Theory of Cross-Phase Evolution and its Impact on ELM Dynamics

P.H. Diamond

CMTFO and CASS, UCSD, USA
WCI Center for Fusion Theory, NFRI, Korea

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A Different Look at ELM Dynamics
→ Thoughts on Selected Issues

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CMTFO and CASS, UCSD, USA
WCI Center for Fusion Theory, NFRI, Korea
KITP, UCSB, USA

2014 US-TTF
Collaborators

- **Pengwei Xi$^{(1,2)}$, X.-Q. Xu$^{(2)}$**
  on cross phase correlation, anomalous dissipation

- **T. Rhee$^{(3)}$, J.M. Kwon$^{(3)}$, W.W. Xiao$^{(3,4)}$**
  on reduced models

- **R. Singh$^{(3)}$**
  on anomalous dissipation

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3) NFRI
4) UCSD
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Caveat Emptor

- Not a professional ELM-ologist
- Perspective is theoretical, and focus is on issues in understanding dynamics
- Perspective is that of a transport theorist
- Aim is to distill elements critical to model building
- Unresolved issues are discussed
- Not a review!
N.B. : Many ideas discussed here are contrary to ‘conventional wisdom’ of ELM-ology

↔ Locale has a history of struggle against group think…
Outline

• The conventional wisdom of ELMs
  – Motivation
  – Mechanism

• Some issues in ELM dynamics
  – How do bursts occur?
  – Mechanism of anomalous dissipation?
  – Assembling the ‘big picture’ → sources and transport effects?
Recent progress on some issues:

i) cross phase coherence and the origin of bursts

ii) phase coherence as leverage for ELM mitigation

iii) hyper-resistivity: single scale or multi-scale!? 

iv) a reduced model of the big picture $\rightarrow$ importance of flux-drive

Conclusions – at this point

Discussion: where should we go next?
**Terra Firma: Conventional Wisdom of ELMs**

- ELMs are ~ quasi-periodic relaxation events occurring at edge pedestal in H-mode plasma

- ELMs
  - Limit edge pedestal
  - Expel impurities
  - Damage PFC

- ELMs → a serious concern for ITER
  - $\Delta W_{ELM} \sim 20\% \, W_{ped} \sim 20 \, MJ$
  - $W_{ELM} / A \sim 10 \times$ limit for melting
  - $\tau_{ri} \sim 200 \, \mu sec$
Terra Firma: Conventional Wisdom of ELMs

- **ELM Types**
  - I, II: $\omega_{ELM} \uparrow$ as $P \uparrow$, greatest concern, related to ideal stability
  - III: $\omega_{ELM} \downarrow$ as $P \uparrow$, closer to $P_{Th}$, unknown $\rightarrow$ resistive ??

- **Physics**
  - Type I, II ELM onset $\rightarrow$ ideal stability limit
  - i.e. peeling + ballooning

![Graph showing Relative ELM Energy Loss vs Pedestal electron Collisionality](image)


**Diagram:**
- Peeling + ballooning
- Edge kink
- Curvature vs bending
- $\delta W$ + Pedestal, geometry
Terra Firma: Conventional Wisdom of ELMs

- Edge pressure gradient is ultimate energy source
  \[ \delta W_p \sim \frac{1}{R_c} \frac{dP}{dr} \xi^2 \quad \text{vs} \quad \delta W_{LB} \]
  ↔ ballooning

- \( J_{bootstrap} \sim \frac{1}{B_\theta (1 + 0.9 \sqrt{v_*})} \frac{dP}{dr} \)
  ↔ peeling
Terra Firma: Conventional Wisdom of ELMs

- Some relation of ELM drive character to collisionality is observed
  - Low collisionality $\rightarrow$ peeling $\sim$ more conductive
  - High collisionality $\rightarrow$ ballooning $\sim$ more convective

- Pedestal perturbation structure resembles P-B eigen-function structure

- Many basic features of ELMs consistent with ideal MHD peeling-ballooning theory
Some Physics Questions

• What IS the ELM?
  – ELMs single helicity or multi-helicity phenomena?
    Relaxation event ↔ pedestal avalanche?
  – How and why do actual bursts occur?
    Why doesn’t turbulence force $\nabla P \sim \nabla P_{\text{crit}}$ oscillations?
  – Pedestal turbulence develops during ELM. Thus, how do P-B modes interact with turbulence? – either ambient or as part of MH interaction?
  – Does, or even should, the linear instability boundary define the actual ELM threshold?
Some Physics Questions

• Irreversibility?
  – Peeling-Ballooning are ideal modes. What is origin of irreversibility? How does fast reconnection occur?
  – If hyper-resistivity is the answer (Xu et al, 2010), what is its origin – ambient micro-turbulence or P-B’s themselves? Can P-B modes drive the requisite hyper-resistivity?
  – What is the relation between hyper-resistivity, reconnection and heat transport, especially for ‘conductive’ ELMs?
Some Physics Questions

• How do the pieces fit together?
  – Do ELM events emerge from a model which evolves profiles with pedestal turbulence?
  – What profiles are actually achieved at the point of ELMs?
  – What is the minimal model in which ELMs emerge?
  – What are the necessary ingredients in a full model?

• How exploit dynamical insight for ELM mitigation?
I) Basic Notions of ELMs:
ELM Bursts and Thresholds as
Consequence of Stochastic Phase Dynamics

→ See P.W. Xi, X.-Q. Xu, P.D.; PRL 2014
P.W. Xi, X.-Q. Xu, P.D.; PoP 2014 in press
Simulation model and equilibrium in BOUT++

- 3-field model for nonlinear ELM simulations
  - Including essential physics for the onset of ELMs

Peeling-ballooning instability
Resistivity
Hyper-resistivity
Ion diamagnetic effect

\[
\begin{align*}
\frac{d\omega}{dt} &= B\nabla \parallel J \parallel + 2b_0 \times \kappa \cdot \nabla \tilde{P} + \mu_{i,\parallel} \partial_{\parallel}^2 \omega \\
\frac{d\tilde{P}}{dt} + \mathbf{V}_E \cdot \nabla P_0 &= 0 \\
\frac{\partial A}{\partial t} + \partial_{\parallel}^2 \phi_T &= \frac{\eta}{\mu_0} \nabla^2 A_{\parallel} + \frac{\eta_H}{\mu_0} \nabla^4 A_{\parallel} \\
\omega &= \frac{m_in_0}{B} \left( \nabla^2 \phi + \frac{1}{en_0} \nabla^2 \tilde{P}_i \right) \\
\end{align*}
\]

\[d / dt = \frac{\partial}{\partial t} + \mathbf{V}_{ET} \cdot \nabla, \mathbf{V}_{ET} = \frac{1}{R} \mathbf{b}_0 \times \nabla \phi_T, \phi_T = \phi_0 + \phi, \nabla \parallel f = B\partial \parallel \frac{f}{R}, \partial_{\parallel} = \partial_{\parallel}^0 + \partial _\mathbf{b} \cdot \nabla, \partial _\mathbf{b} = \frac{1}{B} \nabla A_{\parallel} \times \mathbf{b}_0, J_{\parallel} = J_{\parallel 0} + \tilde{J}_{\parallel}, \tilde{J}_{\parallel} = -\nabla^2 A_{\parallel} / \mu_0\]
Comparison:

Single vs Multi-Mode Dynamics
3D structure of pressure perturbation: filaments–helical coherent perturbation with outward radial motion

Images generated with VisIt
Contrastive perturbation evolution (1/5 of the torus)

- **Single mode**: Filamentary structure is generated by linear instability;
- **Multiple modes**: Linear mode structure is disrupted by nonlinear mode interaction and no filamentary structure appears.
Single mode: ELM crash || Multiple modes: P-B turbulence

- ELM size larger for SMS
  \[ \Delta_{ELM} = \frac{\Delta W_{ped}}{W_{ped}} = \frac{\int dx^3 (P_0 - \langle P \rangle_\zeta)}{\int dx^3 P_0} \]

- SMS has longer duration linear phase than MMS

Nonlinear Mode excitation
Relative Phase (Cross Phase) Dynamics and Peeling-Ballooning Amplification
Peeling-Ballooning Perturbation Amplification is set by Coherence of Cross-Phase

i.e. schematic P.B. energy equation:

\[ \frac{\partial}{\partial t} E_k = \langle \tilde{\phi} 2 \hat{b}_0 \times \hat{k} \cdot \nabla \tilde{P} \rangle_{\tilde{k}} \approx \langle \tilde{\nu}_r \tilde{P} \rangle \rightarrow \text{energy release from} \nabla \langle P \rangle \rightarrow \text{quadratic} \]

\[ + \sum_{\tilde{k}', \tilde{k}''} \tau_{c_{\tilde{k}}} C(\tilde{k}', \tilde{k}'') E_{\tilde{k}'} E_{\tilde{k}''} - \sum_{\tilde{k}, \tilde{k}'} \tau_{c_{\tilde{k}}} C(\tilde{k}', \tilde{k}) E_{\tilde{k}} E_{\tilde{k}'} - \text{dissipation} \]

NL effects
- energy couplings to transfer energy (weak)
- response scattering to de-correlate \( \tilde{\phi}, \tilde{P} \rightarrow \text{regulate drive} \)
Growth Regulated by Phase Scattering

\[ \tau_c \rightarrow \text{phase coherence time} \]

\[ k \]

\[ \nabla P_0 \rightarrow \tilde{P} \]

\[ \langle \tilde{v}_r \tilde{P} \rangle \rightarrow \text{net growth} \rightarrow \text{intensity field} \rightarrow \text{crash?} \]

\[ \text{phase scattering} \]

\[ \text{transfer} \rightarrow \text{dissipation (weak)} \]

Critical element: relative phase
\[ \delta \phi = \arg \left[ \hat{p}_n / \hat{\phi}_n \right] \]

Phase coherence time sets growth

Critical element: relative phase
\[ \delta \phi = \arg \left[ \hat{p}_n / \hat{\phi}_n \right] \]
Cross Phase Exhibits Rapid Variation in Multi-Mode Case

- Single mode case → coherent phase set by linear growth → rapid growth to ‘burst’
- Multi-mode case → phase de-correlated by mode-mode scattering → slow growth to turbulent state
Key Quantity: Phase Correlation Time

- Ala’ resonance broadening (Dupree ‘66):

\[
\frac{\partial}{\partial t} \hat{P} + \vec{v} \cdot \nabla \hat{P} + \langle v \rangle \cdot \nabla \hat{P} - D \nabla^2 \hat{P} = -\vec{v}_r \frac{d}{dr} \langle P \rangle
\]

Nonlinear scattering \hspace{1cm} \text{Linear streaming (i.e. shear flow)} \hspace{1cm} \text{Ambient diffusion}

\[
\hat{P} = Ae^{\hat{\phi}} \hspace{3cm} \text{Relative phase ↔ cross-phase}
\]

\[
\hat{v} = B \hspace{3cm} \text{Velocity amplitude}
\]

\[\Rightarrow \partial_t \hat{\phi} + \vec{v} \cdot \nabla \hat{\phi} + \langle v(r) \rangle \cdot \nabla \hat{\phi} - D \nabla^2 \hat{\phi} - \frac{2D}{A} \nabla A \cdot \nabla \hat{\phi} = 0\]

NL scattering \hspace{1cm} \text{shearing}

\[\partial_t A + \vec{v} \cdot \nabla A + \langle v(r) \rangle \cdot \nabla A + D \left( \nabla \hat{\phi} \right)^2 A - D \nabla^2 A = -B \frac{d}{dr} \langle P \rangle \]

Damping by phase fluctuations
Phase Correlation Time

• Stochastic advection:

\[
\frac{1}{\tau_{ck}} = \vec{k} \cdot D_\phi \cdot \vec{k} + k^2 D
\]

\[
D_\phi = \sum_{k'} \tau_{d_{ck}} |\vec{v}_{\perp k'}|^2
\]

• Stochastic advection + sheared flow:

\[
\frac{1}{\tau_{ck}} \approx (k_{\perp}^2 (D_\phi + D) \langle v_{\perp} \rangle^2)^{1/3} \quad \Rightarrow \text{Coupling of radial scattering and Shearing shortens phase correlation}
\]

• Parallel conduction + diffusion:

\[
\frac{1}{\tau_{ck}} \approx \left[ \frac{\hat{s}^2 k_{\perp}^2}{(Rq)^2} \chi_{\parallel} (D_\phi + D) \right]^{1/2} \quad \Rightarrow \text{Coupling of radial diffusion and conduction shortens phase correlation}
\]
What is actually known about fluctuations in relative phase?

- For case of P.-B. turbulence, a broad PDF of phase correlation times is observed.
Implications for: i) Bursts vs Turbulence

ii) Threshold
Bursts, Thresholds

- P.-B. turbulence can scatter relative phase and so reduce/limit growth of P.-B. mode to large amplitude
- Relevant comparison is:

\[ \gamma_k^L \text{ (linear growth) vs } \frac{1}{\tau_{ck}} \text{ (phase de-correlation rate)} \]

- Key point: Phase scattering for mode \( \vec{k} \) set by ‘background modes \( \vec{k}'' \) i.e. other P.-B.’s or micro-turbulence

\( \Rightarrow \) is the background strong enough??
The shape of growth rate spectrum determines burst or turbulence

\[ \gamma(n) \tau_c(n) < \ln 10 \]

**P-B turbulence**

\[ \gamma(n) \tau_c(n) > \ln 10, n = n_{\text{dom}} \]

\[ \gamma(n) \tau_c(n) < \ln 10, n \neq n_{\text{dom}} \]

**Isolated ELM crash**
So When Does it Crash?
Modest $\gamma(n)$ Peaking $\Rightarrow$ P.-B. turbulence

- Evolution of P-B turbulence
  - No filaments
  - Weak radial extent

$$\alpha = -2\mu_0 R P_0 q^2 / B^2$$
Stronger Peaking $\gamma(n) \rightarrow$ ELM Crash

- ELM crash is triggered
- Wide radial extension

$\alpha = -2\mu_0 R P_0' q^2 / B^2$
\( \gamma(n) \) Peaking VERY Sensitive to Pressure Gradient

\[ \alpha = -2\mu_0 R P'_0 q^2 / B^2 \]

- Higher pressure gradient
  - Larger growth rate;
  - Peaking of growth rate spectrum;
Filamentary structure may not correspond to that of the most unstable mode due to nonlinear interaction.

$\alpha = 2.29$

P-B turbulence

$\alpha = 2.44$

ELM crash

- Triggering ELM and the generation of filamentary structure are different processes!
  - ELM is triggered by the most unstable mode;
  - Filamentary structure depends on both linear instability and nonlinear mode interaction.
What is the Threshold for a Crash?
Linear criterion for the onset of ELMs $\gamma > 0$ is replaced by the nonlinear criterion

$$\gamma > \gamma_c \sim 1/\tau_c$$

- Criterion for the onset of ELMs
  $$\gamma \tau_c > \ln 10 \Rightarrow \gamma > \frac{\ln 10}{\tau_c} \equiv \gamma_c$$

- Linear limit
  $$\lim_{\tau_c \to \infty} \Rightarrow \gamma > 0$$

- $\gamma_c$ is the critical growth rate which is determined by nonlinear interaction in the background turbulence

- N.B. $1/\tau_c$ - and thus $\gamma_{\text{art}}$ - are functionals of $\gamma_L(n)$ peakedness.
Nonlinear Peeling-ballooning model for ELM:

- $\gamma < 0$:
  Linear stable region
- $0 < \gamma < \gamma_c$:
  Turbulent region
  Possible ELM-free regime
  Special state: EHO, QCM (?!)
- $\gamma > \gamma_c$:
  ELMy region

Different regimes depend on both linear instability and the turbulence in the pedestal.

Including all relevant linear physics (not only ideal P-B with $\omega_s$)
Resistivity / Electron inertia /...

→ Turbulence can maintain ELM-free states
Partial Summary

• Multi-mode P.-B. turbulence or ~ coherent filament formation can occur in pedestal
• Phase coherence time is key factor in determining final state and net P.-B. growth
• Phase coherence set by interplay of nonlinear scattering with ‘differential streaming’ in $\hat{P}$ response
• Key competition is $\gamma_L$ vs $1 / \tau_c \rightarrow$ defines effective threshold
• Peekedness of $\gamma(n)$ determines burst vs turbulence
How can these ideas be exploited for ELM mitigation and control?
ELMs can be controlled by reducing phase coherence time

\[
\frac{\partial \omega}{\partial t} + C_R \frac{\mathbf{b} \times \nabla \phi}{B} \cdot \nabla \omega = \text{RHS}
\]

i.e. scan \( C_R \) for fixed profiles

- ELMs are determined by the product \( \gamma(n) \tau_c(n) \);
- Reducing the phase coherence time can limit the growth of instability;

Different turbulence states lead to different phase coherence times and, thus different ELM outcomes
Keys to $\tau_c$

- Scattering field
- ‘differential rotation’ in $\hat{P}$ response to $\hat{\nu}_r$

→ enhanced phase de-correlation

Knobs:

- ExB shear
- Shaping
- Ambient diffusion
- Collisionality

Mitigation States:

- QH mode, EHO
- RMP
- SMBI
- ...

NFRI National Fusion Research Institute
Scenarios

- **QH-mode**
  - enhanced ExB shear $\frac{1}{\tau_c} \rightarrow (k_{\perp}^2 D \left<V_E\right>^2)^{1/3}$
  - Triangularity strengthens shear via flux compression
  - Enhanced de-correlation restricts growth time

Also:

- Is EHO peeling/kink + reduced $\tau_c$?
- $\left<V_E\right>'$ works via $\gamma_L$ and $\tau_c$

N.B. See Bin Gui, Xu; this meeting for more on shearing effects
Scenarios

- **RMP**

\[ - \frac{1}{\tau_c} = \left( \frac{k_1^2 \delta^2}{(Rq)^2} \chi_\parallel D \right)^{1/2} \]

\[ D = D_\phi + D_{am b} \]

- RMP \( \rightarrow D_{am b} \uparrow \rightarrow \) enhanced de-correlation

or

- Enhanced flow damping \( \rightarrow \) enhanced turbulence \( \rightarrow \) increased \( D_\phi \)

- **SMBI**

- enhanced \( D_\phi \) \( \rightarrow \) reduced \( \tau_c \) ?

and/or

- Disruption of pedestal avalanches?
II) Reconnection and Hyper-resistivity
Some Basics

• P.-B.’s are ideal modes ↔ frozen-in law… !?
• ELM phenomena requires irreversibility for:
  – field-fluid decoupling, reconnection
  – Transport, cross field
  – Magnetic stochastization
• What is mechanism of fast reconnection for ELM? Resistivity unlikely…
• $S \geq 10^8$ in pedestal → hyper-resistivity becomes natural candidate
• Hyper-resistivity!? – Electron momentum transport

\[ E_\parallel = \eta J_\parallel + \nabla_\perp \mu_H \nabla_\perp J_\parallel \]

\( \perp \) transport of parallel current  
- ambient micro-turbulence
- P.-B. turbulence

• Xu et al 2010 → Hyper-resistivity \( \sim \chi_e \) needed to dissipate current sheets, so as allow ELM crash

• Hyper-resistivity generally can trigger fast reconnection

i.e. Sweet-Parker: - resistive:

\[ \frac{V}{V_A} \sim \frac{1}{\sqrt{R_m}} \]

- hyper-resistive:

\[ \frac{V}{V_A} \sim \frac{1}{(R_{m,H})^{1/4}} \]

• Origin?
• Simplest Approach: Electron inertia + MHD

i.e. Ohm’s law becomes:

\[ \frac{m_e}{e} \frac{d}{dt} \tilde{v}_{\parallel e} + \tilde{E}_{\parallel} = \eta \tilde{J}_{\parallel} \]

Electron inertial effect \( \Rightarrow \) electron momentum

• Scale : \( \frac{c}{\omega_{pe}} \)

Low \( n \) \( \Rightarrow \rho_i \sim c/\omega_{pe} \)

\( \Rightarrow \) significant effect on linear growth for \( k_{\perp} \left( \frac{c}{\omega_{pe}} \right) \sim O(1) \)

P.-B. \( \Rightarrow \) ‘hyper-resistivity’ ballooning mode…

• Examine impact on nonlinear evolution… \( \Rightarrow \) self-consistent crash?
Electron inertia and P-B turbulence cannot generate enough current relaxation for ELM crash

Micro-turbulence is needed to generate enough current relaxation

The self-consistent nonlinear ELM simulation is a multiple scale issue.
• Interesting candidate for hyper-resistivity

\[ \Rightarrow \text{ETG turbulence in pedestal?!} \]

• ETG indicated by pedestal micro-stability studies \( \Rightarrow \) survive \( \langle V_E \rangle' \)

• Mechanism is advection of electron momentum

\[
\begin{align*}
\chi_\phi &\sim \chi_i \\
\mu_H &\sim \chi_e
\end{align*}
\]

\[ \Rightarrow \text{hyper-resistivity linked to pedestal electron heat transport} \]

\[ \begin{align*}
\mu_H &\approx \left( \frac{c}{\omega_{pe}} \right)^2 C D_{GBe} \\
D_{GBe} &\leftrightarrow L_{Te}^{-1}
\end{align*}\]

\[ \text{anisotropy factor} \]

• Modulation of driving \( \nabla T_e \) by P.-B.’s crucial effect
Anomalous dissipation

ETG

Gradient modulation

P.-B.

Approach as disparate – scale modulation problem via gradient evolution due P.-B.
Partial Summary

- Hyper-resistivity required for dissipation of P.-B. current sheets, and crash
- P.B. + electron inertia insufficient to trigger fast reconnection
- Multi-scale approach to current dissipation is required
- ETG is interesting candidate for origin of $\mu_H$
- Considerable further work required
III) Towards a ‘Big Picture’
- Is there a ‘minimal’ model of ELMs?
- What are the key ingredients?
- Might this help us understand ELM-related phenomena better?

→ See: W. Xiao, et al; NF 2013
   T. Rhee, et al; PoP 2012
Needed: Simple Model...

N.B. full ELM phenomena far beyond "First Principle" Simulations!

• Minimal Model of Pedestal Dynamics

• Necessary Ingredients:
  
  – Bi-stable flux $\rightarrow$ capture turbulence, transport, $L \rightarrow H$ transition
  
  – Fixed ambient diffusion $\rightarrow$ capture (neoclassical) transport in H-mode pedestal
    
    N.B. key: how does system actually organize profiles for MHD activity??
  
  – Hard stability limit $\rightarrow$ capture MHD constraint on local profile. Can be local. (i.e. ballooning $\leftrightarrow \nabla P$) or integrated (i.e. peeling $\leftrightarrow J_{BS} \sim \int dr \nabla P \sim P_{ped, top}$
N.B. Transport vs ‘hard stability’?

\[ Q \sim C \left( \frac{L_P \alpha t}{L_p} - 1 \right)^\alpha \quad : \quad c, \alpha \text{ large for ‘hard stability limit’} \]

**Sandpile (Cellular Automata) Model**

- **Toppling rule:** \( Z_i - Z_{i+1} > Z_{\alpha t} \)  \( \rightarrow \) topple \( Y_i \) cells  \( \rightarrow \) move adjacent

- **Bi-stable toppling:**

\[ Z_i - Z_{i+1} > Z_{\alpha t_1} \quad \rightarrow \text{toppling, threshold, transport} \]

\[ Z_i - Z_{i+1} > Z_{\alpha t_2} , Z_{\alpha t_2} > Z_{\alpha t_1} \quad \rightarrow \text{no toppling, transport bifurcation} \]

(turbulence exciting)

(micro-turbulence, flipping)  (stable by ExB shear flow)  (MHD event, toppling)
Sandpile Model, cont’d:

- Constant diffusion $\Rightarrow$ neoclassical transport (discretized)
- N.B. Bi-stable toppling + diffusion $\Rightarrow$ S-curve model of flux

![Graph of Q vs. -\nT showing L mode, H-mode (suppression of turbulent transport), and Macro MHD instability]

- Hard Limit $\Rightarrow Z_i - Z_{i+1} > Z_{\text{hard}}$ $\Rightarrow$ topple excess $Z_i$ according to rule
- Drive:
  - Random grain deposition, throughout
  - Additional “active grain injection” in pedestal, to model SMBI
L→H Transition

• Now try bi-stable toppling rule, i.e. if $Z_i - Z_{i+1}$ large enough
  \[ \rightarrow \text{reduced or no toppling} \]

• Obvious motivation is $Q = -\frac{\chi V_P}{1 + \alpha V_P^2}$ and $V_E \approx \frac{c}{eB} \frac{V_P}{n}$

• Hard gradient limit imposed

• Transitions happen, pedestal forms!

Gruzinov PRL2002
Note

- Critical deposition level required to form pedestal ("power threshold")
- Pedestal expands inward with increasing input after transition triggered
- Now, including ambient diffusion (i.e. neoclassical)
  - \( N_F \) threshold evident
  - Asymmetry in \( L \rightarrow H \) and \( H \rightarrow L \) depositions
Hysteresis Happens!

- Hysteresis loop in mean flux-gradient relation appears for $D_0 \neq 0$
- Hysteresis is consequence of different transport mechanisms at work in “L” and “H” phases
- Diffusion ‘smoothes’ pedestal profiles, allowing filling limited ultimately by large events

$\Gamma(R) = \text{Flux}$

$Z(R) = \text{Mean Slope}$

Gruzinov PoP2003
ELMs and ELM Mitigation

- ELMs happen!
- Quasi-periodic Edge Relaxation Phenomena (ELM) self-organize. Hard limit on $\nabla Z$ ($\nabla P$) is only MHD ‘ingredient’ here
- ELM occurs as out $\rightarrow$ in and in $\rightarrow$ out toppling cascade
ELM Properties

• Periodic with period $\sim 10^{-2} \tau_p$. $\tau_p$ = grain confinement time
• ELM flux $\sim 500$ diffusive-flux
• ELMs span pedestal
• Period $\leftrightarrow$ pedestal re-fill (approximate)

The What and How of ELMs?

What?
• ELMs are a burst sequence of avalanches, triggered by toppling of ‘full pedestal’
• ELMs are not global (coherent) eigen-modes of pedestal
The What and How of ELMs?

How?

- Toppling cascade:
  - Void forms at boundary, at hard limit
  - Propagates inward, to top of pedestal, triggering avalanche
  - Reflects from top of pedestal, becomes a bump
    (N.B. core is subcritical $\rightarrow$ void cannot penetrate)
  - Bump propagates out, causing further avalanching
  - Bump expelled, pedestal refills
N.B. ELM phenomena appear as synergy of H-phase, diffusion, hard limit

**With Active Grain Injection (AGI):**

- AGI works by adding a group of grains over a period $\tau_{dep}$
- Can repeat at $\tau_{rep}$
  
- Obviously, model cannot capture dynamics of actual SMBI, time delay between injection and mitigation. See Z. H. Wang for injection model
- Model can vary strength, duration, location
Results with AGI

- AGI clearly changes avalanche distribution, and thus ELM ejection distribution

- Mitigation due fragmentation of large avalanches into several smaller ones

- Injection destroys coherency of large avalanches by triggering more numerous small ones

- Consistent with intuition
Edge Flux Evolution (in lieu $D_\alpha$)

- $A/A_0$ drops, $f/f_0$ increases
- An “influence time” $\tau_I$ is evident → duration time of mitigated ELM state
- $\tau_I \sim 5 \tau_{ELM}$
AGI tends to reduce gradient at deposition region

- Drive triggers local toppling → prevents recovery of local gradient
- ‘flat spot’ is effective beach, upon which avalanches break
- $\tau_I$ is recovery time of deformed local gradient
- Related to question of optimal deposition location
Which deposition location is optimal?

- Clue: deep deposition, at top of pedestal, allows avalanches to re-establish coherence ‘behind’ deposition zone.
- Clearly desirable to prevent large avalanches from hitting the boundary.
  ➔ points toward deposition at base of pedestal as optimal.
Results of Study on Deposition

- Study suggests optimal location slightly inside pedestal base
- Here $20 \leq i \leq 100 \rightarrow$ pedestal domain
- Here $\rightarrow$ optimal location $\sim 80$

Results of model study point toward optimal deposition near pedestal base

Color: Red high
Purple low

$X \rightarrow$ location
$Y \rightarrow$ injection intensity

$\delta n/n_0$ [%]
$\frac{f}{f_0}$

$A/A_0 = 0.5$
$A/A_0 = 0.2$

Injection location
Summary of Reduced Model Results

- ELM phenomena emerge from synergy of bi-stable turbulence, ambient diffusion and hard gradient limit. ELM appears as result of avalanche in pedestal.
- N.B. Multi-mode interaction necessarily triggers avalanching.
- SMBI mitigation may be understood as a consequence of fracturing of pedestal-spanning avalanches.
Conclusions – Coarse Grained
Conclusions

- ELM phenomena are intrinsically multi-mode and involve turbulence
- P.-B. growth regulated by phase correlation
  - determines crash + filament vs turbulence
- Phase coherence can be exploited for ELM mitigation
- Hyper-resistivity dissipation is likely a multi-scale phenomena
- ELMs appear as pedestal avalanching in reduced model
Where to Next?
• Simulations MUST move away from IVP – even if motivated by experiment – and to dynamic profile evolution, with:

  – sources, sinks i.e. flux drive essential
  – pedestal transport model
  – anomalous electron dissipation

i.e. → - what profiles are actually achieved?

  - how evolve near P.-B. marginality?
• Should characterize:
  – pdf of phase fluctuations, correlation time
  – Dependence on $\tau_c$ control parameters
  – Threshold for burst

• Need understand feedback of P.-B. growth on turbulent hyper-resistivity

• Continue to develop and extend reduced models.