\omega^-_{q-k})}C_{\mathbf{q},-\mathbf{k}_1}\overline{\phi}_{\mathbf{q},\Omega}I_{k_1,\omega_1}. \tag{30}$$

We have assumed above that the zeroth-order turbulence is delta-correlated,

$$\langle\hat{\psi}^{(0)}_{\mathbf{k},\omega}\hat{\psi}^{(0)}_{\mathbf{k}_1,\omega_1}\rangle=I_{k,\omega}\delta(\mathbf{k}+\mathbf{k}_1)\delta(\omega+\omega_1). \tag{31}$$

In the weak turbulence case when the mode broadening is small, $\Delta\omega\ll\omega$, the fluctuations spectrum can be represented in the form

$$I_{k_1-q,\omega_1-\Omega}=I_{k_1-q}\delta(\omega_1-\Omega-\omega^+_{\mathbf{k}_1}), \tag{32}$$

$$I_{k_1,\omega_1}=I_{k_1}\delta(\omega_1-\omega^+_{\mathbf{k}_1}). \tag{33}$$

In the weak turbulence approximation we also have

$$\frac{i}{\Omega-\omega_1-\omega^+_{\mathbf{q}-\mathbf{k}_1}+i\Delta\omega}\rightarrow\pi\delta(\Omega-\omega_1-\omega^+_{\mathbf{q}-\mathbf{k}_1}), \tag{34}$$

$$\frac{i}{\omega_1-\omega^+_{\mathbf{k}_1}+i\Delta\omega}\rightarrow\pi\delta(\omega_1-\omega^+_{\mathbf{k}_1}), \tag{35}$$

where the $i\Delta\omega$ term in (34) and (35) represents a nonlinear decorrelation rate due to the mode coupling. Then we obtain

$$\mathcal{L}=\frac{1}{2}\pi\delta(\Omega-\omega_1-\omega^+_{\mathbf{q}-\mathbf{k}_1})\delta(\omega_1-\omega^+_{\mathbf{k}_1})\overline{\phi}_{\mathbf{q},\Omega}C_{\mathbf{q},\mathbf{k}_1} \times\left[\left(1+\frac{\omega_{*k_1}}{\omega^+_{k_1}}\right)I_{k_1-q}-\left(1+\frac{\omega_{*(q-k_1)}}{\omega^+_{q-k_1}}\right)I_{k_1}\right]. \tag{36}$$

Substituting (36) in Eq. (20) we find

$$M_{q,\Omega}=-\frac{\pi c}{B_0}\overline{\phi}_{\mathbf{q},\Omega}\int d^2k(\hat{\mathbf{z}}\cdot\mathbf{k}\times\mathbf{q})^2(\mathbf{q}\cdot\mathbf{k})\delta(\Omega-\mathbf{q}\cdot\mathbf{V}_g) \times\left(1+\frac{\omega_{*k}}{\omega_k^+}\right)^2\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{k}}\left(\frac{I_k}{1+\omega_{*k}/\omega_k^+}\right). \tag{37}$$

From Eqs. (37) and (11) and restoring a contribution of the Reynolds stress we obtain the growth rate of the large scale flow with $\mathbf{q}=\hat{\mathbf{x}}q$,

$$\gamma=-i\Omega=\pi\frac{v_{\text{the}}^2c^2}{\omega_{pe}^2B_0^2}q^4\int d^2k\,\delta(\Omega-\mathbf{q}\cdot\mathbf{V}_g)k_y^2k_x \times\frac{\partial}{\partial k_x}\left(\frac{(1-S^{-1})I_k}{1+\omega_{*k}/\omega_k^+}\right). \tag{38}$$

In fact, the resonance broadening is important, so that the response function is

$$\delta(\Omega-\mathbf{q}\cdot\mathbf{V}_g)\rightarrow R(\Omega,\mathbf{q}\cdot\mathbf{V}_g,\Delta\omega)=\frac{\Delta\omega}{(\Delta\omega)^2+(\Omega-\mathbf{q}\cdot\mathbf{V}_g)^2}. \tag{39}$$

Note that for $S>1$, the condition

$$\frac{\partial}{\partial k_x}\left(\frac{I_k}{1+\omega_{*k}/\omega_k^+}\right)>0 \tag{40}$$

is required for the instability, which is the expected result for Alfvén wave type turbulence.[14] We show in Sec. V that the quantity in brackets represents the generalized wave action invariant $N_k=I_k/(1+\omega_{*k}/\omega_k^+)$.[18]

One concludes from (38) and (40) that typically the Maxwell stress is a stabilizing factor while the Reynolds stress is destabilizing for $\partial N_k/\partial k_x<0$. Thus, when the magnetic stress dominates, the population inversion is required for the zonal flow instability which is an unlikely condition for the drift wave type turbulence.

V. WAVE KINETIC EQUATION AND GENERALIZED WAVE ACTION INVARIANT

The wave kinetic equation is an effective tool to describe the modulations of high frequency turbulence in the presence of slowly varying processes. Initially introduced to describe the interaction of Langmuir (fast) and ion-sound (slow)

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waves,[22] the wave kinetic equation was recently effectively used in the problem of interaction of drift wave turbulence with large scale structures, in particular, in the problem of the generation of large scale flows.[7,16,18,23] It has been noted that for drift wave type fluctuations in the presence of the mean flow the wave action may be different from the standard definition of the wave action as the ratio of the wave energy to the wave momentum. It has been shown that the exact form of the conserved wave-action-like quantity (generalized wave action) depends on the form of the coupling matrix describing the interaction of small scale fluctuation and the mean flow.[17,18] This property has long been noted in the theory of waves in fluids[24,25] and can be traced to a certain ambiguity in the definition of the wave momentum.[25] The ambiguity is related to the contribution of the mean flow to the total momentum in the medium. We have developed a formal approach that allows one to calculate a generalized wave action invariant for a given form of the wave-mean-flow interaction coefficient.[17,18] In this section we apply this method for the electromagnetic short-wavelength Alfvén-wave fluctuations to derive the proper form of the wave action invariant. Then we use it to calculate the zonal flow growth rate and show that we get exactly the same result as from the direct calculations in Sec. IV.

Evolution of the wave packets is described by wave kinetic equation in the standard form

$$\frac{\partial N_k}{\partial t}+\frac{\partial \omega}{\partial \mathbf{k}}\cdot\frac{\partial N_k}{\partial \mathbf{x}}-\frac{\partial \omega}{\partial \mathbf{x}}\cdot\frac{\partial N_k}{\partial \mathbf{k}}=2\Gamma_k N_k-\Delta\omega N_k^2/N_0, \quad (41)$$

where Γ_k is the linear instability growth rate, and the term $\Delta\omega N_k^2/N_0$ describes the nonlinear damping due to "nonadiabatic" wave interactions. In what follows, we consider deviations from the saturated state.

The wave action quanta density N_k is modulated by the large scale perturbations due to the variations of the eigenfrequency in the presence of the mean plasma flow. Linearizing Eqs. (9) and (10) one gets

$$(\omega+\omega_{*e})(\omega-\omega_{*e}-2\omega_E)=k_\parallel^2 v_e^2 k_\perp^2 d^2, \quad (42)$$

where $v_e^2=2T_e/m_e$, $d^2=c^2/\omega_{pe}^2$. Then the modulation of the eigen-frequency is described by

$$\frac{\partial \omega}{\partial \omega_E}=\left(1+\frac{\omega_{*k_1}}{\omega_{k_1}}\right). \quad (43)$$

From the wave kinetic equation (41) we find

$$-i(\Omega-\mathbf{q}\cdot\mathbf{V}_g)\tilde{N}_k-\frac{\partial}{\partial \mathbf{x}}\left(\frac{\partial \omega}{\partial \omega_E}\,\delta\omega_E\right)\cdot\frac{\partial N_{0k}}{\partial \mathbf{k}}=0, \quad (44)$$

and

$$-i(\Omega-\mathbf{q}\cdot\mathbf{V}_g)\tilde{N}_k-\left(1+\frac{\omega_{*k_1}}{\omega_{k_1}}\right)\frac{\partial}{\partial \mathbf{x}}(\mathbf{k}\cdot\mathbf{V}_E)\cdot\frac{\partial N_{0k}}{\partial \mathbf{k}}=0. \quad (45)$$

As discussed previously, the general form of the wave kinetic equation is standard, however the exact expression for the wave invariant may differ from the standard definition.[18,19] To find the wave action density in our case, we follow the approach of Ref. 18.

We start from the basic equations where the spatial coordinate is Fourier transformed

$$-i(\omega-\omega_{*k})\hat{\psi}_{\mathbf{k}}+2ick_z\hat{\phi}_{\mathbf{k}}=\int d^2p\,C_{\mathbf{p},\mathbf{k}-\mathbf{p}}\overline{\phi_{\mathbf{p}}}\hat{\psi}_{\mathbf{k}-\mathbf{p},\omega}, \quad (46)$$

$$\frac{\partial}{\partial t}\hat{\psi}_{\mathbf{k}}+i\omega_k^+\hat{\psi}_{\mathbf{k}}=\frac{1}{2}\left(1+\frac{\omega_{*k}}{\omega_k^+}\right)\int d^2p\,C_{\mathbf{p},\mathbf{k}-\mathbf{p}}\overline{\phi_{\mathbf{p}}}\hat{\psi}_{\mathbf{k}-\mathbf{p},\omega}. \quad (47)$$

In the spirit of the scale separation we separate the field into the large and small scale components; the large scale component $\overline{\phi_{\mathbf{p}}}$ is a mean flow potential which equals zero outside a shell $|\mathbf{k}|<\varepsilon\ll 1$, small scale fluctuations $\hat{\psi}_k=0$ for $|\mathbf{k}|<\varepsilon$,

$$L_{\mathbf{p},\mathbf{k}-\mathbf{p}}=-\frac{1}{2}\left(1+\frac{\omega_{*k}}{\omega_k^+}\right)C_{\mathbf{p},\mathbf{k}-\mathbf{p}}$$
$$=-\frac{c}{B_0}\left(1+\frac{\omega_{*k}}{\omega_k^+}\right)\hat{\mathbf{z}}\cdot\mathbf{k}\times(\mathbf{k}-\mathbf{p}). \quad (48)$$

The small scale turbulence is described by the spectral function (Wigner function) $I_k(\mathbf{x},t)$, and defined as follows:

$$\int d^2q\langle\hat{\psi}_{-k+q}\hat{\psi}_k\rangle\exp(i\mathbf{q}\cdot\mathbf{x})=I_k(\mathbf{x},t). \quad (49)$$

Following Ref. 17 we obtain

$$\frac{\partial}{\partial t}I_k(\mathbf{x},t)+i\int d^2q$$
$$\times\exp(i\mathbf{q}\cdot\mathbf{x})(\omega_k+\omega_{-k+q})\langle\hat{\psi}_{-k+q}\hat{\psi}_k\rangle+S_1+S_2=0, \quad (50)$$

where

$$S_1=\int\int d^2p\,d^2q\,\exp(i\mathbf{q}\cdot\mathbf{x})\langle\hat{\psi}_{-k+q}\hat{\psi}_{k-p}\rangle L_{p,k-p}\overline{\phi_p}, \quad (51)$$

$$S_2=\int\int d^2p\,d^2q\,\exp(i\mathbf{q}\cdot\mathbf{x})$$
$$\times\langle\hat{\psi}_{-k+q-p}\hat{\psi}_k\rangle L_{p,-k+q-p}\overline{\phi_p}. \quad (52)$$

The second term in (50) leads to

$$i\int d^2q\,\exp(i\mathbf{q}\cdot\mathbf{x})(\omega_k+\omega_{-k+q})\langle\hat{\psi}_k\hat{\psi}_{-k+q}\rangle$$
$$=\frac{\partial\omega_k}{\partial\mathbf{k}}\cdot\frac{\partial}{\partial\mathbf{x}}I_k(\mathbf{x},t)-2\Gamma_k I_k, \quad (53)$$

where Γ_k is the linear growth rate, and only the real part of the frequency is presumed for ω_k on the right-hand side of this equation.

The ensemble average in S_1 and S_2 can be transformed by using the inverse of (49),

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$$\langle \hat{\psi}_{-k+q}\hat{\psi}_{k-p}\rangle = \langle \hat{\psi}_{k-p}\hat{\psi}_{-(k-p)+q-p}\rangle = \int d^2x' I_{k-p}(x')\exp(-i(\mathbf{q}-\mathbf{p})\cdot\mathbf{x}'). \quad (54)$$

By using (54) and expanding in $\mathbf{p}\ll\mathbf{k}$ the expression for S_1 is transformed to

$$S_1 = S_1' + S_1'', \quad (55)$$

$$S_1' = \int d^2p \exp(i\mathbf{p}\cdot\mathbf{x}) L_{p,k-p} I_k(\mathbf{x}) \overline{\phi_p}, \quad (56)$$

$$S_1'' = -\int d^2p \exp(i\mathbf{p}\cdot\mathbf{x}) L_{p,k-p}\mathbf{p}\cdot\frac{\partial I_k(\mathbf{x})}{\partial\mathbf{k}}\overline{\phi_p}. \quad (57)$$

Similarly, by using the identity analogous to (54) and expanding the interaction coefficient $L_{p,k-p}$ in $\mathbf{p}\ll\mathbf{k}$, we transform S_2 to the form

$$S_2 = S_2' + S_2'', \quad (58)$$

$$S_2' = I_k(x)\int d^2p \exp(i\mathbf{p}\cdot\mathbf{x}) L_{p,-k}\overline{\phi_p}, \quad (59)$$

$$S_2'' = -i\int d^2p \exp(i\mathbf{p}\cdot\mathbf{x})\frac{\partial L_{p,-k}}{\partial(-\mathbf{k})}\cdot\frac{\partial I_k}{\partial\mathbf{x}}\overline{\phi_p}. \quad (60)$$

It results in

$$S_1' + S_2' = I_k\frac{\partial}{\partial\mathbf{k}}\left[\frac{\omega_{*k}}{\omega_k^+}\right]\cdot\frac{\partial}{\partial\mathbf{x}}(\mathbf{k}\cdot\mathbf{V}_E), \quad (61)$$

where

$$S_1'' = -\left(1+\frac{\omega_{*k}}{\omega_k^+}\right)\frac{\partial}{\partial\mathbf{x}}(\mathbf{k}\cdot\mathbf{V}_E)\cdot\frac{\partial I_k}{\partial\mathbf{k}}, \quad (62)$$

$$S_2'' = \frac{\partial}{\partial\mathbf{k}}\left[\left(1+\frac{\omega_{*k}}{\omega_k^+}\right)(\mathbf{k}\cdot\mathbf{V}_E)\right]\cdot\frac{\partial I_k}{\partial\mathbf{x}}. \quad (63)$$

Combining (55), (58), and (61) we obtain

$$\frac{\partial}{\partial t}\frac{I_k}{1+\omega_{*k}/\omega_k^+} + \frac{\partial}{\partial\mathbf{k}}\left(\omega_k^+ + \mathbf{k}\cdot\mathbf{V}_E\left(1+\frac{\omega_{*k}}{\omega_k^+}\right)\right)\cdot\frac{\partial}{\partial\mathbf{x}} \times\frac{I_k}{1+\omega_{*k}/\omega_k^+} - \frac{\partial}{\partial\mathbf{x}}\left(\mathbf{k}\cdot\mathbf{V}_E\left(1+\frac{\omega_{*k}}{\omega_k^+}\right)\right)\cdot\frac{\partial}{\partial\mathbf{k}} \times\frac{I_k}{1+\omega_{*k}/\omega_k^+} = 0. \quad (64)$$

This corresponds to the standard form of the wave kinetic equation with a generalized wave action invariant

$$N_k = \frac{I_k}{1+\omega_{*k}/\omega_k^+}. \quad (65)$$

From (11) and (45) one can see that Eq. (64) leads to the same expression for the growth rate as the one obtained from direct calculations (38).

VI. GENERALIZED WAVE ACTION INVARIANT IN A SYSTEM WITH SLOWLY VARYING MAGNETIC FIELD

As was discussed previously the form of the wave action invariant depends on the way the background equilibrium is modified. Formally it is determined by the couping matrix that decsribes the interaction of fast and slow motion. We have derived earlier the wave kinetic equation (and respective wave action invaraint) for the magnetic turbulence modulated by a zonal flow. For the problem of the large scale magnetic generation we need to determine the response of the turbulence to slow modulations of the background magnetic field.

The basic equation has the conservation form in the $(\mathbf{k,x})$ phase space:

$$\frac{\partial N_k}{\partial t} + \frac{\partial\omega}{\partial\mathbf{k}}\cdot\frac{\partial N_k}{\partial\mathbf{x}} - \frac{\partial\omega}{\partial\mathbf{x}}\cdot\frac{\partial N_k}{\partial\mathbf{k}} = 0. \quad (66)$$

Here the eigen-frequency varies in space due to the slow perturbation of the parallel wave vector $\delta k_\parallel$,

$$\delta k_\parallel = -\frac{1}{B_0}\hat{\mathbf{z}}\cdot\nabla\overline{\psi}\times\mathbf{k}, \quad (67)$$

so that the perturbation of the eigen-frequency r is

$$\delta\omega = \frac{\partial\omega}{\partial k_\parallel}\delta k_\parallel = \delta k_\parallel\frac{k_\parallel}{\omega}v_A^2 k_\perp^2\rho_i^2. \quad (68)$$

The form of the wave action invariant will be determined next.

Making Fourier transformation in Eqs. (9) and (10),

$$-i(\omega-\omega_{*k})\hat{\psi}_\mathbf{k} + 2ick_z\hat{\phi}_\mathbf{k} - \frac{2c}{B_0}\int d\omega d^2p\,\hat{\mathbf{z}}\cdot i\mathbf{p}i(\mathbf{k}-\mathbf{p})\overline{\psi_\mathbf{p}}\hat{\phi}_{\mathbf{k}-\mathbf{p},\omega} = 0, \quad (69)$$

$$-i(\omega+\omega_{*k})\frac{e\hat{\phi}_\mathbf{k}}{T} + \frac{ick_zk_\perp^2}{4\pi en_0}\hat{\psi}_\mathbf{k} - \frac{c}{4\pi en_0B_0}\int d\omega\, d^2p\,\hat{\mathbf{z}}\cdot i\mathbf{p} \times i(\mathbf{k}-\mathbf{p})(\mathbf{k}-\mathbf{p})_\perp^2\overline{\psi_\mathbf{p}}\hat{\psi}_{\mathbf{k}-\mathbf{p},\omega} = 0. \quad (70)$$

By using $\hat{\phi}_\mathbf{k}$ from (70) in (69) we reduce it to a standard form

$$\frac{\partial}{\partial t}\hat{\psi}_\mathbf{k} + i\omega_k^+\hat{\psi}_\mathbf{k} + \int d^2p L_{\mathbf{p},\mathbf{k}-\mathbf{p}}\overline{\psi_\mathbf{p}}\hat{\psi}_{\mathbf{k}-\mathbf{p}} = 0, \quad (71)$$

where $L_{\mathbf{p},\mathbf{k}-\mathbf{p}}$ in this case is

$$L_{\mathbf{p},\mathbf{k}-\mathbf{p}} = \frac{1}{B_0}\frac{k_zv_e^2(k-p)_\perp^2d^2}{2\omega_k^+} \times\left(1+\frac{\omega_k^+ + \omega_{*k}}{\omega_{k-p}^+ + \omega_{*(k-p)}}\right)\hat{\mathbf{z}}\cdot\mathbf{p}\times(\mathbf{k}-\mathbf{p}). \quad (72)$$

Then for $L_{\mathbf{p},-\mathbf{k}}$ we have

$$L_{\mathbf{p},-\mathbf{k}} = -\frac{1}{B_0}\frac{k_zv_e^2k^2d^2}{2\omega_{p-k}^+}\left(1+\frac{\omega_{p-k}^+ + \omega_{*(p-k)}}{\omega_{-k}^+ + \omega_{*(-k)}}\right)\hat{\mathbf{z}}\cdot\mathbf{p}\times(-\mathbf{k}). \quad (73)$$

Using the same approach as in Sec. V we obtain

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$$S_1 = S_1' + S_1'', \tag{74}$$

where

$$S_1' = \int d^2p \exp(i\mathbf{p}\cdot\mathbf{x}) L_{p,k-p} I_k(\mathbf{x}) \overline{\psi}_p, \tag{75}$$

$$S_1'' = -\int d^2p \exp(i\mathbf{p}\cdot\mathbf{x}) L_{p,k-p} \mathbf{p}\cdot \frac{\partial I_k(\mathbf{x})}{\partial \mathbf{k}} \overline{\psi}_p. \tag{76}$$

Similarly,

$$S_2 = S_2' + S_2'', \tag{77}$$

$$S_2' = I_k(x) \int d^2p \exp(i\mathbf{p}\cdot\mathbf{x}) L_{p,-k} \overline{\psi}_p, \tag{78}$$

$$S_2'' = -i \int d^2p \exp(i\mathbf{p}\cdot\mathbf{x}) \frac{\partial L_{p,-k}}{\partial(-\mathbf{k})} \cdot \frac{\partial I_k}{\partial \mathbf{x}} \overline{\psi}_p. \tag{79}$$

Collecting terms we obtain

$$\begin{aligned} S_1' + S_1'' = & -\frac{1}{B_0} \frac{k_z v_e^2 d^2}{\omega_k^+} \left(1 + \frac{\omega_{*k}}{\omega_k^+}\right) \int d^2p \exp(i\mathbf{p}\cdot\mathbf{x}) \\ & \times I_k(\mathbf{x}) \times \hat{\mathbf{z}}\cdot\mathbf{p} \times \mathbf{k}\mathbf{p}\cdot \frac{\partial}{\partial \mathbf{k}} \frac{\mathbf{k}^2}{(1+\omega_{*k}/\omega_k^+)} \\ = & -I_k \frac{k_z v_e^2 d^2}{\omega_k^+} \left(1 + \frac{\omega_{*k}}{\omega_k^+}\right) \\ & \times \frac{\partial}{\partial \mathbf{x}} (\delta k_\parallel) \cdot \frac{\partial}{\partial \mathbf{k}} \frac{\mathbf{k}^2}{(1+\omega_{*k}/\omega_k^+)}, \end{aligned} \tag{80}$$

$$S_1'' = -k_z v_e^2 d^2 \frac{\mathbf{k}^2}{\omega_k^+} \left(1 + \frac{\omega_{*k}}{\omega_k^+}\right) \frac{\partial}{\partial \mathbf{x}} (\delta k_\parallel) \cdot \frac{\partial I_k}{\partial \mathbf{k}}, \tag{81}$$

$$S_2'' = \frac{\partial}{\partial \mathbf{k}} \left[k_z v_e^2 d^2 \frac{\mathbf{k}^2}{\omega_k^+} \delta k_\parallel \right] \cdot \frac{\partial I_k}{\partial \mathbf{x}}. \tag{82}$$

The resulting wave kinetic equation takes the form

$$\begin{aligned} & \frac{\partial}{\partial t} \frac{k^2 I_k}{1+\omega_{*k}/\omega_k^+} + \frac{\partial}{\partial \mathbf{k}} \left(\omega_k^+ + \delta k_\parallel \frac{\delta\omega}{\delta k_\parallel} \right) \cdot \frac{\partial}{\partial \mathbf{x}} \\ & \quad \times \frac{k^2 I_k}{1+\omega_{*k}/\omega_k^+} - \frac{\partial}{\partial \mathbf{x}} \left(\delta k_\parallel \frac{\delta\omega}{\delta k_\parallel} \right) \cdot \frac{\partial}{\partial \mathbf{k}} \frac{k^2 I_k}{1+\omega_{*k}/\omega_k^+} = 0. \end{aligned} \tag{83}$$

Equation (83) describes modulations of the quanta density I_k in response to slow variations of the magnetic field $\overline{\psi}$. The generalized wave action invariant for this case is

$$N_k = \frac{k^2 I_k}{1+\omega_{*k}/\omega_k^+}. \tag{84}$$

Equation (83) will be used in Sec. VII to study the instability of the large scale magnetic field.

VII. INSTABILITY OF A LARGE SCALE MAGNETIC FIELD

The generation of large scale magnetic field in the drift-Alfvén wave turbulence may be an important factor controlling dynamics of the short wavelength turbulence in magnetized plasmas, in particular, in a tokamak.[2] Magnetic structures produced via this mechanism can directly affect the electron transport in such plasmas. Though our study is primarily motivated by the problem of the anomalous electron transport from short wavelength turbulence, it may be relevant also to the general problem of the magnetic field generation in astrophysical and geophysical environments. The fast dynamo mechanism considered in this section is an alternative to the kinematic dynamo that may be quenched in the presence of the large scale field as shown recently.[26,27]

The mean magnetic field is driven by the nonlinear electromotive force due to fluctuating electric and magnetic field. By averaging the Ohm's law (10) we obtain

$$\frac{\partial}{\partial t} \overline{\psi} + \frac{2c}{B_0} \hat{\mathbf{z}}\cdot \overline{\boldsymbol{\nabla} \hat{\phi} \times \boldsymbol{\nabla} \hat{\psi}} = 0, \tag{85}$$

or

$$\begin{aligned} \frac{\partial}{\partial t} \overline{\psi} = & -\frac{2c}{B_0} \frac{\partial}{\partial X} \left(\overline{\hat{\phi} \frac{\partial}{\partial y} \hat{\psi}} \right) + \frac{2c}{B_0} \frac{\partial}{\partial Y} \left(\overline{\hat{\phi} \frac{\partial}{\partial x} \hat{\psi}} \right) \\ = & \frac{2c}{B_0} \frac{\partial}{\partial X} \left(\overline{\sum \hat{\phi}_k i k_y \hat{\psi}_{-k}} \right) \\ & - \frac{2c}{B_0} \frac{\partial}{\partial Y} \left(\overline{\sum \hat{\phi}_k i k_x \hat{\psi}_{-k}} \right). \end{aligned} \tag{86}$$

From Eqs. (85) and (86) summation it is obvious that a finite phase between ϕ and ψ is required for the generation of the large scale magnetic field. At the linear stage such a phase shift occurs due to mode grow/damping associated with dissipation effects; in the saturated turbulent state the phase shift occurs due to the nonlinear mode broadening. The relation between ϕ and ψ that is found from the contunuity equation is

$$\phi_k = \frac{cT}{4\pi e^2 n_0} \frac{k_z k_\perp^2 (\omega_k + \omega_{*e} - i\delta_k)}{(\omega_k + \omega_{*e})^2 + \delta_k^2} \psi_k, \tag{87}$$

where ω_k is a real part of the eigen-frequency (includes a nonlinear frequency shift), and δ_k is the imaginary part of the mode frequency which may include Landau and nonlinear interactions effects. In nonlinear saturated state, the δ_k parameter is negative[28] and is of the order of the nonlinear turn-over time.[19,28–30] In the weakly nonlinear (quasilinear) regime δ_k can be simply estimated from the Landau damping effect $\delta_k/\omega_{*e} = -i\sqrt{\pi}\omega_{*e}/(2k_\parallel v_e)\eta_e$.

Then we have

$$\overline{\sum \hat{\phi}_k i k_y \hat{\psi}_{-k}} = \frac{cT}{4\pi e^2 n_0} \sum \frac{k_z k_\perp^2 \delta_k k_y}{(\omega_k + \omega_{*e})^2 + \delta_k^2} |\psi_k|^2. \tag{88}$$

Note that the driving term is proportional to the $k_z k_y |\psi_k|^2$ factor, which requires a finite helicity.[13,15] Equation for the evolution of the magnetic field is closed via the response of $|\psi_k|^2$ to the variations of the mean vector potential $\overline{\psi}$ that can be found by using the wave kinetic equation (83),

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$$\tilde{N}_k = -\frac{i}{(\Omega - qV_{gx})}\frac{k_y k_z v_{te}^2 k_\perp^2 d^2}{\omega}\frac{1}{B_0}\frac{\partial \overline{\psi}}{\partial Y}\frac{\partial N_k^0}{\partial k_x}. \tag{89}$$

Using it and (88) in (86) we find the following dispersion equation for the magnetic field instability:

$$\Omega(\Omega - qV_{gx}) = -iq^3 \frac{v_{te}^4 d^4}{B_0^2}\sum \frac{\delta_k k_y^2 k_z^2 k_\perp^2}{\omega((\omega_k + \omega_{*e})^2 + \delta_k^2)}\frac{\partial N_k^0}{\partial k_x}. \tag{90}$$

An interesting feature of this dispersion equation is that the instability occurs for any sign of the derivative of the wave action spectrum contrary to the generation of the mean plasma flow.

In (90) we consider the case $\mathbf{q} = \hat{\mathbf{e}}_x q$. A similar instability occurs also for the perturbations with $\mathbf{q} = \hat{\mathbf{e}}_y q$ but the corresponding growth rate is smaller due to the effect of the wave group velocity. One can readily see from (90) that the term qV_{gx} reduces the growth rate rather than provides the threshold against the instability. Thus, the effect of the wave group velocity decreases the growth rate for larger $\mathbf{q}\cdot\mathbf{V}_g$. Thus due to the condition $V_{gy} > V_{gx}$ the most unstable modes will have $q_x > q_y$, which corresponds to the poloidally elongated magnetic structures (magnetic islands).

VIII. SUMMARY

Recently interest in short wavelength electron temperature gradient turbulence has been renewed in relation to the problem of the anomalous electron energy transport.[31–35] Various theories of ion temperature gradient driven turbulence give a reasonable level of ion energy transport, and much progress has been made recently to achieve qualitative agreement. It appears that the electron transport is well above the values based on a simple mixing length estimate for the electrostatic ETG modes. It has been conjectured that the electron transport can be significantly increased due to the nonlinear excitation of radially elongated structures (streamers).[35,33] It has been suggested[35] that in the short wavelength turbulence the generation of zonal flow is much less effective than in the ion temperature gradient driven modes (ITG) leading to a larger level of fluctuations and, subsequently, to the larger anomalous transport. A number of aspects of ETG turbulence can be similar to a generic case of the short wavelength electromagnetic drift wave turbulence that is considered in the present paper. As our analysis has shown there are several reasons for the decrease of the zonal flow generation in the shorter wavelength region. One is due to the less effective source term: electron polarization current replaces the ion polarization current which is absent for $k^2\rho_i^2 > 1$. We have to note that in the intermediate regime $k^2\rho_i^2 \geqslant 1$, one has to compare the electron polarization current with a contribution of the nonadiabatic ion response. By using an order of magnitude estimate for the ion velocity $V_i \simeq V_E \Gamma_0$, we obtain that the electron polarization current dominates for sufficiently short wavelengths, $k_\perp \rho_i > (m_e/m_i)^{1/3}$. A second reason, as noted in Sec. III, is related to a different plasma density response to the long wavelength component $\overline{\phi}$ of the electrostatic potential in the $k^2\rho_i^2 > 1$ regime: it is still Boltzmann even for $\overline{\phi}$ (assuming that the radial scale $k_r^2\rho_i^2 > 1$). As a result the generation of large scale component (ZF) due to the electrostatic fluctuations is reduced compared to the longer wavelength fluctuations (such as ITG). Thus the saturation controlled by the ZF will occur at the higher amplitude of the background turbulence. In this sense the generation of ZF in the short wavelength turbulence is similar to that given by the pure Hasegawa–Mima or quasigeostrophic model. As investigated in the present paper, further reduction of the zonal flow generation is expected due to the electromagnetic effects (Maxwell stress) which partially cancel the Reynolds stress. As typical for zonal flow generation, the instability is sensitive to the sign of the derivative of the wave action spectrum.

In the present paper we have also investigated the instability of a large scale magnetic field. We have shown that electromagnetic short wavelength turbulence is robustly unstable with respect to the growth of the large scale magnetic structure that is somewhat similar to the magnetic islands. Analogous to the conventional magnetic islands, a finite radial component of the magnetic field may affect the radial electron energy transport. Contrary to the zonal flow instability, the magnetic field instability does not depend on the sign of the derivative of the wave action spectrum, though it does require the spectra anisotropy.

In electromagnetic turbulence the electron energy flux in general consist of two parts: one is electrostatic due to the $\mathbf{E}\times\mathbf{B}$ drift, $\langle \tilde{\mathbf{V}}_E \tilde{T}\rangle$, and the other is due to the radial component of the perturbed magnetic field, $\langle \tilde{\mathbf{B}}_r / B_0 \int d^3v \tilde{f}_e m_e v^2 v_\parallel /2\rangle$. In a particular regime considered in this work, $\omega \leqslant k_\parallel v_e$, the energy transport is predominantly electrostatic due to the $\mathbf{E}\times\mathbf{B}$ drift while the magnetic perturbations remain important in the field dynamics.[36] It is interesting to note that the electrostatic part of the electron energy flux may be ultimately related to the source of the secondary large scale magnetic field. For $\omega \leqslant k_\parallel v_e$ the temperature fluctuations are mostly determined by fluctuations of the magnetic potential due to the high electron thermal conductivity along the total magnetic field giving $\nabla_\parallel T = 0$ and, respectively, $\hat{T} = -T_0' \hat{\psi} k_y/(k_z B_0)$. Then in the lowest order the quasilinear electrostatic electron energy flux $\langle \tilde{\mathbf{V}}_E \tilde{T}\rangle$ can be expressed in the form $\tilde{V}_{Ex}\tilde{T} = cT_0'/B_0 \Sigma i k_y^2 \hat{\phi}_k \hat{\psi}_{-k}/k_z$. Comparing this to (86) and (88) one observes that a very similar correlator is a source of the magnetic field generation in (86). This suggests an intrinsic coupling of the radial energy transport and magnetic field generation in the electromagnetic short wavelength turbulence.

It has been noted in previous works that the form of the generalized wave action sensitively depends on the structure of the coupling matrix describing interaction of the large and small scale motion. As shown in this work the form of the generalized wave action is also affected by the presence of the slowly varying magnetic field. We have derived the generalized wave action invariants for the case of the electromagnetic turbulence modulated by slow magnetic field.

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