Role of external torque in the formation of ion thermal internal transport barriers

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We present an analytic study of the impact of external torque on the formation of ion internal transport barriers (ITBs). A simple analytic relation representing the effect of low external torque on transport bifurcations is derived based on a two field transport model of pressure and toroidal momentum density. It is found that the application of an external torque can either facilitate or hamper bifurcation in heat flux driven plasmas depending on its sign relative to the direction of intrinsic torque. The ratio between radially integrated momentum (i.e., external torque) density to power input is shown to be a key macroscopic control parameter governing the characteristics of bifurcation. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.3701560]

Realization of advanced tokamak operation in ITER and future reactors requires operating a plasma in an enhanced confinement regime, likely characterized by the existence of core transport barriers.1–3 Necessarily this is re-kindling interest in internal transport barrier (ITB) dynamics and/or the improvement of ion thermal confinement at zero or low external torque, because of the limited capability of driving the necessary torque in a reactor by neutral beam injection (NBI). Most of the ion ITBs in current experiments have been observed when sufficiently large external momentum is delivered by NBI.4,5 We note, however, that the formation of an ion ITB without external momentum input has been recently observed at Alcator C-Mod using off-axis ion cyclotron resonance heating (ICRH).6 In such a low torque, high heat flux-driven plasma, the intrinsic rotation, which is likely due to off-diagonal contributions to the turbulence-driven parallel Reynolds stress,7,8 will be of primary importance in and strongly coupled to barrier dynamics. The role of intrinsic rotation in zero torque ITB dynamics has been studied in a recent article with the help of global gyrofluid simulations.9

If finite external torque is applied to a plasma, it will interact with the self-generated intrinsic torque. A remarkable example of this external-intrinsic torque interaction is the observation of cancellation of co-current intrinsic rotation by the application of counter-current external torque in H-mode plasmas.10 We remark here that the cancellation of intrinsic rotation in ITB plasmas was actually alluded in early JT-6OU experiments,11 even though the authors did not stress that point. Then, the plasma rotation observed in actual experiments will be the outgrowth of this external-intrinsic torque interaction. In this context, it is necessary to elucidate the physics of this external-intrinsic torque interaction to improve our understanding of ITB formation when external momentum is injected into a plasma. A question then naturally arises of how barrier dynamics is affected by external torque (i.e., how a plasma responds to the external torque). This question is closely related to figuring out the role of external momentum input in the improvement of ion thermal confinement (i.e., the de-stiffening of the ion temperature profile).

The main focus of this paper is to study the role of low but finite external torque in heat flux-driven ion ITB formation. It necessarily requires a model that is capable of incorporating the physics of intrinsic rotation, in addition to the self-consistent inclusion of external heat and momentum sources in barrier dynamics. In particular, we will be interested in identifying a key control parameter which influences the barrier formation when finite external torque is applied. In this regard, recent JET experiments highlight the combined role of large rotation and low magnetic shear in de-stiffening the ion temperature profile, hence achieving ITBs or an enhanced confinement mode with a flat q-profile.12 The familiar story of $E \times B$ shear decorrelation of turbulence mostly by the plasma rotation has been applied to explain confinement enhancement in ion thermal channel, without invoking another important player in barrier dynamics—the intrinsic rotation. As mentioned in the previous paragraph, the interaction between external and intrinsic torque should be included in the analysis of experimental data leading to the ion temperature profile de-stiffening by plasma rotation.

To address this problem analytically, we start from a two-field transport model consisting of the conservation of pressure and the mean toroidal flow

\begin{equation}
\frac{3}{2} \frac{\partial P}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Theta) = S(r),
\end{equation}

\begin{equation}
n \frac{\partial v_\phi}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Pi_{\phi}) = U_\phi(r),
\end{equation}

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where $P$, $n$, $v_\phi$, $S(r)$, and $U_\phi(r)$ denote the mean pressure, density, toroidal flow velocity, external power, and toroidal momentum density input, respectively. The fluxes in Eqs. (1) and (2) are given by

$$\Theta(r) = -\left(\chi_0 + \frac{\chi_1}{1 + x r^2}\right) \frac{\partial P}{\partial r}, \tag{3}$$

$$\Pi_{\rho\phi}(r) = \Pi_{\text{diff}} + \Pi_{\text{res}} = - \left(\nu_0 + \frac{\nu_1}{1 + x r^2}\right) \frac{\partial v_\phi}{\partial r} + \Pi_{\text{res}}, \tag{4}$$

where $\chi_0$ ($\nu_0$) and $\chi_1$ ($\nu_1$) represent the neoclassical and turbulent ion thermal diffusivity (viscosity), respectively, $\gamma_E$ is the $E \times B$ shearing rate, and $x$ is a coupling parameter characterizing the strength of $E \times B$ suppression of turbulence. In writing Eq. (4), we decomposed $\Pi_{\rho\phi}$ into the diffusive ($\Pi_{\text{diff}}$) and the residual stress ($\Pi_{\text{res}}$) parts, which is the off-diagonal element of $\Pi_{\rho\phi}$. In this paper, we assume that $\Pi_{\text{res}}$ is given by

$$\Pi_{\text{res}} = - \frac{\nu_1 \zeta}{1 + x r^2} \frac{\partial v_E}{\partial r}, \tag{5}$$

where $\zeta$ is a dimensionless parameter involving the shift of the intensity fluctuation profile due to $E \times B$ shear, $(T_i/T_e)^{1/2}$, and the ratio of density to magnetic shear scaling length. We note that the sign of $\zeta$ can change depending on the type of turbulence modes (i.e., ion temperature gradient (ITG) or electron drift modes). Equation (5), albeit simple, contains the essential physics of intrinsic rotational generation by turbulence (i.e., the intrinsic torque) due to the $E \times B$ shear-driven symmetry breaking. The other off-diagonal element of $\Pi_{\rho\phi}$, the toroidal momentum pinch term that is proportional to the toroidal velocity itself, is neglected for simplicity. Recent global gyrofluid simulations showed that this compressibility effect is indeed small in ITB dynamics.

Equations (1) and (2) are then coupled through $\Pi_{\text{res}}$ and $\gamma_E$. In this paper, we assume that dominant contributions to $\gamma_E$ come from the diamagnetic term and the toroidal mean flow gradient

$$\gamma_E = - \frac{1}{e B n} \frac{\partial P}{\partial r} + \frac{\epsilon}{q} \frac{\partial v_\phi}{\partial r} \equiv f P_1 - \frac{\epsilon}{q} V_1, \tag{6}$$

where $\epsilon$ and $q$ are the inverse aspect ratio and safety factor, respectively, $P_1 = -\partial P/\partial r$, $V_1 = -\partial v_\phi/\partial r$, and $f = (\partial n/\partial r)/e B n^2 = (e B n) n^{-1}$. Contributions to $\gamma_E$ from the poloidal flow and the pressure curvature are neglected for the sake of analytic progress. Neglecting these terms may make the application of the present model dubious to edge transport barrier (ETB) dynamics (i.e., L-H forward and back transition) where the contributions from these terms are crucial. Further, this model of $\gamma_E$ cannot predict the barrier width, even in the core region, due to the neglect of the pressure curvature term. This limitation is mitigated if we focus on local ITB dynamics where the parallel velocity shear term is likely to be prominent in barrier dynamics, as shown in recent simulations and experiments at Alcator C-mod.

Thus, an important advantage of this simple model is that it captures the physics of intrinsic rotation and its influence in ITB formation, via $\partial v_\phi/\partial r$ term in $\gamma_E$, at a given radius.

We assume a quasi-static process such as slow power ramps. Then, substituting Eqs. (3)–(6) into Eqs. (1) and (2) and integrating them from 0 to $r$ yields

$$\left(\chi_0 + \frac{\chi_1}{1 + x r^2}\right) P_1 = Q(r), \tag{7}$$

$$\left(\nu_0 + \frac{\nu_1}{1 + x r^2}\right) V_1 - \frac{G}{1 + x r^2} P_1 = F(r), \tag{8}$$

where $\nu_2 = (\epsilon/q) \zeta \nu_1$, $G = f \zeta \nu_1$, and $Q(r) = \int_0^r r' S(r') dr'$, $F(r) = \int_0^r r' U_\phi(r') dr'$, is the radially integrated power and toroidal momentum sources, respectively. It is easy to eliminate the pressure gradient term in Eq. (8)

$$\left(\chi_1 \nu_0 V_1 + G \chi_0 P_1 + \chi_1 (\nu_1 + \nu_2) \frac{1}{1 + x r^2} V_1 = \chi_1 F(r) + G Q(r). \tag{9}$$

Using Eqs. (7) and (9), one can express $V_1$ as a function $P_1$, $V_1 = P_1 + \frac{G \chi_0 P_1 - (\chi_1 F(r) + G Q(r))}{\chi_1 (\nu_1 + \nu_2) \frac{1}{1 + x r^2} V_1} = \tilde{Q}(r), \tag{10}$

Substitution of Eq. (10) into Eq. (7) leads to the relation

$$P_1 + \frac{\lambda P_1}{1 + \delta^2 P_1^2 [1 - J(Q, P_1, F)]} = \dot{Q}(r), \tag{11}$$

where $\lambda = \chi_1/\chi_0$, $\dot{Q}(r) = \dot{Q}(r)/\dot{Q}_0$, and

$$J(Q, P_1, F) = \lambda_2 \frac{(\dot{Q} - P_1) + (\dot{Q}/G) F}{(\lambda_1 + \lambda_2) \dot{Q} - (\lambda_1 + \lambda_2 - \lambda) P_1} \tag{12}$$

with $\lambda_1, \lambda_2 = \nu_1, \nu_2$. Equation (11) is a key result of this paper. Basically, it describes the well-known S-curve-like transport bifurcation due to $\gamma_E$ (Refs. 17–19) in the presence of intrinsic rotation and external torque.

A drawback of Eq. (11) that it is analytically intractable. To make progress analytically, we further assume that $\lambda_1 + \lambda_2 \gg \lambda$. In tokamaks, this is a good approximation due to the smallness of $\nu_0$ in comparison to $\chi_0$ while $\nu_1 \sim \chi_1$ in ITG turbulence. The validity of this assumption should be checked in the case of $\lambda_2 < 0$ because $\lambda_2$ can have either positive or negative sign depending on turbulence modes. Since $|\lambda_2| = (\epsilon/q)|\zeta| \nu_1$, $\lambda_1 + \lambda_2 \gg \lambda$ is still valid (to order of $\epsilon/q$) even when $\lambda_2 < 0$ and $|\zeta| \sim 1$.

Given this assumption, Eq. (12) becomes

$$J(Q, P_1, F) = \frac{\lambda_2}{\lambda_1 + \lambda_2} \left[1 + \frac{(\lambda_1 G)F}{Q - P_1} \right]. \tag{13}$$
We first consider the case of zero external momentum input \((F = 0)\), i.e., a purely heat flux driven bifurcation. In this case, the \(\nabla V_{||}\) contribution to \(\gamma_{E}\) is provided exclusively by intrinsic rotation, which is strongly coupled to ITB dynamics.\(^9\) Then, Eq. (11) is reduced to

\[
F(g) = g \left( 1 + \frac{\lambda}{1 + \mu^2 g^2} \right) = \dot{Q}(r),
\]

(14)

where \(g^2 = g_0^2 \mu^2\), \(\mu^2 = [\lambda_1/(\lambda_1 + \lambda_2)]^2 = [1 + (\epsilon/q_0^2)]^{-2}\) and \(\dot{Q}(r) = \dot{Q}_0 \chi^2 \chi /f\). Equation (14) describes transport bifurcation in the presence of intrinsic rotation; for a given value of \(\dot{Q}(r)\), the left hand side of Eq. (14) has local maximum and minimum when \(\lambda > 8\).

The effect of intrinsic rotation in transport bifurcation is contained in \(\mu^2\) via \(\lambda_2\). Without intrinsic rotation, \(\lambda_2 = 0\) and Eq. (14) represents the conventional bifurcation curve driven by \(E \times B\) shear suppression of turbulence.\(^1\) If \(\lambda_2 \neq 0\), however, the threshold heat flux, \(\dot{Q}_{th}\), at which the bifurcation occurs, increases or decreases compared to the no-intrinsic rotation case, depending on the sign of \(\lambda_2\). This is illustrated in Fig. 1 where \(F(g)\) is plotted when \(\mu = 1.0\) (black solid line, 1.2 (blue dotted line), and 0.8 (red dotted line). As can be seen in Fig. 1, negative \(\lambda_2\) (i.e., \(\mu > 1\)) reduces \(\dot{Q}\), while positive \(\lambda_2\) increases it. Since the sign of \(\lambda_2\) depends on turbulence modes (e.g., ITG or TEM), it suggests that the characteristics of pre-transition turbulence may facilitate or hamper heat flux driven transport bifurcation, for a given power input, via intrinsic rotation. This observation leads to an interesting prediction that the ITB power threshold for an ECH heated plasma (i.e., TEM dominant) could be different from that of a NBI heated plasma (i.e., ITG dominant) with balanced beam injection.

Now, we consider the effect of finite external torque on bifurcation. Substitution of Eq. (13) into Eq. (11) and rearrangement of the equation gives rise to

\[
C^2 P_1^2 [P_1 - \dot{Q}(1 - \delta)]^2 + (P_1 - Q)(P_1 + \lambda P_1 - Q) = 0,
\]

(15)

where \(C^2 = g_0^2 \mu^2\) and \(\delta = (\epsilon/q_0^2 \chi_0 \chi /f)(\lambda_1/\lambda_1)(\dot{F}/\dot{Q}) = \gamma_0\) with \(\dot{F} = F / \nu_0\), \(\gamma = (\epsilon/q_0^2 \chi_0 \chi /f)(\lambda_1/\lambda_1)\) and \(\theta = \dot{F}/\dot{Q}\). Typically, \(\gamma \leq 0\) because \(\nu_n \leq 0\), while \(\theta\) can have either sign depending on the direction of external momentum input. We assume that the external momentum input is relatively small in the sense that

\[
\delta = (\epsilon/q_0^2 \chi_0 \chi /f)(\lambda_1/\lambda_1)(\dot{F}/\dot{Q}) \simeq (\epsilon/q_0^2 \nu T/S(r)) \times (V_{th}(r)/V_{th} \mu) < 1,
\]

(16)

where \(V_{th}\) is the ion diamagnetic drift velocity. The inequality of Eq. (16) is easily satisfied under tokamak experiments with tangential NBI. Then, we can retain only the first order term in \(\delta\), and Eq. (15) becomes

\[
C^2 P_1^3 + (1 + \lambda) P_1 = \dot{Q} - C^2 \dot{Q} (2 \delta - 1) P_1^2,
\]

from which we obtain

\[
F(g) = g \left[ \frac{\mu^2 g^2 + (1 + \lambda)}{1 + \mu^2 g^2 (1 - 2 \gamma \theta)} \right] = \dot{Q}(r).
\]

(17)

Equation (17) is the main result in this paper. It describes the S-curve-like transport bifurcation in the presence of both intrinsic rotation and relatively low external torque in the sense given in Eq. (16). The effect of external torque is contained in the parameter \(\theta\). Without external torque, \(\theta = 0\) and Eq. (17) is reduced to Eq. (14).

There are two messages in Eq. (17). First, the external momentum source does not affect transport bifurcation independently, but does as a combination with the heat source, namely, through the ratio of strength between momentum and heat sources. Second, the external momentum source can play a role of control parameter (through \(\theta\)) governing transport bifurcation by lowering or raising \(\dot{Q}_{th}\), depending on its sign. The second point is illustrated in Fig. 2 where \(F(g)\) is plotted for \(\theta = 0\) (black solid line), 2.0 (blue dotted line), and \(-2.0\) (red dotted line). In producing Fig. 2, we have used \(\lambda = 20\), \(\gamma = 0.1\), and \(\mu^2 = 1.2\). Comparing with \(\theta = 0\) case, positive \(\theta\) leads to the transport bifurcation with lower heat flux, while negative \(\theta\) requires higher heat flux for bifurcation. In addition to the change of \(\dot{Q}_{th}\), it is also noted that the end-product of bifurcation, namely ITB pressure gradient, becomes higher when \(\theta\) is positive, while it is considerably reduced when \(\theta\) is negative. Exactly the same tendency is observed regardless of the \(\mu^2\) values being used (i.e., independent of turbulence modes), suggesting it is a general feature of the influence of \(\theta\) in transport bifurcation. This suggests the possibility of exploiting \(\theta\) as a control knob for transport barrier formation.

The prediction that \(\theta\) is a control parameter in ITB formation has an interesting experimental implication: the amount and direction of external torque may determine the ITB pressure gradient. Our analytic results suggest that an ITB with a moderate pressure gradient is accessible when

\[
\dot{Q}_0 = \dot{Q}_0, \quad \lambda_1 = \lambda_1, \quad \gamma_0 = \gamma_0
\]

FIG. 1. Plot of \(F(g) = g \left[ (\mu^2 g^2 + (1 + \lambda) \chi_0 \chi /f) \right] \) for three value of \(\mu (0.8:\) red dotted line, 1.0: solid line, and 1.2: blue dotted line) when there is no external torque (i.e., \(\theta = 0\)). Direction of intrinsic rotation is negative when \(\mu > 1\) while it is positive when \(\mu < 1\). \(\lambda = 20\) and \(\gamma = 0.1\) have been used. More precise definitions of parameters involved in \(F(g)\) are given in the main text. The threshold heat flux \(\dot{Q}_{th}\) at which bifurcation occurs increases or decreases depending on the sign of intrinsic rotation, indicating the relevance of bifurcation to pre-transition turbulence modes.
appropriate negative external torque is applied. This can shed light on the realization of steady state ITBs with low external torque, which is a prime candidate scenario for advanced tokamak operation of ITER.

In conclusion, this paper has focused on the role of external momentum input in the formation of ion thermal ITBs and interaction between external and intrinsic torque. A simple two-field transport model was used to derive a simple relation accounting for the interaction of intrinsic rotation with low external torque in transport bifurcations. The main results of this paper are summarized as follows:

1. Intrinsic rotation is an important player in barrier dynamics. It is strongly coupled to ITB dynamics by providing the $\nabla V$ contribution to $\gamma_E$. When finite external torque is applied, a strong external-intrinsic torque interaction occurs, by which the ITB formation is hampered or facilitated.

2. If there is no externally applied torque, the characteristics of pre-transition turbulence may facilitate or hamper heat flux driven transport bifurcation via intrinsic rotation.

3. The ratio between radially integrated external toroidal momentum density (which can be easily converted into a toroidal torque density) to power input $\theta = \int_0^r U_d r dr / \int_0^1 P_{in} r dr$, (where $U_d$ is the external momentum density and $P_{in}$ is the power input) plays a role of a control parameter in the barrier bifurcation. Low positive (i.e., the same direction with intrinsic torque) torque facilitates the barrier formation while negative (i.e., the opposite direction to intrinsic torque) one impedes it.

Ion confinement enhancement or ion temperature profile de-stiffening by externally injected momentum is an interesting current issue. Recent experimental works at JET emphasizes the combined role of flat $q$-profile and plasma rotation on the de-stiffening of ion temperature profile. On this point, our work provides a new perspective on the interpretation of these experimental results. First, it is the radially integrated torque that is important in the barrier formation, not the rotation, as shown in Eq. (17). Second, it is necessary to consider intrinsic torque as well as the external one (i.e., external-inner torque interaction). The actual plasma rotation observed in experiments will be the result of external-intrinsic torque interaction.

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