Collisionless inter-species energy transfer and turbulent heating in drift wave turbulence

L. Zhao¹ and P. H. Diamond^{1,2}

¹Center for Astrophysics and Space Sciences and Department of Physics, University of California at San Diego, La Jolla, California 92093-0424, USA

²WCI Center for Fusion Theory, National Fusion Research Institute, Gwahangno113, Yuseong-gu, Daejeon 305-333, Korea

(Received 11 May 2012; accepted 30 July 2012; published online 17 August 2012)

We reconsider the classic problems of calculating "turbulent heating" and collisionless interspecies transfer of energy in drift wave turbulence. These issues are of interest for low collisionality, electron heated plasmas, such as ITER, where collisionless energy transfer from electrons to ions is likely to be significant. From the wave Poynting theorem at steady state, a volume integral over an annulus $r_1 < r < r_2$, gives the net heating as $\int_{r_1}^{r_2} dr \langle \tilde{E} \cdot \tilde{J} \rangle = -S_r |_{r_1}^{r_2} \neq 0$. Here S_r is the wave energy density flux in the radial direction. Thus, a wave energy flux differential across an annular region indeed gives rise to a net heating, in contrast to previous predictions. This heating is related to the Reynolds work by the zonal flow, since S_r is directly linked to the zonal flow drive. In addition to *net heating*, there is inter-species *heat transfer*. For collisionless electron drift waves, the total turbulent energy source for collisionless heat transfer is due to quasilinear electron cooling. Subsequent quasilinear ion heating occurs through linear ion Landau damping. In addition, perpendicular heating via ion polarization currents contributes to ion heating. Since at steady state, Reynolds work of the turbulence on the zonal flow must balance zonal flow frictional damping $(\sim \nu_{ii} \langle V_{\theta} \rangle^2 \sim |\frac{e \phi}{T}|^4)$, it is no surprise that zonal flow friction appears as an important channel for ion heating. This process of energy transfer via zonal flow has not previously been accounted for in analyses of energy transfer. As an application, we compare the rate of turbulent energy transfer in a low collisionality plasma with the rate of the energy transfer by collisions. The result shows that the collisionless turbulent energy transfer is a significant energy coupling process for ITER plasma. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.4746033]

I. INTRODUCTION

A. A general turbulent energy transfer and transport problem

We reconsider the classic problems of calculating "turbulent heating" and collisionless inter-species transfer of energy in drift wave turbulence. This issue is of interest for near future low collisionality electron heated plasmas, such as ITER, where collisionless energy transfer from electrons to ions is likely to be significant. In the heat balance equation^{1,2}

$$\frac{3}{2}n\frac{\partial T_{\alpha}}{\partial t} + \nabla \cdot Q_{\alpha} = \langle \tilde{E} \cdot \tilde{J}_{\alpha} \rangle \mp n\nu \frac{m_{e}}{m_{i}} (T_{e} - T_{i}) + \cdots, \alpha = e, i,$$
(1)

where Q_{α} is the heat flux, $\langle \tilde{E} \cdot \tilde{J}_{\alpha} \rangle$ is turbulent dissipation and corresponds to turbulent heating, and the last term on the right hand side represents the collisional transfer of energy between particle species. Generally, turbulent energy can be exchanged between electrons and ions by collisional or collisionless energy transfer. The familiar term $n\nu \frac{m_e}{m_i} (T_e - T_i)$ describes collisional energy transfer through electron and ion binary collisions whereas $\langle \tilde{E} \cdot \tilde{J}_{\alpha} \rangle$ describes turbulent heating for a single species, such as electron turbulent cooling $(\langle \tilde{E} \cdot \tilde{J}_e \rangle < 0 \rightarrow \text{electrons lose energy to the drift wave so the wave is destabilized), or ion turbulent heating <math>(\langle \tilde{E} \cdot \tilde{J}_i \rangle > 0 \rightarrow \text{electrons lose})$

ion gain energy from the drift wave and wave is stabilized). If we consider inter-species turbulent heating for electron heated plasmas, electron and ion collisionless energy transfer could occur where the hot electrons act as the local energy "source" and cold ions as the "energy sink." This collisionless turbulent energy transfer is especially important in a burning plasma, since the fuel ions (deuterium, tritium) can be heated by this inter-species turbulent energy transfer process. Also we note that any energetic particles (α particles) produced in the nuclear reaction will heat electrons first. Thus, the energy flow can be transferred from hot electrons to cooler ions again, to allow the reaction to sustain itself. Hence, the electron and ion collisionless energy coupling will be a critical issue for burning plasmas, such as ITER.

To see this, we track the energy exchanged between electrons and ions. There exist two stages of energy transfer processes before and after the nuclear reaction (see Fig. 1). We ignore the energy loss due to electron and ion radiation in these processes. During the first stage, electrons can be heated by the auxiliary energy system, electron cyclotron resonance heating (ECRH) and then lost by electron heat flux Q_e or transferred to ions through collisionless or collisional energy transfer channels. Next, the fuel ions become hot enough and can reach nuclear reaction ignition. Once the energetic particles are produced in the nuclear reaction, the second energy flow will be generated and the process of energy transport or transfer can continue with more and

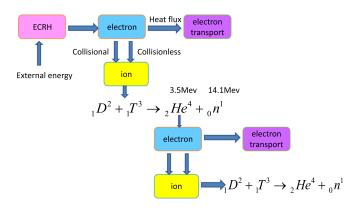


FIG. 1. There are two stages in the energy flow before and after the nuclear reaction.

more reactions occurring. But what is the ultimate fate of the energy? We need to understand the collisionless energy transfer mechanism and what roles turbulent energy transfer and turbulent transport play in the energy budget.

B. Net turbulent heating

Does turbulence heat a given volume of plasma? Manheimer *et al.* argued that there was no net turbulent heating of plasma and only an exchange of energy between electrons and ions. In that calculation, *periodic boundary conditions* in the radial direction were utilized, such that boundary contributions to the turbulent heating vanished. However, we consider the turbulent heating taking place within a region of finite radial extent given by

$$\int dr \langle \tilde{E} \cdot \tilde{J} \rangle = -\tilde{\phi} \tilde{J}_r |_{r_1}^{r_2} + \int dr (\nabla \cdot \tilde{J}) \tilde{\phi} \neq 0, \qquad (2)$$

where the turbulent heating can be written as $\langle \vec{E} \cdot \vec{J} \rangle = \sum_{\alpha=e,i} \langle \tilde{E} \cdot \tilde{J}_{\alpha} \rangle$, \tilde{J}_r is the radial current fluctuation in an annular region, and the width of the annular region is $r_1 < r < r_2$. The $\langle \cdots \rangle$ defines an average in the θ , ϕ direction. The first term on the RHS of Eq. (2) corresponds to a surface term at the annular boundary which can give rise to a *net* turbulent heating. The second term vanishes since $\nabla \cdot \tilde{\bf J} = 0$ (plasma quasi-neutrality). Thus, the boundary effect in a finite annular region will give rise to *net heating*. This can also be understood from the wave energy theorem which is written as 6,7

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{S} + \langle \tilde{E} \cdot \tilde{J} \rangle = 0, \tag{3}$$

where W is wave energy density and S represents wave energy density flux. At steady state, taking a volume integral of this theorem in an annular region, we have at stationarity

$$\int_{r_1}^{r_2} dr \langle \tilde{E} \cdot \tilde{J} \rangle = -S_r|_{r_1}^{r_2}.$$
 (4)

Equation (4) suggests that a wave energy flux differential across an annular region $r_1 < r < r_2$ gives rise to a *net* heating within that region. Aspects of this point have been addressed in Refs. 2 and 3. In addition, this wave energy flux

can be related to the Reynolds stress by noting the wave energy density flux can be written as ^{7,8}

$$S_r = V_{gr,r} W_k = -2 \frac{\rho_s^2 k_r k_\theta v_\star W_k}{(1 + k_\perp^2 \rho_s^2)^2},\tag{5}$$

where $V_{gr,r} = d\omega/d\mathbf{k}$ is the wave group velocity. W_k is the wave energy density in Fourier space. In the electrostatic drift wave, $V_{gr,r} = -(d\chi/d\mathbf{k})/(d\chi/d\omega) = -(2\rho_s^2k_r\omega)/(1+k_\perp^2\rho_s^2)$ and the susceptibility is $\chi(k,\omega) = 1+k_\perp^2\rho_s^2 -\omega_\star/\omega - i\delta_\omega$. The electron drift wave frequency is $\omega = \omega_\star/(1+k_\perp^2\rho_s^2)$ and the electron diamagnetic frequency is $\omega_\star = k_\theta v_\star$. Since the Reynolds stress can be written as $\langle \tilde{V}_r \tilde{V}_\theta \rangle = \sum_k -(c^2/B^2)k_rk_\theta |\tilde{\phi}|^2$, the energy flux differential is seen to be directly proportional to the Reynolds stress, which drives zonal flow generation.

C. Collisionless turbulent energy transfer channels

Now we look at the collisionless interspecies energy transfer channels. The turbulent energy flow channels considered within this manuscript are shown in Fig. 2. Free energy in ∇n and ∇T will drive microturbulence which gives rise to turbulent heating $\langle \tilde{E} \cdot \tilde{J} \rangle$. The dominant parallel components of the turbulent heating are composed of quasilinear electron cooling $\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \rangle^{(2)}$, quasilinear ion heating $\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \rangle^{(2)}$, and nonlinear ion turbulent heating $\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \rangle^{(4)}$. The perpendicular components considered are given by the ion polarization drift and ion diamagnetic drift induced turbulent heating $\langle \tilde{E}_{\perp} \cdot \tilde{J}^{i}_{\perp pol} \rangle$ and $\langle \tilde{E}_{\perp} \cdot \tilde{J}^{i}_{\perp dia} \rangle$. The ion polarization drift induced turbulent heating in an annulus contributes a wave energy flux differential term at the boundary, also a Reynolds work of turbulence on the mean flow. This flux differential, intimately linked to the turbulent Reynolds stress and hence zonal flow formation, will be shown to give rise to net turbulent heating. This process of energy transfer via zonal flows has not previously been accounted for in analyses of energy transfer. On the right

Turbulent Energy flow Channels

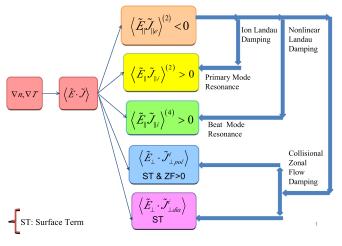


FIG. 2. Turbulent energy flow channels: "energy source" of quasilinear electron cooling $\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \rangle^{(2)}$; "energy sink" of quasilinear ion heating $\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \rangle^{(2)}$, nonlinear ion heating $\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \rangle^{(4)}$, ion polarization drift and ion diamagnetic drift induced turbulent heating $\langle \tilde{E}_{\perp} \cdot \tilde{J}_{\perp pol}^i \rangle$ and $\langle \tilde{E}_{\perp} \cdot \tilde{J}_{\perp dia}^i \rangle$.

side of Fig. 2, we describe three kinds of energy dissipation channels for electron drift wave turbulence. The first two are wave energy dissipated through ion Landau damping at quasilinear order and nonlinear ion Landau damping (the beat mode resonance). The third one is by zonal flow frictional damping at steady state, which is due to the turbulence and zonal flow interaction. Basically, the electrons will lose energy to the wave and the ions will gain energy from the wave, thus providing collisionless energy transfer channels mediated by electron drift waves and zonal flow.

For each turbulent heating term, there will be a corresponding γ_{growth} for wave growth or $\gamma_{damping}$ for wave dissipation in the saturation balance condition. Nonlinear saturation in a turbulent state implies energy transfer from source $(\nabla T_e, \nabla n)$ to sink. Schematically, saturation implies some fluctuation energy balance condition must be satisfied, so we have

$$0 = \gamma = \gamma_L^{electron} + \gamma_L^{ion} + \gamma_{NL}^{ion} + \gamma_{zonalflow}^{ion} + \cdots.$$
 (6)

For quasilinear electron cooling terms, the electrons will lose energy to the drift wave and drive wave instability, which gives rise to $\gamma_L^{electron} > 0$. In contrast, for quasilinear ion heating, the ions gain energy from the wave through ion Landau damping, so $\gamma_L^{ion} < 0$. Also the wave energy can be dissipated through nonlinear ion Landau damping and $\gamma_{NL}^{ion} < 0$. $^{11-13,16}$ In particular, wave energy can be dissipated by zonal flow friction, so that $\gamma_{zonalflow} < 0$ in the saturation balance. Here, $\gamma_{zonalflow}$ represent a nonlinear saturation mechanism. If $\gamma_{zonalflow}$ dominates the saturation balance, 8,17 corresponding to turbulent heating, the energy coupled to the zonal flow must contribute to energy transfer channels as well. (Note: the nonlinear ion turbulent heating and its contribution for turbulent energy transfer channel will be discussed in another paper, for simplicity.)

This paper is organized as follows. In Sec. II, we will calculate the parallel quasilinear turbulent heating terms $\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \rangle^{(2)}$ and $\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \rangle^{(2)}$ which effect a collisionless energy transfer from electrons to ions, mediated by the background waves. In Sec. III, the ion polarization drift and ion diamagnetic drift induced turbulent heating are calculated. $\langle \tilde{E}_{\perp} \cdot \tilde{J}^{i}_{\perp nol} \rangle$ accounts for the energy flux differential term at the boundaries and the net Reynolds work of turbulence on the zonal flow. And $\langle \tilde{E}_{\perp} \cdot \tilde{J}^i_{\perp dia} \rangle$ contributes a heat flux differential at boundary $(\sim \chi_{tur} n \nabla T)$, which transports the turbulent energy through ion diffusion. In Secs. IV and V, we will estimate all of the turbulent heating terms by using a mixing length estimation for $|e\tilde{\phi}/T|^2$. At different collisionalities, we will compare the ratios of the energy dissipation channels through linear Landau damping and zonal flow frictional damping. By applying ITER like parameters, 5,18 we can see that the zonal flow frictional damping can be a significant energy dissipation channel in low collisionality drift wave turbulence in ITER plasmas. In Secs. VI and VII, we will continue to explore the implications of our results for ITER plasmas. For realistic cases, we will extend our discussion to collisionless trapped electron mode (CTEM). 11-13,16 First, we will compare two energy transfer channels: collisional and collisionless. Subsequently, we compare the

collisionless energy transfer with the turbulent energy transport losses through the CTEM heat flux. At the end, we will summarize the turbulent heating and collisionless energy transfer channels in drift wave turbulence.

II. QUASILINEAR TURBULENT HEATING FOR DRIFT WAVE TURBULENCE

Within this section, we calculate the turbulent heating for both electrons and ions by using quasilinear theory. In the limit of vanishing collisionality, the stability of a wave can be determined by computing the transfer of energy between resonant particles and waves. For the case of collisionless drift waves considered below, resonant electrons will lose energy to the wave (resulting in electron cooling), whereas ions generally gain energy (via ion Landau damping), and are thus heated. Any imbalance in the rate of electron cooling versus ion heating will generally lead to wave growth or damping. Generally, the turbulent heating is driven by current fluctuations which can be written as

$$\tilde{J} = \sum_{e,i} e \int d^3v v \tilde{f} = \sigma^t \tilde{E}, \tag{7}$$

where the σ^t is the plasma conductivity due to the turbulence. The calculation of the turbulent heating requires computing a plasma conductivity which related to the particle distribution function \tilde{f} . We will be interested in describing the evolution of collisionless drift waves in an electron heated plasma in a simplified geometry. The evolution of electrons will be described by a drift kinetic equation (DKE), namely,

$$\frac{\partial \tilde{f}_e}{\partial t} - i \frac{c}{B} k_y \frac{\partial \langle f_e \rangle}{\partial x} \tilde{\phi}_k + v_z \frac{\partial \tilde{f}_e}{\partial z} - \frac{e}{m_e} \tilde{E}_z \frac{\partial \langle f_e \rangle}{\partial v_z} = 0.$$
 (8)

The fluctuation of the electron distribution can be separated into adiabatic and non-adiabatic parts

$$\tilde{f}_e = \frac{e\tilde{\phi}_k}{T_e} \langle f_e \rangle + \tilde{g}_k. \tag{9}$$

Substituting Eq. (9) into Eq. (8), yields

$$\frac{\partial \tilde{g}_{k}}{\partial t} + v_{z} \frac{\partial \tilde{g}_{k}}{\partial z} = -\frac{\partial}{\partial t} \frac{e \tilde{\phi}_{k}}{T_{e}} \langle f_{e} \rangle - v_{z} \frac{\partial}{\partial z} \frac{e \tilde{\phi}_{k}}{T_{e}} \langle f_{e} \rangle
- \frac{e}{m_{e}} \frac{\partial}{\partial z} \tilde{\phi}_{k} \frac{\partial \langle f_{e} \rangle}{\partial v_{z}} + i \frac{c}{B} k_{y} \frac{\partial \langle f_{e} \rangle}{\partial x} \tilde{\phi}_{k}. \quad (10)$$

We take the electron equilibrium distribution function as a local Maxwellian in one dimension, i.e., $\langle f_e \rangle = n_0(x) (m_e/2\pi T_e)^{1/2} \exp(-v_z^2/V_{the}^2)$ and $\partial \langle f_e \rangle/\partial v_z = (-2v_z/V_{the}^2) \langle f_e \rangle$. Substituting $\langle f_e \rangle$ into Eq. (10) yields

$$\frac{\partial \tilde{g}_{k}}{\partial t} + v_{z} \frac{\partial \tilde{g}_{k}}{\partial z} = -\frac{\partial}{\partial t} \frac{e \tilde{\phi}_{k}}{T_{e}} \langle f_{e} \rangle + i \frac{c}{B} k_{y} \frac{\partial \langle f_{e} \rangle}{\partial x} \tilde{\phi}_{k}. \tag{11}$$

Fourier transforming Eq. (11) gives

$$\tilde{g}_{k} = \frac{-\omega \frac{e\tilde{\phi}_{k}}{T_{e}} \langle f_{e} \rangle - \frac{c}{B} k_{y} \frac{\partial \langle f_{e} \rangle}{\partial x} \tilde{\phi}_{k}}{\omega - k_{z} v_{z}} = \frac{(\omega_{\star e} - \omega) \frac{e\tilde{\phi}_{k}}{T_{e}} \langle f_{e} \rangle}{\omega - k_{z} v_{z}}.$$
 (12)

Here we define the diamagnetic frequency $\omega_{\star e} = -k_y(T_ec/eB)(1/n)(dn/dx) = (k_y\rho_sC_s)/L_n$, and the scale length for the density gradient is given by $L_n^{-1} = -(1/n)(dn/dr)$, $\rho_s = C_s/\Omega_i$. Then the quasilinear turbulent heating for electrons can be written as

$$\langle \tilde{E}_{||} \cdot \tilde{J}_{||e} \rangle^{(2)} = \sum_{k} -e \int dv_z v_z \tilde{E}_z \tilde{g}_k. \tag{13}$$

Here the contribution of the electron quasilinear turbulent heating is in the parallel electric field direction. Substituting Eq. (12) into Eq. (13), we obtain

$$\langle \tilde{E}_{||} \cdot \tilde{J}_{||e} \rangle^{(2)} = \sum_{k} -e \int dv_{z} v_{z} (ik_{z} \tilde{\phi}_{-k}) \frac{(\omega_{\star e} - \omega)}{\omega - k_{z} v_{z}} \frac{e \tilde{\phi}_{k}}{T_{e}} \langle f_{e} \rangle,$$

$$= \sum_{k} -e \int dv_{z} v_{z} (ik_{z} \tilde{\phi}_{-k}) (\omega_{\star e} - \omega)$$

$$\times \frac{(-i\pi)}{|k_{z}|} \delta(\omega/k_{z} - v_{z}) \frac{e \tilde{\phi}_{k}}{T_{e}} \langle f_{e} \rangle,$$

$$= \sum_{k} \sqrt{\pi} n T_{e} \left| \frac{e \tilde{\phi}_{k}}{T_{e}} \right|^{2} \frac{\omega}{|k_{z}| V_{the}} (\omega - \omega_{\star e})$$

$$\times \exp \left[-\frac{(\omega/k_{z})^{2}}{V_{the}^{2}} \right]. \tag{14}$$

The calculation of the δ function integral $\int dv_z \delta(\omega/k_z - v_z) \langle f_e \rangle = \langle f_e \rangle|_{v_z = \omega/k_z}$ was applied to Eq. (14), where the electron drift waves resonate with the background electrons if the phase velocity satisfies $v_z = \omega/k_z$. We also note that the quasilinear turbulent heating for electrons $\langle \tilde{E}_{||} \cdot \tilde{J}_{||e} \rangle^{(2)}$ is determined by the resonant electron density n, temperature T_e and turbulent intensity $|e\tilde{\phi}_k/T_e|^2$.

Considering the drift wave dispersion relationship $\omega = \omega_{\star e}/(1+k_\perp^2\rho_s^2)$, then we have $\omega(\omega-\omega_{\star e})<0$ which implies a competition between stabilization and destabilization effects for the electron drift wave. On one hand, the ω is the Landau resonance frequency and the wave was stabilized by the Landau damping. On the other hand, free energy was released by the density gradient relaxation related to the diamagnetic frequency $\omega_{\star e}$, which drives wave instability. Here, the electron drift wave was destabilized since $\omega(\omega-\omega_{\star e})<0$ which flips the sign of the electron dissipation and so gives rise to inverse Landau damping. So we obtain the quasilinear electron cooling $\langle \tilde{E}_{||} \cdot \tilde{J}_{||e} \rangle^{(2)} < 0$ and the energy exchange between electrons and drift waves through Landau resonance.

Similarly, we calculate the quasilinear turbulent heating for ions. Utilizing a DKE for ions

$$\frac{\partial f_i}{\partial t} - i \frac{c}{B} k_y \frac{\partial \langle f_i \rangle}{\partial x} \tilde{\phi}_k + v_z \frac{\partial f_i}{\partial z} + \frac{e}{m_e} \tilde{E}_z \frac{\partial \langle f_i \rangle}{\partial v_z} = 0.$$
 (15)

Linearizing Eq. (15), we have

$$\tilde{f}_i = \frac{\frac{e\tilde{\phi}_k}{T_i} k_z v_z \langle f_i \rangle + \frac{e\tilde{\phi}_k}{T_e} \omega_{\star e} \langle f_i \rangle}{\omega - k_z v_z}.$$
 (16)

Substituting Eq. (16) into the turbulent heating for ions, yields

$$\langle \tilde{E}_{||} \cdot \tilde{J}_{||i} \rangle^{(2)} = \sum_{k} e \int dv v_{z} \tilde{E}_{z} \tilde{f}_{i}$$

$$= \sum_{k} \sqrt{\pi} n T_{i} \left| \frac{e \tilde{\phi}_{k}}{T_{e}} \right|^{2} \frac{\omega}{|k_{z}| V_{thi}} \left(\omega + \frac{T_{i}}{T_{e}} \omega_{\star e} \right)$$

$$\times \exp \left[-\frac{(\omega/k_{z})^{2}}{V_{thi}^{2}} \right]. \tag{17}$$

Here the quasilinear ion heating $\langle \tilde{E}_{||} \cdot \tilde{J}_{||i} \rangle^{(2)} > 0$. The ions gain energy from the drift wave through ion Landau damping. In the parallel magnetic field direction, the total quasilinear turbulent heating is $\langle \tilde{E}_{||} \cdot \tilde{J}_{||} \rangle^{(2)} = \langle \tilde{E}_{||} \cdot \tilde{J}_{||e} \rangle^{(2)} + \langle \tilde{E}_{||} \cdot \tilde{J}_{||i} \rangle^{(2)}$. The energy flow is described by $electron \rightarrow drift \ wave \rightarrow ion$. The energy in the hot electrons is lost to the drift wave through inverse Landau damping and the cold ions gain energy via Landau damping. Basically, energy was transferred from electrons to ions via wave-particle resonance (Landau damping). This is one of the wave energy dissipation channels.

So far we have reviewed the calculation of turbulent heating at quasilinear order. We note that the quasilinear electron cooling generally does not cancel the quasilinear ion heating and the energy leftover in the drift wave will drive the wave instability, which is written as $\gamma_L = \gamma_e - \gamma_i > 0$. So, there must be other turbulent heating and energy transfer channels available to the system to dissipate turbulence energy and stabilize the drift wave. In Sec. III, we will show that the ion polarization drift and ion diamagnetic drift induce net ion turbulent heating which contribute another important energy transfer channel in the direction perpendicular to the magnetic field.

III. ION POLARIZATION AND ION DIAMAGNETIC CURRENT CONTRIBUTION TO TURBULENT HEATING

Since the $E \times B$ drifts of the ions and electrons cancel automatically, and $m_e \ll m_i$ (which allows the electron polarization drift to be ignored), we only need to consider the perpendicular turbulent current carried by the ion polarization drift and the ion diamagnetic drift in the direction perpendicular to the magnetic field, which is written as

$$\tilde{\mathbf{J}}_{\perp}^{i} = \tilde{\mathbf{J}}_{\perp nol}^{i} + \tilde{\mathbf{J}}_{\perp dia}^{i}.$$
 (18)

First, we compute the ion polarization contribution to the turbulent heating defined by

$$\tilde{Q}_{\perp pol}^{i} = \langle \tilde{\mathbf{E}}_{\perp} \cdot \tilde{\mathbf{J}}_{\perp pol}^{i} \rangle. \tag{19}$$

The ion polarization drift velocity is given by $\tilde{V}_{\perp pol}^i = (c/B\Omega_i)d\tilde{E}_{\perp}/dt$, and we define $d/dt = \partial/\partial t + \mathbf{V}\cdot\nabla$,

 $V=\langle V_y \rangle + \tilde{V}_\perp, \; E_\perp = \langle E_\perp \rangle + \tilde{E}_\perp, \; {
m so} \; {
m we} \; {
m have} \; {
m the} \; {
m ion}$ polarization current

$$\tilde{\mathbf{J}}_{\perp pol}^{i} = en\tilde{V}_{\perp pol}^{i} = nm_{i}\frac{c^{2}}{B^{2}} \left[\frac{\partial}{\partial t}\tilde{\mathbf{E}}_{\perp} + \langle V_{y} \rangle \frac{\partial}{\partial y}\tilde{\mathbf{E}}_{\perp} + \hat{x}\tilde{V}_{x}\frac{\partial}{\partial x} \langle E_{x} \rangle \right], \tag{20}$$

where we have neglected quadratic terms in fluctuation amplitude for simplicity. Considering the stationary limit, and substituting the ion polarization current into the expression for turbulent heating, yields

$$\langle \tilde{\mathbf{E}}_{\perp} \cdot \tilde{\mathbf{J}}_{\perp pol}^{i} \rangle = \left\langle n_{i} m_{i} \frac{c^{2}}{B^{2}} \tilde{\mathbf{E}}_{\perp} \cdot \frac{\partial \tilde{\mathbf{E}}_{\perp}}{\partial y} \right\rangle \langle V_{y} \rangle + \left\langle n_{i} m_{i} \frac{c^{2}}{B^{2}} \tilde{E}_{x} \tilde{V}_{x} \right\rangle \frac{\partial \langle E_{x} \rangle}{\partial x}. \tag{21}$$

Noting the definition of the $E \times B$ flow $V_y = -(c/B)E_x$ and simplifying, allows

$$\langle \tilde{\mathbf{E}}_{\perp} \cdot \tilde{\mathbf{J}}_{\perp pol}^{i} \rangle = \frac{1}{2} n_{i} m_{i} \frac{c^{2}}{B^{2}} \left\langle \frac{\partial}{\partial y} |\tilde{\mathbf{E}}_{\perp}|^{2} \right\rangle \langle V_{y} \rangle - n_{i} m_{i} \frac{c}{B} \langle \tilde{V}_{y} \tilde{V}_{x} \rangle \frac{\partial \langle E_{x} \rangle}{\partial x}, \tag{22}$$

for the simplified geometry considered here, $n_i = n_i(x)$ and B = B(x). The first term can be seen to vanish identically upon averaging, hence we can write

$$\langle \tilde{\mathbf{E}}_{\perp} \cdot \tilde{\mathbf{J}}_{\perp pol}^{i} \rangle = -n_{i} m_{i} \frac{c}{B} \langle \tilde{V}_{y} \tilde{V}_{x} \rangle \frac{\partial \langle E_{x} \rangle}{\partial x}, \qquad (23)$$

which is Reynolds work on the $E \times B$ flow. This is simply the work associated with the flow generation.

We note that this relation is exact only for the simplified slab geometry considered within this work. A more realistic geometry [i.e., with $B=B(r,\theta)$] will generally give rise to additional geometrical contributions and small corrections. Here we consider the geometry in a local annular region with the center at r_0 (see Fig. 3), so the coordinates can be described by (r,θ,z) . For our simple case, the magnetic field is considered as a constant. Since the Reynolds stress in slab geometry is given by $\langle \tilde{V}_x \tilde{V}_y \rangle = -(c^2/B^2) \langle \partial_x \tilde{\phi} \partial_y \tilde{\phi} \rangle$, then in a local annulus the Reynolds stress can be written as

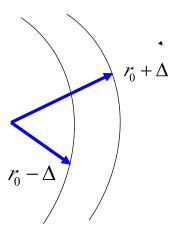


FIG. 3. Energy flux differential in an annular region. The annular width is $2\Delta.\,$

 $\langle \tilde{V}_x \tilde{V}_y \rangle = \langle \tilde{V}_r \tilde{V}_\theta \rangle$. Now we take the mean poloidal velocity to be the mean $\mathbf{E} \times B$ velocity, so $\langle V_\theta \rangle \sim \langle V_E \rangle = -(c/B)$ $\langle E_r \rangle$. Then the turbulent heating can be obtained as

$$\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}_{\perp pol}^{i} \rangle = n_{i} m_{i} \langle \tilde{V}_{r} \tilde{V}_{\theta} \rangle \langle V_{\theta} \rangle'.$$
 (24)

If we define an average over an annular region with the width 2Δ at the local centre r_0 , this average can be written as

$$\langle \cdots \rangle = \int_0^{2\pi R} dz \int_0^{2\pi} r_0 d\theta \int_{r_0 - \Lambda}^{r_0 + \Delta} (\cdots) dr.$$
 (25)

The surface area is a constant and is given by a number $A(r_0)$, $A(r_0) = \int_0^{2\pi R} dz \int_0^{2\pi} r_0 d\theta$. Then, we only need to consider the radial integral to compute the turbulent heating within an annulus, which is

$$\int_{0}^{2\pi R} dz \int_{0}^{2\pi} r_{0} d\theta \int_{r_{0}-\Delta}^{r_{0}+\Delta} \langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}_{\perp pol}^{i} \rangle dr$$

$$= n_{i} m_{i} A \int_{r_{0}-\Delta}^{r_{0}+\Delta} dr \langle V_{\theta} \rangle' \langle \tilde{V}_{r} \tilde{V}_{\theta} \rangle$$

$$= n_{i} m_{i} A (\langle V_{\theta} \rangle \langle \tilde{V}_{r} \tilde{V}_{\theta} \rangle)|_{r_{0}-\Delta}^{r_{0}+\Delta} - \int_{r_{0}-\Delta}^{r_{0}+\Delta} dr \langle V_{\theta} \rangle \frac{\partial}{\partial r} \langle \tilde{V}_{r} \tilde{V}_{\theta} \rangle).$$
(26)

The first term in Eq. (26) is the energy flux differential at the boundary, which gives rise to a *net* turbulent heating. This arises since wave propagation can carry energy through the annular boundary. The finite wave energy flux S_r is proportional to the Reynolds stress (see Eq. (5)) which drives zonal flow formation. So zonal flow generation is the destination of net turbulent heating.

The second term in the RHS of Eq. (26) is the Reynolds work of the turbulence on the mean flow, which is directly linked to the zonal flow drive. For the simplified advective nonlinearity of the incompressible fluid momentum balance equation (including friction) in this local annular region, ^{14,15} we have

$$\frac{\partial}{\partial t} \langle V_{\theta} \rangle = -\frac{\partial}{\partial r} \langle \tilde{V}_r \tilde{V}_{\theta} \rangle - \nu_{col} \langle V_{\theta} \rangle. \tag{27}$$

Since at steady state, Reynolds work on the zonal flow must balance zonal flow friction, $\partial \langle \tilde{V}_r \tilde{V}_\theta \rangle / \partial r = -\nu_{col} \langle V_\theta \rangle$, then the ion polarization drift induced turbulent heating in Eq. (26) can be approximated as

$$\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}_{\perp pol}^{i} \rangle \approx n m_{i} A \int_{r-\Delta}^{r+\Delta} dr \nu_{col} \langle V_{\theta} \rangle^{2},$$
 (28)

which is zonal flow frictional damping $(\sim \nu_{col} \langle V_{\theta} \rangle^2 \sim |e\tilde{\phi}/T|^4)$. Hence, the ion polarization drift induced turbulent heating over an annular region gives rise to a net heating which is ultimately due to the zonal flow friction. Then this net heating can be dissipated by the zonal flow frictional damping at steady state, which gives rise to another electronion energy transfer channel. This process of energy transfer via zonal flow has not previously been accounted for in analyses of energy coupling.

Now we calculate the ion diamagnetic current induced turbulent heating in the direction perpendicular to the magnetic field. We have the diamagnetic drift velocity induced by the gradient of ion pressure fluctuation $\nabla \tilde{P}$ in the direction perpendicular to $\bf B$

$$\tilde{\mathbf{V}}_{\perp dia}^{i} = \frac{c}{en} \frac{\mathbf{B} \times \nabla \tilde{P}_{i}}{B^{2}}.$$
 (29)

The corresponding diamagnetic current is

$$\tilde{\mathbf{J}}_{\perp dia}^{i} = en\tilde{V}_{\perp dia}^{i} = c\frac{\mathbf{B} \times \nabla \tilde{P}_{i}}{B^{2}}.$$
 (30)

Then we obtain the divergence of the diamagnetic current as

$$\nabla_{\perp} \cdot \tilde{\mathbf{J}}_{\perp dia}^{i} = \nabla_{\perp} \cdot \frac{c}{R^{2}} [\tilde{P}_{i} \nabla \times \mathbf{B} - \nabla \times (\tilde{P}_{i} \mathbf{B})] = 0, \quad (31)$$

where **B** is assumed to be a straight magnetic field. Further, the turbulent heating induced by the diamagnetic current can be calculated and is given by

$$\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}_{\perp dia}^{i} \rangle = \langle -\nabla_{\perp} \cdot (\tilde{\phi} \tilde{\mathbf{J}}_{\perp dia}^{i}) + \phi \nabla \cdot \tilde{\mathbf{J}}_{\perp dia}^{i} \rangle,$$

$$= -\frac{1}{r} \frac{\partial}{\partial r} r \langle \tilde{\phi} \frac{c}{B} \frac{1}{r} \frac{\partial \tilde{P}_{i}}{\partial \theta} \rangle = -\frac{1}{r} \frac{\partial}{\partial r} r \langle \tilde{V}_{E \times B} \tilde{P}_{i} \rangle. \quad (32)$$

Averaging over Eq. (32) in an annulus, we obtain

$$\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}_{\perp dia}^{i} \rangle = -A \int_{r-\Delta}^{r+\Delta} dr \frac{1}{r} \frac{\partial}{\partial r} r \langle \tilde{V}_{E \times B} \tilde{P}_{i} \rangle = -A \langle \tilde{V}_{E \times B} \tilde{P}_{i} \rangle |_{r-\Delta}^{r+\Delta}.$$
(33)

Obviously, Equation (33) is the ion heat flux differential across the annulus, which means the ion turbulent energy can be eliminated through a heat flux. Similarly, we can calculate the electron diamagnetic flow induced turbulent heating, which is a electron heat flux differential across the annulus and is given by

$$\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}_{\perp dia}^{e} \rangle = A \langle \tilde{V}_{E \times B} \tilde{P}_{e} \rangle |_{r-\Delta}^{r+\Delta}. \tag{34}$$

Here the total heat flux $(Q = -\chi_T \nabla T - \chi_{neo} \nabla T)$, including the turbulence and neoclassical parts, is a constant. In a turbulent transport dominated annular region, the turbulent heat flux $Q_{tur} = Q^e_{tur} + Q^i_{tur}$ must also be a constant. Then, the diamagnetic flow contribution to the turbulent heat flux differential can be ignored in an annular region. However, if we consider a barrier region [i.e., internal transport barrier (ITB)], the heat flux differential will be finite, and so will result in heating in the barrier region.

So far the whole process of turbulent energy flow due to the electron and ion transfer has been completed. In electron heated plasma, electrons can lose energy to the wave and drive wave instability and turbulence, so electrons are cooled. And the ions gain energy from the wave through wave Landau damping and the ions are heated. Meanwhile, we note that the energy flux differential can contribute a net heating, which drives zonal flow generation in an annular region. This net energy can be dissipated by zonal flow frictional damping, giving rise to another collisionless energy transfer channel (*electrons* \rightarrow *ions*). As for the diamagnetic flow contributed turbulent heating, we can ignore that since the turbulent heat flux is not changed in the annular region of interest.

Now we see that the simplified electron and ion turbulent energy coupling can be written into a form

$$\langle \tilde{E} \cdot \tilde{J} \rangle = A_L I + B_{NL} I^2 + C_{ZF} \frac{\langle V_{\theta} \rangle^2}{C_s^2}.$$
 (35)

Here the turbulence intensity is defined as $I = \sum_k |e\tilde{\phi}_k/T_e|^2$, and the coefficients A_L, B_{NL}, C_{ZF} have dimensions of a power density. $A_L = \sum_k A_k |e\tilde{\phi}_k/T_e|^2 / \sum_k |e\tilde{\phi}_k/T_e|^2$ is set by the electron and ion quasilinear turbulent heating, $B_{NL} = \sum_{k,k'} B_{k,k'} |e\tilde{\phi}_k/T_i|^2 |e\tilde{\phi}_{k'}/T_i|^2 /I^2$ describes the nonlinear ion turbulent heating through the nonlinear Landau damping (here we ignore this nonlinear effect, but will analyze it in a future paper), 9,11–13 and the coefficient C_{ZF} is determined by heating through zonal flow formation. 7,8,15

In relation to turbulent heating, we also analyzed two basic wave energy dissipation channels, which are quasilinear Landau damping and zonal flow frictional damping. In Sec. IV, we will estimate the size of each turbulent heating channel by using the mixing length approximation ^{19,20} and then comparing these two dissipation channels by considering ITER parameters. ^{5,18}

IV. APPLICATION OF THE RESULTS TO ITER

In Table I, we listed all of the turbulent heating terms before and after using the mixing length approximation for fluctuation levels. ^{19,20} Now we will discuss how to estimate each turbulent heating term in detail. For the quasilinear electron cooling term is given by

$$\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \rangle^{(2)} = \sum_{k} \sqrt{\pi} n T_{e} \left| \frac{e \tilde{\phi}_{k}}{T_{e}} \right|^{2} \frac{\omega}{|k_{z}| V_{the}} (\omega - \omega_{\star e})$$

$$\times \exp \left[-\frac{(\omega/k_{z})^{2}}{V_{the}^{2}} \right]. \tag{36}$$

Here, the turbulence intensity can be estimated as $e\tilde{\phi}_k/T_e \sim \rho_{\star}$, ^{19,20} the wave vector $k_z \sim 1/Rq$, and the dispersion relationship is taken as $\omega = \omega_{\star e}/(1 + k_{\perp}^2 \rho_s^2)$ in drift wave turbulence (note: $\omega = \omega_k$). Also the exponential factor

TABLE I. Overview of results: estimation of the turbulent heating contributions.

Turbulent heating contribution	Analytical theory prediction	Scaling, using mixing length approximations for fluctuation levels: $\frac{e\dot{\phi}}{T_e}\sim \rho_\star$
$\begin{split} & {\left \left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)} \right } \\ & \left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)} \\ & \left\langle \tilde{E}_{\perp} \cdot \tilde{J}_{\perp pol}^{i} \right\rangle \end{split}$	$nT_{e} \left \frac{e\bar{\phi}}{T_{e}} \right ^{2} \frac{(\omega - \omega_{\star e})\omega}{ k V_{the}}$ $nT_{i} \left \frac{e\bar{\phi}}{T_{i}} \right ^{2} \frac{(\omega + \frac{T_{i}}{T_{e}}\omega_{\star e})\omega}{ k _{z} V_{thi}} \exp \left(- \left(\frac{\omega}{k V_{thi}} \right)^{2} \right)$ $nm_{i} \nu_{col} \langle V_{\theta} \rangle^{2}$	$nT_e \rho_{\star}^2 \frac{Rq\omega_{\star e}^2}{V_{the}} F_1(k_{\perp}\rho_s)$ $nT_i \rho_{\star}^2 \frac{Rq\omega_{\star e}^2}{V_{thi}} F_2(k_{\perp}\rho_s)$ $n\rho_{\star}^2 \nu_{\star i} \epsilon^{1/2} m_i C_s^2 \frac{V_{thi}}{Rq}$

 $\exp[-(\omega/k_z)^2/V_{the}^2] \approx 1$ since $\omega/k_z \ll V_{the}$. The quasilinear electron cooling then can be estimated as

$$\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \rangle^{(2)} = -\sum_{k} \sqrt{\pi} n T_{e} \frac{Rq}{V_{the}} \rho_{\star}^{2} \omega_{\star e}^{2} \left(\frac{k_{\perp} \rho_{s}}{1 + k_{\perp}^{2} \rho_{s}^{2}} \right)^{2}. \quad (37)$$

We define the function $F_1(k_\perp \rho_s) = [k_\perp \rho_s/(1 + k_\perp^2 \rho_s^2)]^2$ which is a dimensionless number dependent on the finite Larmor effect $k_\perp \rho_s$, so the simplified quasilinear electron cooling can be written as

$$|\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \rangle^{(2)}| \sim n T_e \rho_{\star}^2 \frac{Rq\omega_{\star e}^2}{V_{the}} F_1(k_{\perp} \rho_s). \tag{38}$$

Similarly, the quasilinear ion heating term can be estimated as

$$\begin{split} &\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \rangle^{(2)} \\ &= \sum_{k} \sqrt{\pi} n T_{i} \left| \frac{e \tilde{\phi}_{k}}{T_{i}} \right|^{2} \frac{\omega}{|k_{z}| V_{thi}} \left(\omega + \frac{T_{i}}{T_{e}} \omega_{\star e} \right) \exp \left[-\frac{(\omega/k_{z})^{2}}{V_{thi}^{2}} \right], \\ &= \sum_{k} \sqrt{\pi} n T_{i} \left| \frac{T_{e}}{T_{i}} \right|^{2} \rho_{\star}^{2} \frac{Rq}{V_{thi}} \omega_{\star e}^{2} \left(\frac{1}{1 + k_{\perp}^{2} \rho_{s}^{2}} \right) \left(\frac{1}{1 + k_{\perp}^{2} \rho_{s}^{2}} + \frac{T_{i}}{T_{e}} \right) \\ &\times \exp \left[-\left(\frac{k_{\perp} \rho_{s}}{1 + k_{\perp}^{2} \rho_{s}^{2}} \right)^{2} \left(\frac{Rq}{a} \right)^{2} \frac{T_{e}}{T_{i}} \right], \\ &\approx n T_{i} \rho_{\star}^{2} \frac{Rq \omega_{\star e}^{2}}{V_{thi}} F_{2}(k_{\perp} \rho_{s}), \end{split} \tag{39}$$

where $F_2(k_\perp \rho_s)$ is another dimensionless number which is determined by finite Larmor radius $k_\perp \rho_s$

$$F_{2}(k_{\perp}\rho_{s}) = \left(\frac{1}{1+k_{\perp}^{2}\rho_{s}^{2}}\right) \left(\frac{1}{1+k_{\perp}^{2}\rho_{s}^{2}} + \frac{T_{i}}{T_{e}}\right) \times \exp\left[-\left(\frac{k_{\perp}\rho_{s}}{1+k_{\perp}^{2}\rho_{s}^{2}}\right)^{2} \left(\frac{Rq}{a}\right)^{2} \frac{T_{e}}{T_{i}}\right]. \tag{40}$$

Now we estimate the ion polarization drift and ion diamagnetic drift induced turbulent heating. The energy flux differential in an annular region gives rise to the energy in the zonal flow which can be written as

$$\langle \tilde{E} \cdot \tilde{J}^{i}_{\perp pol} \rangle = n m_i A \int_{r_0 - \Delta}^{r_0 + \Delta} dr \nu_{col} \langle V_{\theta} \rangle^2.$$
 (41)

Here the zonal flow can be treated as an $E \times B$ flow, so $\langle V_{\theta} \rangle \sim \langle V_{E \times B} \rangle \sim -(C/B) \langle E_r \rangle$. At steady state, the radial electric field $E_r \approx (1/n) (\nabla P/e) \approx -(T_e/e) (1/L_n)$. The long wavelength zonal $\mathbf{E} \times \mathbf{B}$ flow can then be approximated as the diamagnetic flow

$$\langle V_E \rangle \sim \frac{c}{B} \frac{T_e}{e} \frac{1}{L_n} = \frac{\rho_s}{L_n} C_s.$$
 (42)

Then an estimate of the energy in the zonal flow can be obtained, which is given by

$$\langle \tilde{E} \cdot \tilde{J}^{i}_{\perp pol} \rangle \approx n m_i \nu_{col} \left(\frac{\rho_s}{L_n} C_s \right)^2 (A \Delta).$$
 (43)

Defining a dimensionless number for collisionality $\nu_{\star i} = \epsilon^{-3/2} \nu_{ii} Rq/V_{thi}$, the effective collisionality $\nu_{col} \approx \nu_{ii}/\epsilon$, the net turbulent heating is given by (note: we assume the volume of annulus to be $A\Delta \approx 1$)

$$\langle \tilde{E} \cdot \tilde{J}^{i}_{\perp pol} \rangle \approx n \nu_{\star i} \rho_{\star}^{2} \epsilon^{1/2} m_{i} C_{s}^{2} \frac{V_{thi}}{Ra}.$$
 (44)

So far we have estimated all of the turbulent heating terms by using mixing length theory (see Table I). The total electron and ion energy coupling in Eq. (35) can also be obtained (note: we did not consider the nonlinear ion turbulent heating term for the reason of simplicity) as

$$\langle \tilde{E} \cdot \tilde{J} \rangle = A_L I + C_{ZF} \frac{\langle V_{\theta} \rangle^2}{C_s^2},$$
 (45)

where $A_L = \sum_k A_k |e\tilde{\phi}_k/T_e|^2/\sum_k |e\tilde{\phi}_k/T_e|^2$, $A_k(n,T_e,T_i,V_{the},V_{thi},\omega_\star,k_\perp\rho_s) = -nT_e(Rq/V_{the})\omega_{\star e}^2F_1(k_\perp\rho_s) + nT_i \; (Rq/V_{thi})$ $\omega_{\star e}^2F_2(k_\perp\rho_s)$, and $C_{ZF}(n,m_i,q,V_{thi},\epsilon,C_s,\nu_{\star i}) = nm_iC_s^2\;\nu_{\star i}\epsilon^{1/2}$ $V_{thi}/(Rq)$. By using ITER parameters, 5,18 we can see the physical implications of results of the calculation in Sec. V.

V. BASIC COMPARISON OF DISSIPATION CHANNELS

In Sec. IV, turbulent heating terms which include the quasilinear electron cooling, quasilinear ion heating, and zonal flow frictional heating have been estimated using the mixing length approximation. 19,20 These calculations are specially important for ITER, as a low collisionality plasma. Then we will use ITER parameters^{5,18} to determine each contribution to turbulent heating, and compare the ratios of the various energy dissipation channels at different collisionalities. There are two kinds of basic turbulent energy dissipation channels in an electron drift wave. One is through linear ion Landau damping and another one is the net turbulent heating, which is due to the zonal flow frictional damping at steady state. Basically, the quasilinear electron cooling plays the role of the "energy source," while the ion turbulent heating channels act as an "energy sink." The ratios of two kinds of energy dissipation channels are listed in Table II.

The following gives the details of the comparisons. The ratio of the energy dissipated by the ion Landau damping to the energy released by electron cooling is given by

$$\frac{\left| \langle \tilde{E} \cdot \tilde{J}_{\parallel i} \rangle^{(2)} \right|}{\left| \langle \tilde{E} \cdot \tilde{J}_{\parallel e} \rangle^{(2)} \right|} = \frac{V_{the}}{V_{thi}} \frac{F_2(k_{\perp} \rho_s)}{F_1(k_{\perp} \rho_s)}.$$
 (46)

For ITER parameters, the electron temperature and ion temperature $T_e \approx T_i = 10 \,\text{keV}$, the major radius $R = 6.2 \,\text{m}$, the

TABLE II. Basic comparison of dissipation channels.

ITER parameters	R = 6.2 m, a = 2 m, q = 2	
$egin{aligned} Ratio &= rac{\langle ilde{\mathcal{E}} ilde{J}_i angle}{\langle ilde{\mathcal{E}} ilde{J}_e angle} \ \langle ilde{\mathcal{E}}_{\parallel} \cdot ilde{J}_{\parallel i} angle^{(2)} \end{aligned}$	Short wavelength: $k_{\perp}\rho_{s}\sim1$ 1.6×10^{-2}	
$\langle { ilde E}_{\perp} \cdot { ilde J}^i_{\perp pol} angle$	$0.8 u_{\star}$	

minor radius r=2 m, and the safety factor is q=2 at the core.^{5,18} Also we consider the influence of finite Larmor radius effects, where the short wave length $k_{\perp}\rho_s \sim 1$ for the most unstable mode. Then, the ratio in Eq. (46) is

$$\frac{\left| \left\langle \tilde{E} \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)} \right|}{\left| \left\langle \tilde{E} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)} \right|} = 1.6 \times 10^{-2}.$$
 (47)

Now we consider the ratio of the turbulent energy dissipated by the zonal flow frictional damping to the electron cooling. This is given by

$$\frac{\left|\langle \tilde{E} \cdot \tilde{J}_{\perp pol}^{i} \rangle\right|}{\left|\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \rangle^{(2)}\right|} = \frac{4}{\pi} \nu_{\star} \epsilon^{\frac{1}{2}} \sqrt{\frac{m_{i}}{m_{e}}} \sqrt{\frac{T_{i}}{T_{e}}} \left(\frac{a}{Rq}\right)^{2} \approx 0.8 \nu_{\star}. \tag{48}$$

The result shows that the energy dissipation through the zonal flow frictional damping channel is also a dimensionless number which is determined by the collisionality $\nu_{\star i}$. So far we estimated the ratios of energy dissipation channels, and now we will compare them at different collisionalities in ITER. Using the collisionality $\nu_{\star i} \approx 10^{-3}$ (ITER parameters were used to calculate $\nu_{\star i}$)^{5,18} in the ratio of the zonal flow frictional damping channel in Eq. (48), we can see that this energy dissipation channel is not big. It is about 5% compared to the ratio of the energy dissipated by the ion linear Landau damping. However, if we consider the collisionaltiy $\nu_{\star i} = 10^{-2}$, zonal flow frictional damping can be a significant energy dissipation channel for the collisionless drift wave and the ratio rises to 50%, as compared to the ion linear Landau damping. The first case corresponds to the collisionality in the core in the ITER and second one is appropriate close to the edge. In addition, we need to clarify why we discuss frictional zonal flow damping for a "collisionless" drift wave. Since $\omega_{\star} \gg \nu_{\star} > 0$, "collisionless" drift wave means the collisionalty is low compare to the drift wave frequency. But even weak collisionality is still significant for zonal flow frictional damping. Since the zonal flow frequency is approximately zero, even low collisionality will have a strong effect on the zonal flow via frictional damping. So zonal flows indeed play a very important role in collisionless energy transfer for low collisionlity ITER plasmas.

VI. IMPLICATIONS: COLLISIONLESS TURBULENT ENERGY TRANSFER IN ITER PLASMA

We calculated all of turbulent heating terms in electron drift wave and then estimated them by using a mixing length approximation. ^{19,20} By using ITER parameters, we compared the ratios of the energy dissipation channels at different collisionalities. We realized that the turbulent heating not only involves energy exchange between electrons and ions at the quasilinear level but also produce net heating which is the energy flux differential in an annulus. The latter is related to the energy in the zonal flow. The net energy dissipated by the zonal flow frictional damping for a collisionless drift wave is a significant energy dissipation channel in ITER

plasma. In the following, we will continue to explore the implication of our results for ITER. For a realistic case, we will extend our discussion to the CTEM. Consider the electron turbulent energy transport process in CTEM, the electron heat balance equation can be written as

$$\frac{3}{2}n\frac{\partial T_e}{\partial t} + \nabla \cdot Q_e = \langle \tilde{E} \cdot \tilde{J}_e \rangle - n\nu \frac{m_e}{m_i} (T_e - T_i) + \cdots. \quad (49)$$

The free energy in the electrons $(\nabla T, \nabla n)$ can be lost through the electron heat flux $\nabla \cdot Q_e$ in the turbulent energy transport process, or energy can be transferred to ions through collisional transfer $n\nu \frac{m_e}{m_i}(T_e-T_i)$ and collisionless energy transfer process $\langle \tilde{E}\cdot \tilde{J}_e\rangle$. The energy flow in the electron turbulent energy transport is shown in Fig. 4. First, we compare two energy transfer processes which are collisional and collisionless energy transfer. Since the collisionality is small $(\nu_\star \sim 10^{-3})$ in ITER, the collisional energy transfer is small and collisionless energy transfer may dominate in the energy transfer process. Then we will compare the energy transfer in the collisionless transfer process with the energy transported by the heat flux Q.

The collisionality is defined as $\nu_{\star e} = \epsilon^{-3/2} \nu_{ee} Rq/V_{the}$. Here ν_{ee} is the collision rate for electrons and is given by

$$\nu_{ee} = 2.9 \times 10^{-6} \times n_e \times \lambda \times T_e^{-3/2} \approx 2.9 \times 10^3 \text{s}^{-1}.$$
 (50)

The plasma density $n_e \approx n_i = 1.1 \times 10^{14} {\rm cm}^{-3}$, the Coulomb logarithm is defined as $\lambda = ln\Lambda \equiv ln(r_{max}/r_{min}) \sim 10$, plasma temperature $T_e \approx T_i = 10 \, {\rm keV}^{.5,18}$ So, the normalized collisionality is obtained as $\nu_{\star e} = 4.3 \times 10^{-3}$ in ITER. Now we find the collisionality ν_{\star} at the crossover of *collisional* turbulent energy *transfer* and *collisionless* turbulent energy *transfer*. The energy exchange in the electron-ion collision process is given by Braginskii's equation²¹

$$W_i = \frac{3n_e m_e}{m_i} \nu_e T_e \left(1 - \frac{T_i}{T_e} \right), \tag{51}$$

which is the energy per unit time transferred from electrons to ions.

Now we estimate the collisionless energy transfer in CTEM. The toroidal geometry of the magnetic surfaces can

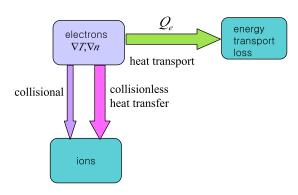


FIG. 4. The electron turbulent energy flow. The energy can be transported by the electron heat flux Q_e or energy can be transferred to the ions by the collisional or collisionless energy transfer channels.

be described by coordinates (r, θ, ξ) , which are the minor radius, the poloidal angle, and the toroidal angle, respectively. Also the equilibrium magnetic field can be written as $\mathbf{B} = B_0 \hat{e}_{\xi} + (\epsilon/q) \hat{e}_{\theta}$, where $\epsilon = r/R$ is the inverse aspect ratio. For trapped electrons, poloidal motion is prohibited but slow precessional motion in the toroidal direction occurs. For electrostatic perturbations, the dynamics of the non-adiabatic response for trapped electrons is given by the bounce averaged kinetic equation, $^{11-13}$ which is

$$\bar{\tilde{g}}_{k,\omega} = -\frac{e}{T_e} \langle f_e \rangle \frac{\omega - \omega_{\star e} \left[1 + \eta_e (\frac{E}{T_e} - \frac{3}{2}) \right]}{\omega - \bar{\omega}_d + i\nu_{eff}} \langle e^{-inq\theta} \tilde{\phi}_{k,\omega} \rangle_b, (52)$$

where $\bar{\omega}_d \approx G(k_\theta \rho_s C_s/R)(E/T_e)$ is the orbit-averaged trapped electron precession frequency, G is the magnetic field geometry effect, and $G \approx 1$ for deeply trapped electrons.²² The effective collision frequency ν_{eff} is ignored in the collisionless regime, and $\eta_e = L_n/L_{T_e}$. The $\langle \cdots \rangle_b$ means bounce average and was taken as $\langle \cdots \rangle_b = (\oint \frac{d\theta}{\nu_{\parallel}} \cdots)/(\oint \frac{d\theta}{\nu_{\parallel}})$. The bounce averaged potential fluctuation is $\langle e^{-inq\theta} \tilde{\phi}_{k,\omega} \rangle_b \approx \tilde{\phi}_{k,\omega}$, since the finite orbit width effects of trapped electrons are neglected. Similar to the quasilinear electron cooling in electron drift waves, a realistic application is the quasilinear trapped electrons of significant trapped electrons is given by

$$\langle \bar{\tilde{E}} \cdot \bar{\tilde{J}} \rangle_b^{(2)} = \sum_k -e \int d^3 v \bar{v}_d \bar{\tilde{E}} \bar{\tilde{g}}_{k,\omega}, \tag{53}$$

where the bounce averaged curvature and ∇B drift velocity is $\bar{v}_d = \bar{\omega}_d/k_\theta$ and \tilde{E} is the bounce averaged electric field. The velocity integration over the trapped-electron population is written as

$$d^3v \simeq 4\pi \left(\frac{\epsilon}{2}\right)^{\frac{1}{2}}v^2 dv d\kappa^2 \left(\kappa^2 - \sin^2\frac{\theta}{2}\right)^{-1/2},\tag{54}$$

where κ is the pitch angle variable related to the azimuthal angle θ_0 of the turning point of a trapped electron and $\kappa = \sin\theta_0$. We can convert the integral in velocity space into one in energy space (for the kinetic energy $E = m_e v^2/2$), where we find

$$\int d^3v = \left(\frac{\epsilon}{2}\right)^{1/2} \int 4\pi v^2 dv \int_{\kappa_0}^1 d\kappa^2 \left(\kappa^2 - \sin^2\frac{\theta}{2}\right)^{-\frac{1}{2}}$$

$$\approx 2\pi \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{2}{m_e}\right)^{3/2} \int E^{1/2} dE. \tag{55}$$

The fraction of the trapped electrons $f_t = (\epsilon/2)^{1/2}$ $\int_{\kappa_0}^1 d\kappa^2 \left(\kappa^2 - \sin^2\frac{\theta}{2}\right)^{-1/2} \approx (\epsilon/2)^{1/2}$ was approximated in Eq. (55), for simplicity. Then the bounce averaged turbulent heating from the curvature and ∇B drift current is

$$\langle \tilde{E} \cdot \tilde{J} \rangle_{b}^{(2)} = \sum_{k} -2\pi \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{2}{m_{e}}\right)^{3/2} e \int E^{1/2} dE \bar{v}_{d} \tilde{E} \tilde{g}_{k,\omega},$$

$$= \sum_{k} -2\pi \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{2}{m_{e}}\right)^{3/2} e \int E^{1/2} dE \left(\frac{\bar{\omega}_{d}}{k_{\theta}}\right) (ik_{\theta} \tilde{\phi}_{-k}) \left(-\frac{e\tilde{\phi}_{k}}{T_{e}} \langle f_{e} \rangle \frac{\omega - \omega_{\star T}}{\omega - \bar{\omega}_{d}}\right),$$

$$= \sum_{k} 2\pi i \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{2}{m_{e}}\right)^{3/2} T_{e} \left|\frac{e\tilde{\phi}_{k}}{T_{e}}\right|^{2} \int E^{3/2} dE \left(\frac{k_{\theta} \rho_{s} C_{s}}{R T_{e}}\right) \left(\frac{\omega - \omega_{\star T}}{\omega - \frac{k_{\theta} \rho_{s} C_{s}}{R}} \langle f_{e} \rangle\right), \tag{56}$$

where we define $\omega_{\star T} = \omega_{\star e} [1 + \eta_e (E/T_e - 3/2)]$ in Eq. (56). The integral is

$$\int dE \frac{\langle f_e \rangle}{\omega - (k_\theta \rho_s C_s E/RT_e)} = \frac{-i\pi}{|k_\theta \rho_s C_s/RT_e|} \langle f_e \rangle_{E = \frac{\omega RT_e}{k_\theta \rho_s C_s}}.$$

The wave-trapped electron resonance gives $\omega = \bar{\omega}_d$, so energy can be obtain at the resonance $E = (\omega RT_e)/(k_\theta \rho_s C_s)$ $\approx RT_e/2L_n$. The drift wave frequency $\omega = \omega_{\star e}/(1 + k_\perp^2 \rho_s^2)$, the unstable wave number $k_\perp \rho_s \approx 1$, and $\omega_{\star e} = k_\theta \rho_s C_s/L_n \approx C_s/L_n$ were considered for CTEM. Then, the quasilinear turbulent cooling for trapped electron can be estimated as

$$\langle \bar{\tilde{E}} \cdot \bar{\tilde{J}} \rangle_{b}^{(2)} = \sum_{k} 2\pi i \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{2}{m_{e}}\right)^{3/2} T_{e} \left| \frac{e\tilde{\phi}_{k}}{T_{e}} \right|^{2} \left(\frac{RT_{e}}{2Ln}\right)^{3/2} (-i\pi)(\omega - \omega_{\star T}) \langle f_{e} \rangle \Big|_{E=\frac{RT_{e}}{2Ln}}$$

$$= \sum_{k} 2\pi^{1/2} \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{R}{2L_{n}}\right)^{3/2} n_{0} T_{e} \left| \frac{e\tilde{\phi}_{k}}{T_{e}} \right|^{2} \left(\frac{C_{s}}{L_{n}}\right) \left(-\frac{1}{2} - \eta_{e} \frac{R}{2L_{n}} + \eta_{e} \frac{3}{2}\right) \exp\left(-\frac{R}{2L_{n}}\right). \tag{57}$$

Here the electron equilibrium distribution function is still taken to be a local Maxwellian, i.e., $\langle f_e \rangle = n_0(x)$

 $(m_e/2\pi T_e(x))^{3/2} \exp(-E/T_e)$. The quasilinear turbulent cooling for trapped electrons is $\langle \tilde{E} \cdot \tilde{J} \rangle_b^{(2)}(\epsilon, n, R/L_n, R/L_T, R/L_T)$

 $\rho_{\star}, T_e, k_{\perp}\rho_s$), which is a function dependent on the all the parameters.

Now we compare the collisional turbulent energy transfer (51) to the magnitude of quasilinear trapped electron cooling (57), and we can obtain the collisionality at the crossover, which is a dimensionless number given by

$$\nu_{\star} \approx \left(\frac{Rq\epsilon^{-3/2}}{V_{the}}\right) \frac{2}{3} \pi^{1/2} \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{R}{2L_n}\right)^{3/2} \left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \left(\frac{C_s}{L_n}\right)$$

$$\times \left(\frac{1}{2} + \frac{R\eta_e}{2L_n} - \frac{3\eta_e}{2}\right) \frac{m_i}{m_e} \left(1 - \frac{T_i}{T_e}\right)^{-1} \exp\left(-\frac{R}{2L_n}\right)$$

$$= \frac{1}{12} \pi^{1/2} \sqrt{\frac{m_i}{m_e}} \left(\frac{q}{\epsilon}\right) \left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \left(\frac{R}{L_n}\right)^{3/2}$$

$$\times \left(\frac{R}{L_n} + \frac{R^2}{L_n L_T} - 3\frac{R}{L_T}\right) \left(1 - \frac{T_i}{T_e}\right)^{-1} \exp\left(-\frac{R}{2L_n}\right). \quad (58)$$

Here the collisionality $\nu_{\star}(\epsilon,q,R/L_n,R/L_T,\rho_{\star},T_i/T_e)$ can be determined if all parameters are given. For simplicity, we assume $T_i/T_e < 1$ for an electron heated plasma and analyze how the ratio of T_i/T_e affects collisionality ν_{\star} . We also note that the collisionality ν_{\star} at crossover is sensitive to the local parameters R/L_n and R/L_T . ^{23,24} Qualitatively, the relation between them is described in Fig. 5 where we take the ITER parameters (the inverse aspect ratio $\epsilon \approx 1/3$, safety factor q=2), ^{5,18} the mixing length approximation $|e\tilde{\phi}_k/T_e|\sim 10^{-3}$, the temperature ratio $T_i/T_e=1/2$, the ranges $3 < R/L_T < 13$ and $3 < R/L_n < 13$. For typical parameters $R/L_T=10$, $R/L_n=4$, we have

$$\nu_{\star} \approx 1.2 \times 10^{-3} \tag{59}$$

Obviously, the collisionality in ITER $\nu_{\star} \approx 10^{-3}$ is same order as the collisionality at the crossover of collisional energy transfer and collisionless energy transfer. In other words, the collisionality is low enough such that the collisionless turbulent energy transfer and the collisional

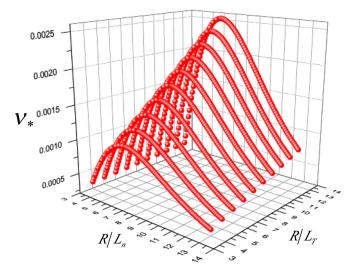


FIG. 5. The collisionality ν_{\star} at crossover depends on the local parameters R/L_T and R/L_n . For $\epsilon \approx 1/3$, q=2, $\rho_{\star}=10^{-3}$, $T_i/T_e=1/2$, $R/L_T=10$, $R/L_n=4$, the crossover collisionality is $\nu_{\star}=1.2\times 10^{-3}$.

inter-species coupling process are *both* important for the energy transfer process!

However, as the ions gain more and more energy, the difference between T_i and T_e will decrease, and the collisional energy transfer will drop. For example, $T_i \approx 0.95T_e$ corresponding to the collisionality at the crossover $\nu_{\star} = 1.2 \times 10^{-2}$, in Eq. (58). Thus, the collisionless energy transfer process will ultimately control electron-ion energy transfer in ITER.

We examined the turbulent energy transfer mechanism for CTEM and found that the collisionless energy transfer is anticipated to be the dominant electron-ion coupling process in the heat balance. Furthermore, in Sec. VII, we will compare the rate of the electron turbulent energy lost by turbulent *transport* through the electron heat flux with the rate of the electron energy *transferred* to the ions in the CTEM.

VII. TURBULENT ENERGY TRANSFER VS TURBULENT ENERGY TRANSPORT

The effective way to compare the rate of the electron energy lost by the turbulent transport to the rate of the collisionless energy transfer by comparison of the volume integral of the electron cooling, to the surface integrated electron heat flux in CTEM. In the electron heat balance equation (49), we ignore the collisional energy transfer term for low collisionality CTEM and have

$$\frac{3}{2}n\frac{\partial T_e}{\partial t} + \nabla \cdot Q_e = \langle \tilde{E} \cdot \tilde{J}_e \rangle + Source + \cdots.$$
 (60)

At steady state, the volume integral of the annulus for Eq. (60), we obtain

$$AQ_e|_{boundary} = \int d^3r \langle \tilde{E} \cdot \tilde{J}_e \rangle,$$
 (61)

where A is a constant surface area and the volume integral over the annular region can be estimated as $\int d^3r = A\Delta r$, where Δr is the annular thickness. Then the equation can be simplified to

$$Q_e|_{boundary} = \Delta r \langle \tilde{E} \cdot \tilde{J}_e \rangle.$$
 (62)

Here Q_e is the electron heat flux, and we consider the quasilinear trapped electron heat flux in CTEM which is given by 13,25

$$Q_e = \langle \tilde{V}_r \tilde{P}_e \rangle = -\frac{c}{B} \sum_k k_\theta \operatorname{Im}(\tilde{P}_e^{(1)} \tilde{\phi}). \tag{63}$$

The pressure fluctuation \hat{P} is written as

$$\begin{split} \tilde{P}_{e}^{(1)} &= \int d^{3}v \frac{1}{2} m_{e} v^{2} \bar{\tilde{g}}_{k,\omega} \\ &= 2\pi \left(\frac{2}{m_{e}}\right)^{3/2} \left(\frac{\epsilon}{2}\right)^{1/2} \int dE E^{3/2} \\ &\times \left(-\frac{e\tilde{\phi}_{k}}{T_{e}} \langle f_{e} \rangle \frac{\omega - \omega_{\star e} \left[1 + \eta_{e} \left(\frac{E}{T_{e}} - \frac{3}{2}\right)\right]}{\omega - \bar{\omega}_{d}}\right). \end{split}$$
(64)

The bounce averaged trapped electron distribution function $\tilde{g}_{k,\omega}$ in Eq. (52) has been used here. Considering the resonance of the wave with trapped electron precession motion (i.e., $\omega = \omega_d = (k_\theta \rho_s C_s/R)(E/T_e)$) and noting the electron diamagnetic frequency $\omega_{\star e} = k_\theta \rho_s C_s/L_n$, the pressure fluctuation is seen to be

$$\operatorname{Im} \tilde{P}_{e}^{(1)} = 2\pi^{2} \left(\frac{2}{m_{e}}\right)^{3/2} \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{\omega R T_{e}}{L_{n} \omega_{\star e}}\right)^{3/2} \left(\frac{e \tilde{\phi}_{k}}{T_{e}}\right) \times \frac{(\omega - \omega_{\star T})}{\left|\frac{L_{n} \omega_{\star e}}{R T_{e}}\right|} \left\langle f_{e} \right\rangle_{E = \frac{R T_{e}}{2 L_{n}}}.$$
(65)

The quasilinear electron heat flux in Eq. (63) can thus be calculated as

$$Q_{e} = \sum_{k} 2\pi^{2} \left(\frac{2}{m_{e}}\right)^{3/2} \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{RT_{e}}{2L_{n}}\right)^{3/2} \left|\frac{e\tilde{\phi}_{k}}{T_{e}}\right|^{2} \frac{V_{the}^{2}k_{\theta}}{\Omega_{e}} \left|\frac{RT_{e}}{L_{n}}\right|$$

$$\times \left[\frac{1}{2} + \eta_{e} \left(\frac{E}{T_{e}} - \frac{3}{2}\right)\right] \langle f_{e} \rangle_{E=\frac{RT_{e}}{2L_{n}}}$$

$$= \sum_{k} 2\pi^{1/2} \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{R}{2L_{n}}\right)^{3/2} n_{0} T_{e} \left|\frac{e\tilde{\phi}_{k}}{T_{e}}\right|^{2} \frac{V_{the}^{2}k_{\theta}}{\Omega_{e}} \left|\frac{R}{L_{n}}\right|$$

$$\times \left[\frac{1}{2} + \eta_{e} \left(\frac{E}{T_{e}} - \frac{3}{2}\right)\right] \exp\left(-\frac{R}{2L_{n}}\right). \tag{66}$$

The trapped electron distribution function at resonance $\langle f_e \rangle_{E=RT_e/2L_n} = n_0(x)(m_e/2\pi T_e(x))^{3/2} \exp(-R/2L_n)$ was used in Eq. (66). Comparing Eq. (57) with Eq. (66), the ratio of the electron energy lost by turbulent transport to the collisionless energy transfer can be written as (note: we estimated $\omega_{\star e} = k_\theta \rho_s C_s/L_n \approx C_s/L_n$ in Eq. (57), as before)

$$\frac{\Delta r \langle \tilde{E} \cdot \tilde{J}_e \rangle}{Q_e|_{boundary}} = \frac{L_n}{R} \frac{\Omega_e}{V_{the}^2 k_{\theta}} \omega_{\star e} \Delta r = \frac{\Delta r}{R} \approx \frac{a}{R}, \tag{67}$$

where the annular width $\Delta r \approx a$, so the radial annular integral can extend to the whole minor radius. Thus, the ratio of transfer to transport loss is given by $a/R \sim o(1)$ which suggests that electron turbulent energy transfer to ions in a collisionless plasma can be the same order as electron heat transport loss! Hence the collisionless electron heat transfer by turbulence is surely a critical element of any transport analysis model for a low collisionality, electron heated plasma. It is necessary to consider the influence of the collisionless energy transfer to determine the total electron heat budget. This issue is especially relevant to ITER plasmas.

VIII. CONCLUSION

In this paper, we considered two problems: the net turbulent heating and inter-species collisionless energy transfer channels in electron drift wave turbulence. The principal results of this analysis are:

(1) We extended the classical calculation of the turbulent heating within the quasilinear framework to the nonlinear level. The turbulent heating includes quasilinear electron cooling, quasilinear ion heating, and ion polarization drift and ion diamagnetic drift induced turbulent heating. The volume integral of the ion polarization drift in an annulus give rises to the net turbulent heating, which occurs through the zonal flow. This net heating is dissipated by the zonal flow frictional damping. Thus, it constitutes an important collisionless energy transfer channel working through zonal flow generation. The process of energy transfer via the zonal flow has not previously been accounted for in analyses of energy transfer from electrons to ions. We ignore the small effect of the ion diamagnetic drift induced heat flux differential in an annulus.

- (2) We identified three kinds of collisionless turbulent energy transfer channels in electron drift wave turbulence. The hot electrons can transfer turbulent energy to cold ions through wave-particle interaction (ion Landau damping, ion Nonlinear Landau damping) and turbulence-zonal flow interaction (zonal flow frictional damping). Here we focus more on the nonlinear turbulent energy transfer through the zonal flow channel, since it can dominate the nonlinear saturation balance. The nonlinear collisionless heat transfer through the nonlinear Landau damping will be discussed in a future paper.
- (3) By using a mixing length approximation, we estimated all of the turbulent heating ratios. Using ITER parameters, we discussed the implication of our results. The comparison of the ratios of the energy dissipation channels showed that the zonal flow frictional damping can be a significant energy dissipation channel for the low collisionality drift wave in ITER plasma.
- (4) We explored the meaning of the collisionless turbulent energy transfer channels in a more realistic case, namely for CTEM. For ITER plasma, the collisionality is low enough such that the collisionless energy transfer may ultimately dominate the energy transfer process. Also we compared the rate of the energy lost through collisionless energy transfer with the electron turbulent energy transport in CTEM. The ratio is order unity, which means the collisionless turbulent energy transfer can be comparable to the turbulent energy transport in the heat balance. Then in future large, collisionless tokamaks, we have to consider the influence of the collisionless energy transfer as well as the turbulent energy transport.

The collisionless electron-ion coupling model may be related to some experimental phenomena, such as the electron temperature profile "stiffness" where the temperature profile reacts weakly to changes in auxiliary heating deposition. Cone of the possible causes of such behavior is the nondiffusive term in the heat flux, which is an inward flow and carries energy from edge to the core. Another reason may due to electron-ion energy transfer in the core, where the electron energy is dissipated through collisionless energy transfer. These two effects are different and independent. Both must be examined to see which one is more efficient in future experiments. The proper analysis of these two effects will be presented in a future paper.

ACKNOWLEDGMENTS

We would like to thank L. Wang, Y. Kosuga, C. J. McDevitt, and F. L. Hinton for helpful discussions

This work was supported by the Department of Energy under Award No. DE-FG02-04ER54738 and the Ministry of Education, Science and Technology of Korea via the WCI Project 2009-001.

- ¹⁰R. Z. Sagdeev and A. A. Galeev, *Nonlinear Plasma Theory* (Benjamin, New York, 1969), p. 103.
- ¹¹F. Y. Gang and P. H. Diamond, Phys. Fluids B 2(12), 2976 (1990).
- ¹²F. Y. Gang, P. H. Diamond, and M. N. Rosenbluth, Phys. Fluids B 3(1), 68 (1991).
- ¹³T. S. Hahm and W. M. Tang, Phys. Fluids B **3**(4), 989 (1991).
- ¹⁴M. N. Rosenbluth and F. L. Hinton, *Phys. Rev. Lett.* **80**, 4 (1998).
- ¹⁵P. H. Diamond and Y.-M. Liang, Phys. Rev. Lett. **72**, 16 (1994).
- ¹⁶L. Chen, R. L. Berger, J. G. Lominadze, M. N. Rosenbluth, and P. H. Rutherford, Phys. Rev. Lett. 39, 754 (1977).
- ¹⁷Z. Lin, T. S. Hahm, W. W. Lee, W. M. Tang, and R. B. White, Science 281, 1835 (1998).
- ¹⁸B. J. Grenn, the ITER International Team, and Participant Teams, Plasma Phys. Controlled Fusion 45, 687 (2003).
- ¹⁹B. B. Kadomtsev, Sov. J. Plasma Phys. **1**, 295 (1975).
- ²⁰B. B. Kadomtsev, *Tokamak Plasma: A Complex Physical System* (Institute of Physics, Bristol, 1992).
- ²¹S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovitch, translated by H. Lashinsky (Consultants Bureau, New York, 1965), Vol. I, p. 205.
- ²²J. Li and Y. Kishimoto, Plasma Phys. Controlled Fusion **44**, A479 (2002).
- ²³J. Lang, Y. Chen, and S. E. Parker, Phys. Plasmas **14**, 082315 (2007).
- ²⁴J. Lang, S. E. Parker, and Y. Chen, Phys. Plasmas **15**, 055907 (2008).
- ²⁵L. Wang and P. H. Diamond, Nucl. Fusion **51**, 083006 (2011).
- ²⁶C. C. Petty and T. C. Luce, Nucl. Fusion **34**, 121 (1994).

¹W. M. Manheimer, E. Ott, and W. M. Tang, Phys. Fluids **20**, 806 (1977).

²F. L. Hinton and R. E. Waltz, Phys. Plasmas **13**, 102301 (2006).

³R. E. Waltz and G. M. Staebler, Phys. Plasmas **15**, 014505 (2008).

⁴ITER Physics Expert Group on Energetic Particles, Heating and Current Drive, ITER Physics Basis Editors, and ITER EDA, Nucl. Fusion 39, 2471 (1999)

⁵ITER Physics Basis Editors, ITER Physics Expert Group Chairs and Co-Chairs, ITER Joint Central Team and Physics Integration Unit, ITER EDA, Nucl. Fusion **39**, 2137 (1999).

⁶L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, Oxford, 1984).

⁷P. H. Diamond and Y. B. Kim, Phys. Fluids B 3, 2050 (1991).

⁸P. H. Diamond, S.-I. Itoh, K. Itoh, and T. S. Hahm, Plasma Phys. Controlled Fusion 47, R35 (2005).

⁹W. M. Manheimer and T. H. Dupree, Phys. Fluids **11**, 2709 (1968).