Geodesic Acoustic Eigenmodes

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The eigenmode of a geodesic acoustic mode in the presence of a temperature gradient is discussed. Eigenmodes are obtained and the characteristic wavelength scales as \( \rho_i^{2/3} L_T^{1/3} \) (\( \rho_i \): ion gyroradius, \( L_T \): temperature gradient scale length). The direction of propagation is discussed.

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Zonal flows have attracted attention owing to their essential role in the turbulent transport of magnetically confined plasmas[1]. The geodesic acoustic mode (GAM) is a kind of zonal flow, which has finite real frequency owing to the geodesic curvature of a toroidal magnetic field [2], and is driven by microscopic turbulence [3, 4]. Measurements of GAMs have been recently reported [5–11]. It has been known that the GAMs have real frequency \( \omega_G = \sqrt{2} c_i / R \) in tokamaks (\( c_i \): ion sound velocity, \( R \): major radius). [The coefficient \( \sqrt{2} \) depends on the model of plasma dynamics [1], but this is not an issue addressed in the present article.] In tokamaks and other toroidal plasmas, the plasma temperature changes in the radial direction, so that the dispersion relation \( \omega = \omega_G \), which is provided by the local theory, predicts different frequencies at different radii. In contrast, fluctuations with a common frequency are observed within a region which has a substantial width in radial direction [10, 11]. This indicates that the GAM oscillation appears as an eigenmode. In this article, we discuss the eigenmode of GAM oscillation in the presence of a temperature gradient. Due to the finite ion gyroradius, local oscillations on different magnetic surfaces interfere with one another so as to constitute a radial eigenmode. The characteristic wavelength is found to scale as \( \rho_i^{2/3} L_T^{1/3} \) (\( \rho_i \): ion gyroradius, \( L_T \): temperature gradient scale length) and propagates outward if the temperature decreases towards the edge.

The dispersion relation of GAMs, \( \omega = \omega_G \), is derived by balancing the cross-field current \( J_{Dz} \) (due to the magnetic field curvature) and the ion polarization current \( J_P \) under the imposition of an electrostatic perturbation that has a form \( \hat{\phi} \exp(ikr - i\omega t) \) in the leading order [12–15]. In order to study the radial eigenmode with analytic transparency, we take a simple collisionless limit with \( T_e \gg T_i \) and \( k\rho_i \ll 1 \). In the limit of \( T_e \gg T_i \), the relation \( \nu_{th1}/R \ll \omega \) holds for \( \omega \sim \omega_G \), and \( J_{Dz} \) is dominated by the electron response (\( \nu_{th1} \): ion thermal velocity)[14]. Therefore, \( J_{Dz} \) is not significantly influenced by the finite gyroradius effect. In contrast, the ion polarization current, which is in proportion to \( \omega \), is screened by the factor \( 1 - k^2 \rho_i^2 \) owing to the finite gyroradius effect. Thus, the relation \( J_{Pz} + J_{Dz} = 0 \) provides

\[
(1 - k^2 \rho_i^2) \omega^2 = \omega_G^2, \tag{1}
\]

where the lowest order finite-gyroradius correction is included (see [12–16] for a more detailed derivation). We consider the case in which the temperature decreases in radius, and choose the radius \( r_0 \) where \( \omega^2 = \omega_G^2(r_0) \) holds. Taking the radial gradient of temperature into account, we write \( \omega_G^2(r) = \omega_G^2(r_0) \left[ 1 - (r - r_0)/L_T \right] \). The dispersion relation (1) can be rewritten as an eigenmode equation

\[
\rho_i^2 \frac{d^2}{dr^2} \phi(r) + \frac{r - r_0}{L_T} \phi(r) = 0, \tag{2}
\]

by the replacement \( k^2 \rho_i^2 \rightarrow -\rho_i^2 d^2/dr^2 \). Equation (2) has a characteristic scale length,

\[
A = \rho_i^{2/3} L_T^{1/3}, \tag{3}
\]

and is normalized as

\[
\frac{d^2}{dx^2} \phi(x) + x \phi(x) = 0, \tag{4}
\]

where \( x = (r - r_0) \lambda^{-1} \). Equation (4) is readily solved by employing the Airy function:

\[
\phi(x) = A \text{Ai}(x). \tag{5}
\]

The result seen in Eq. (5) shows that the eigenmode peaks near the region \( x \approx 0 \), propagates in the lower-temperature region \( (x > 0) \), and is evanescent in the higher

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temperature region ($x < 0$). Figure 1 illustrates the radial eigenfunction. The wave length is a few times $\lambda$. For this solution (5), the finite gyroradius correction has the order of magnitude $k^2 \rho_i^2 \sim \rho_i^{2/3} L_T^{-2/3}$, and is much smaller than unity if $\rho_i \ll L_T$ holds. The assumption $k \rho_i \ll 1$ is verified a posteriori. We note that, in the limit of $\rho_i \to 0$, an eigenmode is localized to a magnetic surface.

In summary, the GAM oscillation was found to exist in a form of radial eigenmode when the temperature is inhomogeneous. This is consistent with the observation that GAM oscillations are observed as radial eigenmodes [11]. The radial wavelength has a dependence of $\rho_i^{2/3} L_T^{-1/3}$, showing that GAMs are mesoscale fluctuations.

One can extend this analysis in a couple of ways. The extension to a more general profile of temperature $T(r)$ is possible. When $T_e$ comes closer to $T_i$, the screening owing to the finite-gyroradius effect also appears in $J_{0,D}$ as was explained in [12–15], so that the coefficient to $k^2 \rho_i^2$ in Eq. (1) becomes smaller (i.e., the radial wavelength becomes shorter). As was pointed in [17], the finite ion gyroradius effect can lead to the collisionless ion damping even in the limit of $k || n_{th,i} |\ll \omega_i$, such collisionless damping having recently been studied theoretically [15]. When a small but finite damping rate is introduced, the eigenfunction shows an oscillation in the region of $x < 0$. Details of these investigations are left for future research.

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