On Imaging of Plasma Turbulence

K. ITOH, A. FUJISAWA, Y. NAGASHIMA1), S.-I. ITOH1), M. YAGI1), P.H. DIAMOND2), A. FUKUYAMA3) and K. HALLATSCHEK4)

National Institute for Fusion Science, Toki 509-5292, Japan
1) Research Institute for Applied Mechanics, Kyushu University, Kasuga 816-8580, Japan
2) Department of Physics, University of California San Diego, San Diego, CA 92093-0319, U.S.A.
3) Department of Nuclear Engineering, Kyoto University, Kyoto 606-8501, Japan
4) Max-Planck-Institute für Plasmaphysik, D-85748 Garching, Germany

(Received 4 December 2006 / Accepted 23 March 2007)

We discuss aspects of imaging of plasma turbulence, taking the standpoint that the imaging is a path for induction of law from complex signals. An example of induction of symbol from signals is illustrated. Then the image of plasma turbulence is extended, putting an emphasis on the identification of the nonlinear interaction in turbulence. By employing the bispectral method, magnitudes of interactions among excited modes are estimated. Interactions between microscopic turbulence and large-scale structures are demonstrated, providing our understanding of structure formation in turbulent plasmas.

© 2007 The Japan Society of Plasma Science and Nuclear Fusion Research

Keywords: plasma turbulence, induction of symbol, nonlinear interaction, bispectral analysis, disparate-scale interaction

DOI: 10.1585/pfr.2.S1003

1. Introduction

Plasmas turbulence research has driven a number of active scientific areas. In nuclear fusion research, the achievements of devices such as JT-60 and LHD have lead to a new era of burning plasma experiment ITER (international thermonuclear experimental reactor). In space physics and in astro-physics, numerous data from measurements have been explored. In addition, plasmas play important roles in developing new materials with special functions. Among these research areas, an important issue exists, namely, the structural formation of turbulent plasma associated with electro-magnetic field formation and its selection rules. That is, the generation of structure from turbulence (e.g., zonal flows and transport barriers in magnetic confinement of plasmas, tachocline in solar convection zone, dynamo magnetic field, acceleration in accretion of mass in stellar dynamics, etc.) attracts considerable interest of research. This research has been established based upon the rapid progress in the precise knowledge of spatio-temporal evolution of plasmas. Thus, imaging of plasmas has been a strong driving force for the progress of modern science.

‘Imaging’ is composed of various processes in human intellectual activities. In one of thoughts, the visualization of the subject is a central theme, but in the others, emphasis is placed upon the process of comparing the signals and icons with the models, to establish understanding. As more and more precise data are provided, a goal of imaging (i.e., the induction of image and law from complicated signals) comes closer to us. When a threshold in resolution is surpassed, our understanding makes a stepwise jump to a new level. By the progress of the imaging technologies and of image processing, the induction of working hypothesis (or the test of present hypothesis) becomes possible (Figure 1). A view is chosen here that imaging is a path for induction of law from complex signals.

The fundamental issue in studying (thus in making the imaging of) turbulence is to identify the nonlinear interactions among excited degrees of freedoms, and to establish a law. The structure formation has attracted attention in the study of turbulent plasmas [1]. In plasma turbulence, turbulent fluctuations are composed of (nonlinear) collective modes and incoherent granulations. Collective modes are considered to interact through wave-wave (W-W) interactions and wave-particle (W-P) interactions [2]. In the

Fig. 1 Induction of image and law is a goal of imaging, and becomes possible when a threshold is overcome in imaging technology and image processing.
end, both of the nonlinear interactions must be measured and integrated into our understanding. Experimental studies are more abundant for W-W interactions. This interaction is dominated by the three-wave coupling. Therefore, the analysis of the third order spectrum is a useful method to measure the nonlinear interactions in plasma turbulence. Quantitative studies of nonlinear interactions in plasma turbulence are now possible, as a result of the rapid progress in capabilities in data acquisition and computation.

In this article, we first discuss one fundamental aspect of imaging, i.e., the induction of symbols from complicated signals. Then we discuss the image of plasma turbulence, by reviewing identification of the nonlinear interaction in turbulence. This is given by the higher order spectral analysis. By employing the bipspectral method [3], magnitudes of interactions among excited modes are measured and comparisons with theoretical modeling are performed. Interactions between microscopic turbulence, mesoscale fluctuations [4–6] and large-scale structures are also demonstrated [1, 7, 8], providing our understanding of ‘how global structures are generated by turbulence’. This is a new step in establishing imaging of plasma turbulence.

2. From Signals to Symbols

We begin the description of imaging as a path of induction of law (images, symbols) from complex signals, by illustrating the example of the recognition of the vortex. The vortex is an essential element in the physics of turbulence, and its image is indispensable in understanding the turbulence.

Vortices have been observed more than million of years by human being. The first (and symbolized) record of our recognition of the essence of vortices is the hieroglyph in ancient China (Figure 2). This hieroglyphic character for vortex represents ‘the concave vacancy of water’. The feature that the water surface is concave (when the vortex is excited) had been considered the most essential element of various observations of vortices, and was symbolized.

This induction of the symbol illustrates the wisdom of mankind. After the use of this character for more than tens of centuries, a mathematical description has been invented. The equation of motion is given as

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + \mathbf{g},
\]

where \( \mathbf{v}, \rho \) and \( p \) are the velocity, mass density and pressure of fluids, respectively, and \( \mathbf{g} \) is the acceleration by gravity. In the presence of a vortex, the centrifugal force (the second term of the LHS) balances with the change of pressure (the first term of the RHS). The reduction of the pressure, in the presence of gravity, causes the concave deformation of the water surface. This symbol of vortices in a hieroglyph, depletion of pressure, is not only proved relevant by modern science, but also drives new progress in today’s research. One can quote the identification of complex vortices in a large-scale nonlinear simulation. By finding the minima of pressure, axes of swirling vortices are identified. (See Fig. 3 from [9] as an example.) Thus, the induction of a rule through imaging is again propelling progress in the imaging technology and image processing under much more general the circumstance.

3. Dynamics and Causality

3.1 Turbulence and structure

We now proceed to the central problem, i.e., the structural formation of turbulent plasma associated with electromagnetic field formation. Our knowledge of structures in turbulent plasmas includes examples, e.g., the zonal flows, zonal field and transport barriers in magnetic confinement of plasmas, the tachocline in the solar convection zone, the dynamo magnetic field in solar and astrophysical plasmas, the acceleration in accretion of mass in stellar dynamics, etc. These stimulating phenomena are considered to be driven by turbulence. Imaging of plasma turbulence is illustrated here, explaining the measurements of nonlinear interactions that generate structures.

Two prototypical examples are given in Figs. 4 and 5.
In high temperature toroidal plasmas, which are hot in center and cold near the edge, the convection of heat is induced by microscopic fluctuations. Microscopic fluctuations are shown by small-scale corrugations of the colour contours in Fig. 4 (a). (See [10] for description of nonlinear simulation.) The microscopic fluctuations, at the same time, drive the zonal flows, which are constant on toroidal surfaces but change their directions in radius (shown by bold arrows in Fig. 4 (b)). The zonal flows in turn suppress the microscopic fluctuations, thus reducing the heat transport from the hot center to the cold periphery [4–8, 10–14]. In addition to this coupling between micro fluctuations and zonal flows, there are many nonlinear mechanisms that induce structured flow of plasmas. The nonlinearities in the relations between the gradient, structured flow, turbulent fluctuations, and turbulent transport govern the spatio-temporal evolution of the global features of toroidal plasmas (including the spatial interface, barriers, and structural transitions) [1].

In the dynamics of high temperature astrophysical plasmas, the structure formation of turbulent plasmas also plays essential roles. One example is quoted from the studies of the accretion disk (Fig. 5) [15]. Microscopic turbulence causes turbulent viscosity that enhances the rate of mass accretion to the gravitational center. The turbulence, at the same time drives the structured magnetic field (which may have the form of a torus in the disc and have cylindrical shape in the jets), i.e., the dynamo process.

The driven magnetic field regulates the momentum transport, and strongly influences the accretion of mass. Thus, the stellar evolution is governed by the mutual interaction of microscopic turbulence and structured magnetic field (Fig. 5 (b)).

These constitute the central working hypothesis in modelling the turbulent plasmas in laboratory and in nature. The essential issues are the identification of the structured flow (vorticity) and magnetic field, and confirmation of the nonlinear interaction between them and microscopic turbulence. This area of research has been propelled noticeably, owing to the improved knowledge of spatio-temporal data with high resolution, to the advanced analysis of data, and to the integration of theoretical, simulational and experimental research.

3.2 Zonal flows and microscopic fluctuations in toroidal plasmas

The discovery of the zonal flows in toroidal plasmas (experiments on CHS – compact helical system) is illustrated in Fig. 6 [16]. Dual heavy ion beam probes (HIBPs) are installed on CHS. This provides information on the radial electric field at multiple points (at two different
Fig. 6 Installment of dual HIBP system on CHS torus (a). Time series of raw data from one of the HIBP (b). The power spectrum (red) of fluctuations and cross-correlation between signals from two HIBPs (blue), (c). The high cross-correlation at low frequency range indicates the presence of the zonal flow.

cross spectrum of signals from two HIBPs. The coherence between the low-frequency components from two HIBPs is calculated (blue line in Fig. 6(c)). The signals from two HIBPs show high coherence, indicating that the perturbation in the low frequency range is nearly homogeneous on the magnetic surface. The relative radial distance of two HIBPs is varied, and it was found that (1) the radial wavelength of the zonal flows is one or two cm, and that (2) the correlation length of the zonal flows is longer than a few cm. (A peak in the power spectrum around 20 kHz is an oscillating zonal flow, which has been named geodesic acoustic modes (GAMs) [17].)

3.3 Measurement of nonlinear interaction

The nonlinear interaction among excited components in the turbulence was measured by use of bicoherence analysis. The method, bispectral analysis, is a relevant tool for the understanding of nonlinear interactions in plasma turbulence. The nonlinear dynamical equation may be written in a form

$$\frac{\partial}{\partial t} g + (-\gamma + iL_0) g = \sum N_{gg},$$

(2)

where $g$ is a dynamical variable of interest, $\gamma$ is a linear growth rate, $L_0$ represents the linear frequency, $N$ and denotes the coefficient of nonlinear interaction (which symbolizes the nonlinear interactions such as the $\mathbf{v} \cdot \nabla \mathbf{v}$ term in Eq. (1)). Thus, the triplet correlation $(g_{gg})$ determines the evolution of the quadratic moment $(g_{g})$ (e.g., the power spectrum). When a variable $g(t)$ is analyzed, we introduce Fourier components as

$$g(t) = \sum_{\omega} g_{\omega} \exp(-i\omega t)$$

(3)

The bispectrum estimator $\hat{B}(\omega, p)$, the squared bicoherence $\hat{b}^2(\omega, p)$, and the summed-bicoherence $\sum \hat{b}^2$ are defined as

$$\hat{B}(\omega, p) = (g_{\omega}^* g_{p-\omega} g_{p}),$$

(4)

$$\hat{b}^2(\omega, p) = \frac{|\hat{B}(\omega, p)|^2}{(g_{\omega}^* g_{p-w} g_{p})},$$

(5)

and

$$\sum \hat{b}^2(\omega) = \sum_p \hat{b}^2(\omega, p).$$

(6)

The biphase is defined by the relation

$$\hat{B}(\omega, p) = |\hat{B}(\omega, p)| e^{i\theta}.$$  

(7)

We see that this bispectrum estimator $\hat{B}(\omega, p)$ is in proportion to the projection of the response $g_p$ to the nonlinear force $N_{gg} g_{\omega} g_{p-w}$. Thus, the bicoherence represents the nonlinear interactions. The bispectrum in turbulent plasmas has been theoretically evaluated in [18].

The studies on nonlinear interaction among drift wave turbulence and zonal flows have been widely reported. (See [19, 20], cluster papers on “Experiments of zonal flow and turbulence” [21] and refs. [22–29].) The case
Fig. 7 Time series of raw data from the measurement of edge turbulence on JFT-2M tokamak (a). The power spectrum of fluctuations indicates the presence of low frequency zonal flow, oscillating zonal flow (GAMs) near 15 kHz, and high frequency microscopic fluctuations (b). The bicoherence of the signal together with the biphase is shown in (c). The prominent peak of the interaction between GAMs and micro fluctuations are clearly demonstrated. The biphase, associated with this GAMs-micro fluctuations coupling, shows only weak dependence on the frequency of micro fluctuations. In contrast, the coupling between the microscopic fluctuations indicates the randomly varying biphase.

Fig. 7 Time series of raw data from the measurement of edge turbulence on JFT-2M tokamak (a). The power spectrum of fluctuations indicates the presence of low frequency zonal flow, oscillating zonal flow (GAMs) near 15 kHz, and high frequency microscopic fluctuations (b). The bicoherence of the signal together with the biphase is shown in (c). The prominent peak of the interaction between GAMs and micro fluctuations are clearly demonstrated. The biphase, associated with this GAMs-micro fluctuations coupling, shows only weak dependence on the frequency of micro fluctuations. In contrast, the coupling between the microscopic fluctuations indicates the randomly varying biphase.

of the JFT-2M tokamak is introduced here as an example. Figure 7 shows the process of imaging in the study of plasma turbulence. The time series of raw data (taken by Langmuir probes at the plasma edge, in this example) is given in Fig. 7(a). A rapidly changing signal indicates strong broadband turbulence. As a standard procedure, the power spectrum is deduced from the time series of signals. It is apparent that there are three characteristic activities in the turbulence. The high frequency, broadband fluctuations (higher than 20 kHz), a sharp peak at around 15 kHz, and the peak in the low frequency range (1 kHz) are observed. The broadband fluctuations are considered the drift wave turbulence in this example. The peak in the intermediate frequency is the GAMs (oscillating zonal flows), and the low frequency peak is considered to be the zonal flow.

The causal relations among excited fluctuations are now identified by use of nonlinear analysis of observations. Figure 7c shows the result of the bicoherence analysis of the spectrum in Fig. 7(b) [20, 28]. The squared bicoherence, \( \hat{b}^2(f_1, f_2) \), and the biphase, \( \Theta(f_1, f_2) \), are given in Fig. 7c. It is evident that, when \( f_1 \) or \( f_2 \) or \( f_1 + f_2 \) correspond to the frequency of the GAM oscillation, the bicoherence has a large value, indicating the strong nonlinear interaction between the GAM oscillation and two component in the broadband fluctuations. The biphase for this combination of interactions (one of three is the GAM oscillation) takes a nearly constant value, showing that the microscopic fluctuations belonging to the broadband fluctuations coherently interact with the GAM oscillation. These observations demonstrate that the microscopic fluctuations and the meso-scale perturbation (GAMs) interact through modulational nonlinear coupling. This process has been theoretically pointed out as the driving mechanism of zonal flows in plasma turbulence [4, 6, 8, 30, 31]. Further studies have been done for the magnitude of nonlinear coupling, and experiments have given confirmation to the theoretical predictions (within the experimental error) if the convergence study is carefully performed [32]. The interaction between the zonal flow (in the rage of 1 kHz in Fig. 7(b)) and broadband components are also observed in the bispectral analysis, Fig. 7c. The coupling is less prominent for zonal flows, in comparison with that of GAMs. This is partly because of the limitation of the number of realizations in the statistical average. The real frequency of the zonal flow is much smaller than that of GAMs, and the number of realizations for the statistical average is smaller for zonal flows than for GAMs. The nonlinear coupling among microscopic fluctuations is also shown in Fig. 7c. In contrast to the cases of zonal flows and GAMs, the coupling between the microscopic fluctuations indicates the randomly varying biphase. This is consistent with the theoretical prediction of nonlinear interaction of drift wave turbulence [18]. Structural transition in turbulent plasmas has been known [1, 7], and a new type of it has been pointed out by introducing the coupling between micro, meso and macro perturbations [33, 34].

Theories have also predicted that the zonal magnetic field can also be excited by microscopic turbulence [8]. The first report of the observation of the zonal field (i.e., the
that the ansatz ‘all things flow’ has taught evolution of nature. As is illustrated in Fig. 8, traditional and are generated by turbulence. This image (structured axial vector field (vorticity or magnetic field), elements of activities are captured by the power spectrum, and the magnitudes of interactions among excited modes are given by employing the bispectral method.

From the progress of studies of turbulence, it has become clear that a substantial part of the observed ‘structures’ in nature and in laboratories are accompanied by the structured axial vector field (vorticity or magnetic field), and are generated by turbulence. This image (understanding) is a scientific answer to the human recognition of the evolution of nature. As is illustrated in Fig. 8, traditional wisdom in oriental and occidental civilizations has taught that the law ‘all things flow’ has been confirmed ubiquitously. The law ‘all things flow’ has been confirmed by mankind, because the structures (which do not last) have been continuously generated. The rule of generation and passage of structures started to be formulated.

As is illustrated in this article, the progress of plasma physics induced a paradigm shift from the previous ‘linear, local and deterministic’ view of turbulent transport to the new ‘nonlinear, no-local (in real and wavenumber spaces), statistical’ view of turbulent transport. For recent development of statistical theory in turbulent plasmas, readers are referred to Refs. [1, 6, 36-40] The fluctuating force from background broadband turbulence can induce stochastic transition in global dynamics. This is one of the processes that determine the lifetime of structures.

Surrounded by increasing information, the basic understanding of plasmas is necessary. “Knowledge should be developed into Understanding”; such research trends spread all over the world. The understanding of turbulent plasma is a central theme of this century.

**Acknowledgements**

Authors acknowledge discussions with many colleagues, especially Prof. A. Yoshizawa, Prof. Y. Miura, Prof. G.R. Tynan, Dr. T. Ido and members of Specially-Promoted Research (16002005) of MEXT. We are grateful to Dr. H. Miura and Prof. R. Matsumoto for permitting reproduction of figures and to Prof. O. Motojima for encouragement.

This work was partly supported by the Grant-in-Aid for Specially-Promoted Research (16002005) and the Grant-in-Aid for Scientific Research (15560495), by the collaboration programmes of NIFS (NIFS06KDA005, NIFS03KKMD001) and of the RIAM of Kyushu University, and by Asada Eiichi Research Foundation.

---

**Fig. 8** Traditional wisdom in human recognition of nature in oriental and occidental civilizations.

---

**References**


